

QCD und Jet Physik an e^+e^- Beschleunigern

- Geschichte der Starke Wechselwirkung
- QCD; confinement; asymptotic freedom
- Hadronisierung und Hadron-Jets
- Quark-Spin
- Gluon-Spin
- Selbstkopplung des Gluons
- Asymptotische Freiheit aus Jetraten
- Messungen von α_s

QCD an Hadron-Beschleunigern: \rightarrow WS

Geschichte der Starken Wechselwirkung (1)

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1932: Entdeckung des **Neutrons**

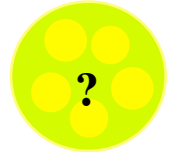


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1933: $\vec{\mu} \cong 2.5 \frac{e}{2 m_p} \vec{\sigma} \Rightarrow$ **Substruktur** des Protons

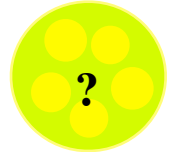


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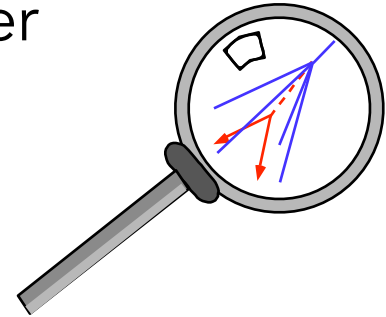
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1947: Entdeckung der π -Mesonen und langlebiger V-Teilchen (K^0 , Λ) in **Höhenstrahlung**

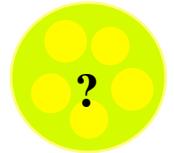


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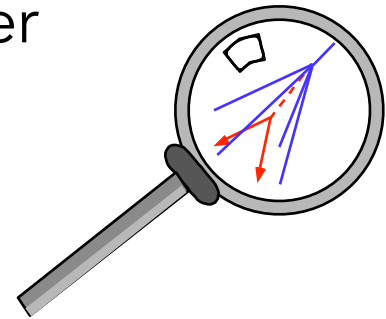


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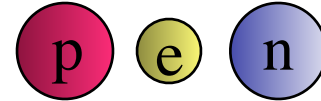
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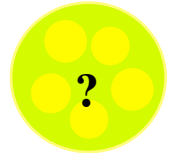


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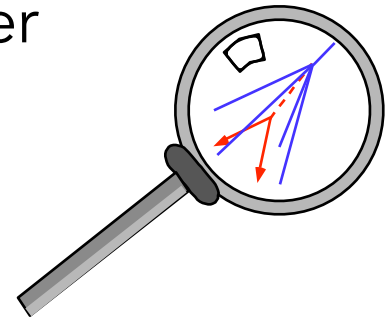
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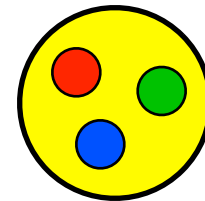


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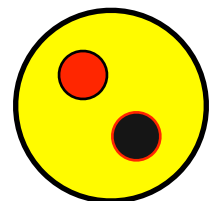


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Baryon
(p,n, Λ ,...)

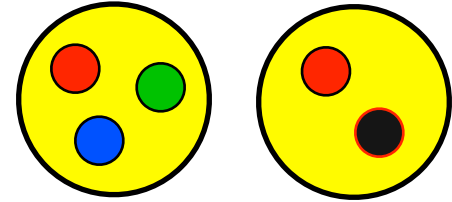


Meson
(π ,K,...)

Geschichte der Starken Wechselwirkung (2)

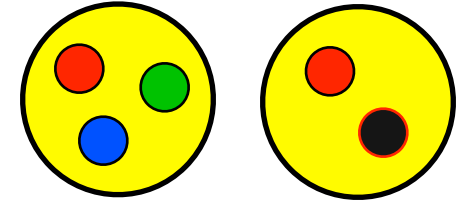
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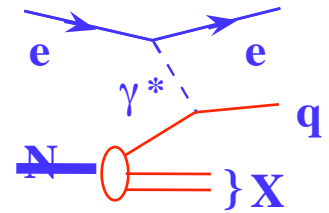


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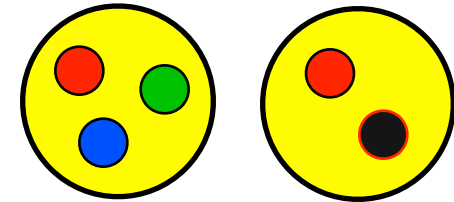


1969: Dynamisches **Partonenmodell**:

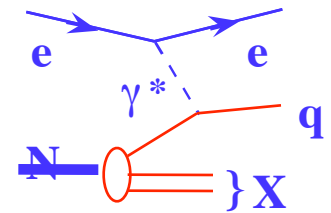


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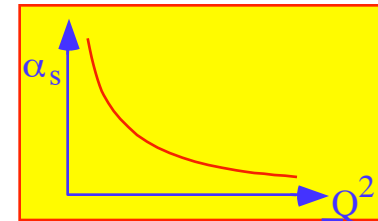
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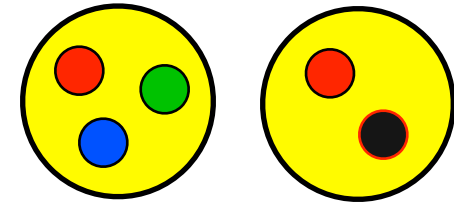


1973: Konzept der **Asymptotischen Freiheit** ;
Quanten **C**hromo **D**ynamik.

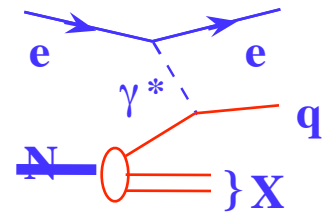


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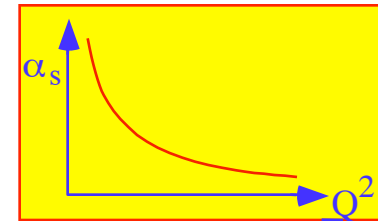
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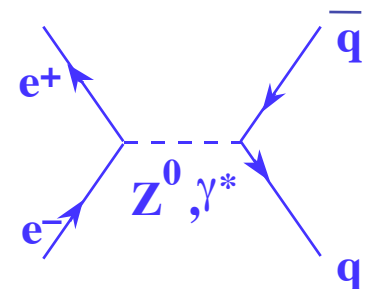
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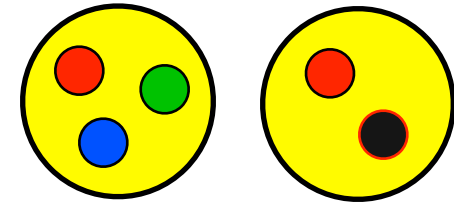


1975: **2-Jet Struktur** in $e^+ e^-$ -Vernichtung:
Bestätigung **Quark-Parton-Modell**.

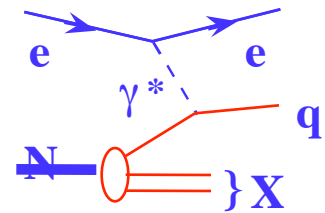


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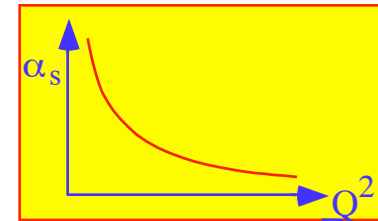
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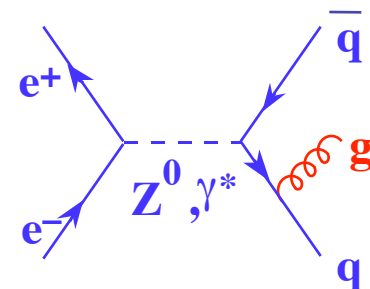
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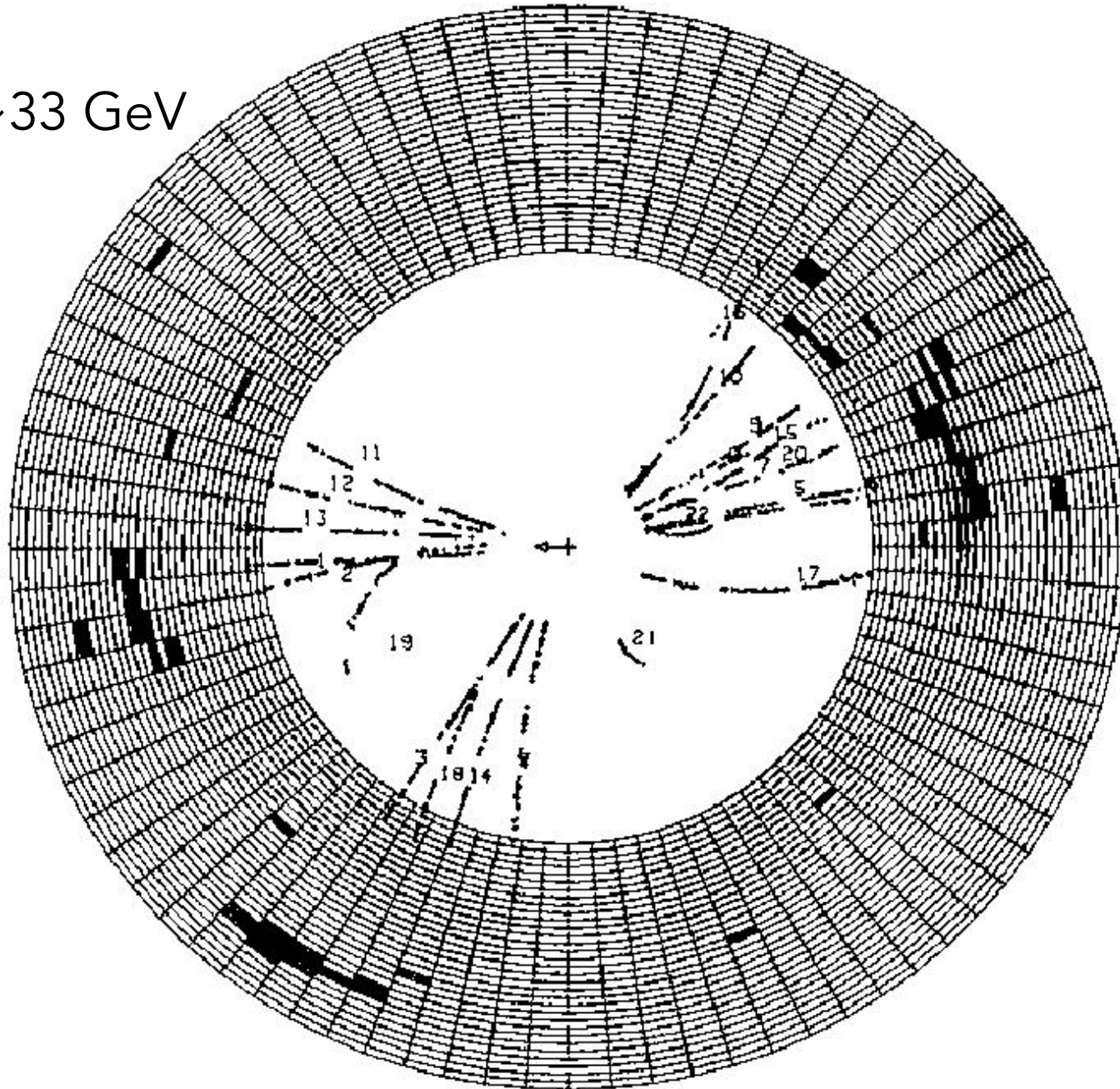
1975: **2-Jet Struktur** in $e^+ e^-$ -Vernichtung:
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1979: Entdeckung des **Gluons** in **3-Jet-**
Ereignissen der $e^+ e^-$ -Vernichtung.

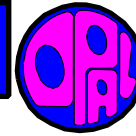
3-Jet Ereignis gemessen mit dem JADE Detektor (1979-1986)

$E_{cm} \sim 33 \text{ GeV}$

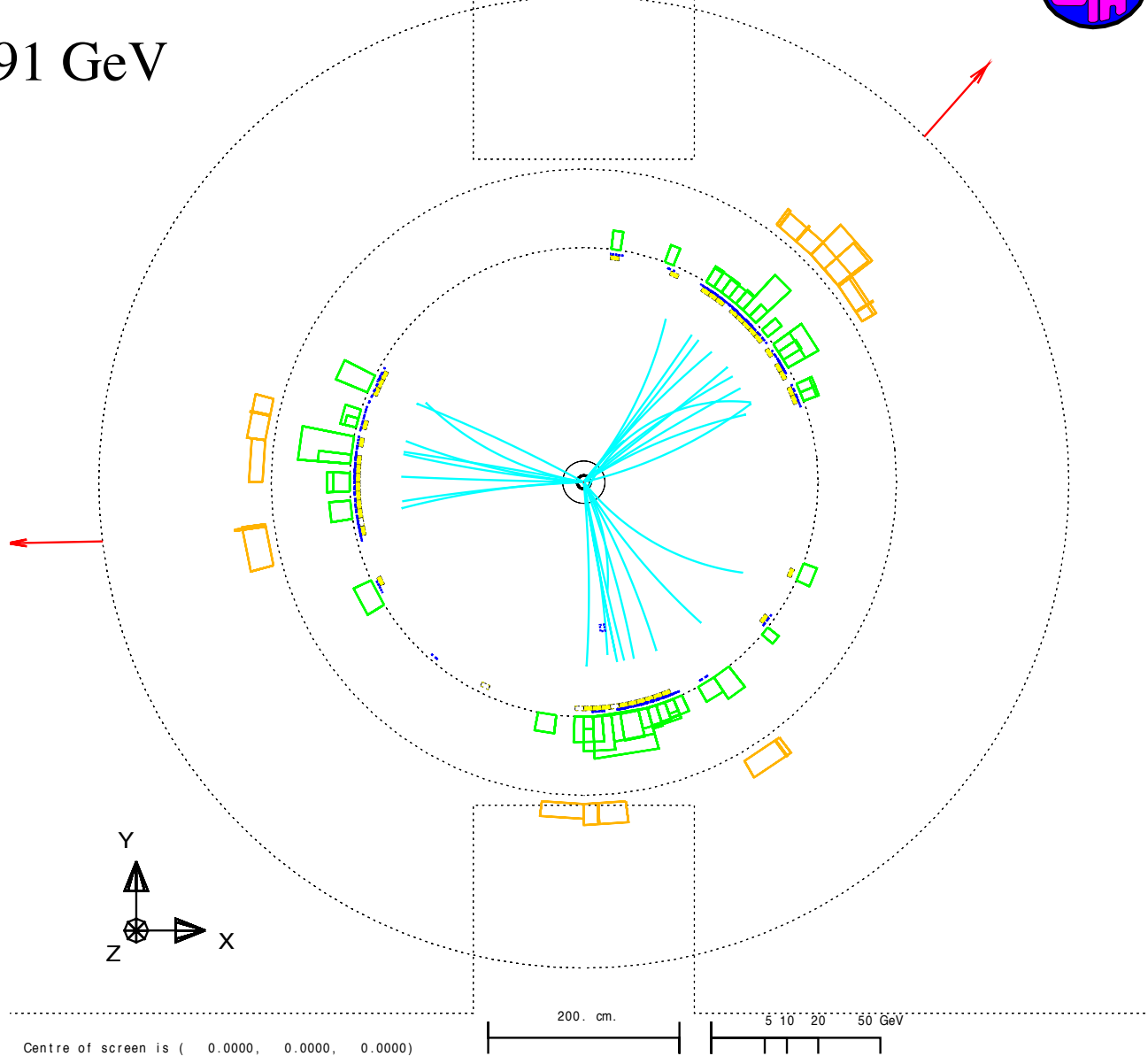


3-Jet Ereignis gemessen mit dem OPAL Detektor (1989-2000)

```
Run:event 2513: 61702 Date 910910 Time 85656 Ctrk(N= 37 Sump= 65.7) Ecal(N= 55 SumE= 44.8) Hcal(N=19 SumE= 8.6)
Ebeam 45.613 Evis 90.2 Emiss 1.1 Vtx ( -0.09, 0.10, -0.22) Muon(N= 2) Sec Vtx(N= 3) Fdet(N= 0 SumE= 0.0)
Bz=4.350 Thrust=0.6788 Aplan=0.0381 Oblat=0.4248 Spher=0.6273
```



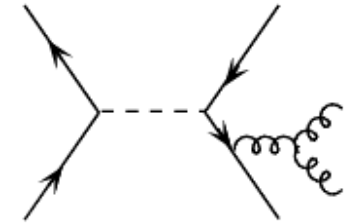
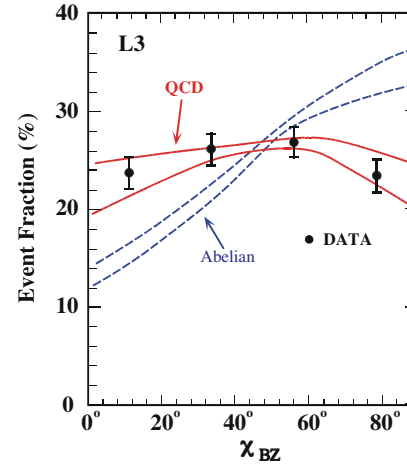
$E_{cm} = 91 \text{ GeV}$



Geschichte der Starken Wechselwirkung (3)

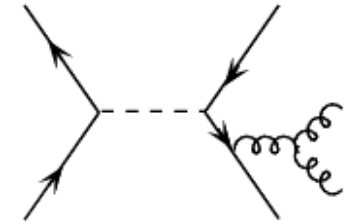
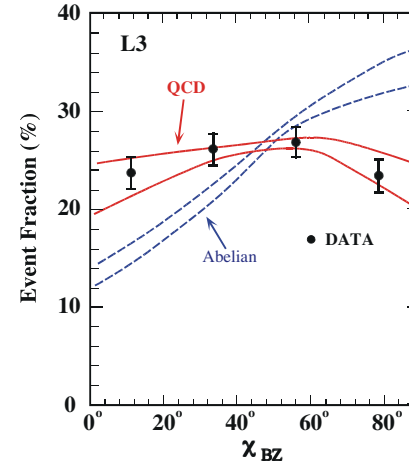
Geschichte der Starken Wechselwirkung (3)

1991: exp. Signatur der
Gluon-Selbstkopplung

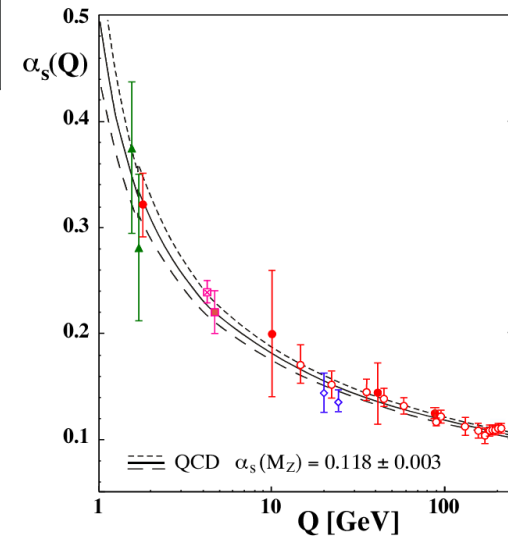


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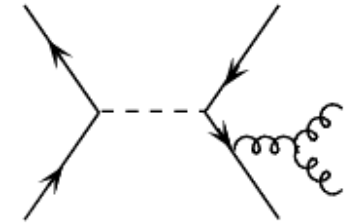
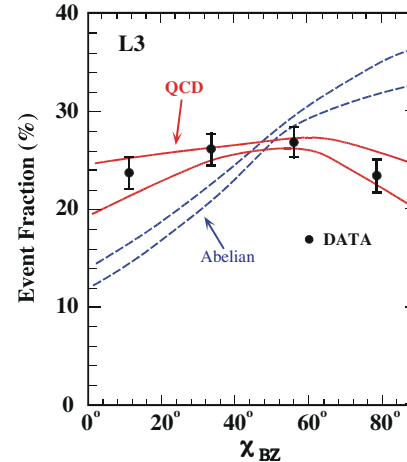


