$B \to D^{(*)}$ form factors at large recoil from QCD Light-Cone Sum Rules

Christoph Klein

based on a publication [arXiv: 0809.0222 (hep-ph)] with

Sven Faller
Thomas Mannel
Alexander Khodjamirian

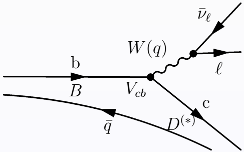
Theoretische Physik 1
Universität Siegen

Ringberg Workshop 28.04.2009

Outline

Semileptonic decays and form factors

Exclusive semileptonic decays of $B \to D^{(*)}\ell\nu_\ell$
$\to$ important for determination of V_{cb}



$$\frac{d\Gamma_{(B\to D^{(*)}\ell\nu_\ell)}(q^2)}{dq^2} \sim |V_{cb}|^2 \cdot \left|\langle D^{(*)}|\bar{c}\,\gamma^\mu(1-\gamma_5)b|B\rangle\right|^2$$

Transition matrix elements are parametrized by form factors:

$$\langle D(p')|\bar{c}\,\gamma^\mu b|B(p)\rangle = (p^\mu + p'^\mu)\cdot f_+(q^2) + (p^\mu - p'^\mu)\cdot f_-(q^2)$$

$$\langle D^*(p',\epsilon)|\bar{c}\,\gamma^\mu b|B(p)\rangle = \frac{2V(q^2)}{m_B + m_{D^*}}\,\epsilon^{\mu\nu\alpha\beta}\,\epsilon^*_\nu\,p_\alpha p'_\beta \qquad (q^\mu = p^\mu - p'^\mu)$$

$$\begin{aligned}\langle D^*(p',\epsilon)|\bar{c}\,\gamma^\mu\gamma_5\,b|B(p)\rangle &= i\epsilon^{*\mu}(m_B + m_{D^*})\cdot A_1(q^2)\\ &- i(\epsilon^*\cdot q)\,(p^\mu + p'^\mu)\cdot\frac{A_2(q^2)}{m_B + m_{D^*}}\\ &- i(\epsilon^*\cdot q)\,(p^\mu - p'^\mu)\cdot\frac{2\,m_{D^*}}{q^2}(A_3(q^2) - A_0(q^2))\end{aligned}$$

Decay rate for $B \to D$:

$$\frac{d\Gamma_{(B\to D\ell\nu_\ell)}(q^2)}{dq^2} \overset{m_\ell=0}{=} \frac{G_F^2|V_{cb}|^2}{192\pi^3 m_B^3} \cdot \left|f_+(q^2)\right|^2 \cdot \left((q^2 - m_B^2 - m_D^2)^2 - 4\, m_B^2\, m_D^2\right)^{3/2}$$

$\to$ Only form factor $f_+(q^2)$ is important for $B \to D$,

$f_-(q^2)$ suppressed by lepton mass $\to$ cannot yet be seen in experiment

Decay rate for $B \to D$:

$$\frac{d\Gamma_{(B\to D\ell\nu_\ell)}(q^2)}{dq^2} \overset{m_\ell=0}{=} \frac{G_F^2|V_{cb}|^2}{192\pi^3 m_B^3} \cdot \left|f_+(q^2)\right|^2 \cdot \left((q^2 - m_B^2 - m_D^2)^2 - 4\, m_B^2\, m_D^2\right)^{3/2}$$

$\to$ Only form factor $f_+(q^2)$ is important for $B \to D$,

$f_-(q^2)$ suppressed by lepton mass $\to$ cannot yet be seen in experiment

Form-factor definition convenient in HQET:

$$\left(w = v \cdot v' = \frac{1}{2\, m_B\, m_{D(*)}}(m_B^2 + m_{D(*)}^2 - q^2)\right)$$

$$\frac{\langle D(v')|V^\mu|\bar{B}(v)\rangle}{\sqrt{m_B m_D}} = h_+(w)(v+v')^\mu + h_-(w)(v-v')^\mu \qquad \frac{\langle D^*(v',\epsilon)|V^\mu|\bar{B}(v)\rangle}{\sqrt{m_B m_D^*}} = h_V(w)\epsilon^{\mu\nu\alpha\beta}\,\epsilon^*_\nu\, v'_\alpha v_\beta$$

