NNLO CORRECTIONS TO SEMILEPTONIC AND HADRONIC B DECAYS

[GUIDO BELL]

UNIVERSITÄT KARLSRUHE

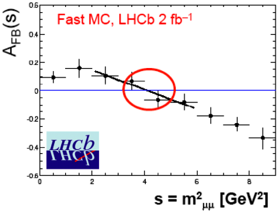
Karlsruhe Institute of Technology

Precision B physics?

Expect precision measurements from current + future B physics experiments

Some goals:

- $\delta\mathrm{Br}(B \to X_s\gamma) \sim 5\%$
- $\delta V_{ub} \sim 5\%$
- $\delta\gamma \sim 5^\circ$
- $A_{FB}(B \to K^*\ell^+\ell^-) : \delta s_0 \sim 10\%$



Theory is challenged to match the experimental precision

- construct observables that are almost free of hadronic uncertainties
- need progress from non-perturbative methods
- work out subleading corrections: NNLO and $1/m_b$

B physics at the NNLO frontier

NNLO programme complete for weak effective Hamiltonian

$$\mathcal{H}_{\text{eff}} = \sum C_i \, Q_i$$

- 2-loop / 3-loop matching corrections [Bobeth, Misiak, Urban 99; Misiak, Steinhauser 04]
- 3-loop / 4-loop anomalous dimensions [Gorbahn, Haisch 04; Gorbahn, Haisch, Misiak 05; Czakon, Haisch, Misiak 06]

$\rightarrow$ need hadronic matrix elements to same level of precision

Some decay modes that are currently investigated at NNLO:

$B \to X_s \gamma$

$B \to X_s \ell^+ \ell^-$

$B \to X_u \ell \nu$

$B \to V \gamma$

$B \to MM$

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$B \to V \gamma$

$B \to MM$

In this talk:

Part 1: $B \to X_u \ell \nu$, $B \to M \ell \nu$

Part 2: $B \to MM$

Factorization in semileptonic decays

Inclusive decays: $B \to X_u \ell \nu$

- experiments impose cuts to suppress background from $B \to X_c \ell \nu$
- measurements restricted to shape-function region: $E_X \sim m_b,\ m_X^2 \sim m_b \Lambda_{QCD}$

[Korchemsky, Sterman 94]

$$W^{\mu\nu} \sim \sum_{i,j} \underbrace{H_{ij}(n_+ p)}_{m_b} \int d\omega \underbrace{J(p_\omega^2)}_{\sqrt{m_b \Lambda_{QCD}}} \underbrace{S(\omega)}_{\Lambda_{QCD}}$$

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Exclusive decays: $B \to \pi \ell \nu$ at large recoil

- symmetry relations emerge in large energy limit $E_\pi \sim m_b$ [Charles et al. 98]
- soft Feynman mechanism vs. hard scattering

[Beneke, Feldmann 00]

$$F_i(E) \sim \underbrace{C_i(E)}_{m_b}\, \underbrace{\xi(E)}_{\sqrt{m_b \Lambda_{QCD}},\, \Lambda_{QCD}} + \int d\omega \int du\, \underbrace{T_i(E,\omega,u)}_{m_b,\, \sqrt{m_b \Lambda_{QCD}}}\, \underbrace{\phi_B(\omega)\, \phi_M(u)}_{\Lambda_{QCD}}$$

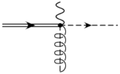
Heavy-to-light currents in SCET

[Bauer, Fleming, Pirjol, Stewart; Beneke, Feldmann; Chay, Kim; Hill, Neubert; . . . 00+]

2-body operators

3-body operators

$$\bar{q}\,\Gamma b \;=\; \sum_i \int ds\; \tilde{C}_i^{(A)}(s)\; O_i^{(A)}(s) \;+\; \sum_i \int ds_1 ds_2\; \tilde{C}_i^{(B)}(s_1,s_2)\; O_i^{(B)}(s_1,s_2) \;+\; \dots$$



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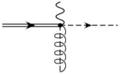

