The Photon Energy Spectrum in $B \to X_s \gamma$

Frank Tackmann

Massachusetts Institute of Technology

Ringberg Workshop on New Physics, Flavors and Jets
April 30, 2009

[Ligeti, Stewart, FT: PRD 78 (2008) 114014 [arXiv:0807.1926]]

[Ligeti, Stewart, FT: work in progress]

[Bernlochner, Lacker, Ligeti, Stewart, FT, Tackmann: work in progress]

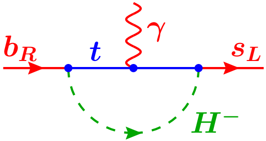
Outline

Outline

One-Page Introduction to $B \to X_s\gamma$

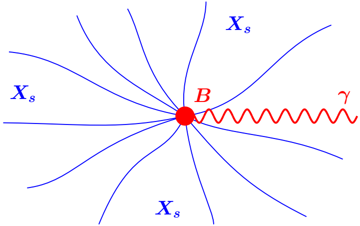
Sensitive to new physics even without new flavor violation

- Strong bounds on many new physics models
- Major efforts to compute SM prediction for $\mathcal{B}(B \to X_s\gamma)$ at NNLO [see Mikolaj's talk]



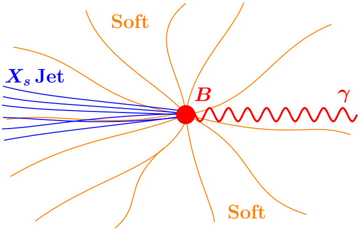
Experiments measure photon energy spectrum

$$m_B = E_X + E_\gamma\,, \quad \vec{p}_X = -\vec{p}_\gamma \quad \Rightarrow \quad 0 \le p_X^+ = m_B - 2E_\gamma \le m_B$$



Large backgrounds require cut on photon energy $E_\gamma > E_{\mathrm{cut}}$

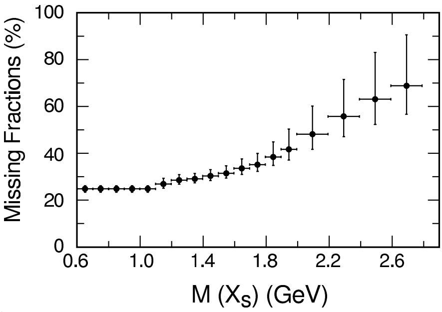
$\Rightarrow$

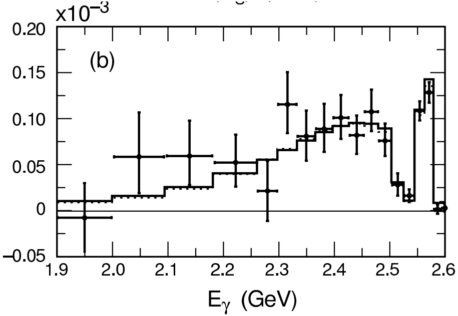


$E_\gamma \to m_B/2$ or $p_X^+ \to 0$, jet-like X_s gives cleaner signal

Sum Over Exclusive Modes Measurement

Babar, $81.9\ \mathrm{fb}^{-1}$ [Phys. Rev. D 72, 052004 (2005)]





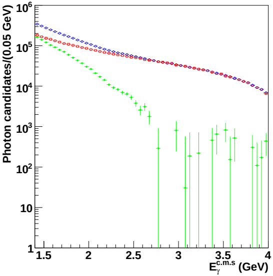
Reconstruct X_s by summing over exclusive modes $B \to (K + n\pi)\gamma$

- Very good resolution since signal B is completely reconstructed
- Large systematic uncertainties from missing modes and hadronization
  - Get worse for larger m_X = smaller E_γ

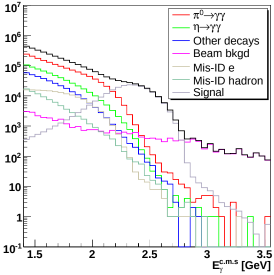
Fully Inclusive Measurement

Belle, $605\ \mathrm{fb}^{-1}$ [arXiv:0804.1580]