$$\frac{\langle D^*(v',\epsilon)|A^\mu|\bar{B}(v)\rangle}{\sqrt{m_B m_D^*}} = -i\epsilon^{*\mu}(w+1)h_{A_1}(w) + i(\epsilon^*\cdot q)\, v^\mu\, h_{A_2}(w) + i(\epsilon^*\cdot q)\, v'^\mu\, h_{A_3}(w)$$

HQET-prediction in heavy-mass limit: $m_b, m_c \to \infty$:

$h_+(w) = h_V(w) = h_{A_1}(w) = h_{A_3}(w) = \xi(w)$ $\quad\to$ Isgur-Wise-function $\xi(w)$

$h_-(w) = h_{A_2}(w) = 0$ $\quad$ zero-recoil-point $w = 1 \quad \to \xi(1) = 1$

- Today best prediction of $B \to D^{(*)}$ form factors from heavy-quark symmetry:
  $\to$ Luke's theorem $\left(\xi(w=1) = 1 + \mathcal{O}(1/m^2) \right)$
- Moreover: estimates from lattice-QCD near $w = 1$.

- Today best prediction of $B \to D^{(*)}$ form factors from heavy-quark symmetry:
  $\to$ Luke's theorem $\left(\xi(w=1) = 1 + \mathcal{O}(1/m^2)\right)$
- Moreover: estimates from lattice-QCD near $w = 1$.

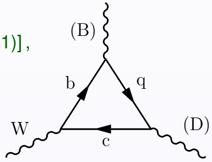
- No description of the form factors far from $w = 1$
  $\to$ Analytic continuation to $w > 1$ is done.
- Experimental data on the $B \to D^{(*)}$ form factor shape available.
  [BABAR('08)],[Belle('02)]

$\to$ Direct theoretical prediction at $w > 1$ desirable.

Outline

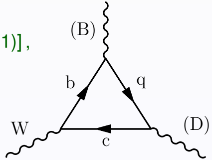
Method of QCD sum rules

- Early work (beginning of 90s): three-point sum rules
  [Ball('92)] , [Baier,Grozin('90)] , [Colangelo,Nardulli,Ovchinnikov,Paver('91)] , [Ovchinnikov,Slobodenyuk('89)] , [Radyushkin('91)] and many more . . .
- Several calculation with finite or infinite (HQET) heavy quark masses.
- Alternative approach: Light-cone sum rules (LCSR)



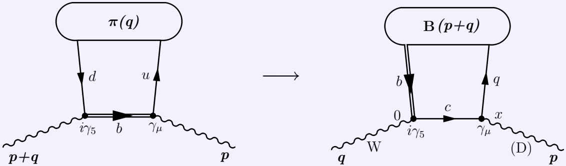
Method of QCD sum rules

- Early work (beginning of 90s): three-point sum rules
  [Ball('92)] , [Baier,Grozin('90)] , [Colangelo,Nardulli,Ovchinnikov,Paver('91)] , [Ovchinnikov,Slobodenyuk('89)] , [Radyushkin('91)] and many more . . .
- Several calculation with finite or infinite (HQET) heavy quark masses.
- Alternative approach: Light-cone sum rules (LCSR)



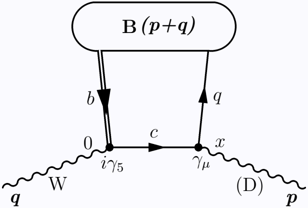
- Today very elaborate LCSR-calculation for heavy-to-light transitions ($B \to \pi$, etc.)
- Well-known pion-distribution amplitudes are used.

$\longrightarrow$ Now new approach: use sum rules with B-meson-distribution amplitudes.



