$\mathcal{N} = 4$ SYM within Dyson-Schwinger equations (DSE's)

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Yang-Mills around for 53 years, **non-abelian** gauge theory

$$\begin{aligned}\mathcal{L} &= \frac{1}{4} F_{\mu\nu a} F_{\mu\nu a} \quad \text{with} \\ F_{\mu\nu a} &= \partial_\mu A_{\nu a} - \partial_\nu A_{\mu a} + g f^{abc} A_{\mu b} A_{\nu b}\end{aligned}$$

Why still investigate

- Non-perturbative gluon behaviour
- High temperature QCD

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- Multiplet structure (gluon,fermion,scalar)
- Testing functional methods ($\beta = 0$)
- Maldacena's conjecture $\rightarrow$ AdS/CFT

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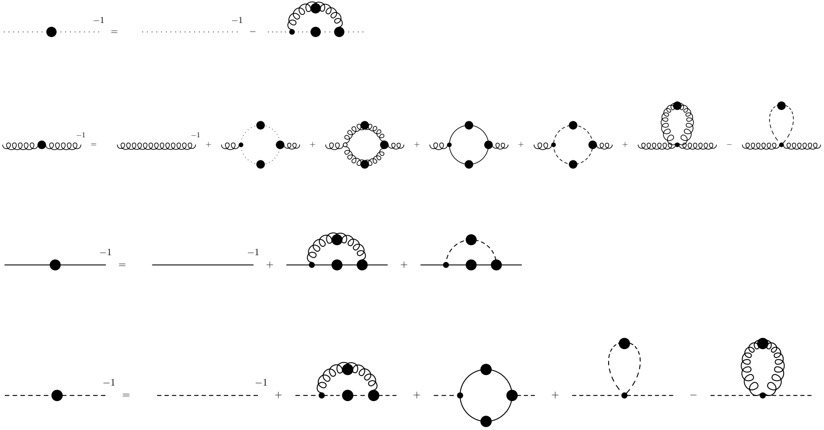
- n-point fct. depend on higher n-point fcts.
- Tower of coupled integral equations $\rightarrow$ truncation

To derive DSE's a Lagrangian is needed

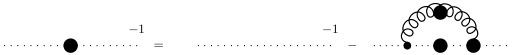
$$\begin{aligned}\mathcal{L} \;\; = \;\; & \overline{c}_a \partial_\mu D_\mu c_b - \frac{1}{4} F_{a\mu\nu} F_{a\mu\nu} + \frac{g^2 \theta}{64\pi^2} \varepsilon_{\mu\nu\rho\sigma} F_{a\mu\nu} F_{a\rho\sigma} \\ & - \frac{1}{2} [\psi^T_{iaL} \epsilon (\not{D} \psi^i_R)_a - \psi^{iT}_{aR} \epsilon (\not{D} \psi_{iL})_a] \\ & + \frac{1}{2} (D_\mu \phi^{ij})_a (D_\mu \phi^{ij})^*_a - \frac{1}{8} | f^{abc} \phi^{ij}_b \phi^{kl}_c |^2 \\ & - \sqrt{2} Re f^{abc} \phi^{ij}_a (\psi^T_{ibL} \epsilon \psi_{jcL})\end{aligned}$$

- $SU(4)$-R-symmetry (Yukawa)
- Gauge-symmetry (e.g. SU(N))
- Conformal-symmetry

What one ends up with:



Equation diagrammatically



and mathematically

$$D_G^{ab}(p^2)^{-1} = -\,\delta^{ab}p^2$$
$$-\,igf^{ade}\int \frac{d^dq}{(2\pi)^d} D^{ef}_{\mu\nu}(p-q) D_G^{dh}(q) \Gamma_\nu^{c\bar{c}A;bfh}(-p,q,p-q)$$

Propagators look like

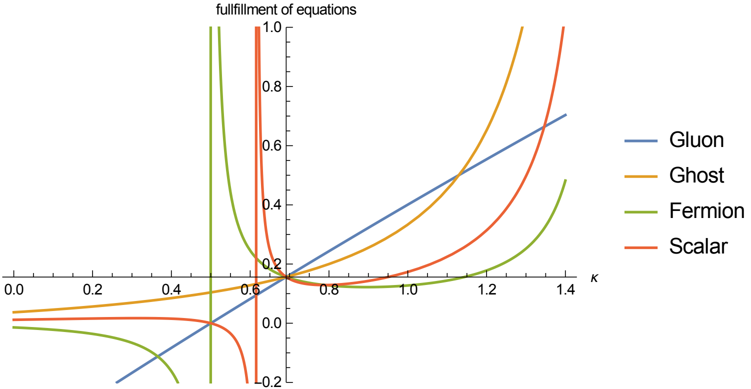
$$D(p^2) = D_0(p^2)A(p^2)$$

Complicated structure hidden in dressing fct. $A(p^2)$'s
Ansatz for propagator dressing fct., due to conformality **powerlaws**

$$\begin{aligned} G(x) &= ax^{\kappa_1} \qquad \text{ghost} \\ Z(x) &= bx^{\kappa_2} \qquad \text{gluon} \\ F(x) &= fx^{\kappa_3} \qquad \text{fermion} \\ S(x) &= sx^{\kappa_4} \qquad \text{scalar} \end{aligned}$$

$\ldots x = p^2$

What we get $\rightarrow$ a single κ
Crossing of all propagators



$\kappa \approx 0.69 \qquad \kappa_{YM} \approx 0.59$

All couplings show conformal behaviour

$$\alpha(x) = \alpha(\mu) A^2(x) B(x) = const.$$

Correspond to

$$\beta(\alpha) = 0$$

Dressing fct. given by

$$\begin{aligned} G(x) &= a x^{-\kappa} \\ Z(x) &= 3s(a) x^{2\kappa} \\ F(x) &= f(a) x^{-\kappa} \\ S(x) &= s(a) x^{2\kappa} \end{aligned}$$

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Pure non-perturbative problem

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- What we want $\rightarrow$ fundamental modular region (FMR)
- Conjectured FMR $=$ 1.Gribov region for $\mathcal{N} = 4$ SYM
- Perturbative expansion around global minimum (FMR)

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And outlook

- Improvement of truncation, higher order vertex modelling

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- Improvement of truncation, higher order vertex modelling
- Simulate on lattice

To emphasise it again, first full solution

Gauge sector of $\mathcal{N} = 4$ SYM behaves as YM gauge sector

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Thank you