Inclusive decays:

leading contribution

$$\langle O_i^{(A)} \rangle \;\to\; J \otimes S$$

power-correction

$$\langle O_i^{(B)} \rangle \;\to\; \sum_i j_i \otimes s_i$$

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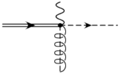

Inclusive decays: leading contribution $\langle O_i^{(A)} \rangle \;\to\; J \otimes S$

power-correction $\langle O_i^{(B)} \rangle \;\to\; \sum_i j_i \otimes s_i$

Exclusive decays: leading contribution $\langle O_i^{(A)} \rangle \;\to\; \xi$

soft-overlap contribution

leading contribution $\langle O_i^{(B)} \rangle \;\to\; J_{||} \otimes \phi_B \otimes \phi_M$

hard spectator scattering

Heavy-to-light currents in SCET

[Bauer, Fleming, Pirjol, Stewart; Beneke, Feldmann; Chay, Kim; Hill, Neubert; . . . 00+]

2-body operators 3-body operators

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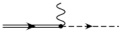

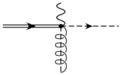

Hard coefficients $\mathcal{O}(\alpha_s)$: 1-loop

QCD $\rightarrow$ SCET$_I$ [Beneke,Feldmann 00; Bauer, Fleming, Pirjol, Stewart 00]

$\mathcal{O}(\alpha_s^2)$: 2-loop

$\mathcal{O}(\alpha_s^2)$: 1-loop

[Beneke, Kiyo, Yang 04; Becher, Hill 04]

$\rightarrow$ 2-loop matching required for NNLO analysis of inclusive and exclusive B decays!

Heavy-to-light currents in SCET

[Bauer, Fleming, Pirjol, Stewart; Beneke, Feldmann; Chay, Kim; Hill, Neubert; ... 00+]

2-body operators 3-body operators

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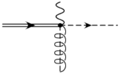


Status of 2-loop matching calculation

- $\Gamma = \gamma^\mu$ $\quad B \to X_u \ell\nu,\; B \to \pi \ell \nu$ [Bonciani, Ferroglia 08; Asatrian, Greub, Pecjak 08; Beneke, Huber, Li 08; GB 08]
- $\Gamma = 1, \sigma^{\mu\nu}$ $\quad B \to X_s \ell^+\ell^-,\; B \to K^* \ell^+ \ell^-$ [GB, Beneke, Huber, Li in preparation]

NDR-scheme relates currents with and without γ_5

Outline of matching calculation

Heavy-to-light currents at leading power

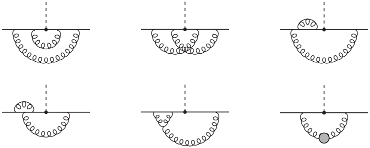
$$\bar{q}\,\Gamma\, b \;\simeq\; \sum_i \int ds\; \tilde{C}_i^{(A)}(s)\; [\xi W_{hc}](s n_+)\, \Gamma_i'\, h_v$$

Strategy: compute $\langle q| \ldots |b\rangle$ in QCD and SCET

matching simplifies in Dimensional Regularization with on-shell quarks

$\rightarrow$ SCET diagrams are scaleless and vanish

Main task: 2-loop QCD calculation



$$p_b^2 = m_b^2$$

$$p^2 = 0$$

$$q^2 = (1-u)m_b^2$$

Multi-loop techniques

Automatized reduction algorithm

- integration-by-parts identities [Chetyrkin, Tkachov 81]
- solve large system of equations efficiently [Laporta 00]

$\rightarrow$ reduce $\mathcal{O}(1.000)$ scalar 2-loop integrals to 14 Master Integrals

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Calculation of Master Integrals

- method of differential equations [Kotikov 91; Remiddi 97]
- harmonic polylogarithms [Remiddi, Vermaseren 00]
- Mellin-Barnes techniques [Smirnov 99; Tausk 99]
- method of sector decomposition [Binoth, Heinrich 04]