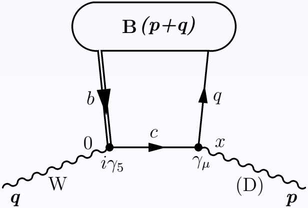
Starting point of any QCD sum rule → Correlation function:

$$F_{ab}(p,q) = i\int d^4x\, e^{ipx}\,\langle 0 \,|\, T\,\{\bar{q}(x)\Gamma_a\, c(x), \bar{c}(0)\Gamma_b\, b(0)\}|\, B\,\rangle$$



Starting point of any QCD sum rule $\rightarrow$ ***Correlation function:***

$$F_{ab}(p,q) = i \int d^4x\, e^{ipx} \langle 0 \,|\, T\,\{\bar{q}(x)\Gamma_a\, c(x), \bar{c}(0)\Gamma_b\, b(0)\}|\, B \rangle$$



Do two steps:

$\rightarrow$ Represent by hadronic quantities.

$\rightarrow$ Calculate via OPE.

Hadronic representation
Choose e.g. $\Gamma_a = i\gamma_5, \Gamma_b = \gamma_\mu$ to get $B \to D$-form factors

$$F_\mu(p,q) = i \int d^4x \, e^{ipx} \langle 0 | T \{ \bar{q}(x) i\gamma_5 \, c(x), \bar{c}(0)\gamma_\mu \, b(0) \} | B \rangle$$

$\to$ Putting in a sum over hadronic intermediate states $1 = \sum_h |h\rangle\langle h|$:

$$F_\mu(p,q) = \frac{\langle 0 | \bar{q} i\gamma_5 \, c | D \rangle \langle D | \bar{c}\gamma_\mu \, b | B \rangle}{m_D^2 - p^2} + \int_{s_0^h}^{\infty} ds \frac{\rho(s)}{s - p^2} \sim \frac{f_D \cdot ((2 p_\mu + q_\mu) f^+(q^2) + q_\mu f^-(q^2))}{m_D^2 - p^2} + \cdots$$

Hadronic representation

Choose e.g. $\Gamma_a = i\gamma_5, \Gamma_b = \gamma_\mu$ to get $B \to D$-form factors

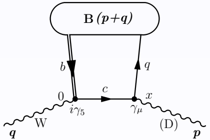
$$F_\mu(p,q) = i \int d^4x\, e^{ipx} \langle 0 \,|\, T\,\{\bar{q}(x) i\gamma_5\, c(x), \bar{c}(0)\gamma_\mu\, b(0)\}\,|\, B \rangle$$

$\to$ Putting in a sum over hadronic intermediate states $1 = \sum_h |h\rangle\langle h|$:

$$F_\mu(p,q) = \frac{\langle 0 \,|\bar{q} i\gamma_5\, c|\, D \rangle \langle D \,|\, \bar{c}\gamma_\mu\, b|\, B \rangle}{m_D^2 - p^2} + \int_{s_0^h}^{\infty} ds \frac{\rho(s)}{s - p^2} \sim \frac{f_D \cdot ((2\,p_\mu + q_\mu) f^+(q^2) + q_\mu f^-(q^2))}{m_D^2 - p^2} + \cdots$$

Calculation via operator product expansion (OPE)

- B-Meson: Transition to HQET: $|B(p+q)> \to |B_v>,\; b(0) \to h_v(0)$



Hadronic representation
Choose e.g. $\Gamma_a = i\gamma_5, \Gamma_b = \gamma_\mu$ to get $B \to D$-form factors

$$F_\mu(p,q) = i \int d^4x\, e^{ipx} \langle 0 | T\{\bar{q}(x) i\gamma_5 c(x), \bar{c}(0)\gamma_\mu b(0)\} | B \rangle$$

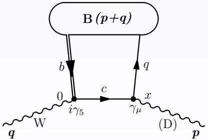
$\to$ Putting in a sum over hadronic intermediate states $1 = \sum_h |h\rangle\langle h|$:

$$F_\mu(p,q) = \frac{\langle 0 | \bar{q} i\gamma_5 c | D \rangle \langle D | \bar{c}\gamma_\mu b | B \rangle}{m_D^2 - p^2} + \int_{s_0^h}^{\infty} ds \frac{\rho(s)}{s - p^2} \sim \frac{f_D \cdot ((2p_\mu + q_\mu) f^+(q^2) + q_\mu f^-(q^2))}{m_D^2 - p^2} + \cdots$$