1990-2000: Bestätigung der
Asymptotischen Freiheit



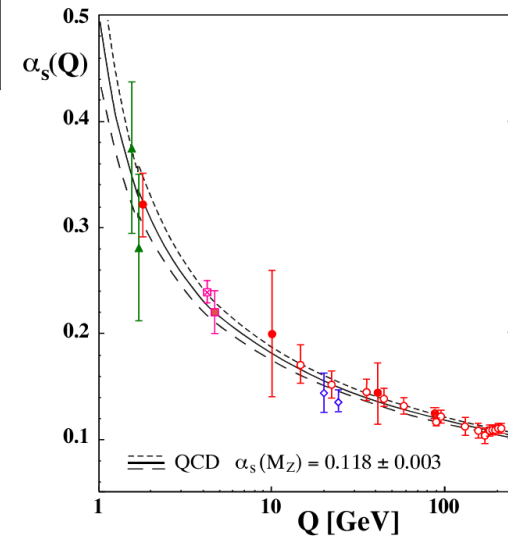
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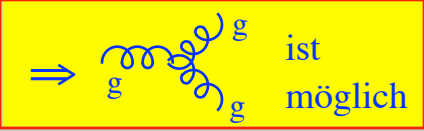
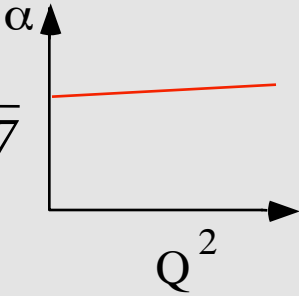
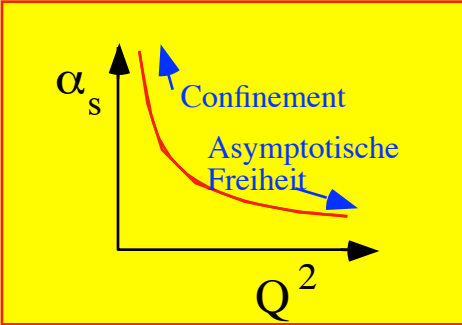
2004: Nobelpreis (Konzept der A.F.) an
D. Gross, H.D. Politzer und F. Wilczek



QCD:

- Eich-Feldtheorie der Starken Wechselwirkung
- zugrunde liegende Eichgruppe: $SU(3)$; nicht-abelsch
- „Kraft“- oder Austausch-Teilchen: Gluonen
- Selbstwechselwirkung der Gluonen
- renormierte Kopplungskonstante α_s ist energieabhängig:
- α_s groß bei kleinen Energien (grossen Abständen):
Confinement der Quarks
- α_s klein bei grossen Energien (kleinen Abständen)
Asymptotische Freiheit der Quarks

Eigenschaften der QED und der QCD:

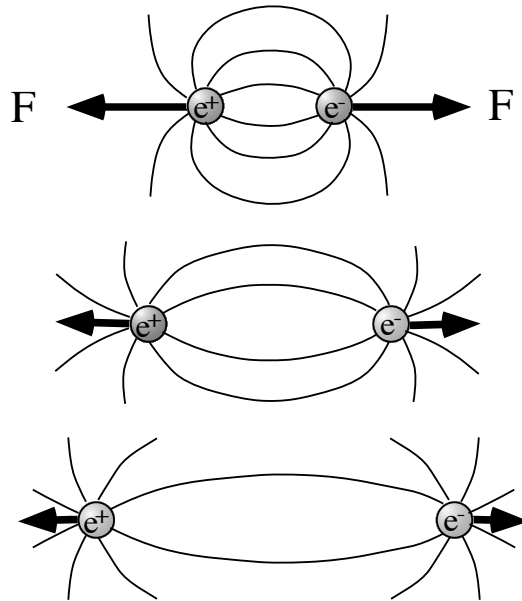
| | QED | QCD |
|-----------------------------------|--|--|
| <i>Fermionen</i> | <i>Leptonen (e, μ, τ)</i> | <i>Quarks (u, d, s, c, b, t)</i> |
| <i>Kraft koppelt an:</i> | <i>elektrische Ladung</i> | 3 Farb-Ladungen |
| <i>Austausch-quantum</i> | <i>Photon (γ) (trägt keine Ladung)</i> | <i>Gluonen (g) (tragen 2 Farbladungen)</i> ⇒  ist möglich |
| <i>Kopplungs- "Konstante"</i> | $\alpha(Q^2=0) = \frac{1}{137}$  | $\alpha_s(Q^2 = M_Z^2) \approx 0.12$  |
| <i>Freie Teilchen</i> | <i>Leptonen (e, μ, τ)</i> | (Farbneutrale, gebundene Zustände von \bar{q} and q) Hadronen |
| <i>Theorie</i> | <i>Störungstheorie bis zur $O(\alpha^5)$</i> | <i>Störungstheorie bis $O(\alpha_s^4)$</i> |
| <i>Erreichte Präzision</i> | $10^{-6} \dots 10^{-7}$ | 1% ... 20% |

Warum gibt es keine freien Quarks?

QED

Elektrische Ladungen:

Kraft $F \propto 1/r^2$; Energiedichte $\propto 1/r$



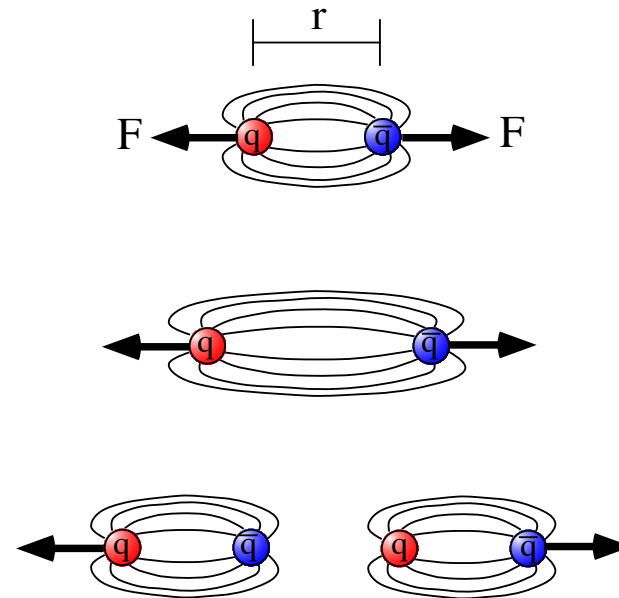
Kraft- und Energiedichte zwischen Ladungsträgern nimmt ab.

⇒ Träger elektrischer Ladung sind freie Teilchen

QCD

Farbladungen:

Kraft $F \propto \text{const}$; Energiedichte $\propto r$



Kraft- und Energiedichte steigen an, bis ein neues Quark- Antiquark-Paar aus dem Vakuum erzeugt wird.

⇒ Träger von Farbladung kommen nur in gebundenen, 'farbneutralen' Zuständen vor.

"Confinement"

Energieabhängigkeit der Kopplungs-“Konstanten“:

Renormalisation Group Equation (“ β -function”)

- in führender Ordnung Störungstheorie:

$$\mu \frac{d}{d\mu} \alpha_i(\mu) = -\beta_0 \alpha_i^2 \quad \text{mit} \quad \beta_0 = \frac{1}{2\pi} \left[\frac{11}{3} \begin{pmatrix} N_c \equiv 0 \\ N_c \equiv 2 \\ N_c \equiv 3 \end{pmatrix} - \frac{4}{3} \begin{pmatrix} N_{fam} \\ N_{fam} \\ N_f / 2 \end{pmatrix} - N_{Higgs} \begin{pmatrix} \frac{1}{10} \\ \frac{1}{6} \\ 0 \end{pmatrix} \right] \begin{matrix} \leftarrow \text{QED} \\ \leftarrow \text{weak} \\ \leftarrow \text{QCD} \end{matrix}$$

Energieabhängigkeit der Kopplungs-“Konstanten“:

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- Integration \Rightarrow

$$\alpha_i(q^2) = \frac{\alpha_i(\mu^2)}{1 + \frac{\beta_0}{2} \alpha_i(\mu^2) \ln \frac{q^2}{\mu^2}}$$

or

$$\alpha_i(q^2) = \frac{2}{\beta_0 \ln \frac{q^2}{\Lambda^2}}$$

$$\text{with } \Lambda^2 = \frac{\mu^2}{e^{2/\beta_0 \alpha_s(\mu^2)}}$$

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← QED
 ← weak
 ← QCD

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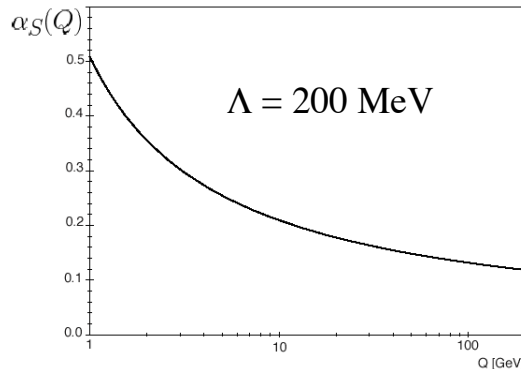
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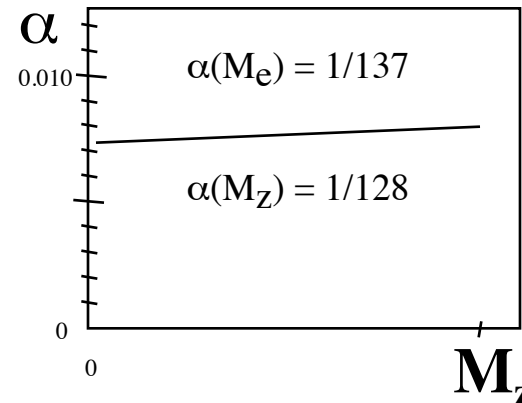
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QCD: $N_c = 3$; $\beta_0 = \frac{23}{6\pi}$

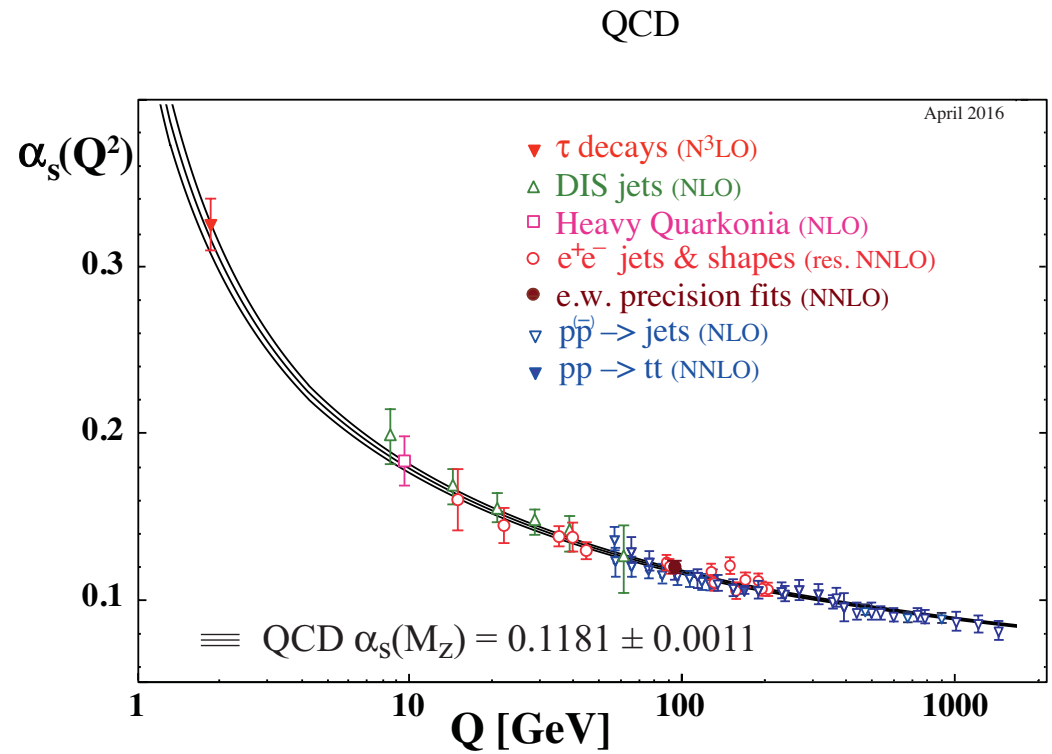
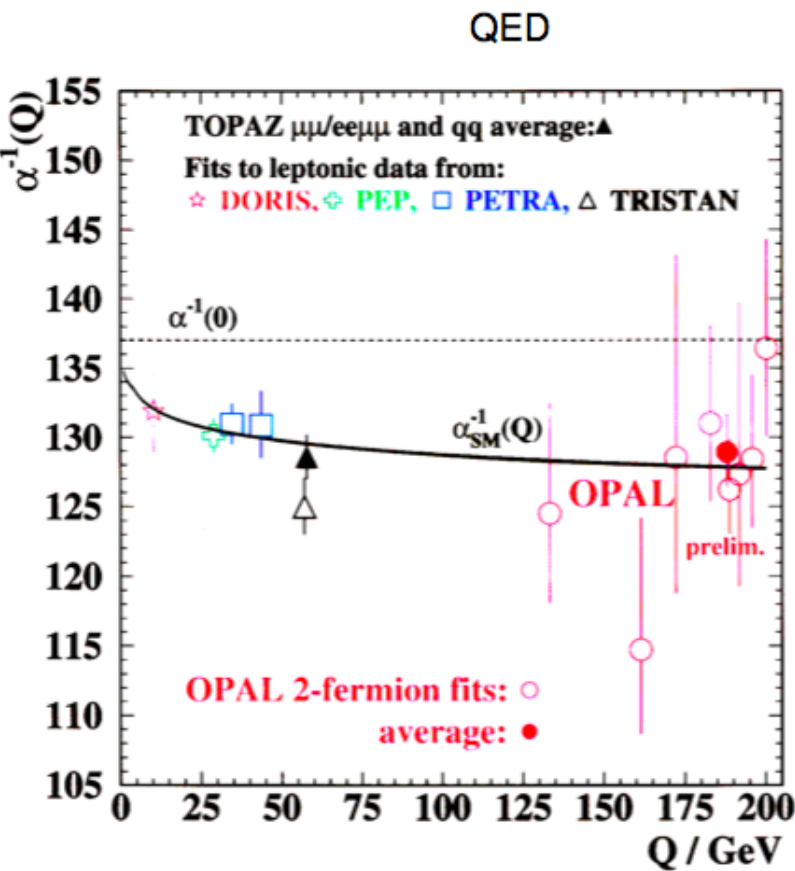


QED: $N_c = 0$; $N_{fam} = 3$ $\beta_0 = -\frac{12}{6\pi}$

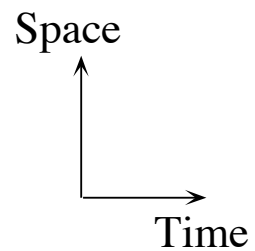


Energieabhängigkeit der Kopplungs“konstanten“:

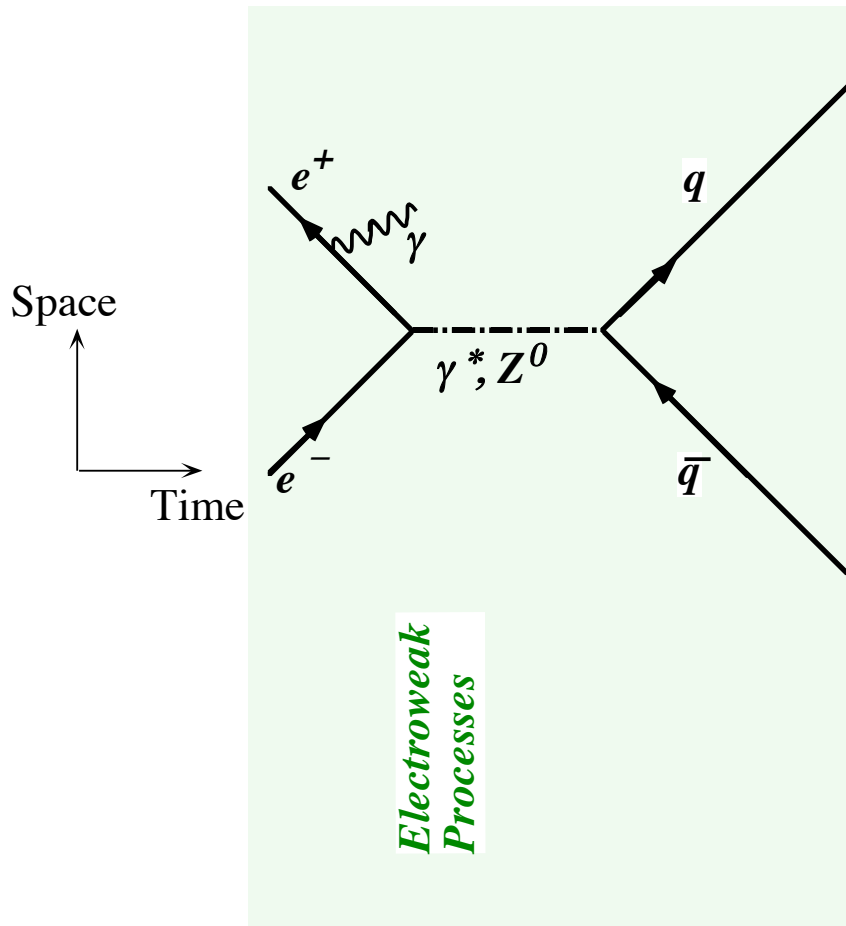
- experimentell mit hoher Genauigkeit verifiziert



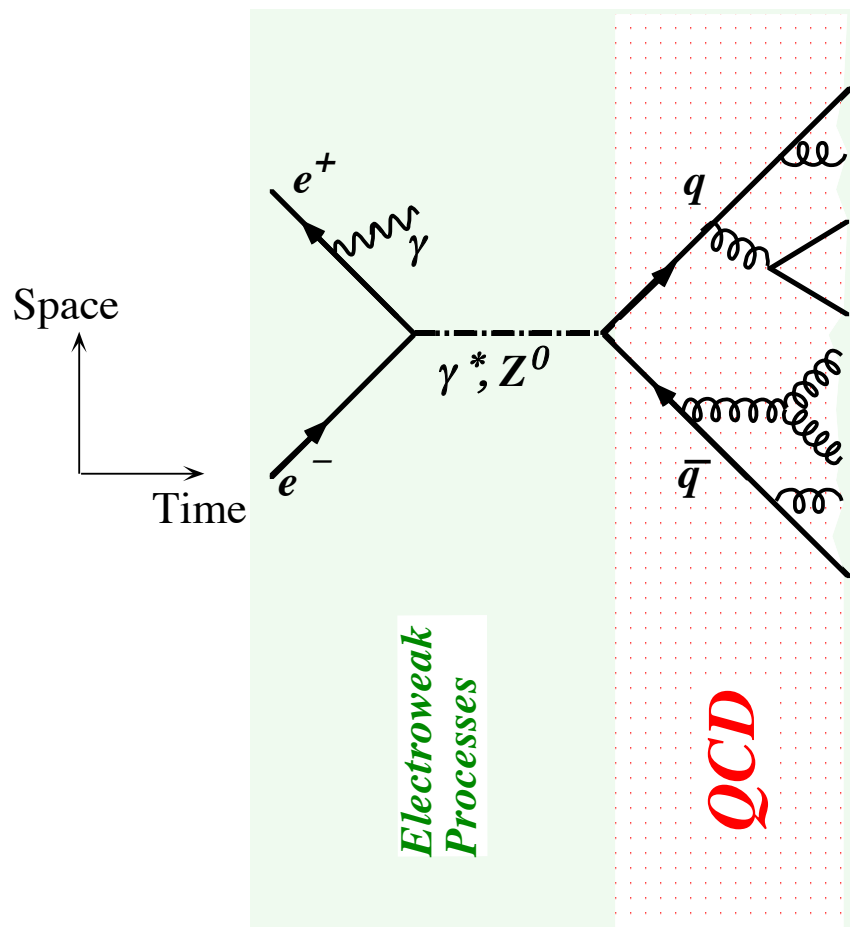
Anatomy of hadronic events in e^+e^- annihilation