$\rightarrow$ analytical results known from 2-loop analysis in $B \rightarrow \pi\pi$ [GB 07]

confirmed by several independent calculations [Bonciani, Ferroglia 08; Asatrian, Greub, Pecjak 08; Beneke, Huber, Li 08; Huber 09]

Wilson coefficients in NNLO

$\Gamma = \gamma^\mu \quad \rightarrow \; C_{V,1}, C_{V,2}, C_{V,3}$

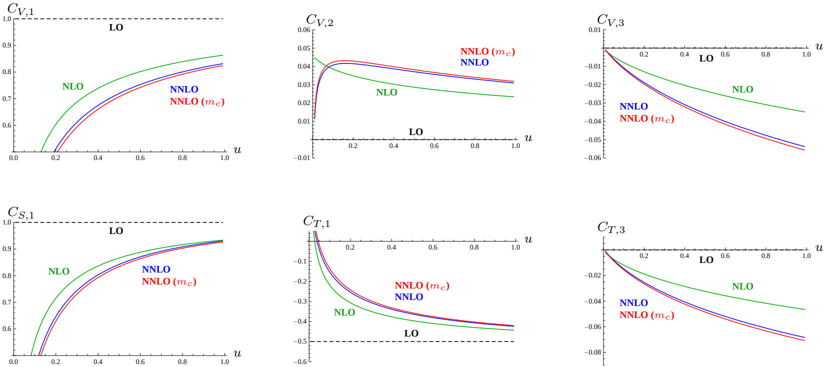
$\Gamma = 1 \quad \rightarrow \; C_{S,1}$

$\Gamma = i\sigma^{\mu\nu} \quad \rightarrow \; C_{T,1}, C_{T,3} \quad (C_{T,2} = C_{T,4} = 0)$

momentum transfer $q^2 = (1-u) m_b^2$

$$u = \frac{n_+ p}{m_b} = \frac{2E}{m_b}$$

$$\mu = m_b$$

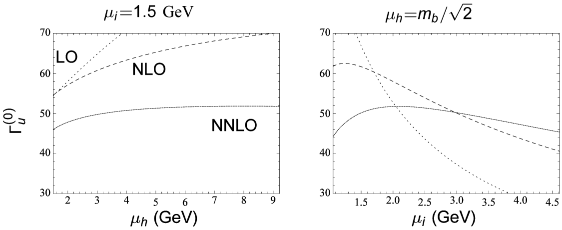


Application I: $B \to X_u \ell \nu$ in the SF region

[Ben Pecjak @ SCET 09]

Partial rate with cut on $P_+ = E_X - |\vec{P}_X| < 0.66$ GeV

$$\Gamma_u\big|_{\text{cut}} = |V_{ub}|^2 \left[\Gamma_u^{(0)} + \frac{1}{m_b}\Gamma_u^{(1)} + \frac{1}{m_b^2}\Gamma_u^{(2)} + \dots \right]$$



- reduced scale dependence at NNLO
- large negative shift between NLO and NNLO

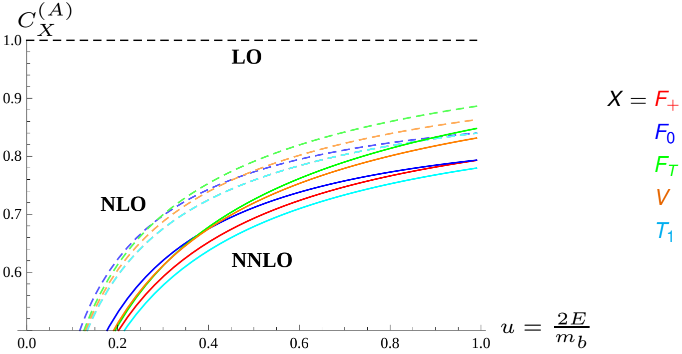
$\to$ $|V_{ub}|_{\text{inclusive}}$ goes up by $\sim 10\%$ compared to NLO

[Greub, Neubert, Pecjak preliminary]