Calculation via operator product expansion (OPE)

- B-Meson: Transition to HQET: $|B(p+q)> \to |B_v>, \; b(0) \to h_v(0)$
- Light-cone dominance in a certain kinematic region: $x^2 \simeq 0$

$\to$ Contraction of $c(x)\, \bar{c}(0)$ to free quark propagator
(OPE: ***light-cone expansion***)



$$F_{ab}(p,q) = i \int d^4x \int \frac{d^4k}{(2\pi)^4} e^{i(p-k)x} \left[\Gamma_a \frac{(\not{k} + m_c)}{k^2 - m_c^2} \Gamma_b \right]_{\beta\alpha} \langle 0 | T\{\bar{q}_\alpha(x) h_{v\beta}(0)\} | B_v \rangle$$

Light-cone dominance

$$F_{ab}(p,q) = i \int d^4x \int \frac{d^4k}{(2\pi)^4}\, e^{i(p-k)x} \left[\Gamma_a \frac{(\not k + m_c)}{k^2 - m_c^2} \Gamma_b\right]_{\beta\alpha} \langle 0 | T\left\{\bar q_\alpha(x) h_{v\beta}(0)\right\} | B_v\rangle$$

- Light-cone expansion: only applicable for *c*-quark far enough off-shell

$\longrightarrow$ translated into kinematical region: q^2 far enough from zero-recoil point q^2_{max}

- Opposite edge of phase space than Heavy Quark Symmetry and Lattice QCD

decay	$B \to D$	$B \to D^*$
kinematical region for q^2 $[GeV^2]$	$0 - 11.6$	$0 - 10.7$
applicable region in LCSR $[GeV^2]$	$q^2 \lesssim 8 \leftrightarrow w \gtrsim 1.3$	

Calculation of the correlation function

$$F_{ab}(p,q) = i \int d^4x \int \frac{d^4k}{(2\pi)^4}\, e^{i(p-k)x} \left[\Gamma_a \frac{(\not{k} + m_c)}{k^2 - m_c^2} \Gamma_b\right]_{\beta\alpha} \langle 0 | T\left\{\bar{q}_\alpha(x) h_{v\beta}(0)\right\} | B_v\rangle$$

- The remaining matrix element is parametrized by two nonperturbative B-meson light-cone distribution amplitudes $\phi_+(\omega), \phi_-(\omega)$.

$$\left\langle 0 \left| \bar{q}_\alpha(x)\, h_{v\beta}(0) \right| B_v \right\rangle = -\frac{i f_B m_B}{4} \int_0^\infty d\omega\, e^{-i\omega\, v\cdot x} \left[(1+\not{v}) \left\{ \phi_+(\omega) - \frac{\phi_+(\omega) - \phi_-(\omega)}{2\, v\cdot x} \not{x} \right\} \gamma_5 \right]_{\beta\alpha}$$

- One (dimensionful) variable ω:
  projection of the light-quark momentum onto the light-cone

Summary

Approximating of hadronic representation and OPE-calculation → sum rule.

Correlation function:

$$F_{ab}(p,q) = i \int d^4x \, e^{ipx} \langle 0 | T\{\bar{q}(x)\Gamma_a c(x), \bar{c}(0)\Gamma_b b(0)\} | B \rangle$$

hadronic dispersion relation ⟷ OPE & light-cone expansion

$$F_{ab}(p,q) = \frac{\langle 0 | \bar{q}\Gamma_a c | D \rangle \langle D | \bar{c}\Gamma_b b | B \rangle}{m_D^2 - p^2} + \dots$$

$$F_{ab}(p,q) \sim \langle 0 | \bar{q}(x) b(0) | B \rangle$$

↓ $\sim f_+, f_-$ — form factors

↓ $\sim \phi_+, \phi_-$ — distribution amplitudes

form factors ⟷ distribution amplitudes

→ quark-hadron-duality (parameter s_0)
→ Borel-transformation (parameter M^2)