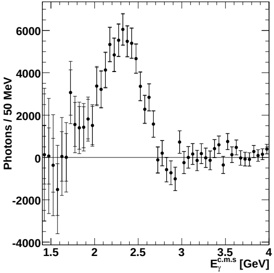
Subtract continuum bkgd



⇒ Subtract $B\bar{B}$ background



⇒ Raw spectrum



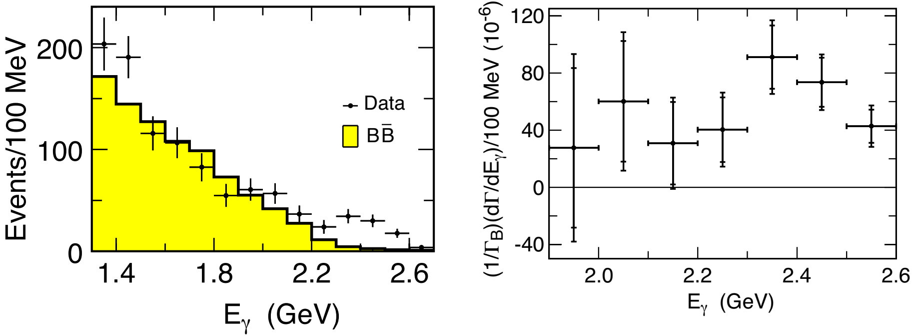
Fully inclusive (untagged): Only look for a photon

- Large statistics
- Huge background from both continuum and $B\bar{B}$ events
- Uncertainties on $\mathcal{B}(E_\gamma > E_{\mathrm{cut}})$:

E_{cut} / GeV	1.7	1.8	1.9	2.0
σ_{stat}	5.7%	5.2%	4.8%	4.8%
σ_{sys}	11.2%	7.4%	5.2%	4.1%

Inclusive Tagged Measurement

Babar, $210\ \mathrm{fb}^{-1}$ [Phys. Rev. D 77, 051103 (2008)]



Inclusive tagged: Fully reconstruct and remove one B (tag) in the event

- Eliminates background from continuum and mostly from tagged B
- Much smaller statistics (due to small tagging efficiencies $< 1\%$)
- Cannot eliminate background from remaining signal B

$\Rightarrow$ Pushing E_{cut} much below $2.0\,\mathrm{GeV}$ causes large systematic uncertainties

Sources of Non-Experimental Uncertainties

Theoretical uncertainties

- Neglected higher order terms in theoretical expansions (perturbative, nonperturbative, kinematic)
- Large E_{cut} drastically enhances corrections
  - Perturbative: Double logarithms $\ln^2 p_X^+/m_B$
  - Nonperturbative: Spectrum is sensitive to b-quark PDFs in B meson

    [Bigi et al., Neubert (1993); Bauer, Luke, Mannel (2001)]

Uncertainties from input parameters

- m_b, λ_1, m_c, α_s, ...,
- Leading and subleading shape functions

Extrapolation down to $E_{\mathrm{cut}} = 1.6\,\mathrm{GeV}$

- Depends on theory and input parameters
- Should not be lumped into experimental uncertainties

Strategy Towards Precision Test of $B \to X_s\gamma$

Measure and compare $\mathcal{B}(E_\gamma > E_{\text{cut}})$ for fixed E_{cut}

- Choice of E_{cut} only trades between systematic and theoretical uncertainties (both are equally non-rigorous)
- Not clear what "optimal" value is

Combine all information we have

- Perturbative information (normalization & shape)
- Existing constraints on input parameters (e.g. m_b, λ_1 from $B \to X_c\ell\nu$)
- Experimental information (normalization & shape)

Perform global fit to all available data

- Simultaneously determines normalization and input parameters
- Constrains (some) non-experimental uncertainties by data
- m_b, SF are also important input for inclusive $|V_{ub}|$ determination