Application II: Heavy-to-light form factors

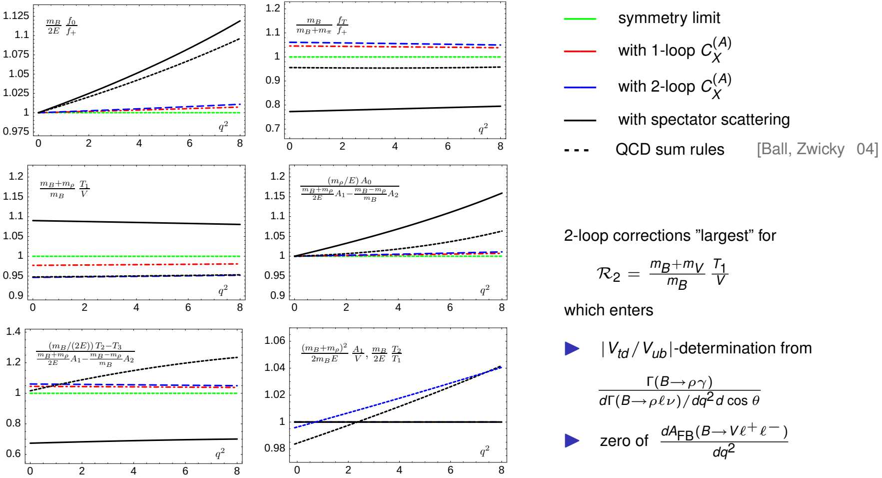
[GB, Beneke, Huber, Li preliminary]

$$F_X^{B\to M}(E) \simeq C_X^{(A)}(E)\,\xi_M(E) + \int d\omega \int du\; T_X(E,\omega,u)\,\phi_B(\omega)\,\phi_M(u)$$



→ "universal" corrections to heavy-to-light form factors

Symmetry-breaking corrections to form factor ratios



2-loop corrections "largest" for

$$\mathcal{R}_2 = \frac{m_B+m_V}{m_B}\,\frac{T_1}{V}$$

which enters

- $|V_{td}/V_{ub}|$-determination from

$$\frac{\Gamma(B\to\rho\gamma)}{d\Gamma(B\to\rho\ell\nu)/dq^2\,d\cos\theta}$$

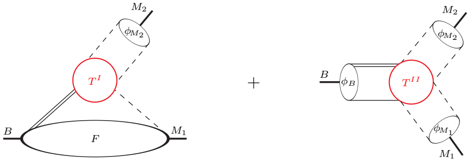
- zero of $\frac{dA_{\rm FB}(B\to V\ell^+\ell^-)}{dq^2}$

Factorization in hadronic decays

Hadronic matrix elements factorize in heavy quark limit $m_b \to \infty$

[Beneke, Buchalla, Neubert, Sachrajda 99,01]

$$\langle M_1 M_2 | Q_i | \bar{B} \rangle \simeq F^{BM_1}(0) \int du \; T_i^I(u) \; \phi_{M_2}(u) + \int d\omega \, du \, dv \; T_i^{II}(\omega, u, v) \; \phi_B(\omega) \; \phi_{M_1}(v) \; \phi_{M_2}(u)$$

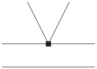


vertex corrections $T^I = \mathcal{O}(1)$ spectator-scattering $T^{II} = \mathcal{O}(\alpha_s)$

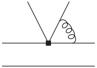
- valid to all orders in $\alpha_s(m_b)$ and to leading power in $1/m_b$
- may include resummation of $\ln m_b/\Lambda_{QCD}$ via RGEs in SCET

Structure of perturbative expansion

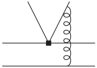
LO: "naive" factorization $\langle M_1 M_2 | Q_i | \bar{B} \rangle = F^{BM_1}(0)\, f_{M_2}$



NLO: 1-loop vertex corrections

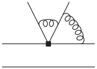


tree-level spectator scattering

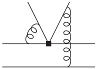


[Beneke, Buchalla, Neubert, Sachrajda 01]