$\Longrightarrow$ *Sum rule:*

$$f_+(q^2), f_-(q^2) = f\left[\phi_+(\omega), \phi_-(\omega)\right]$$

B-meson 2-particle distribution amplitudes

- B-meson 2-particle distribution amplitudes
  $\phi_+(\omega)$ and $\phi_-(\omega)$ in HQET:

$\longrightarrow$ Nonperturbative quantities, difficult to calculate

$\longrightarrow$ Asympotics from two-point-sum rules $\rightarrow$ *construction of models necessary*

B-meson 2-particle distribution amplitudes

- B-meson 2-particle distribution amplitudes
  $\phi_+(\omega)$ and $\phi_-(\omega)$ in HQET:

$\longrightarrow$ Nonperturbative quantities, difficult to calculate

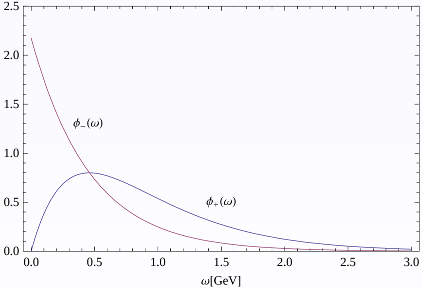
$\longrightarrow$ Asympotics from two-point-sum rules $\rightarrow$ *construction of models necessary*

- Here: Exponential model [Grozin and Neubert (1997)]

$$\phi_+(\omega) = \frac{\omega}{\omega_0^2} e^{-\frac{\omega}{\omega_0}}$$

$$\phi_-(\omega) = \frac{1}{\omega_0} e^{-\frac{\omega}{\omega_0}}$$

$$\frac{1}{\lambda_B} = \int_0^\infty d\omega \frac{\phi_+(\omega)}{\omega} \qquad \omega_0 = \lambda_B$$



- Still big uncertainty: $\lambda_B = (0.46 \pm 0.11)\ GeV$
  from 2-point sum rules [Braun,Ivanov,Korchemsky('03)]

B-meson 3-particle distribution amplitudes

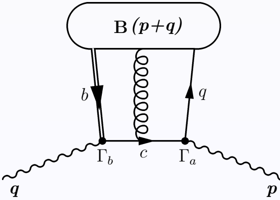
Higher-order correction: 3-particle Fock-state

$$\langle\, 0 \,|\, \bar{q}_{2\alpha}(x)\, G_{\lambda\rho}(ux) h_{v\beta}(0) |\, \bar{B}_v \,\rangle$$

$$= \frac{f_B m_B}{4} \int_0^\infty d\omega \int_0^\infty d\xi\, e^{-i(\omega+u\xi)\, v\cdot x} \Big[(1+\not{v})$$

$$\Big\{ (v_\lambda \gamma_\rho - v_\rho \gamma_\lambda)(\Psi_A(\omega,\xi) - \Psi_V(\omega,\xi)) - i\sigma_{\lambda\rho}\Psi_V(\omega,\xi)$$

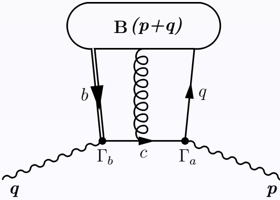
$$- \left(\frac{x_\lambda v_\rho - x_\rho v_\lambda}{v\cdot x} \right) X_A(\omega,\xi) - \left(\frac{x_\lambda v_\rho - x_\rho v_\lambda}{v\cdot x} Y_A(\omega,\xi) \right) \Big\} \gamma_5 \Big]_{\beta\alpha}$$