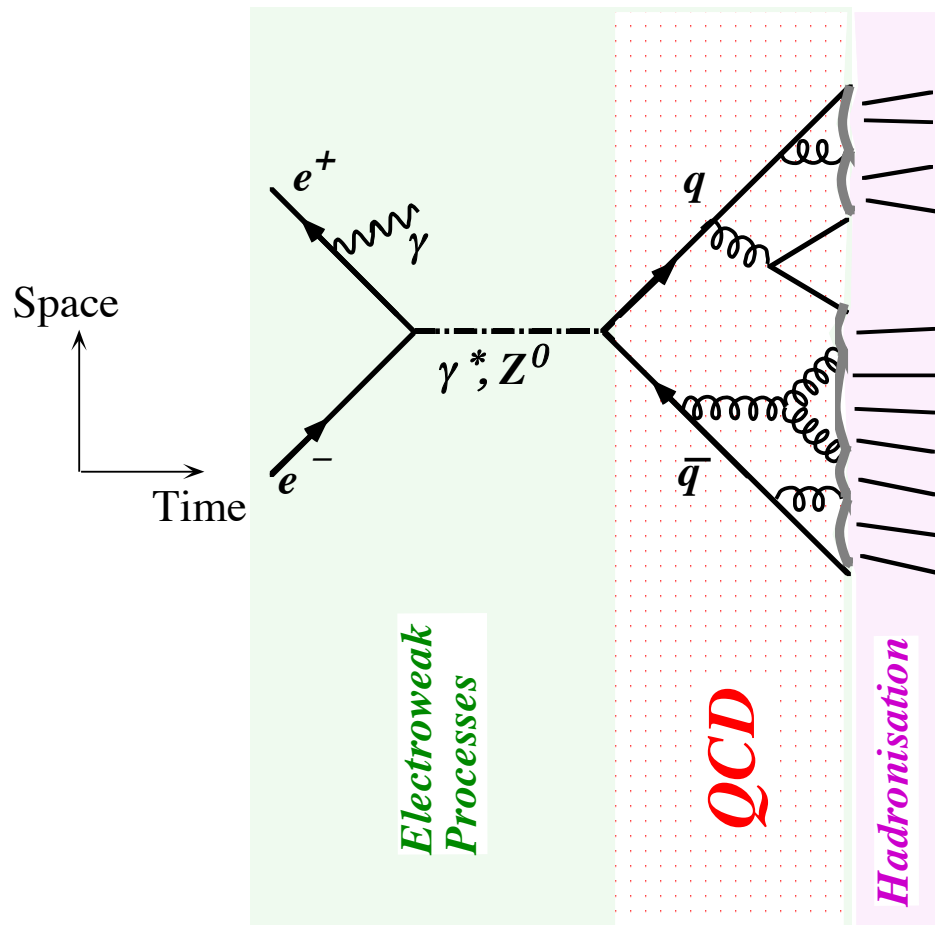
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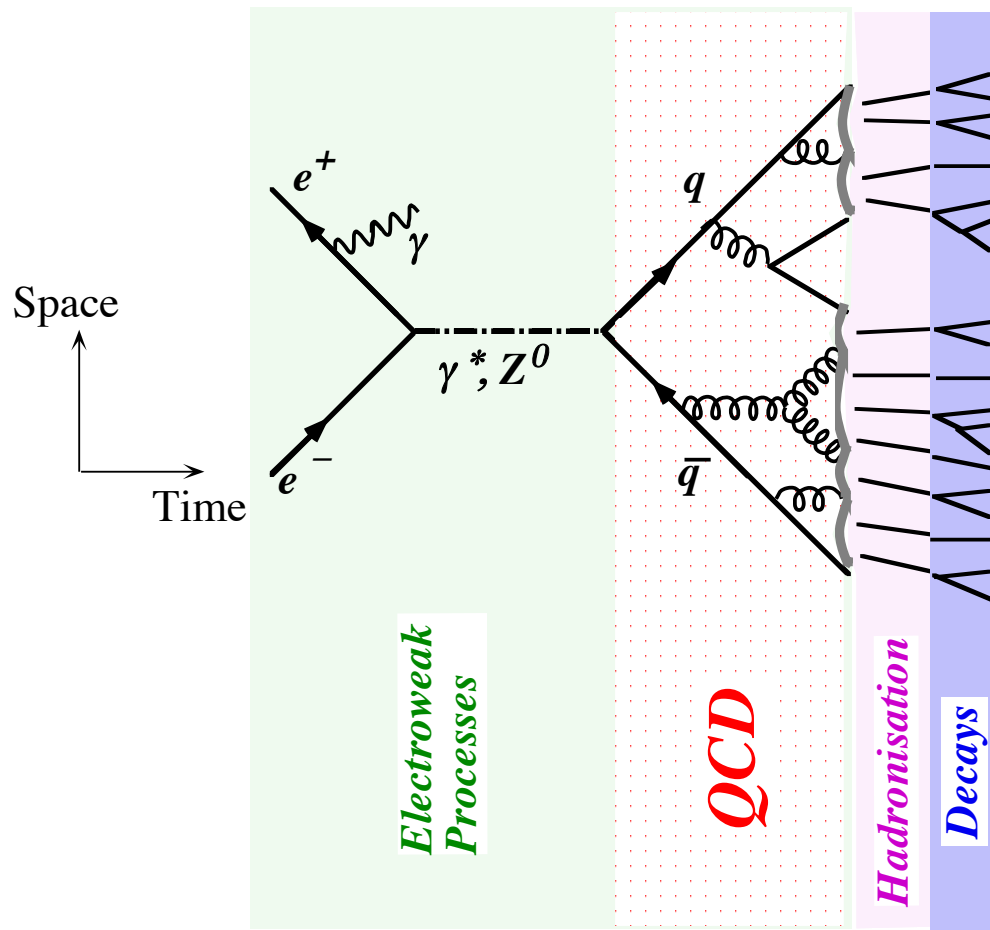
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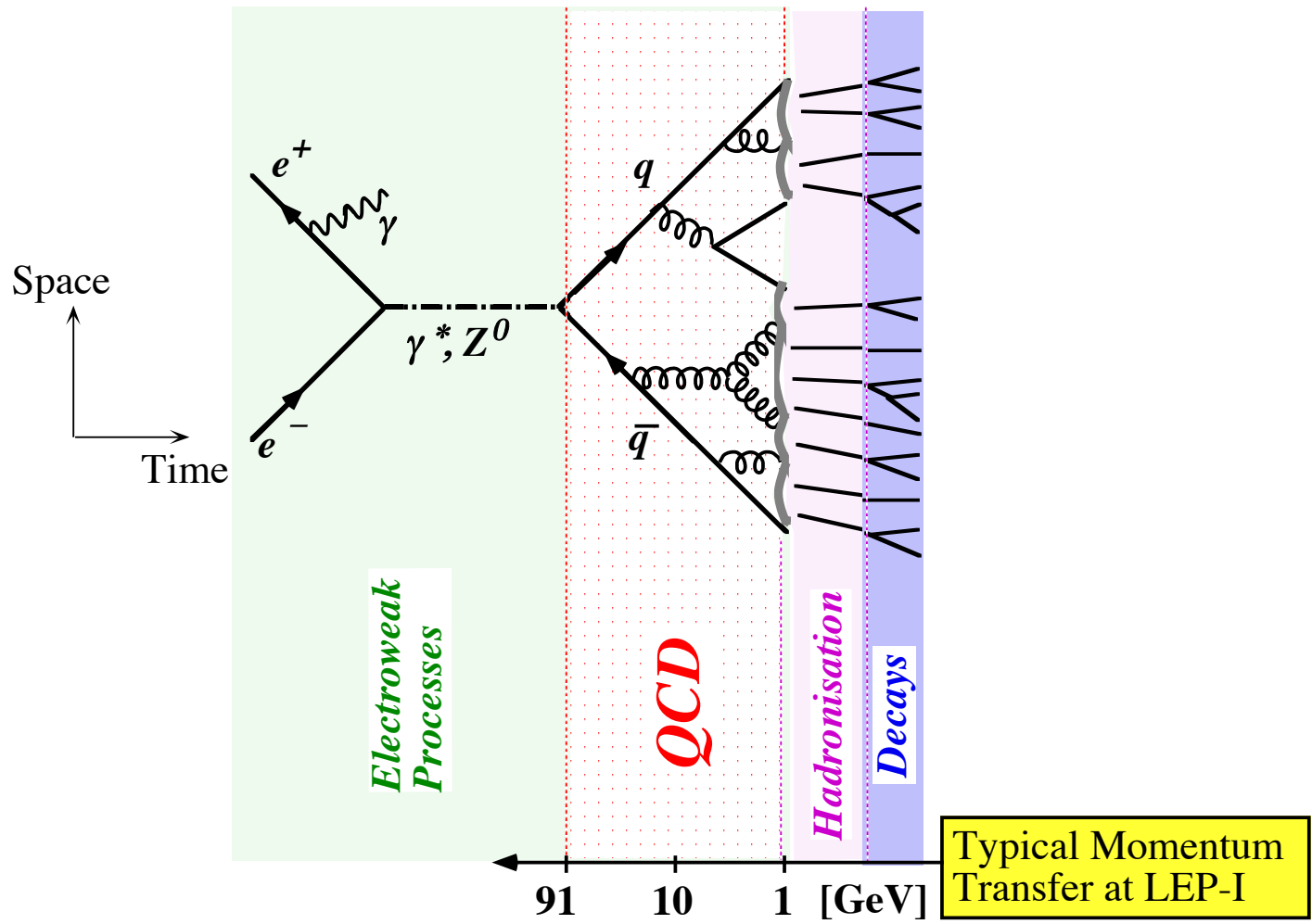
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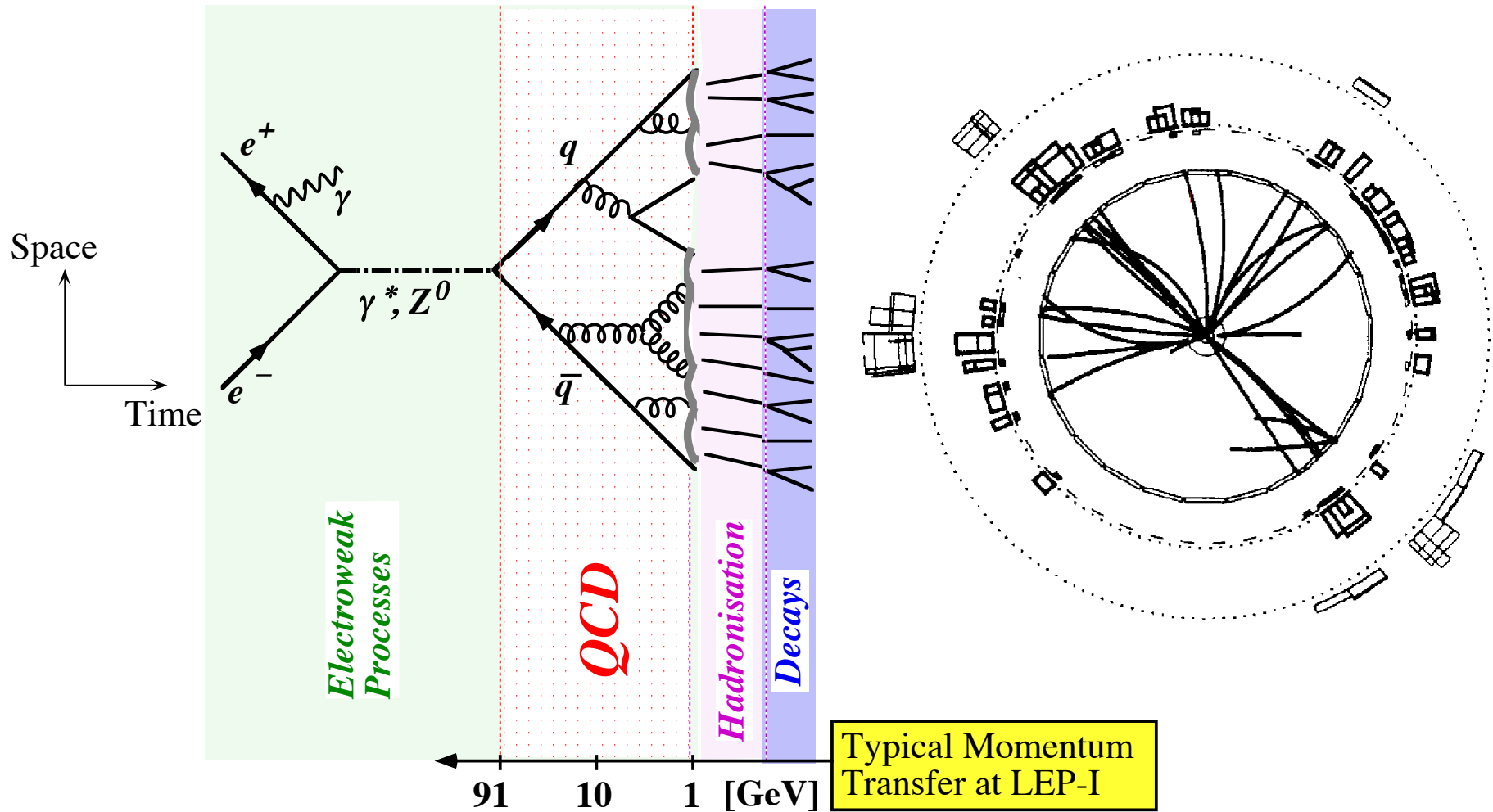
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Anatomy of hadronic events in e^+e^- annihilation



Anatomy of hadronic events in e^+e^- annihilation



- QCD: shower development calculated in perturbation theory (fixed order; (N)LLA)
- Hadronisation: phenomenological models of string-, cluster- or dipole fragmentation
- Decays: randomized according to experimental decay tables

Physik der Hadronen-Jets

Zum Vergleich von Hadronen-Jets
mit analytischen QCD -Rechnungen
(Quark- und Gluondynamik)

muß man

auflösbare Teilchenjets

Theorie und Praxis definieren.

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muß man
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Theorie und Praxis definieren.



Dazu benötigt man:

- Definition eines Auflösungskriteriums
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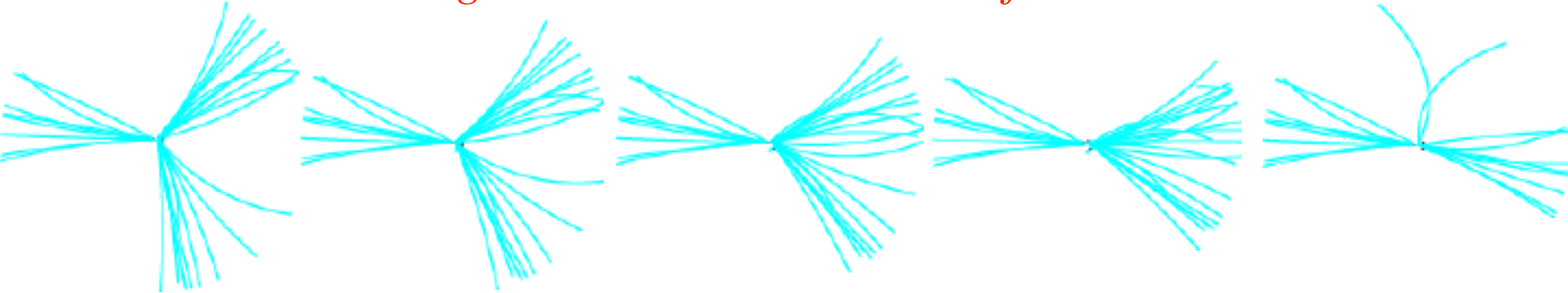


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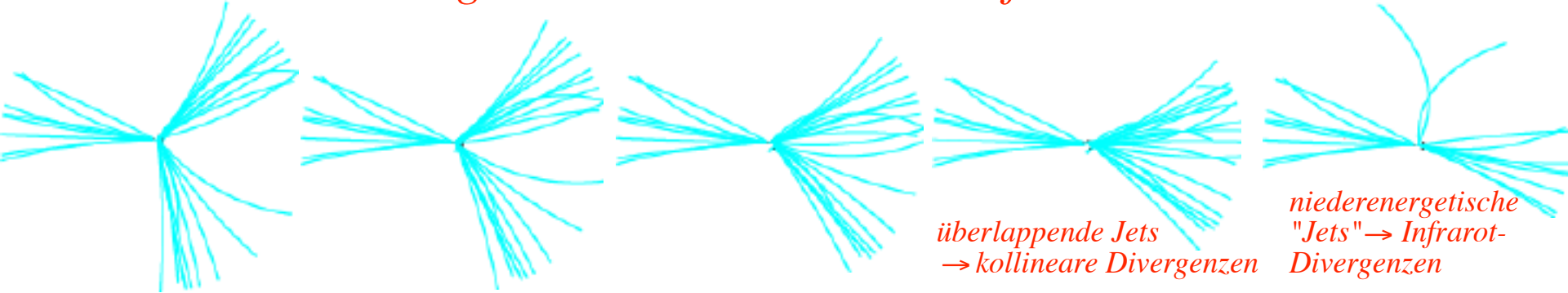


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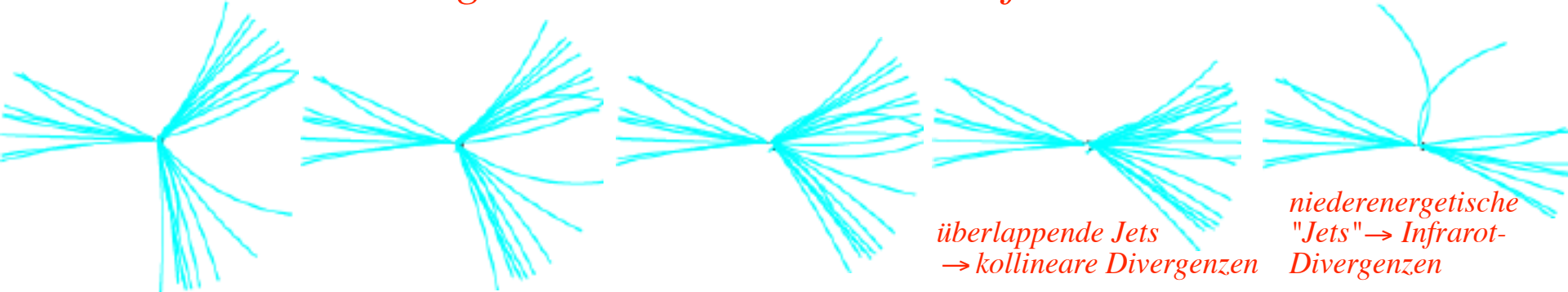


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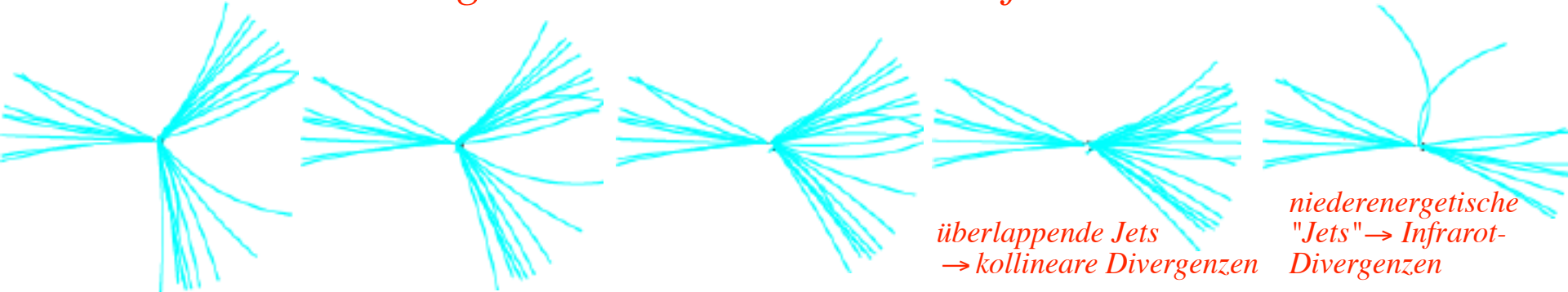


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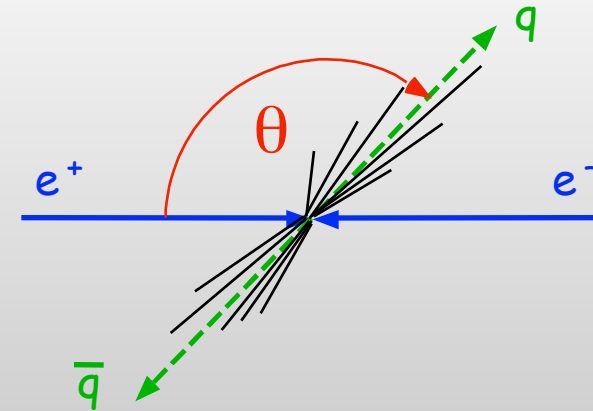
2 Gruppen von Teilchen, i und j , können aufgelöst werden falls für die minimale transversale Energie der 4er-Vektoren, $y_{ij} = 1/2 \min(E_i^2, E_j^2) \cdot (1 - \cos(\theta_{ij}))$, gilt: $y_{ij} \geq y_{cut}$

Falls $y_{ij} < y_{cut}$, werden die 'Proto-jets' i und j von einem neuen, einzelnen (Proto-) Jet k ersetzt (Rekombination): $p_k = p_i + p_j$ (rekursives Verfahren, bis alle $y_{ij} \geq y_{cut}$).