NNLO: 2-loop vertex corrections

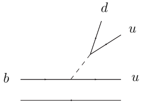


1-loop spectator scattering

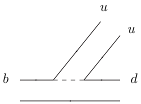


Status of NNLO calculation

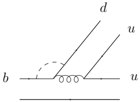
Topological amplitudes



colour-allowed tree α_1



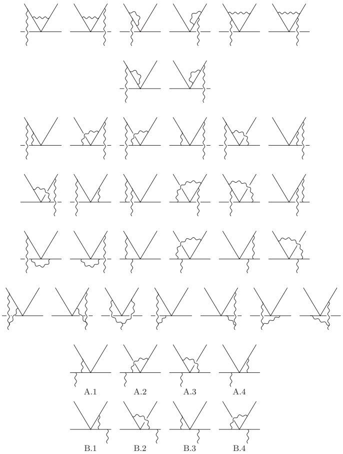
colour-suppressed tree α_2



QCD penguins α_4

	2-loop vertex corrections	1-loop spectator scattering
Trees	[GB 07, 09]	[Beneke, Jäger 05] [Kivel 06] [Pilipp 07]
Penguins	in progress	[Beneke, Jäger 06] [Jain, Rothstein, Stewart 07]

1-loop spectator scattering



Characterization

scales $\mu_h \sim m_b, \ \mu_{hc} \sim \sqrt{m_b \Lambda_{QCD}}$

hard-scattering kernels factorize

$$T_i^{II} = H_i^{II}(\mu_h) \otimes J_{||}(\mu_{hc})$$

jet function $J_{||}$ known to NLO

[Becher, Hill 04, Kirilin 05, Beneke, Yang 05]

Tree amplitudes

H_i^{II} from QCD $\rightarrow$ SCET$_I$ matching

[Beneke, Jäger 05; Kivel 06]

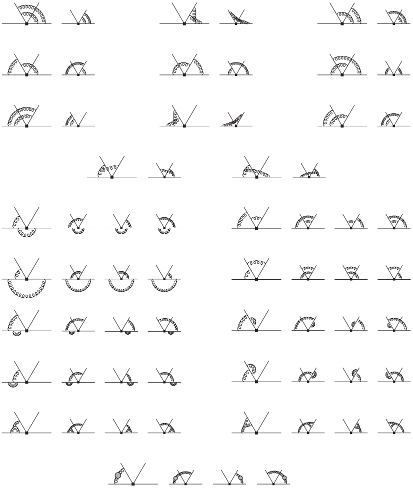
T_i^{II} confirmed by QCD calculation

[Pilipp 07]

Penguin amplitudes

[Beneke, Jäger 06; Jain, Rothstein, Stewart 07]

2-loop vertex corrections



Characterization

- 2-loop calculation with 4 legs
- essentially QCD calculation
- issue: evanescent operators
  - (complicated for α_2)

Tree amplitudes

- imaginary part [GB 07]
- real part [GB 09]

Penguin amplitudes

- not yet . . .

NNLO result for tree amplitudes

Input parameters for default scenario

μ_h	μ_{hc}	$F_+^{B\pi}(0)$	$\lambda_B(1\text{ GeV})$	$a_2^\pi(1\text{ GeV})$	$10^3\,\lvert V_{ub}\rvert$	γ
$4.8^{+4.8}_{-2.4}$	$1.5^{+1.5}_{-0.7}$	0.26 ± 0.04	0.40 ± 0.15	0.25 ± 0.15	3.95 ± 0.35	70 ± 20

Tree amplitudes

$$\begin{aligned}
\alpha_1(\pi\pi) &= [\,1.008\,]_{V_0} + [\,0.022 + 0.009\,i\,]_{V_1} + [\,0.024 + 0.026\,i\,]_{V_2} \\
&\quad - [\,0.014\,]_{S_1} - [\,0.016 + 0.012\,i\,]_{S_2} - [\,0.008\,]_{1/m_b} \\
&= 1.015^{+0.020}_{-0.029} + (0.023^{+0.015}_{-0.015})\,i \\
\alpha_2(\pi\pi) &= [\,0.224\,]_{V_0} - [\,0.174 + 0.075\,i\,]_{V_1} - [\,0.029 + 0.046\,i\,]_{V_2} \\
&\quad + [\,0.084\,]_{S_1} + [\,0.037 + 0.022\,i\,]_{S_2} + [\,0.052\,]_{1/m_b} \\
&= 0.194^{+0.130}_{-0.095} - (0.099^{+0.057}_{-0.056})\,i
\end{aligned}$$