B-meson 3-particle distribution amplitudes

Higher-order correction: 3-particle Fock-state

$$\langle 0 \,|\, \bar{q}_{2\alpha}(x)\, G_{\lambda\rho}(ux) h_{v\beta}(0) |\, \bar{B}_v \,\rangle$$
$$= \frac{f_B m_B}{4} \int_0^\infty d\omega \int_0^\infty d\xi\, e^{-i(\omega+u\xi)\, v\cdot x} \Big[(1+\not{v}) \Big\{ (v_\lambda\gamma_\rho - v_\rho\gamma_\lambda)(\Psi_A(\omega,\xi) - \Psi_V(\omega,\xi)) - i\sigma_{\lambda\rho}\Psi_V(\omega,\xi)$$
$$- \left(\frac{x_\lambda v_\rho - x_\rho v_\lambda}{v\cdot x}\right) X_A(\omega,\xi) - \left(\frac{x_\lambda v_\rho - x_\rho v_\lambda}{v\cdot x} Y_A(\omega,\xi)\right) \Big\} \gamma_5 \Big]_{\beta\alpha}$$



New sum-rule based exponential model: [Khodjamirian, Mannel and Offen (2007)]

$$\Psi_A(\omega,\xi) = \Psi_V(\omega,\xi) = \frac{\lambda_E^2}{6\omega_0^4}\,\xi^2 e^{-\frac{(\omega+\xi)}{\omega_0}}$$

$$X_A(\omega,\xi) = \frac{\lambda_E^2}{6\omega_0^4}\,\xi(2\omega - \xi) e^{-\frac{(\omega+\xi)}{\omega_0}} \qquad Y_A(\omega,\xi) = -\frac{\lambda_E^2}{24\omega_0^4}\,\xi(7\omega_0 - 13\omega + 3\xi) e^{-\frac{(\omega+\xi)}{\omega_0}}$$

Outline

Results for the form factors

Example for the sum rule structure: $f_+(q^2)$

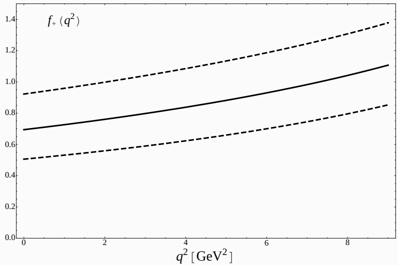
M^2 : Borel-parameter, s_0 : duality-parameter

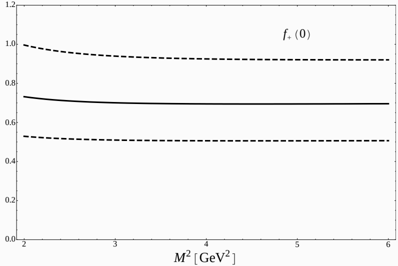
$$
\begin{aligned}
f_+(q^2) = &\frac{f_B m_c}{2 f_D m_D^2} \int_0^{\omega_0(q^2,s_0)} d\omega \exp\left(\frac{-s\left(\omega,q^2\right) + {m_D}^2}{M^2}\right) \\
&\times \left\{ \frac{(m_B - \omega + m_c)\,\phi_+(\omega) - \Phi_\pm(\omega)}{(1 - \frac{\omega}{m_B})} \right. \\
&- \frac{1}{(m_B - \omega)^2 + m_c^2 - q^2} \left(\frac{2 m_B m_c (m_B - \omega)\,(m_B - \omega + m_c)}{(m_B - \omega)^2 + m_c^2 - q^2} \Phi_\pm(\omega) \right. \\
&\left.\left. + m_B m_c \Big((m_B - \omega + m_c) \left(\phi_+(\omega) - \phi_-(\omega)\right) - \Phi_\pm(\omega) \Big) \right) \right\} \\
&+ \left(\Delta f_+(q^2)\right)_{\text{3-particle}}
\end{aligned}
$$

$$
s(\omega,q^2) = \omega m_B + \frac{m_B m_c^2 - \omega q^2}{m_B - \omega} \qquad \Phi_\pm(\omega) = \int_0^\infty d\tau \left(\phi_+(\tau) - \phi_-(\tau)\right)
$$