Test of basic quantum numbers (q-, g-spin):

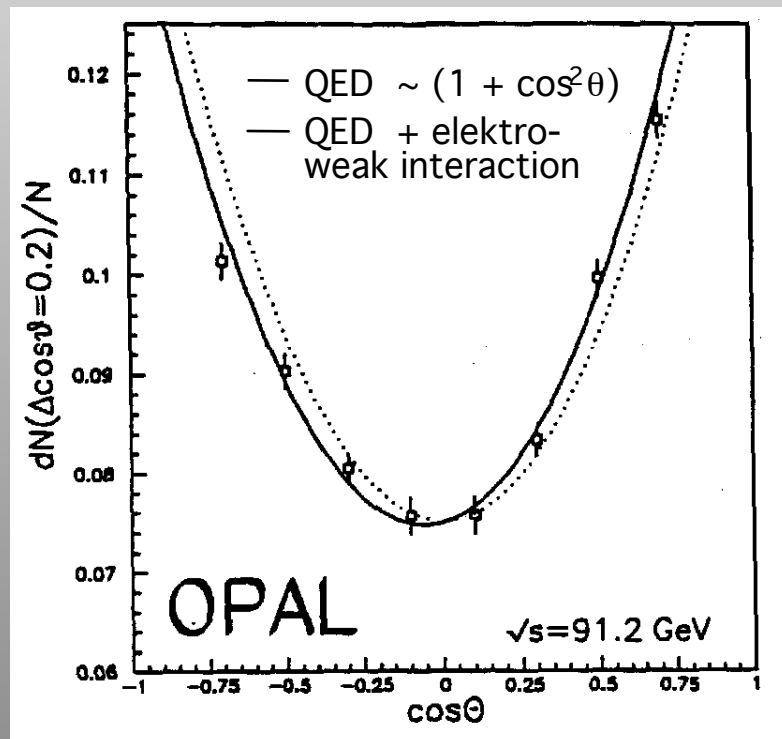
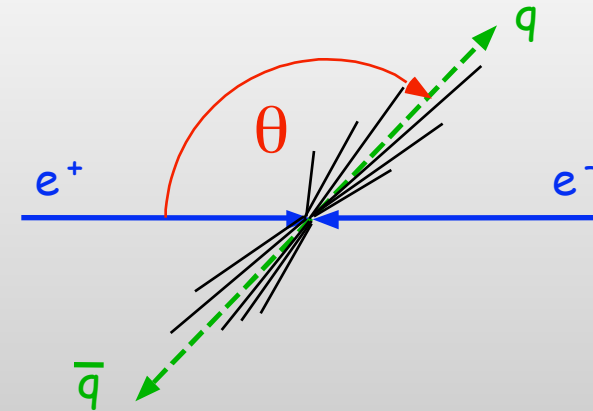
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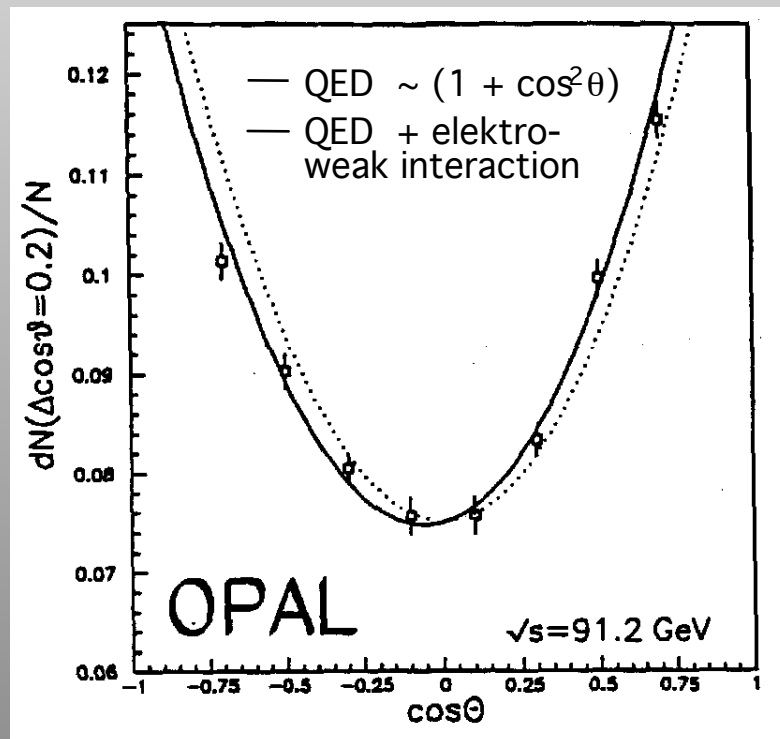
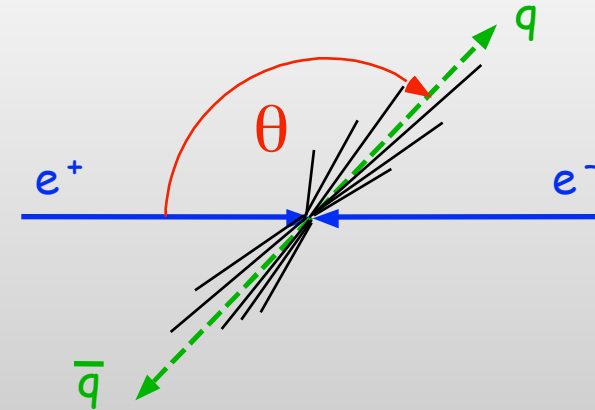
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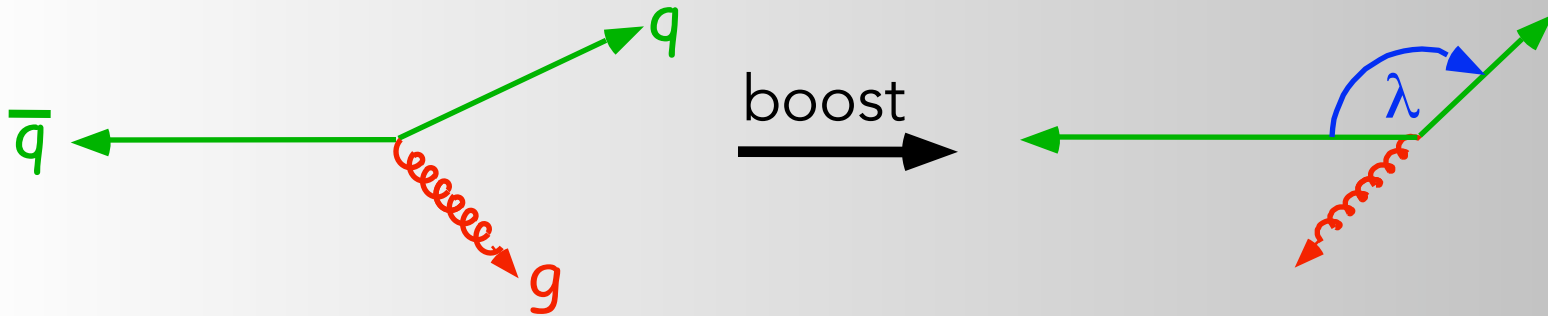
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fine structure: deviation from $1 + \cos^2\theta$ is due to electro-weak interference contributions of 4.5%;
 $\sin^2\theta_w = 0.2255 \pm 0.00212$

Orientation of Gluon-Jets in 3-Jet-Events:

