Damping rate of a scalar field at high temperatures

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- real scalar field performing small oscillations around its potential minimum
- interactions with other fields lead to damping of oscillations $\rightarrow$ decay
- produced particles constitute a plasma which has effects on further decay

- homogeneous scalar field drives inflation
- slowly rolls down its potential
- after reaching minimum: oscillation
- $\rightarrow$ production of particles

- influence of plasma makes damping rate temperature dependent
- this effects the temperature evolution in the early universe
- maximal temperature can have implications for production of dark matter candidates

- density matrix ρ determines thermal and quantum mechanical average of operators: $\langle B \rangle = \mathrm{tr}(\rho B)$
- von Neumann equation $\frac{\partial \rho}{\partial t} = [\rho, H]$
- consider free Hamiltonian + small perturbation $H = H_0 + V(t)$, where $V(t) = \int \mathrm{d}^3\mathbf{x} J(t, \mathbf{x}) A(\mathbf{x})$
- integration and expanding in J gives to first order:

$$\langle B \rangle = \langle B \rangle_{\mathrm{eq.}} - \int \mathrm{d}^4 x' J(x') \Delta^{\mathrm{R}}_{BA}(x - x')$$

- $\langle \dots \rangle_{\mathrm{eq.}}$ can be calculated in thermal field theory

- use this formula for the field operator ϕ ($A = B = \phi$)
- "kick" the field homogeneously: $J(x) = J_0 \delta(t)$

$$\Rightarrow \delta\phi = -\frac{J_0}{2M_{\text{th}}} \sin(M_{\text{th}} t) e^{-\gamma t}$$

with

$$\gamma \approx -\frac{\text{Im}\Pi(M_{\text{th}} + i\epsilon, \mathbf{0})}{2M_{\text{th}}}$$

- thermal mass squared $M_{\text{th}}^2 = m^2 +$ corrections
- $\Pi =$ self energy calculated from thermal field theory

High temperature regime
A simple model

Consider the following model:

$$\mathcal{L} = \mathcal{L}_{\mathsf{KG},\phi} + \mathcal{L}_{\mathsf{KG},\chi} - \frac{\lambda}{4!}\chi^4 - g\phi\chi^2$$

- thermal masses correspond to frequencies of oscillation
- at high temperatures thermal masses are dominated by corrections
- assume self coupling of plasma to be much larger than coupling of the decay-channel
- from the point of view of the plasma: ϕ-field barely oscillates, therefore:

$$\to \gamma \approx \lim_{\omega\to 0} -\frac{\mathrm{Im}\Pi(\omega + \mathrm{i}\epsilon, \mathbf{0})}{2\omega}$$

- What does Π look like?
- to leading order in g and "all orders" in λ one has

$$\mathrm{Im}\Pi(\omega + \mathrm{i}\epsilon, \mathbf{k}) \propto g^2 \Delta^{\mathrm{R}}_{\chi^2\chi^2}(\omega, \mathbf{k})$$

$$\rightarrow \gamma \propto \lim_{\omega\rightarrow 0} -\frac{\Delta^{\mathrm{R}}_{\chi^2\chi^2}(\omega, \mathbf{0})}{2\omega}$$

In a slightly different model one has

$$\gamma \propto \lim_{\omega \to 0} -\frac{\Delta^{\mathrm{R}}_{(\partial\chi)^2(\partial\chi)^2}(\omega, \mathbf{0})}{2\omega}.$$

and

$$\partial_\mu \chi \partial^\mu \chi \propto T^\mu_{\ \mu}$$

$$\to \gamma \propto \lim_{\omega \to 0} -\frac{\Delta^{\mathrm{R}}_{T^\mu_{\ \mu} T^\nu_{\ \nu}}(\omega, \mathbf{0})}{2\omega}$$

[D. Boedeker, JCAP 0606 (2006) 027]

Use Kubo formula from hydrodynamics involving bulk viscosity ζ

$$\zeta = \frac{1}{9} \lim_{\omega \to 0} \frac{1}{\omega} \Delta^{\mathrm{R}}_{T^\mu_\mu T^\nu_\nu}(\omega, \mathbf{0})$$

$$\to \gamma \propto \zeta$$

Summary

- real scalar field performs damped oscillations around potential minimum
- damping rate has influence on temperature evolution in the early Universe for example
- formula for damping rate can be obtained from linear response
- damping rate can be related to a hydrodynamic quantity (up to now only in one model)