V_0 : naive factorization
V_1 : 1-loop vertex corrections
V_2 : 2-loop vertex corrections

S_1 : tree level spectator scattering
S_2 : 1-loop spectator scattering
$1/m_b$: power corrections (estimate)

$B \to \pi\pi$ branching ratios

Decay amplitudes with NNLO trees + NLO penguins

$$\sqrt{2}\,\mathcal{A}(B^- \to \pi^-\pi^0) \sim V_{ub}V_{ud}^*\left[\alpha_1 + \alpha_2\right] G_F m_B^2 f_\pi F_+^{B\pi}(0)$$

$$\mathcal{A}(\bar{B}^0 \to \pi^+\pi^-) \sim \left\{V_{ub}V_{ud}^*\left[\alpha_1 + \alpha_4^u\right] + V_{cb}V_{cd}^*\,\alpha_4^c\right\} G_F m_B^2 f_\pi F_+^{B\pi}(0)$$

$$-\,\mathcal{A}(\bar{B}^0 \to \pi^0\pi^0) \sim \left\{V_{ub}V_{ud}^*\left[\alpha_2 - \alpha_4^u\right] - V_{cb}V_{cd}^*\,\alpha_4^c\right\} G_F m_B^2 f_\pi F_+^{B\pi}(0)$$

CP-averaged branching ratios with default input parameters

		[HFAG 08]
$10^6\ \mathrm{Br}(B^- \to \pi^-\pi^0)$	$= 6.23^{+1.15}_{-1.05}(\mathrm{CKM})^{+2.34}_{-1.68}(\mathrm{had})^{+0.16}_{-0.18}(\mathrm{scale})^{+0.43}_{-0.42}(\mathrm{power})$	$[5.59\ ^{+0.41}_{-0.40}]$
$10^6\ \mathrm{Br}(\bar{B}^0 \to \pi^+\pi^-)$	$= 8.90^{+1.84}_{-1.87}(\mathrm{CKM})^{+3.01}_{-2.67}(\mathrm{had})^{+0.16}_{-0.20}(\mathrm{scale})^{+1.23}_{-0.70}(\mathrm{power})$	$[5.16 \pm 0.22]$
$10^6\ \mathrm{Br}(\bar{B}^0 \to \pi^0\pi^0)$	$= 0.40^{+0.19}_{-0.17}(\mathrm{CKM})^{+0.37}_{-0.11}(\mathrm{had})^{+0.03}_{-0.03}(\mathrm{scale})^{+0.28}_{-0.08}(\mathrm{power})$	$[1.55 \pm 0.19]$

Consider scenario where some parameters lie at edges of error range ("S4/G")

$$|V_{ub}|F_+^{B\pi}(0)\Big|_{\mathrm{fit}} = 0.81 \times |V_{ub}|F_+^{B\pi}(0)\Big|_{\mathrm{default}}$$

$$\frac{\hat{f}_B}{F_+^{B\pi}(0)\lambda_B}\Bigg|_{\mathrm{fit}} = 2.07 \times \frac{\hat{f}_B}{F_+^{B\pi}(0)\lambda_B}\Bigg|_{\mathrm{default}}$$

$\longrightarrow$

$$10^6\ \mathrm{Br}(B^- \to \pi^-\pi^0) = 5.59$$
$$10^6\ \mathrm{Br}(\bar{B}^0 \to \pi^+\pi^-) = 5.16$$
$$10^6\ \mathrm{Br}(\bar{B}^0 \to \pi^0\pi^0) = 0.71$$