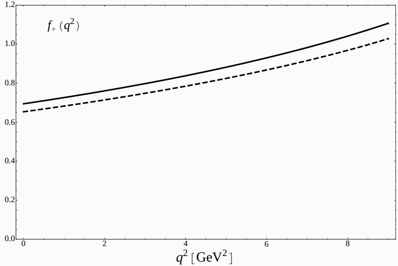
$$
\omega_0(q^2,s_0) = \frac{1}{2 m_B} \left(m_B^2 - q^2 + s_0 - \sqrt{4\left(m_c^2 - s_0\right) m_B^2 + \left(m_B^2 - q^2 + s_0\right)^2} \right)
$$

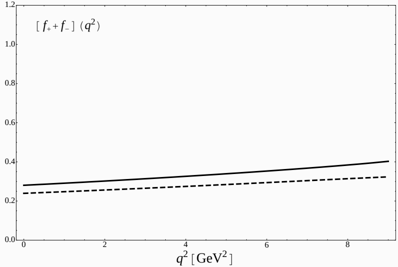
$f_+(q^2)$ with uncertainty (left) and M^2-dependence (right)





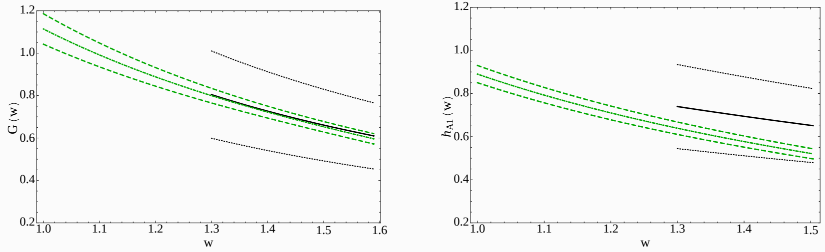
$B \to D$-form factors (straight line) and only 2-particle contributions (dashed)





Comparison with experiment (BABAR '08)

Form factors $\mathcal{G}(w)$, $h_{A_1}(w)$ [LCSR: $(w = 1.3 - w_{max})$, exp.: $(w = 1 - w_{max})$]



Sum rules only applicable far enough from $w = 1$: $w \simeq 1.3 - w_{max}$

Fit to experimental data with parametrization: [Caprini, Lellouch, Neubert '97]

$$h_{A_1}(w) = h_{A_1}(1)\Big(1 - 8\rho^2 z(w) + (53\rho^2 - 15) z(w)^2 - (231\rho^2 - 91) z(w)^3\Big)$$

$$f_+(q^2) \mathrel{\hat{=}} \mathcal{G}(w) = \mathcal{G}(1)\Big(1 - 8\rho^2 z(w) + (51\rho^2 - 10) z(w)^2 - (252\rho^2 - 84) z(w)^3\Big)$$

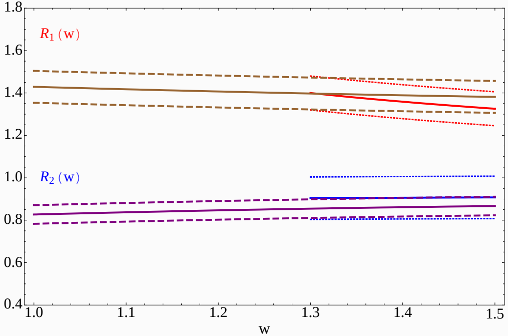
$\rightarrow$ Normalization for exp. data $h_{A_1}(1), \mathcal{G}(1)$ from lattice-QCD

$$z(w) = \left(\frac{\sqrt{w+1} - \sqrt{2}}{\sqrt{w+1} + \sqrt{2}}\right)$$

Comparison with experiment (BABAR '08)

$$R_1(w) = \frac{h_V(w)}{h_{A_1}(w)} \qquad R_2(w) = \frac{\frac{m_{D^*}}{m_B} h_{A_2}(w) + h_{A_3}(w)}{h_{A_1}(w)}$$

Ratios $R_1(w)$, $R_2(w)$ [LCSR: ($w = 1.3 - 1.5$), exp.: ($w = 1 - 1.5$)]



$$R_1(w) = R_1(1) - 0.12(w-1) + 0.05(w-1)^2$$

$$R_2(w) = R_2(1) + 0.11(w-1) - 0.06(w-1)^2$$

In the ratios the normalizations cancel: $\rightarrow$ more precise prediction

Discussion of the $B \to D^{(*)}$-results

- Relatively large uncertainty due to uncertainty of input parameters, mainly λ_B (B-meson-DAs) and f_D, f_B.
- Good agreement with experiment:
  $\to$ important cross-check of other methods and good test of sum rule concept.