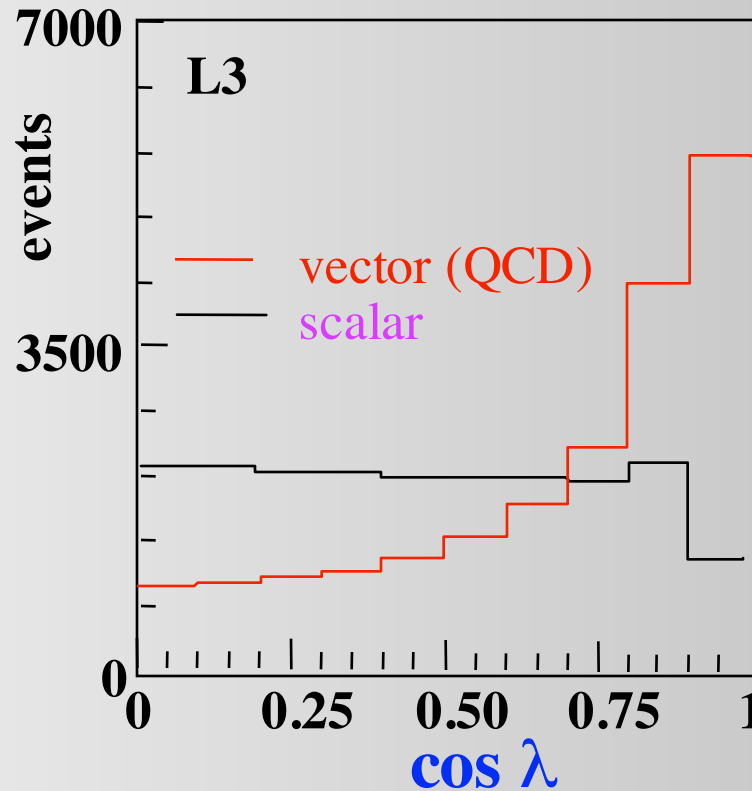
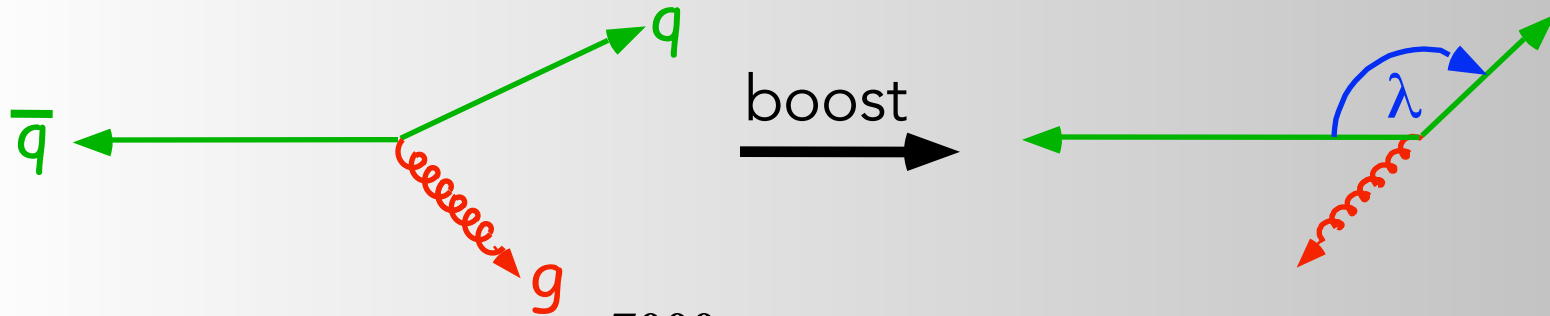
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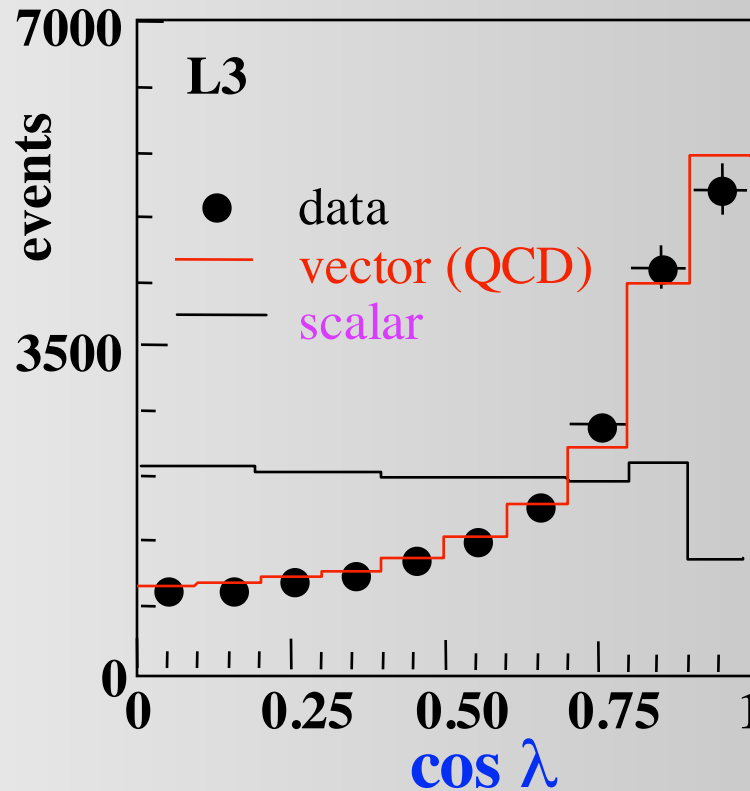
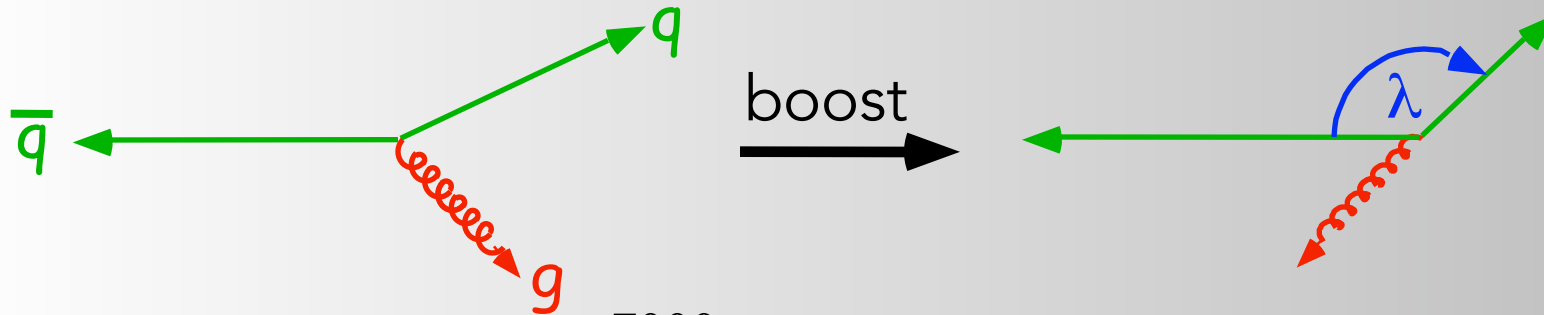
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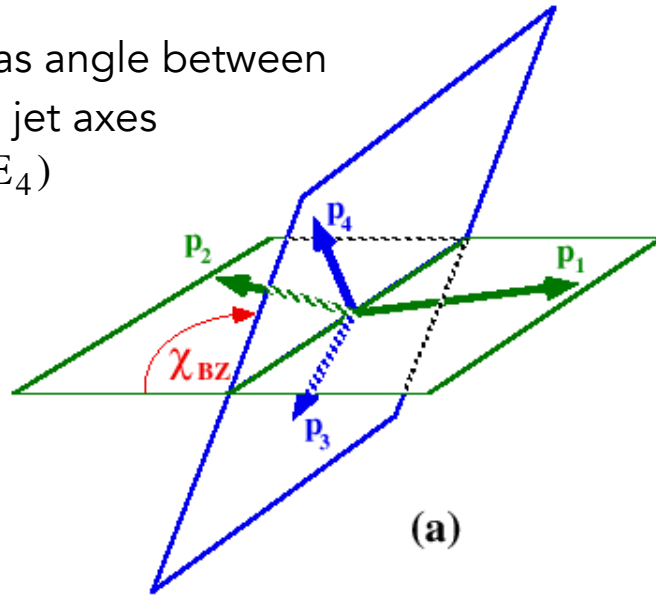
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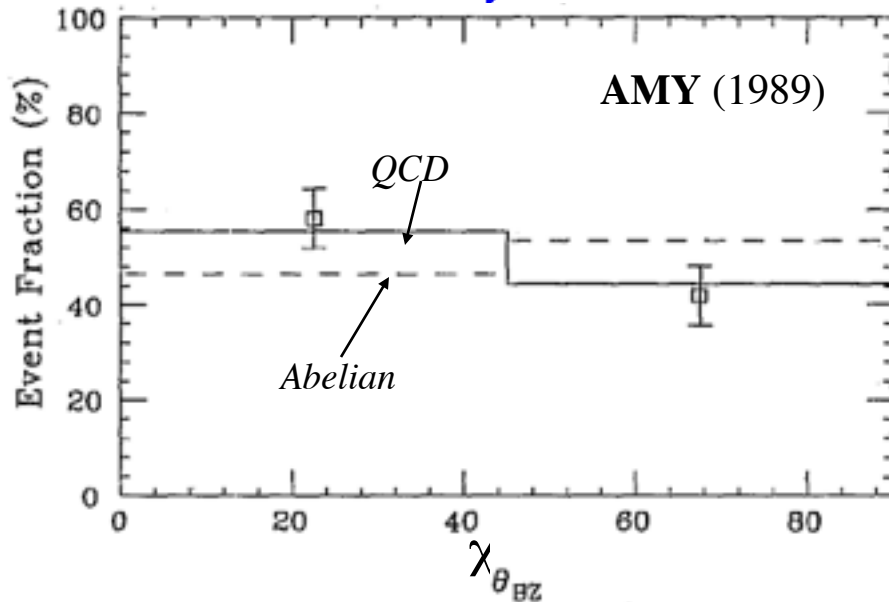
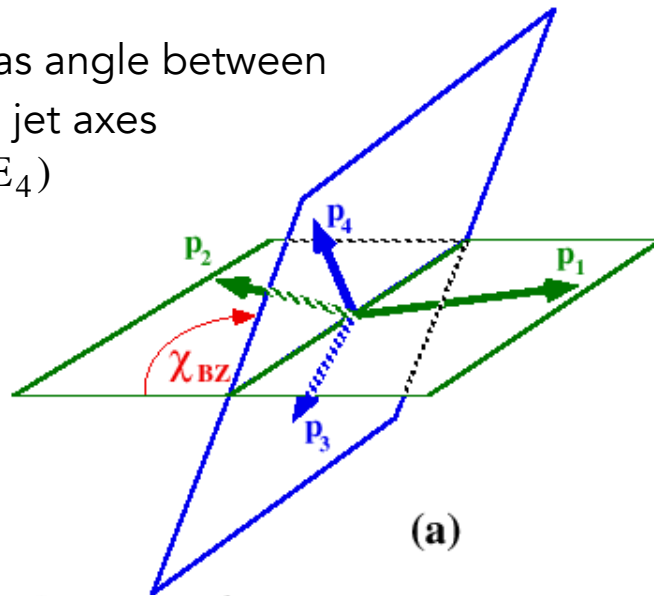
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Bengtson-Zerwas angle between
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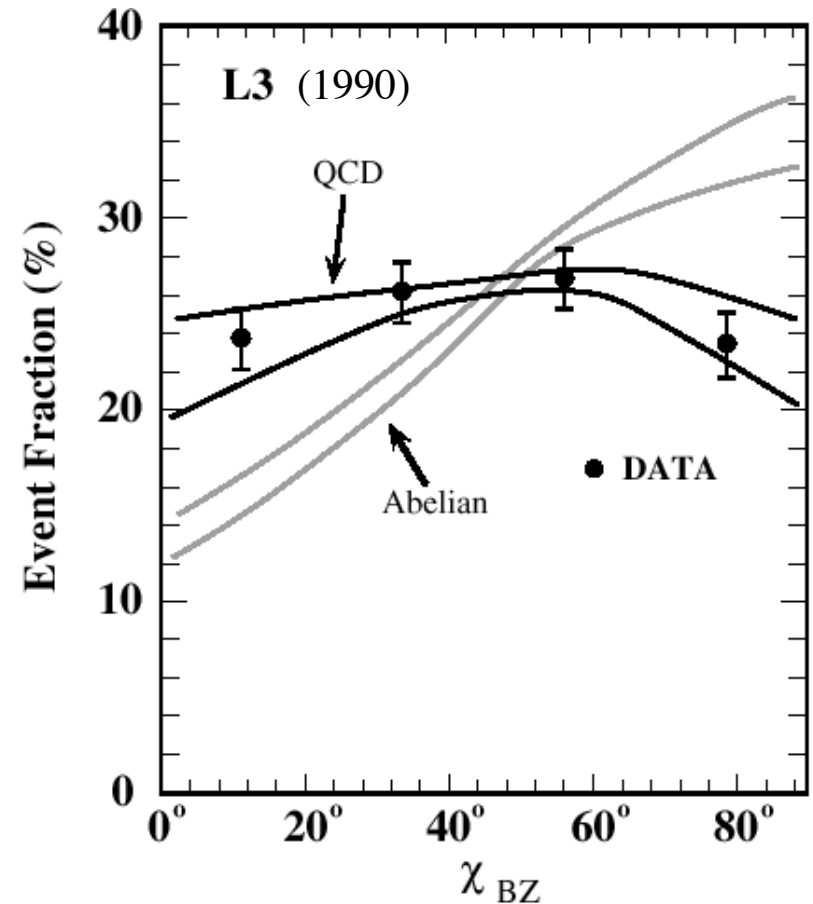
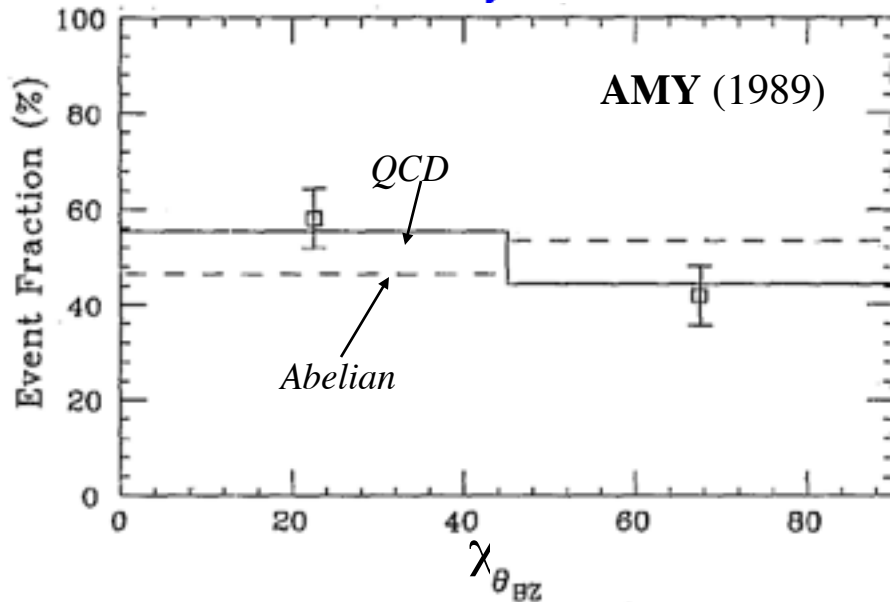
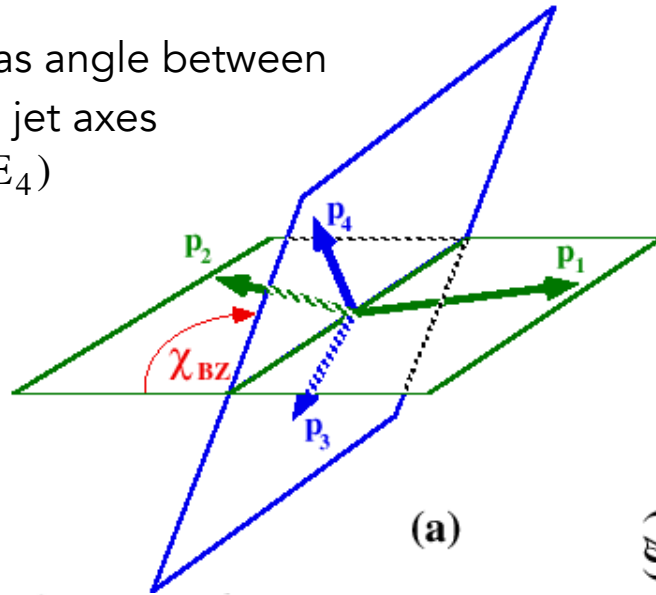
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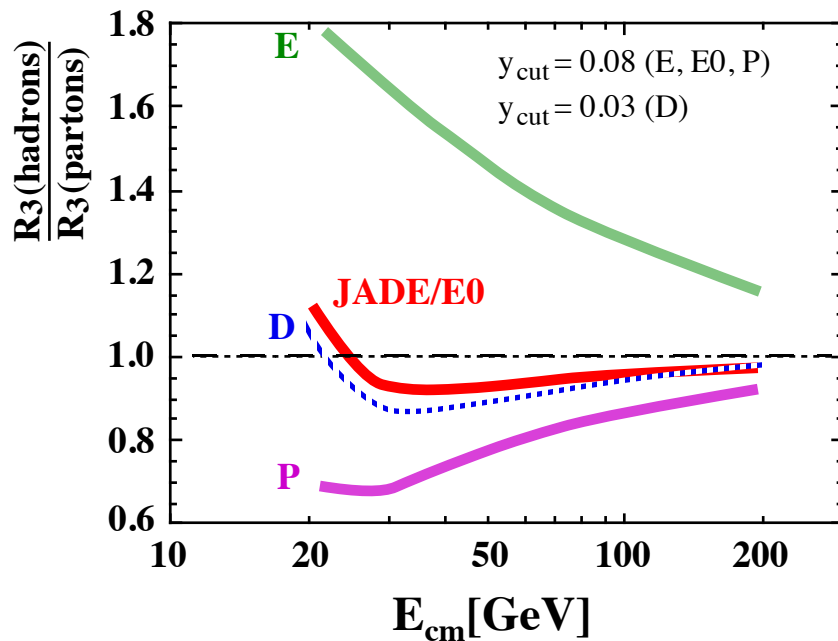
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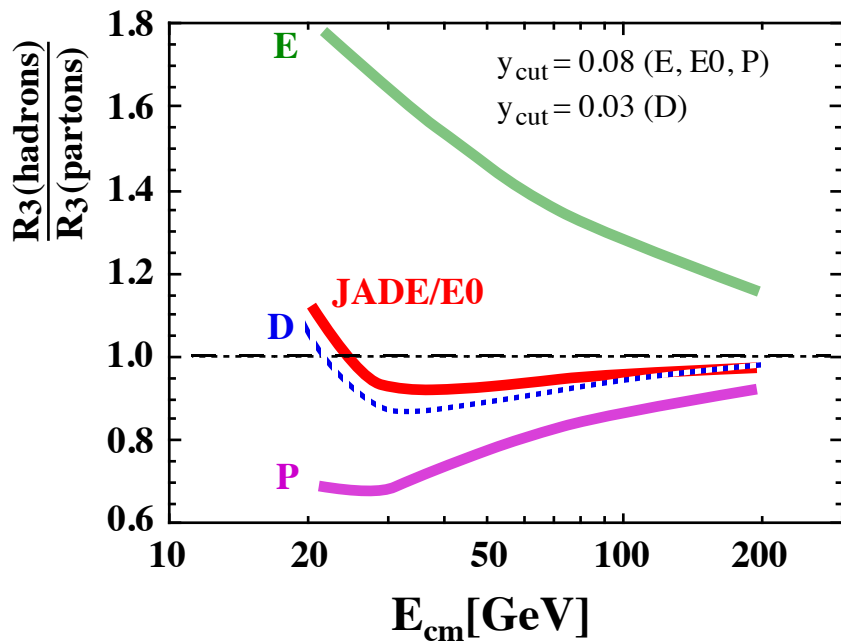
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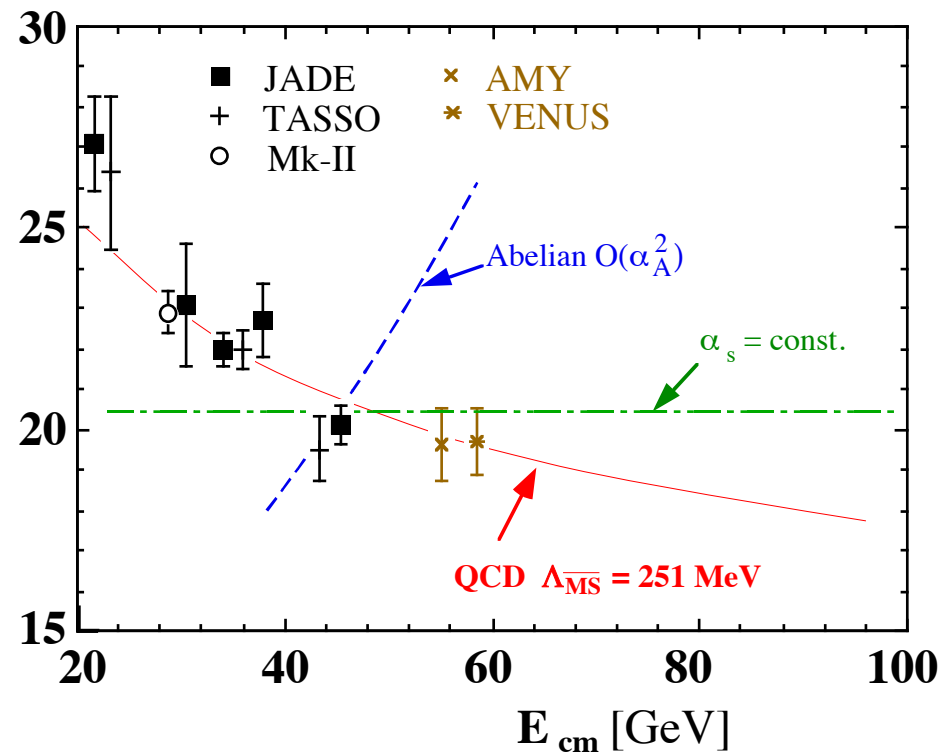
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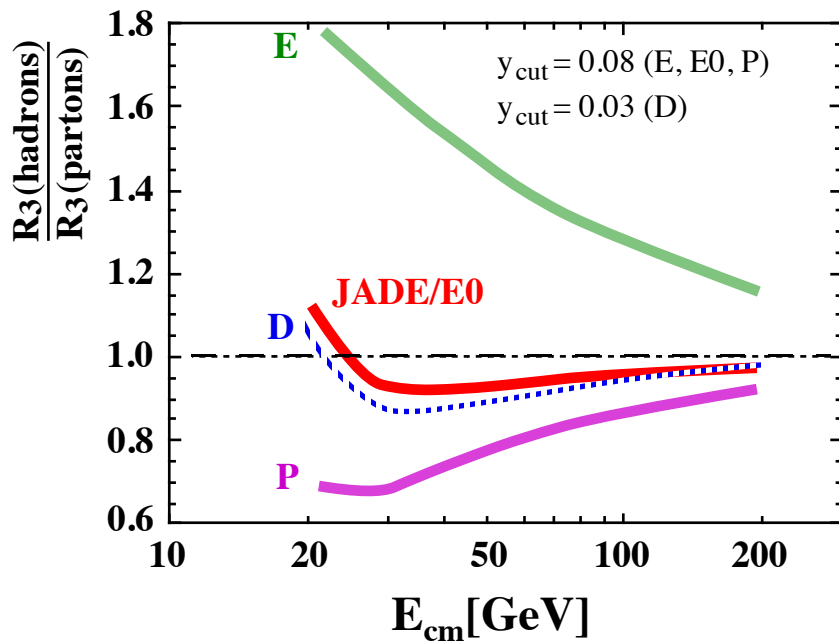
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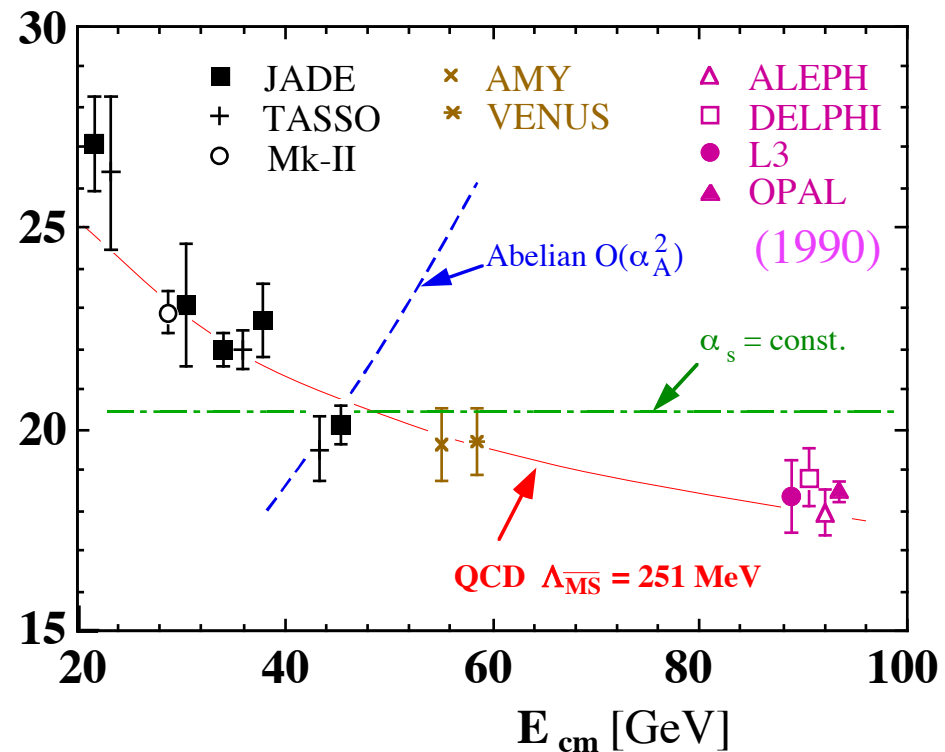
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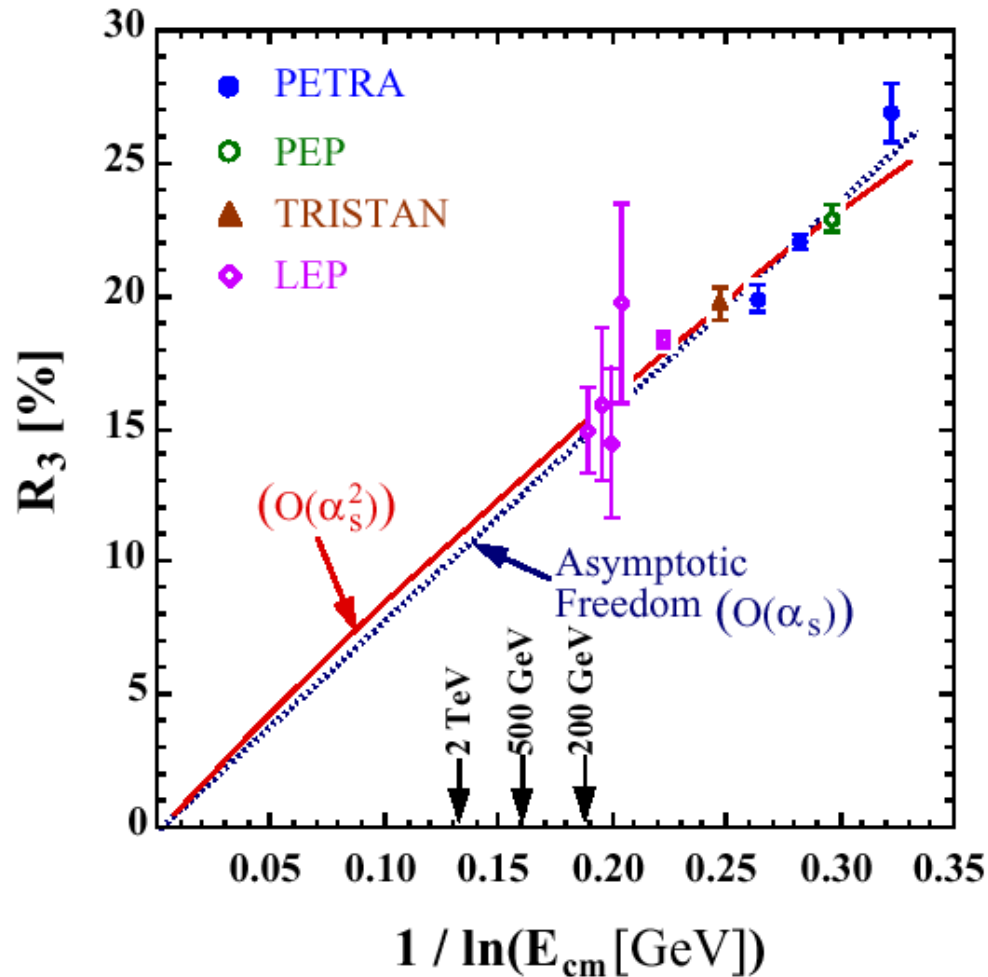


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Asymptotic Freedom from jet rates

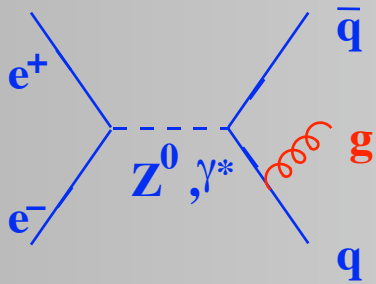
$$R_3 \equiv \frac{\sigma_{3\text{-jet}}}{\sigma_{\text{tot}}} \propto \alpha_s(E_{\text{cm}}) \propto \frac{1}{\ln E_{\text{cm}}}$$



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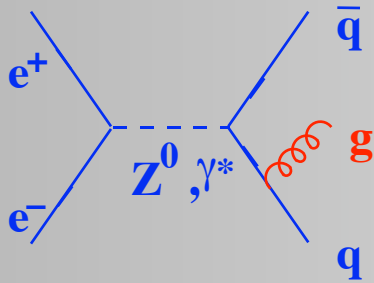
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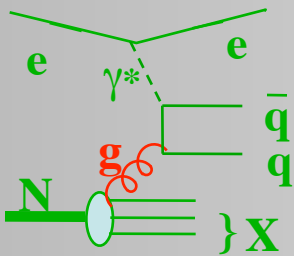
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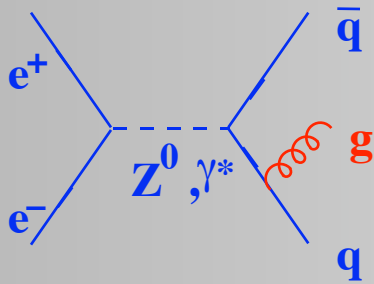
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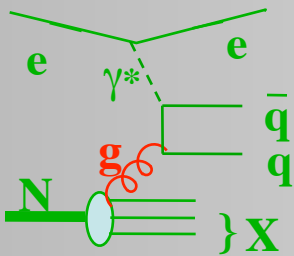
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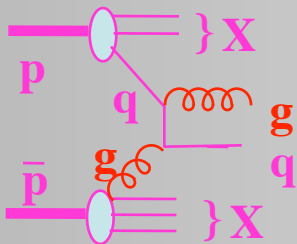
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- proton-(anti-)proton collisions
 - jet rates
 - photoproduction
 - t-quark production cross section

running α_s up to 4th order:

$$Q^2 \frac{\partial \alpha_s(Q^2)}{\partial Q^2} = \beta(\alpha_s(Q^2))$$

$$\beta(\alpha_s(Q^2)) = -\beta_0 \alpha_s^2(Q^2) - \beta_1 \alpha_s^3(Q^2) - \beta_2 \alpha_s^4(Q^2) - \beta_3 \alpha_s^5(Q^2) + \mathcal{O}(\alpha_s^6)$$

$$\beta_0 = \frac{33 - 2N_f}{12\pi},$$

$$\beta_1 = \frac{153 - 19N_f}{24\pi^2},$$

$$\beta_2 = \frac{77139 - 15099N_f + 325N_f^2}{3456\pi^3},$$

$$\beta_3 \approx \frac{29243 - 6946.3N_f + 405.089N_f^2 + 1.49931N_f^3}{256\pi^4}$$

$$\alpha_s(Q^2) = \frac{1}{\beta_0 L} - \frac{1}{\beta_0^3 L^2} \beta_1 \ln L$$

$$+ \frac{1}{\beta_0^3 L^3} \left(\frac{\beta_1^2}{\beta_0^2} (\ln^2 L - \ln L - 1) + \frac{\beta_2}{\beta_0} \right)$$