Outline

Organization of Perturbative Corrections

Match onto effective currents at $\mu = m_b$

$$\left\langle X_s\gamma \middle| \sum_{i=1}^{8} C_i O_i \middle| B \right\rangle = \frac{e}{4\pi^2}\varepsilon_\mu \Big[\underbrace{C_7^{\mathrm{incl}} \langle X_s | J_7^\mu | B\rangle + \cdots}_{\gamma \text{ factorizes}} + \underbrace{C_{q\bar q} \otimes \langle X_s | J_{q\bar q}^\mu | B\rangle + \cdots}_{\text{Non-local } \gamma \text{ vertex}} \Big]$$

[see Mikolaj's talk]

- $J_7^\mu = 2m_b\, \bar s\, \mathrm{i} q_\nu \sigma^{\mu\nu} P_R b \Big|_{\mu=m_b}$
- $C_7^{\mathrm{incl}}(\mu_0) = \mathcal{C}_7 - \dfrac{\alpha_s C_F}{4\pi} \displaystyle\sum_{i=1-6,8} C_i f_i^{(7)} + \cdots$
- Other finite bremsstrahlung from currents with additional gluon fields

Concentrate on leading J_7^μ current ($p_X^+ = m_B - 2E_\gamma$)

$$\frac{\mathrm{d}\Gamma_s}{\mathrm{d}E_\gamma} = \frac{2\Gamma_{0s}}{m_b^3} |C_7^{\mathrm{incl}}|^2 (m_B - p_X^+) W(p_X^+) \qquad W(p_X^+) \sim \langle B | J_{7\mu}^\dagger(x) J_7^\mu(0) | B \rangle$$

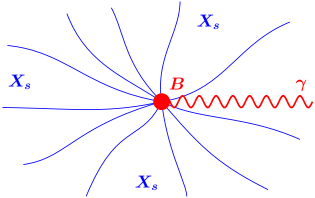
Regions of Phase Space

$$\frac{\mathrm{d}\Gamma_s}{\mathrm{d}E_\gamma} = \frac{2\Gamma_{0s}}{m_b^3}|C_7^{\mathrm{incl}}|^2(m_B - p_X^+)W(p_X^+) \qquad (p_X^+ = m_B - 2E_\gamma)$$

Local OPE region: $\Lambda_{\mathrm{QCD}} \ll p_X^+ \sim m_B$

$$W(p_X^+) = \int \mathrm{d}^4x\, e^{-\mathrm{i}q\cdot x}\langle b|J_{7\mu}^\dagger(x)J_7^\mu(0)|b\rangle$$

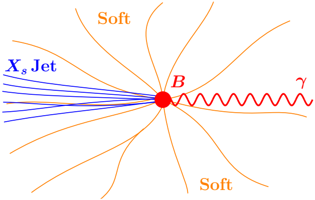
- Corrections: $(\Lambda_{\mathrm{QCD}}/p_X^+)^2$, $(\Lambda_{\mathrm{QCD}}/m_B)^2$



SCET region (SF region): $\Lambda_{\mathrm{QCD}} \sim p_X^+ \ll m_B$

$$W(p_X^+) = (m_B - p_X^+)^2 h(m_b) \times \int \mathrm{d}\omega\, m_b J(m_b\omega)\, S(p_X^+ - \omega)$$

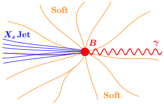
- Corrections: p_X^+/m_B, $\Lambda_{\mathrm{QCD}}/m_B$



$\Rightarrow$ Optimal theory description/expansion depends on phase space region

What to Expand and What to Resum

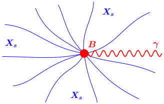
How to treat region $E_\gamma \sim (1.6 - 2)\,\mathrm{GeV} \Rightarrow p_X^+ \sim (1-2)\,\mathrm{GeV}$?