Discussion of the $B \to D^{(*)}$-results

- Relatively large uncertainty due to uncertainty of input parameters, mainly λ_B (B-meson-DAs) and f_D, f_B.
- Good agreement with experiment:
  $\to$ important cross-check of other methods and good test of sum rule concept.

- Expansion in powers of $\frac{1}{m_b}$ and $\frac{1}{m_c}$:

$\longrightarrow$ Analysis of heavy mass corrections.
Sum rules fulfill the Heavy Quark symmetry relations for $m_b, m_c \to \infty$.

$\to$ Allows determination of the Isgur-Wise-Function.

Limit of infinitely heavy quarks

$\rightarrow$ Limit $m_c, m_b \rightarrow \infty, \kappa := \frac{m_c}{m_b} = const$ for the sum rules:

$$h_+(w) = h_V(w) = h_{A_1}(w) = h_{A_3}(w) = \xi(w) = \int_0^{\beta_0/w} d\rho\, e^{\left(\frac{\bar{\Lambda}-\rho\, w}{\tau}\right)} \left[\frac{1}{2w}\phi_-^B(\rho) + (1 - \frac{1}{2w})\phi_+^B(\rho)\right]$$

$$h_-(w) = h_{A_2}(w) = 0$$

scale parameter: $m_B = m_b + \bar{\Lambda}$, $m_D = m_c + \bar{\Lambda}$, $s_0^D = m_c^2 + 2m_c\beta_0$, $M^2 = 2m_c\tau$

Limit of infinitely heavy quarks

$\rightarrow$ Limit $m_c, m_b \rightarrow \infty, \kappa := \frac{m_c}{m_b} = const$ for the sum rules:

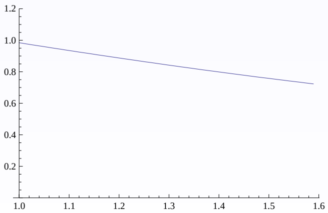
$$h_+(w) = h_V(w) = h_{A_1}(w) = h_{A_3}(w) = \xi(w) = \int_0^{\beta_0/w} d\rho\, e^{\left(\frac{\bar\Lambda - \rho\, w}{\tau}\right)} \left[\frac{1}{2w}\phi_-^B(\rho) + (1 - \frac{1}{2w})\phi_+^B(\rho)\right]$$

$$h_-(w) = h_{A_2}(w) = 0$$

scale parameter: $m_B = m_b + \bar\Lambda,\ m_D = m_c + \bar\Lambda,\ s_0^D = m_c^2 + 2m_c\beta_0,\ M^2 = 2m_c\tau$

- Independent of mass ratio κ
- Numerical (within uncertainty) the right value:

  $$\xi(1) \simeq 1$$
- Deviation to finite-mass values (at $w = w_{max}$) $\sim 20 - 25\%$ $\rightarrow 1/m$-corrections



Summary and Outlook

- New exploratory study for description of $B \longrightarrow D^{(*)}$-decays.
- Important cross-check for the determination of V_{cb}
  $\longrightarrow$ possible future applications with more precise input values.
- Applicable far from zero-recoil point, where other methods are not optimal.
- Encouraging results:
  agreement with experiment and right behaviour for $m_c, m_b \longrightarrow \infty$

- In the future: study of perturbative corrections.
- Also possible issues:
  $\longrightarrow$ consideration of further dirac structures
  (tensor form factor, $B \longrightarrow D^{**}, \ldots$).
  $\longrightarrow$ further analysis of heavy quark mass expansion.