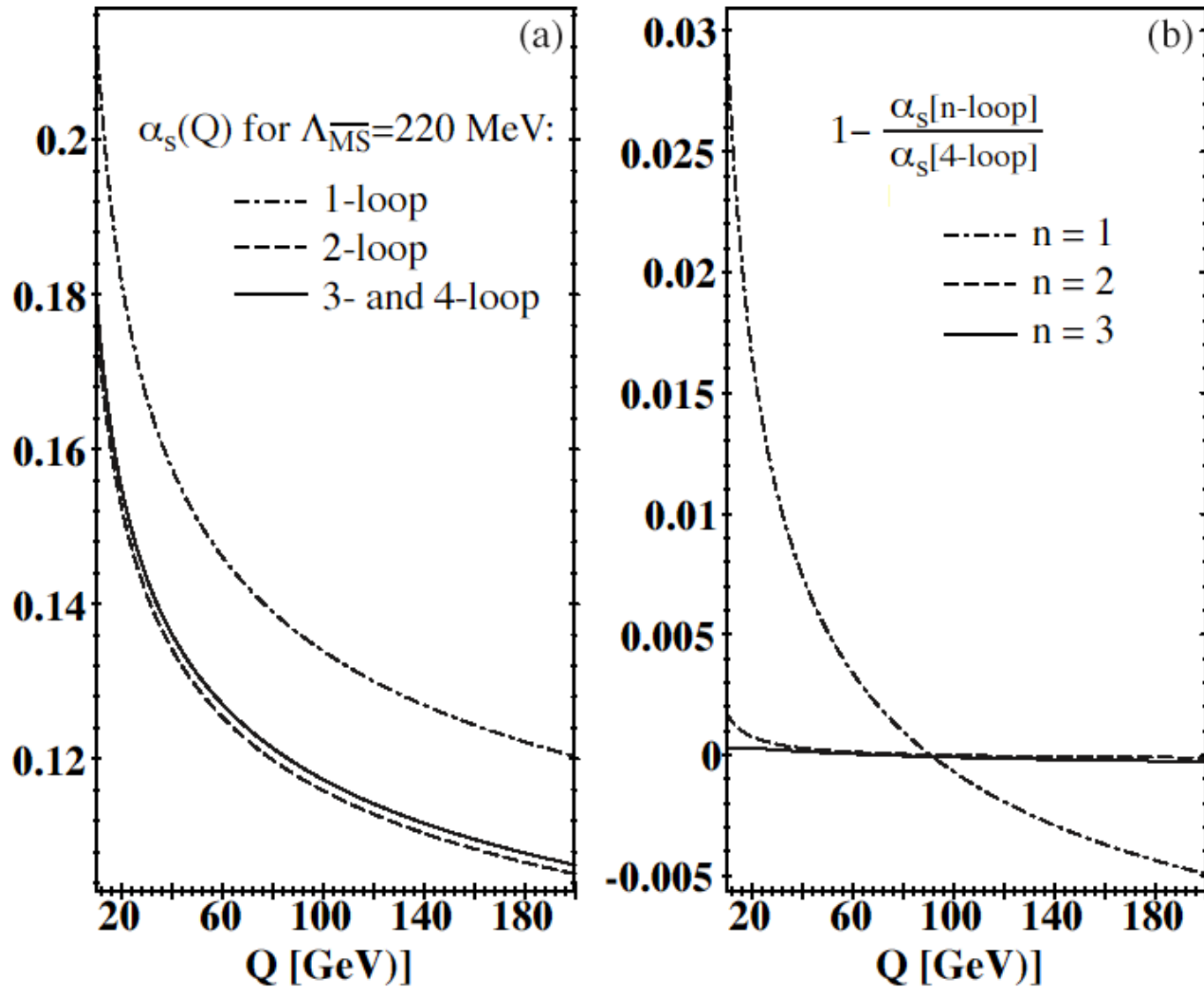
$$+ \frac{1}{\beta_0^4 L^4} \left(\frac{\beta_1^3}{\beta_0^3} \left(-\ln^3 L + \frac{5}{2} \ln^2 L + 2 \ln L - \frac{1}{2} \right) - 3 \frac{\beta_1 \beta_2}{\beta_0^2} \ln L + \frac{\beta_3}{2\beta_0} \right) \quad L = \ln \frac{Q^2}{\Lambda_{\overline{MS}}^2}$$

Ritbergen,
Vermaseren,
Larin

β_0 and β_1 do not depend on renormalisation scheme; β_2 and β_3 ... do !

choose \overline{MS} scheme for all of the following discussion.

relative size of higher order corrections



heavy quark threshold matching

Matching conditions for the choice $\mu^{(N_f)} = M_Q$ (pole mass definition):

$$\frac{a'}{a} = 1 + C_2 a^2 + C_3 a^3 \quad (\text{with } a' = \alpha_s^{(N_f-1)}/\pi; \quad a = \alpha_s^{(N_f)}/\pi)$$

$$C_2 = -0.291667 \text{ and } C_3 = -5.32389 + (N_f - 1) \cdot 0.26247$$

(3-loop condition; Chetyrkin, Kniehl, Steinhauser)

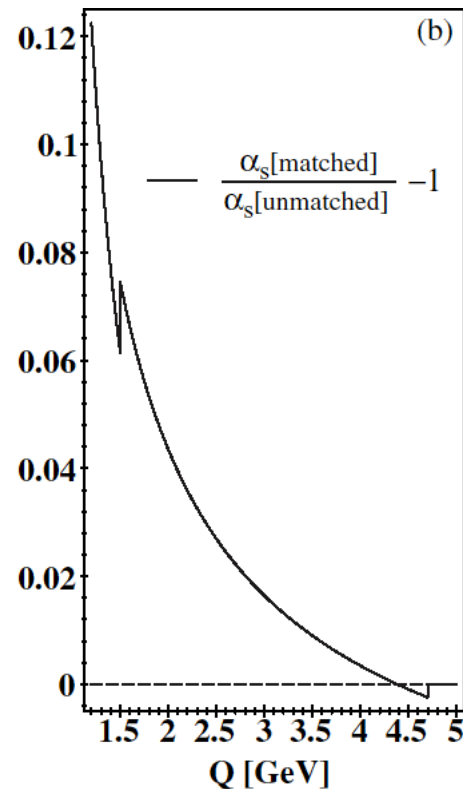
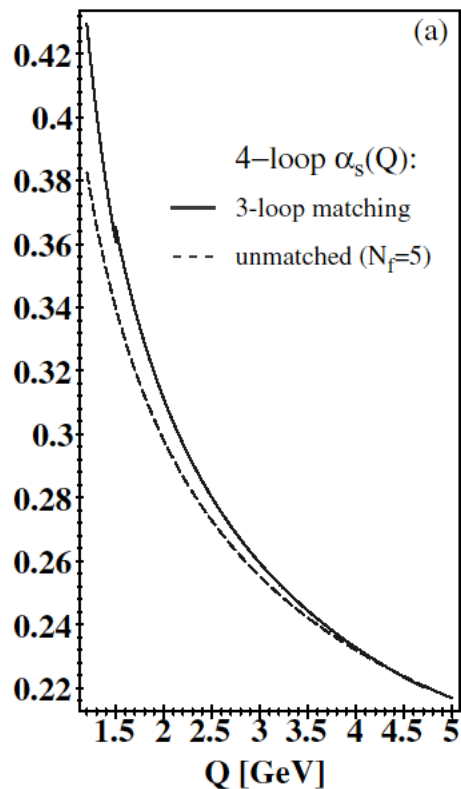
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$$\begin{aligned}\mathcal{R}(Q^2) &= P_l \sum_n R_n \alpha_s^n \\ &= P_l (R_0 + R_1 \alpha_s(\mu^2) + R_2 (Q^2/\mu^2) \alpha_s^2(\mu^2) + \dots)\end{aligned}$$

in n^{th} order perturbation theory

R_1 : “leading order coefficient” (lo)

R_2 : “next to leading coefficient” (nlo)

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Resummation of logs arising from soft and collinear singularities:

$$\Sigma(\mathcal{R}) \equiv \int_0^{\mathcal{R}} \frac{1}{\sigma} \frac{d\sigma}{d\mathcal{R}} d\mathcal{R} = C(\alpha_s) \exp[G(\alpha_s, L)] + D(\alpha_s, \mathcal{R}) \quad L = \ln(1/\mathcal{R}) \quad C(\alpha_s) = 1 + \sum_{n=1}^{\infty} C_n \hat{\alpha}_s^n$$

$$\begin{aligned} G(\alpha_s, L) &= \sum_{n=1}^{\infty} \sum_{m=1}^{n+1} G_{nm} \hat{\alpha}_s^n L^m \\ &\equiv Lg_1(\alpha_s L) + g_2(\alpha_s L) + \alpha_s g_3(\alpha_s L) + \alpha_s^2 g_4(\alpha_s L) \dots \end{aligned}$$

| | Leading logs | Next-to-Leading logs | Subleading logs | Non-log. terms | |
|-----------------------------|---------------------------------|---------------------------------|---|-------------------------------|---------------------------|
| $\ln \Sigma(\mathcal{R}) =$ | $G_{12} \hat{\alpha}_s L^2$ | $+ G_{11} \hat{\alpha}_s L$ | | $+ \alpha_s \mathcal{O}(1)$ | $\mathcal{O}(\alpha_s)$ |
| | $+ G_{23} \hat{\alpha}_s^2 L^3$ | $+ G_{22} \hat{\alpha}_s^2 L^2$ | $+ G_{21} \hat{\alpha}_s^2 L$ | $+ \alpha_s^2 \mathcal{O}(1)$ | $\mathcal{O}(\alpha_s^2)$ |
| | $+ G_{34} \hat{\alpha}_s^3 L^4$ | $+ G_{33} \hat{\alpha}_s^3 L^3$ | $+ G_{32} \hat{\alpha}_s^3 L^2 + \dots$ | $+ \dots$ | $\mathcal{O}(\alpha_s^3)$ |
| | $+ \dots$ | $+ \dots$ | $+ \dots$ | $+ \dots$ | \vdots |
| $=$ | $Lg_1(\alpha_s L)$ | $+ g_2(\alpha_s L)$ | $+ \dots$ | $+ \dots$ | |

renormalisation scale dependence

$$\mathcal{R} \equiv \mathcal{R}(Q^2/\mu^2, \alpha_s); \quad \alpha_s \equiv \alpha_s(\mu^2)$$

since choice of μ is arbitrary, physical observables \mathcal{R} should not depend on μ

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→

$$R_0 = \text{const.} ,$$

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renormalisation scale dependence

$$\mathcal{R} \equiv \mathcal{R}(Q^2/\mu^2, \alpha_s); \quad \alpha_s \equiv \alpha_s(\mu^2)$$

since choice of μ is arbitrary, physical observables \mathcal{R} should not depend on μ

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Perturbative QCD coefficients beyond leading order become renormalisation scale dependent !

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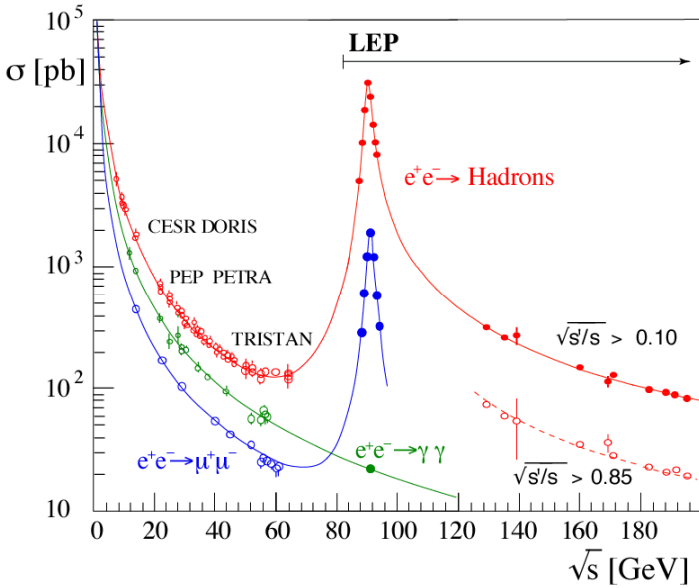
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Perturbative QCD coefficients beyond leading order become renormalisation scale dependent !

This dependence is used to quantify theoretical uncertainties due to unknown higher orders.

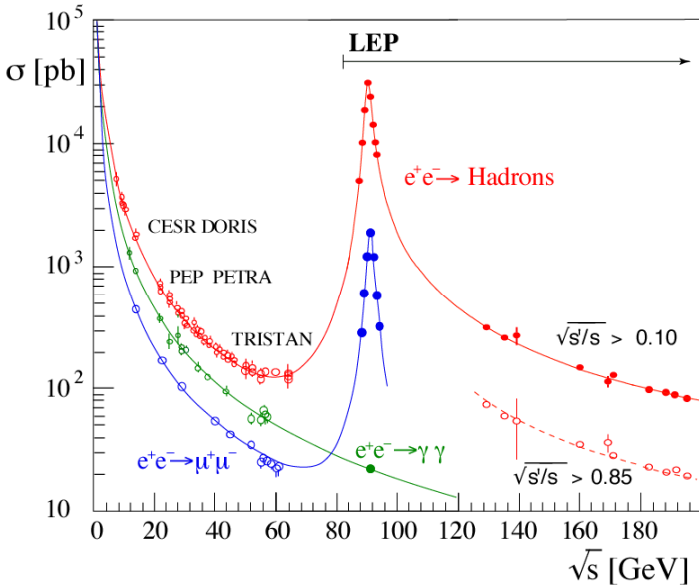
hadronische Breite des Z^0 Boson



$$R_Z = \frac{\Gamma(Z^0 \rightarrow \text{hadrons})}{\Gamma(Z^0 \rightarrow \text{leptons})} = 20.768 \pm 0.0024$$

$$R_Z = 19.934 \left[1 + 1.045 \frac{\alpha_s(\mu)}{\pi} + 0.94 \left[\frac{\alpha_s(\mu)}{\pi} \right]^2 - 15 \left[\frac{\alpha_s(\mu)}{\pi} \right]^3 \right]$$

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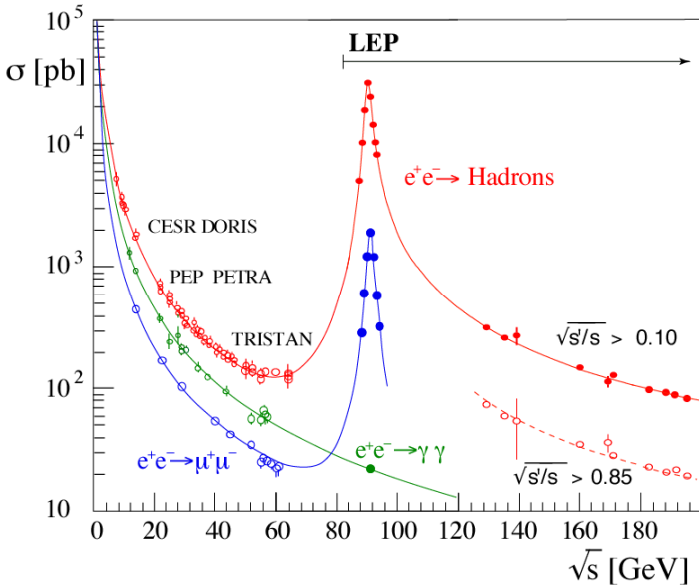


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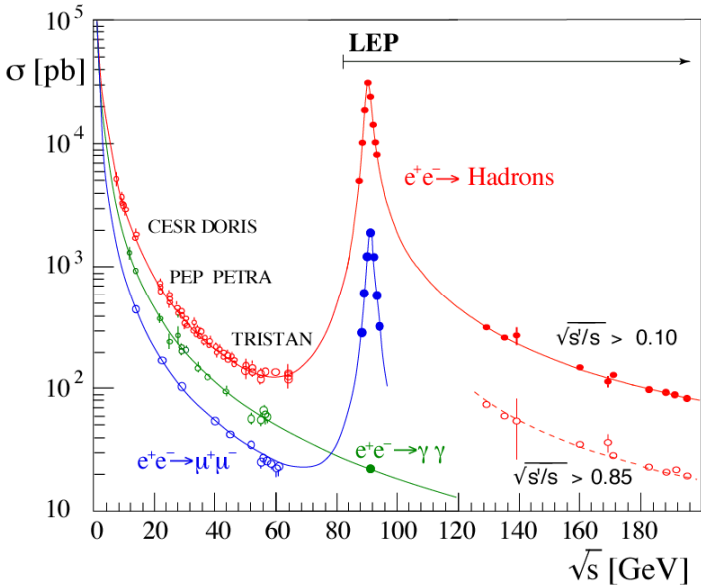
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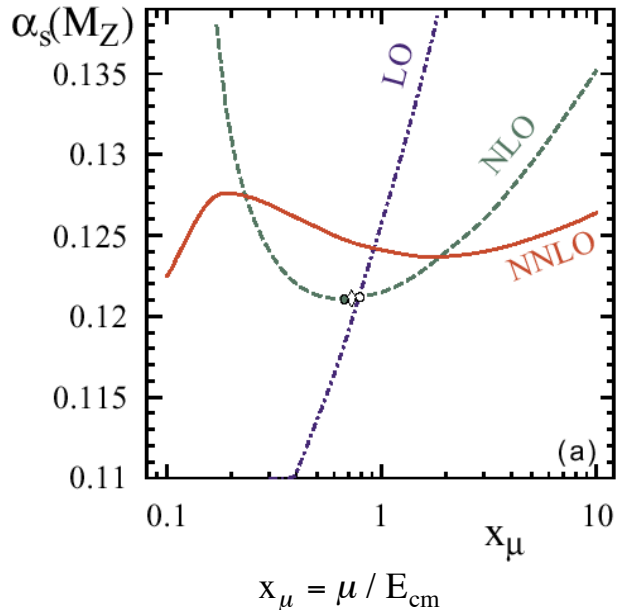
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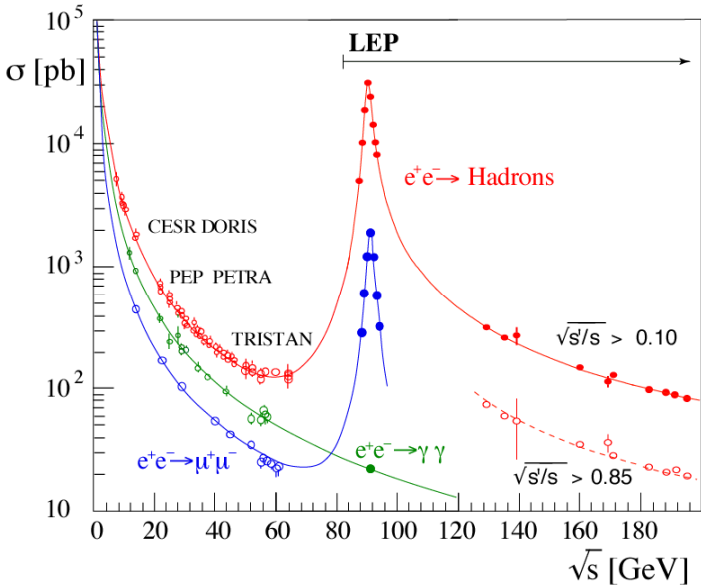
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$$x_\mu = \mu / E_{\text{cm}}$$