Testing factorization with $B^- \to \pi^-\pi^0$

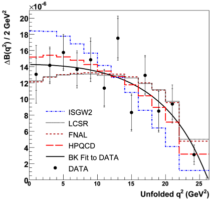
$B^- \to \pi^-\pi^0$ is a pure tree decay $\quad\to\quad$ completely determined at NNLO

no weak annihilation, no dependence on γ

Test QCD dynamics with ratio [Beneke, Neubert 03]

$$\frac{\Gamma(B^- \to \pi^-\pi^0)}{d\Gamma(\bar{B}^0 \to \pi^+\ell^-\bar{\nu})/dq^2|_{q^2=0}} = 3\pi^2|V_{ud}|^2 f_\pi^2 |\alpha_1 + \alpha_2|^2$$

NNLO in QCDF/SCET: $\quad |\alpha_1 + \alpha_2| = \begin{cases} 1.211^{+0.098}_{-0.066} & \text{(default)} \\ 1.415^{+0.132}_{-0.133} & \text{(fit)} \end{cases}$ [GB, Pilipp preliminary]



HFAG: $10^6\ \mathrm{Br}(B^- \to \pi^-\pi^0) = 5.59\ ^{+0.41}_{-0.40}$

Babar: $|V_{ub}|F_+^{B\pi}(0) = (9.6 \pm 0.3 \pm 0.2) \times 10^{-4}$

$\to \quad |\alpha_1 + \alpha_2| = 1.226^{+0.069}_{-0.066}$ (exp)

Summary

Many theory analysis on the way beyond NLO:

- NNLO prediction of $|V_{ub}|_{\text{inclusive}}$ to appear soon
- more complicated for exclusive decays since spectator scattering $\sim \frac{\hat{f}_B}{F_+^{B\pi}(0)\lambda_B}$

Status of $B \to MM$:

- 1-loop spectator scattering: complete
- 2-loop vertex corrections: trees finished, penguins missing

First NNLO result in QCDF/SCET for $B^- \to \pi^- \pi^0$

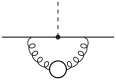
- don't have to tune any parameters to describe clean observable

$$\frac{\Gamma(B^- \to \pi^- \pi^0)}{d\Gamma(\bar{B}^0 \to \pi^+ \ell^- \bar{\nu})/dq^2|_{q^2=0}} \quad \to \quad |\alpha_1 + \alpha_2| = \begin{cases} 1.211^{+0.098}_{-0.066} & \text{(default)} \\ 1.226^{+0.069}_{-0.066} & \text{(exp)} \end{cases}$$

Backup slides

Charm mass effects

charm quark enters at 2-loop through fermion bubble



$\rightarrow$ not tremendously important, but naively $\frac{m_c^2}{m_b^2} \ln \frac{m_b^2}{m_c^2} \sim 0.20 \gtrsim \frac{\alpha_s(m_b)}{\pi}$

Choose power-counting

- ▶ $m_c \sim \mu_{hc} \sim (\Lambda_{QCD} m_b)^{1/2}$

  IR-scale in hard matching ($m_c = 0$)

  m_c-dependence in jet-function

- ▶ $m_c \rightarrow \infty$, $m_b \rightarrow \infty$, m_c/m_b fixed

  m_c-dependence in hard matching

  jet function with 3 massless quarks

Adopt second scenario

$\rightarrow$ 4 new Master Integrals, modified UV- and IR-subtractions, numerical results

Motivation for NNLO

Phenomenological need

- strong phases $\sim \mathcal{O}(\alpha_s)$ $\rightarrow$ direct CP asymmetries known to LO only

  suffer from large (scale) uncertainties
- C/T seems to be too small $\rightarrow$ large cancellation in LO+NLO

  particularly sensitive to NNLO

Conceptual aspects

- spectator scattering with $\alpha_s(\mu_{hc})$ $\rightarrow$ scale uncertainties even more important

  PT well-behaved at $\mu_{hc} \sim 1.5$ GeV ?
- factorization proof ? $\rightarrow$ does factorization hold at all ?

Last but not least

- systematic framework $\rightarrow$ compute systematic corrections !

Scale dependence