$\Lambda_{\mathrm{QCD}} \sim p_X^+ \ll m_B$

? ⟷

$\Lambda_{\mathrm{QCD}} \ll p_X^+ \ll m_B$



$\Lambda_{\mathrm{QCD}} \ll p_X^+ \sim m_B$

	expand in	can resum logs	keep all powers
Model $S(\omega, \mu)$	$\frac{p_X^+}{m_B}$	$\ln^n \frac{\mu}{m_B}$	$\left(\frac{\Lambda_{\mathrm{QCD}}}{p_X^+}\right)^n$
MSOPE [Becher, Neubert]	$\frac{p_X^+}{m_B}$ & $\frac{\Lambda_{\mathrm{QCD}}}{p_X^+}$	$\ln^n \frac{\mu}{m_B}$ & $\ln^n \frac{\mu}{p_X^+}$	
local OPE [Misiak et al.]	$\frac{\Lambda_{\mathrm{QCD}}}{p_X^+}$		$\left(\frac{p_X^+}{m_B}\right)^n$

- Double logs $\ln^2(p_X^+/m_B) \sim 1$ but exponentiate $e^{\ln(p_X^+/m_B)} = p_X^+/m_B$
- $1 \sim \Lambda_{\mathrm{QCD}}/p_X^+ \ll 1$? $1 \sim p_X^+/m_B \ll 1$? $\Rightarrow$ Combine both limits

New and Improved Approach to Shape Function

[Ligeti, Stewart, FT, 0807.1926]

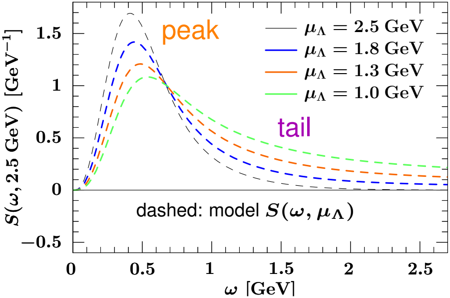
Start with $1 \sim \Lambda_{\mathrm{QCD}}/p_X^+ \ll 1$

Nonpert. peak: $\omega \sim \Lambda_{\mathrm{QCD}}$

Perturbative tail: $\omega \gg \Lambda_{\mathrm{QCD}}$

$\Rightarrow$ Non-exponential tail from RGE running [Balzereit, Mannel, Kilian (1998)]

$S(\omega, \mu)$ run up to $\mu = 2.5$ GeV



$$S(\omega, \mu_\Lambda) = \frac{\langle B | O_0(\omega, \mu_\Lambda) | B \rangle}{2 m_B}$$

New and Improved Approach to Shape Function

[Ligeti, Stewart, FT, 0807.1926]

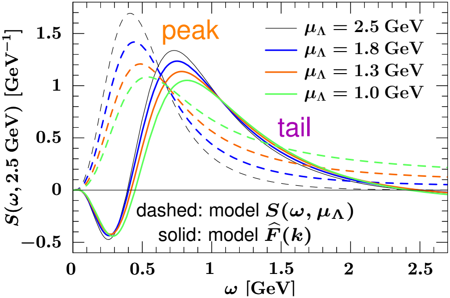
Start with $1 \sim \Lambda_{QCD}/p_X^+ \ll 1$

Nonpert. peak: $\omega \sim \Lambda_{QCD}$

Perturbative tail: $\omega \gg \Lambda_{QCD}$

$\Rightarrow$ Non-exponential tail from RGE running [Balzereit, Mannel, Kilian (1998)]

$S(\omega, \mu)$ run up to $\mu = 2.5$ GeV



Derive factorized form of SF

$$S(\omega, \mu_\Lambda) = \frac{\langle B|O_0(\omega, \mu_\Lambda)|B\rangle}{2m_B} = \int dk\, \widehat{C}_0(\omega - k, \mu_\Lambda)\, \widehat{F}(k)$$

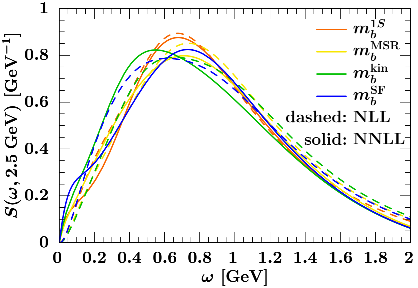
- Perturbative $\widehat{C}_0(\omega, \mu_\Lambda)$ (partonic SF) gives tail consistent with RGE Known to NNLO [Bauer, Manohar (2003); Becher, Neubert (2006)]
- Peak determined by purely nonperturbative function $\widehat{F}(k)$ Moments of $\widehat{F}(k)$ given by HQE parameters (m_b, λ_1) to all orders in α_s