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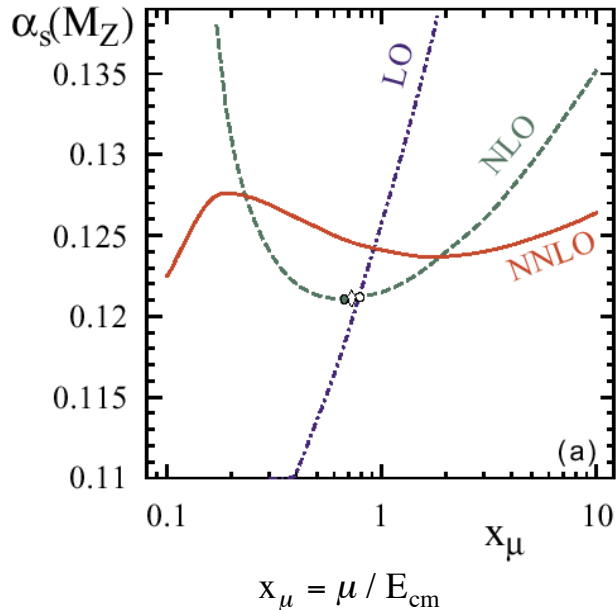
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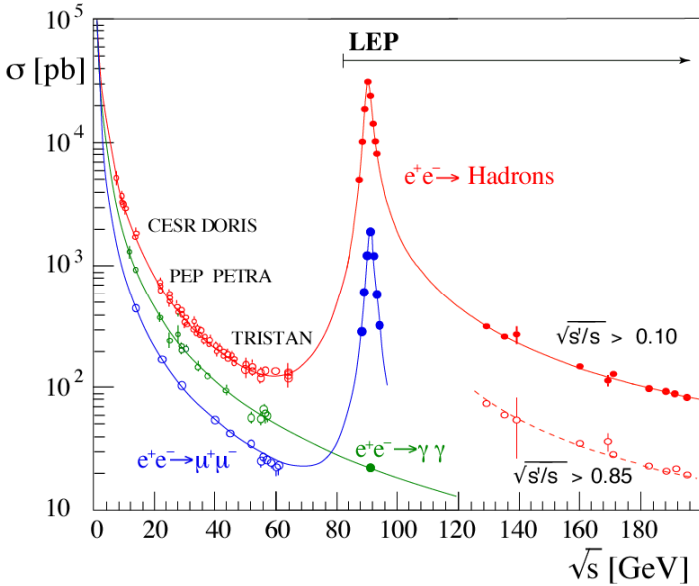
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| error source | $\Delta\alpha_s(M_{Z^0})$ |
|--|---------------------------|
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| $\Delta M_t = \pm 5 \text{ GeV}$ | ± 0.0002 |
| $M_H = 100 \dots 1000 \text{ GeV}$ | ± 0.0017 |
| $\mu = \left(\frac{1}{4} \dots 4\right) M_{Z^0}$ | $+ 0.0028$ $- 0.0004$ |
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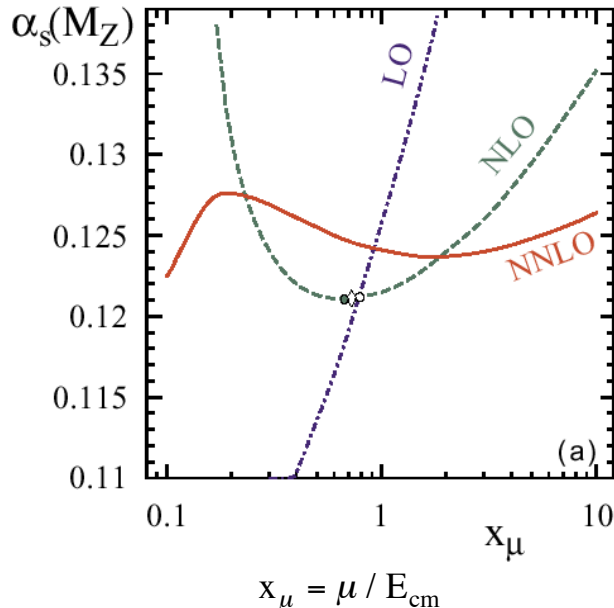
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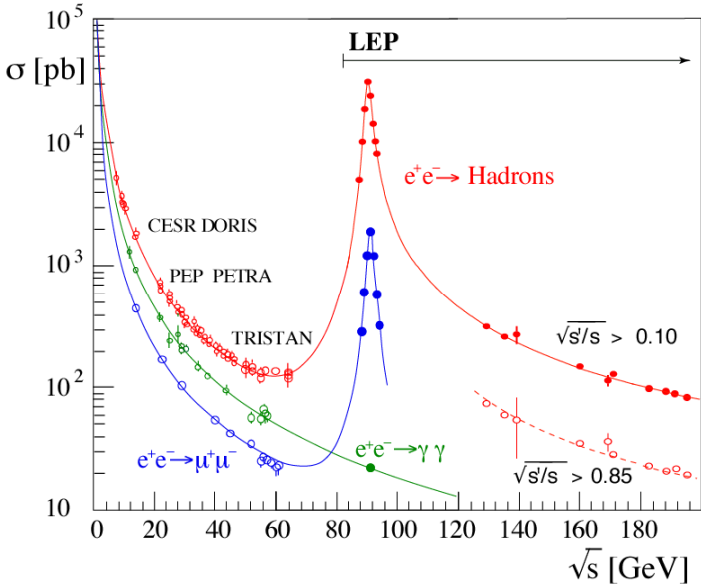
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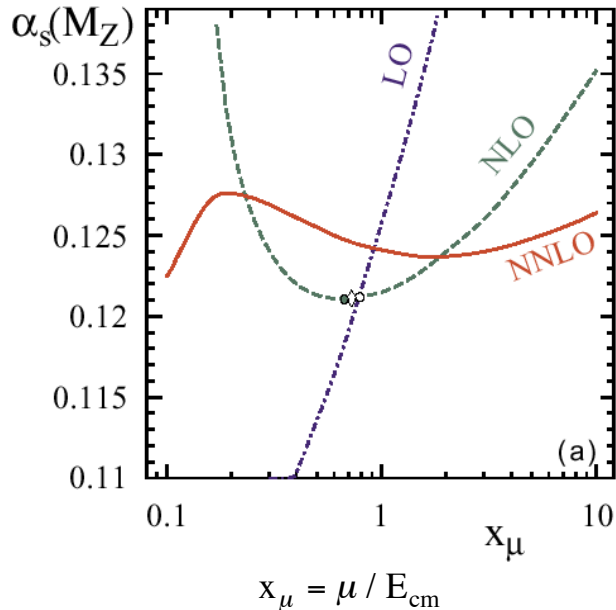
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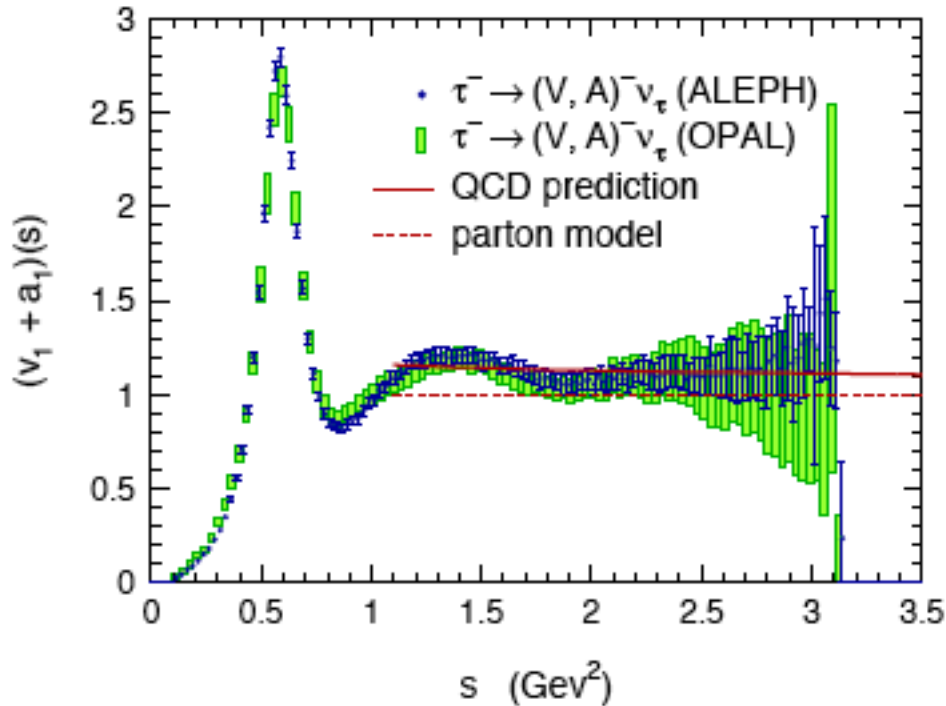
α_s from τ -decays

$$R_\tau = \frac{\Gamma(\tau \rightarrow \text{hadrons } \nu_\tau)}{\Gamma(\tau \rightarrow e \nu_e \nu_\tau)}$$

$$QCD: R_\tau = 3.058(1.001 + \delta_{pert} + \delta_{nonpert})$$




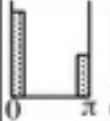

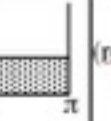



$$\delta_{pert} = \frac{\alpha_s(m_\tau)}{\pi} + 5.20 \left(\frac{\alpha_s(m_\tau)}{\pi} \right)^2 + 26.37 \left(\frac{\alpha_s(m_\tau)}{\pi} \right)^3$$

measurements of R as well as the mass spectra of hadronic τ -decays and comparison with $O(\alpha_s^3)$ perturbative QCD results in $\alpha_s(M_\tau)$ also provides an independent determination of the leading nonperturbative contributions $\delta_{nonpert}$

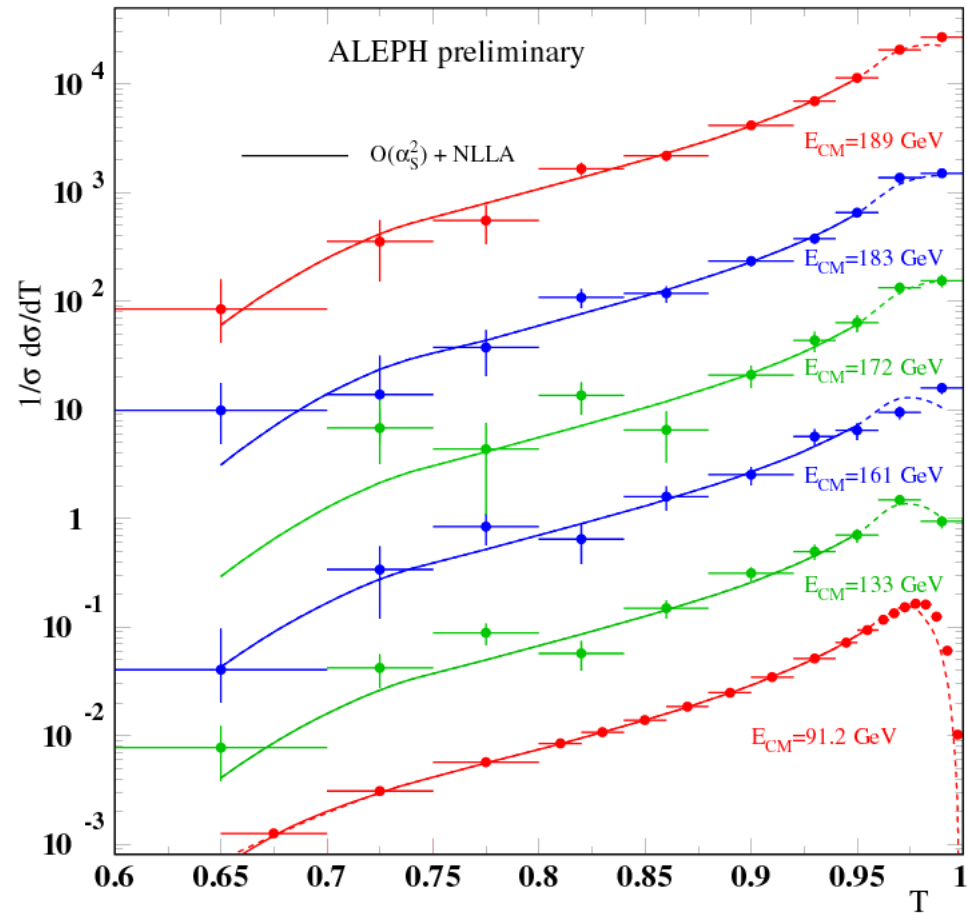
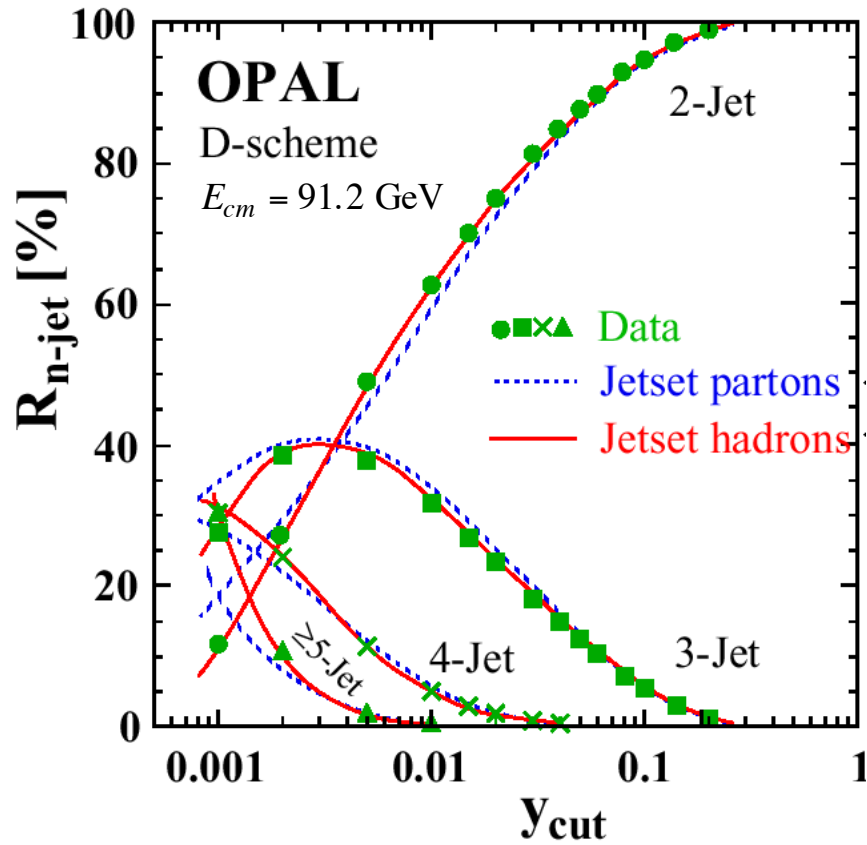


$$\alpha_s(M_Z) = 0.1213 \pm 0.0006_{\text{exp}} \pm 0.0010_{\text{theo}}$$

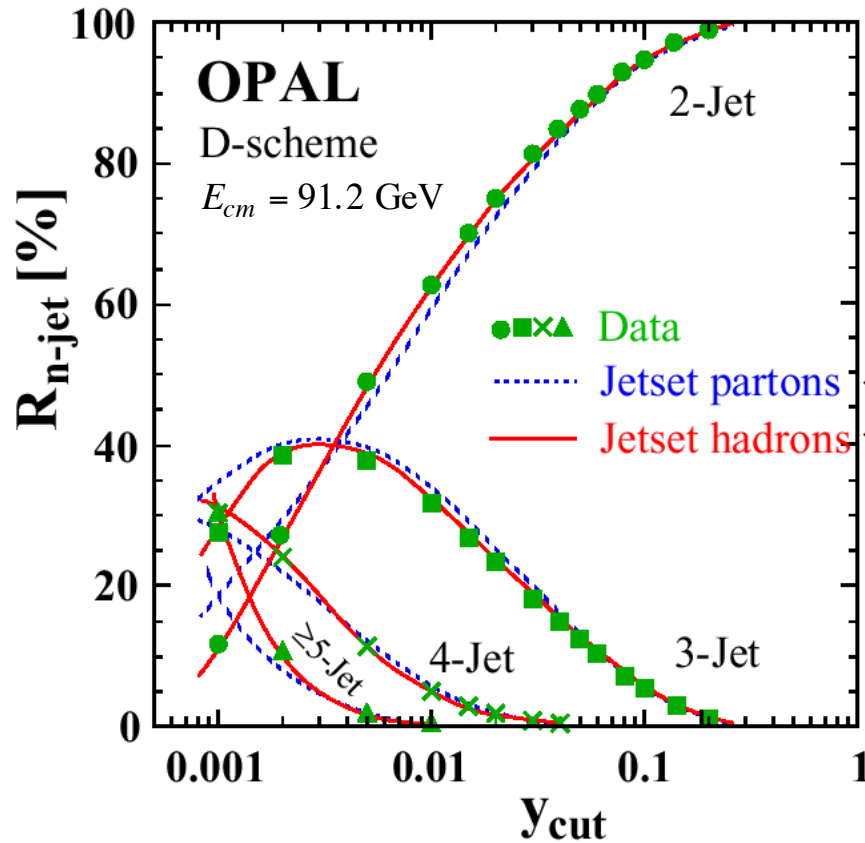
Event Shape Observables

| Name of Observable | Definition | Typical Value for: | | | QCD calculation |
|----------------------------|---|---|---|---|-------------------------------|
| | |  |  |  | |
| Thrust | $T = \max_{\vec{n}} \left(\frac{\sum_i \vec{p}_i \vec{n} }{\sum_i \vec{p}_i } \right)$ | 1 | $\geq 2/3$ | $\geq 1/2$ | (resummed) $O(\alpha_s^2)$ |
| Thrust major | Like T, however T_{maj} and \vec{n}_{maj} in plane $\perp \vec{n}_T$ | 0 | $\leq 1/3$ | $\leq 1/\sqrt{2}$ | $O(\alpha_s^2)$ |
| Thrust minor | Like T, however T_{min} and \vec{n}_{min} in direction \perp to \vec{n}_T and \vec{n}_{maj} | 0 | 0 | $\leq 1/2$ | $O(\alpha_s^2)$ |
| Oblateness | $O = T_{maj} - T_{min}$ | 0 | $\leq 1/3$ | 0 | $O(\alpha_s^2)$ |
| Sphericity | $S = 1.5 (Q_1 + Q_2)$; $Q_1 \leq \dots \leq Q_3$ are Eigenvalues of $S^{\alpha\beta} = \frac{\sum_i p_i^\alpha p_i^\beta}{\sum_i p_i^2}$ | 0 | $\leq 3/4$ | ≤ 1 | none (not infrared safe) |
| Aplanarity | $A = 1.5 Q_1$ | 0 | 0 | $\leq 1/2$ | none (not infrared safe) |
| Jet (Hemisphere) masses | $M_{\pm}^2 = (\sum_i E_i^2 - \sum_i \vec{p}_i^2)_{i \in S_{\pm}}$ (S_{\pm} : Hemispheres \perp to \vec{n}_T) $M_{H1}^2 = \max(M_{+}^2, M_{-}^2)$ $M_D^2 = M_{+}^2 - M_{-}^2 $ | 0 | $\leq 1/3$ | $\leq 1/2$ | (resummed) $O(\alpha_s^2)$ |
| Jet broadening | $B_{\pm} = \frac{\sum_{i \in S_{\pm}} \vec{p}_i \times \vec{n}_T }{2 \sum_i \vec{p}_i }$; $B_T = B_+ + B_-$ $B_w = \max(B_+, B_-)$ | 0 | $\leq 1/(2\sqrt{3})$ | $\leq 1/(2\sqrt{2})$ | (resummed) $O(\alpha_s^2)$ |
| Energy-Energy Correlations | $EEC(\chi) = \sum_{\alpha \neq \beta} \int_{\chi - \frac{\Delta\chi}{2}}^{\chi + \frac{\Delta\chi}{2}} \sum_{i,j=1,2} \frac{E_i E_j}{E_{vis}^2} \delta(\chi - \chi_{ij}) d\chi$ |  |  |  | (resummed) $O(\alpha_s^2)$ |
| Asymmetry of EEC | $AEEC(\chi) = EEC(\pi - \chi) - EEC(\chi)$ |  |  |  | $O(\alpha_s^2)$ |
| Differential 2-jet rate | $D_2(y) = \frac{R_2(y - \Delta y) - R_2(y)}{\Delta y}$ | | | | (resummed) $O(\alpha_s^2)$ |

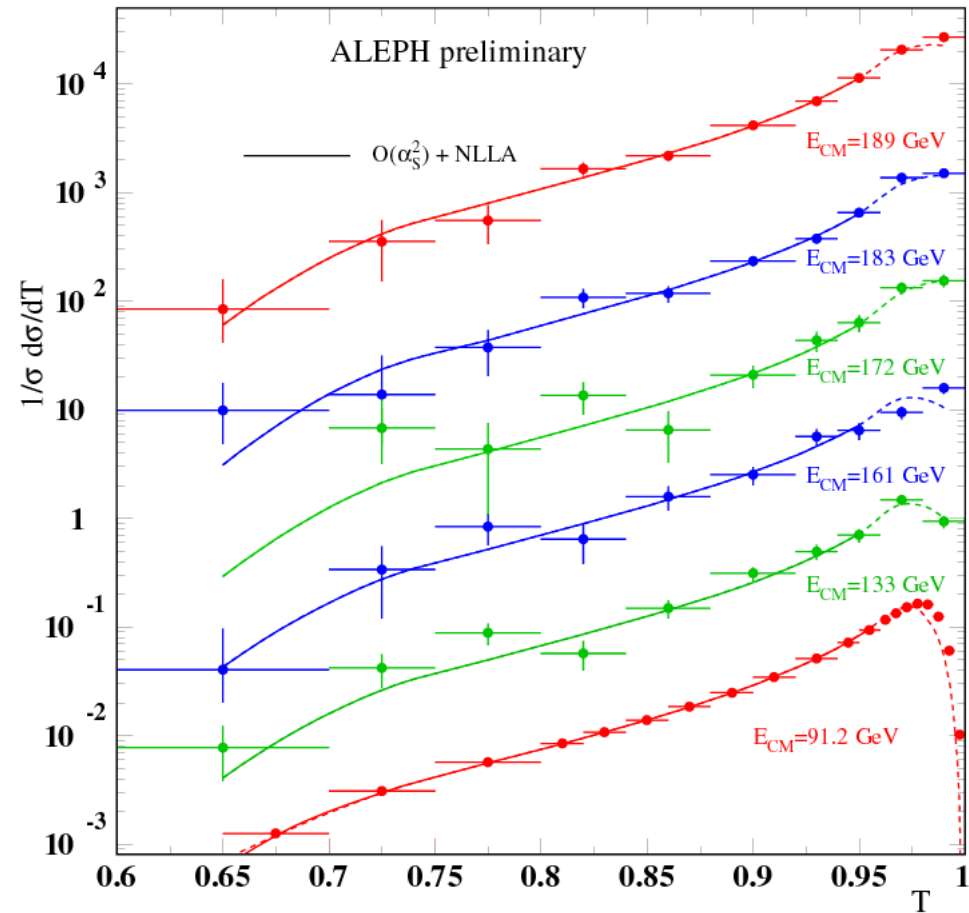
Jet production and hadronic event shapes



Jet production and hadronic event shapes



Pert. QCD



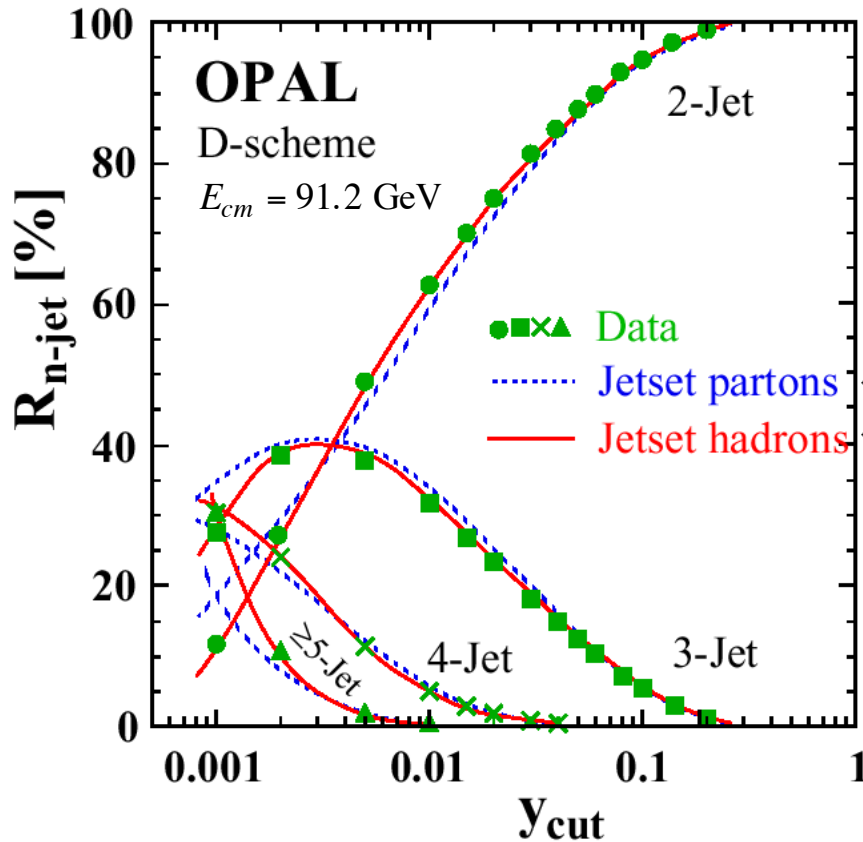
- in NLO: $\frac{1}{\sigma_0} \frac{d\sigma}{dy} = R_1(y) \alpha_s(\mu^2) + R_2\left(y, \frac{\mu^2}{Q^2}\right) \alpha_s^2(\mu^2)$

Ellis, Ross & Terrano (ERT);
Kunszt & Nason, Catani & Seymour

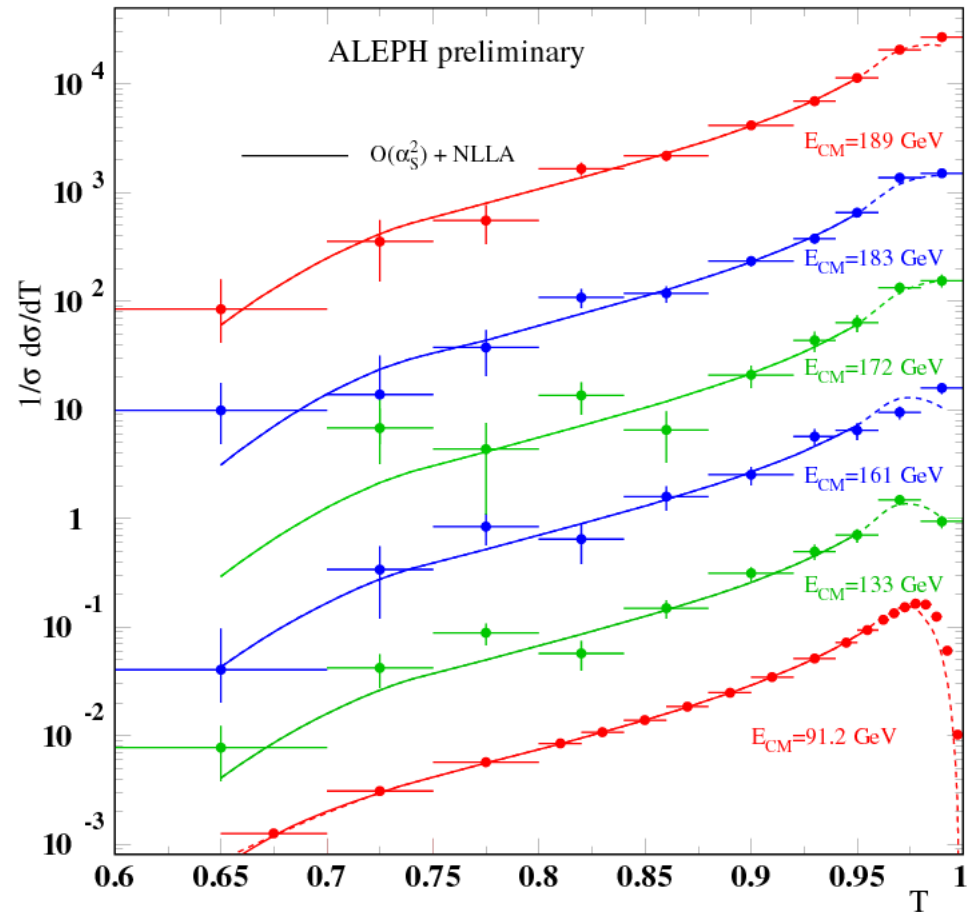
- plus resummation of leading and next-to-leading logarithms (NLLA) → "matching schemes"

Catani, Trentadue, Turnock, Webber

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in NNLO: A. Gehrmann-de Ridder et al., 2007

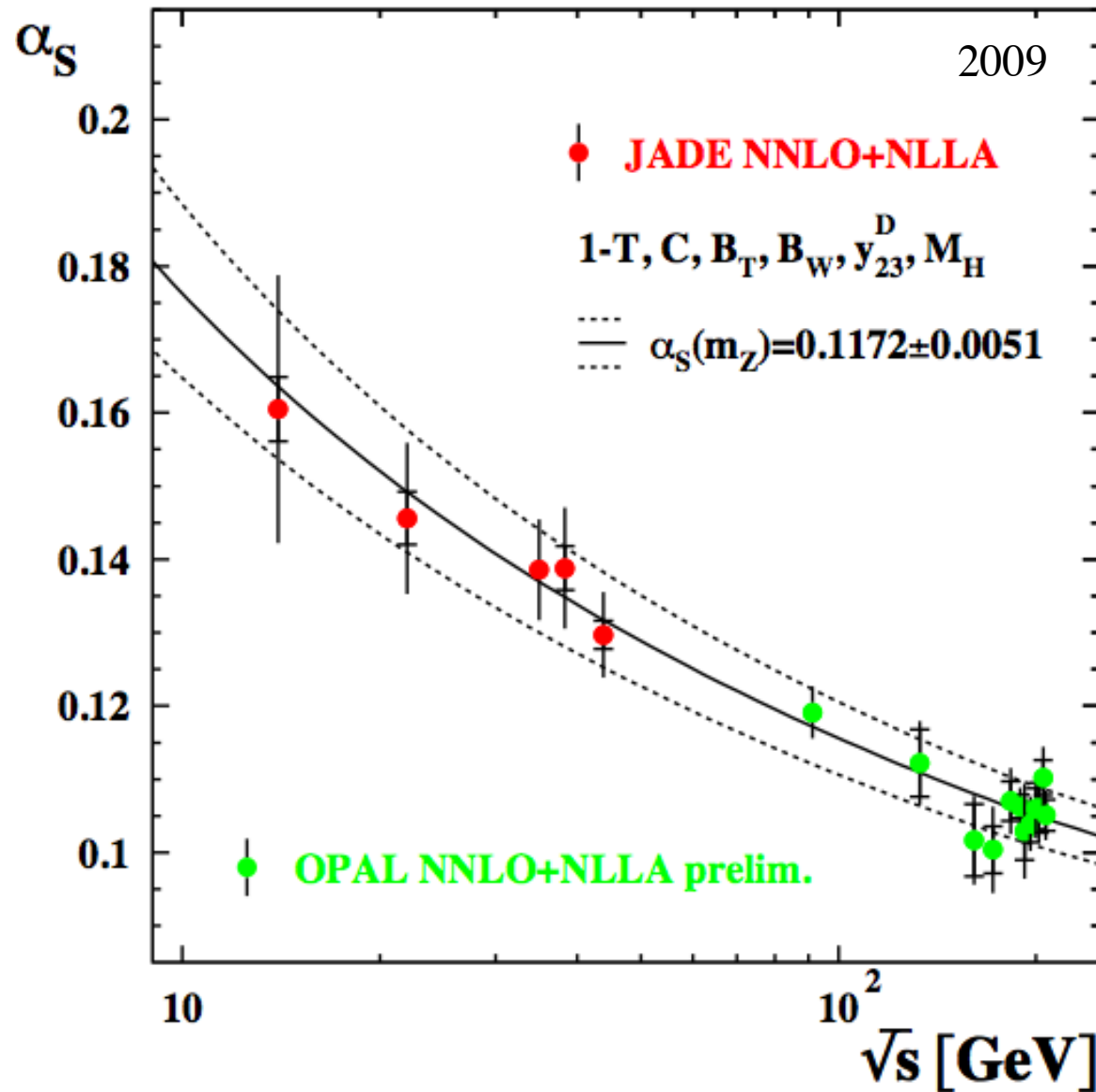
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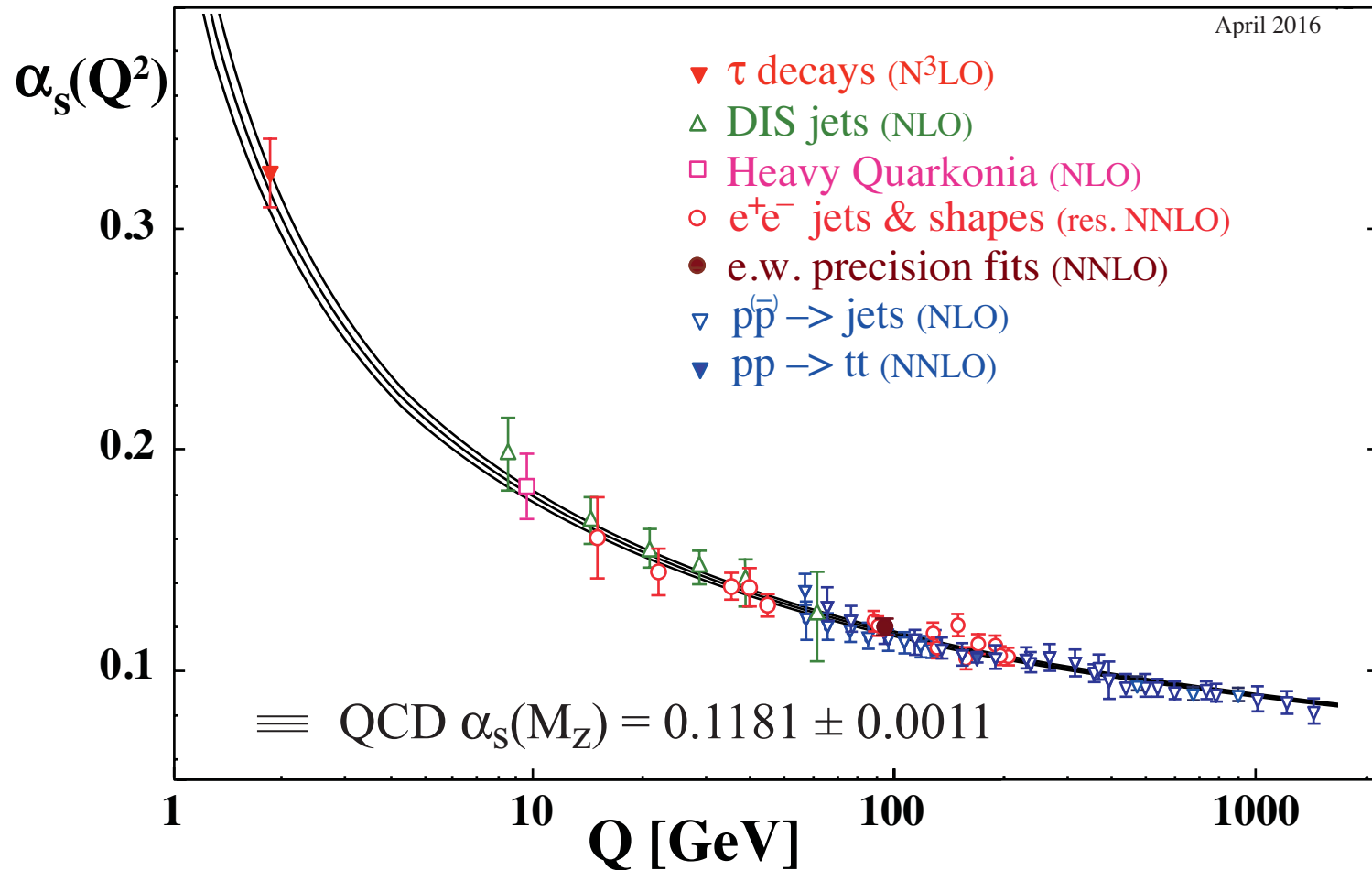
α_s aus Jetraten und event shapes in NNLO QCD:

2009

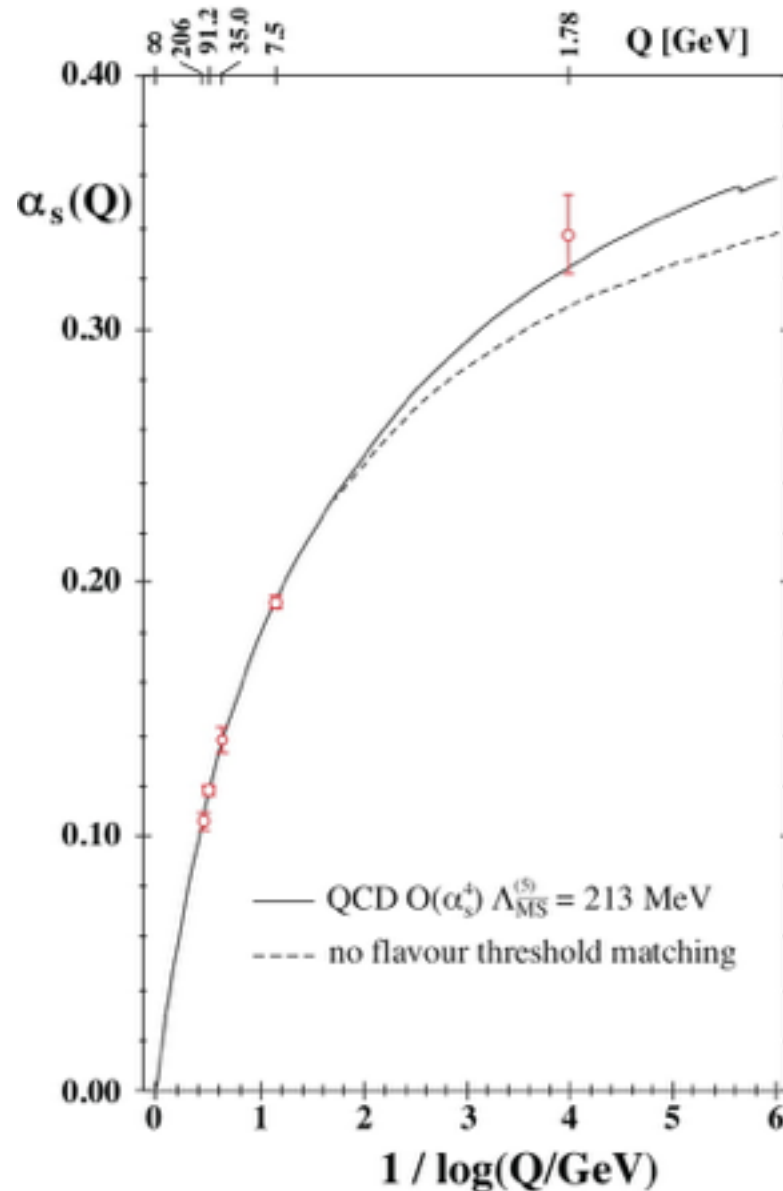
α_s aus Jetraten und event shapes in NNLO QCD:



globale Zusammenfassung der Messungen von α_s



Evidence for Asymptotic Freedom:



Zusammenfassung:

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- QCD als Eichfeldtheorie der Starken Wechselwirkung etabliert:
 - asymptotische Freiheit aus Energieabhängigkeit der Jetraten und von α_s experimentell verifiziert
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- Messung von α_s aus vielen Reaktionen:
 $\alpha_s(M_Z) \sim 0.12 \quad (0.1181 \pm 0.0011)$

Literaturempfehlungen

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