Switching to Short Distance Schemes

$\widehat{C}$ and $\widehat{F}$ defined in generic SD scheme

$$S(\omega) = \int \mathrm{d}k\, C_0^{\mathrm{pole}}(\omega - k)\, F^{\mathrm{pole}}(k)$$
$$= \int \mathrm{d}k\, C_0^{1S}(\omega - k)\, F^{1S}(k)$$
$$= \int \mathrm{d}k\, C_0^{\mathrm{kin}}(\omega - k)\, F^{\mathrm{kin}}(k)$$
$$= \int \mathrm{d}k\, C_0^{\mathrm{SF}}(\omega - k)\, F^{\mathrm{SF}}(k) = \dots$$

- Dip at small ω from renormalon in m_b^{pole} is removed by any SD $\widehat{m}_b$



Switching to Short Distance Schemes

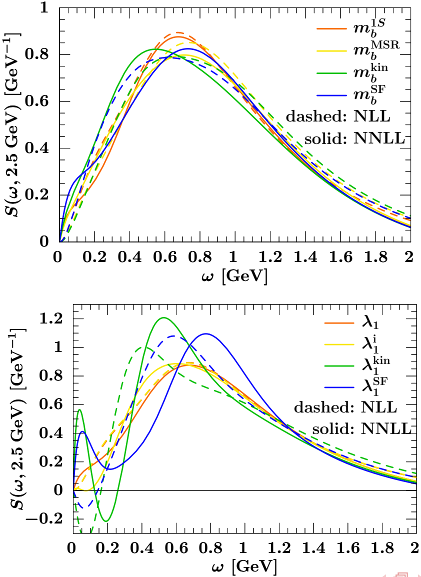
$\widehat{C}$ and $\widehat{F}$ defined in generic SD scheme

$$S(\omega) = \int \mathrm{d}k\, C_0^{\mathrm{pole}}(\omega - k)\, F^{\mathrm{pole}}(k)$$
$$= \int \mathrm{d}k\, C_0^{1S}(\omega - k)\, F^{1S}(k)$$
$$= \int \mathrm{d}k\, C_0^{\mathrm{kin}}(\omega - k)\, F^{\mathrm{kin}}(k)$$
$$= \int \mathrm{d}k\, C_0^{\mathrm{SF}}(\omega - k)\, F^{\mathrm{SF}}(k) = \ldots$$

- Dip at small ω from renormalon in m_b^{pole} is removed by any SD $\widehat{m}_b$

"Invisible" λ_1 renormalon at $\mathcal{O}(\alpha_s^2)$

- kinetic and SF scheme for $\lambda_1 = -\mu_\pi^2$ appear to oversubtract
- Define invisible scheme $\lambda_1^{\mathrm{i}} = \lambda_1 - 0\alpha_s - R^2\alpha_s^2 C$



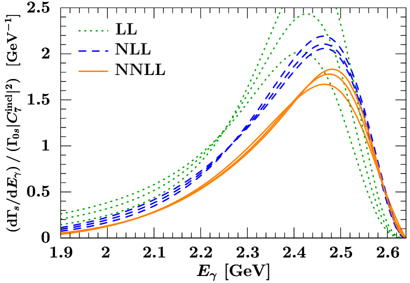
The Spectrum

$$\frac{\mathrm{d}\Gamma_s}{\mathrm{d}E_\gamma} = \frac{2\Gamma_{0s}}{m_b^3} |C_7^{\mathrm{incl}}|^2 (m_B - p_X^+)^3\, h(m_b) \int \mathrm{d}k\, \widehat{P}(m_b, p_X^+ - k)\, \widehat{F}(k)$$

- Factorization determines which perturbative pieces are convoluted with $\widehat{F}(k)$
- Contains all info from $\Lambda_{\mathrm{QCD}} \ll p_X^+$ limit without expanding in $\Lambda_{\mathrm{QCD}}/p_X^+$ ($p_X^+ = m_B - 2E_\gamma$)

$\Rightarrow$ Consistently combines $\Lambda_{\mathrm{QCD}} \sim p_X^+$ (peak) and $\Lambda_{\mathrm{QCD}} \ll p_X^+$ (shoulder)

- Still missing: p_X^+/m_B, $\Lambda_{\mathrm{QCD}}/m_B$



$\mu_h = 4.7$ GeV
$\mu_i = 2.5$ GeV
$\mu_\Lambda = (1.2, 1.5, 1.9)$ GeV

Connecting to Local OPE Region

[Ligeti, Stewart, FT: in preparation]

Next step: Connect

SCET region: $p_X^+ \ll m_B$

Local OPE: $p_X^+ \sim m_B$

Start from SCET result and resum p_X^+/m_B corrections

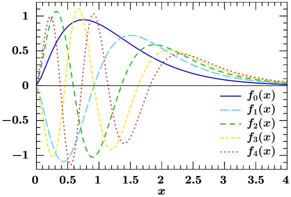
$$\frac{d\Gamma_s}{dE_\gamma} = \frac{2\Gamma_{0s}}{m_b^3} |C_7^{\mathrm{incl}}|^2 (m_B - p_X^+) \int dk\, \widehat{W}_{\mathrm{pert}}(m_b, p_X^+, k)\, \widehat{F}(k)$$

- Nontrivial due to different structure of perturbation series
  - Short-distance scheme
  - Separation $S = \widehat{C} \otimes \widehat{F}$ crucial to get correct result for local OPE
  - Scale up μs to shut off resummation for $p_X^+ \to m_B$ to restore cancellations between singular and non-singular corrections

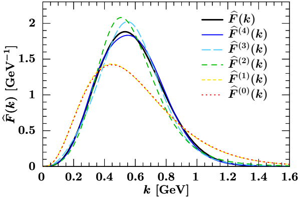
Outline

Designer Orthonormal Basis Functions

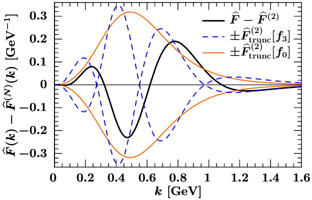
Basis



Expansion of Gaussian $\widehat{F}(k)$



Truncation error $N = 2$



Design suitable orthonormal basis for $\widehat{F}(k)$ (formally model independent)

$$\widehat{F}(\lambda x) = \frac{1}{\lambda}\Big[\sum_{n=0}^{\infty} c_n f_n(x)\Big]^2 \qquad \text{with} \qquad \int \mathrm{d}k\, \widehat{F}(k) = \sum_{n=0}^{\infty} c_n^2 = 1$$

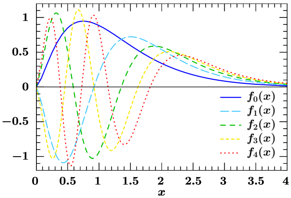
When fitting $\widehat{F}(k)$ from data

- Truncate at $n \leq N$, with N as required by precision of data
- Experimental uncertainties reflected in basis coefficients c_n
- Size of truncation error controlled by $1 - \sum_{n=0}^{N} c_n^2$

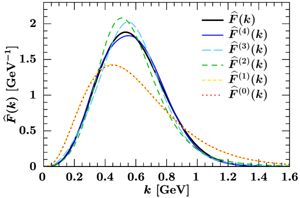
$\Rightarrow$ Systematic and realistic SF uncertainties

Designer Orthonormal Basis Functions

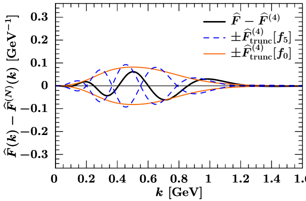
Basis



Expansion of Gaussian $\widehat{F}(k)$



Truncation error $N = 4$



Design suitable orthonormal basis for $\widehat{F}(k)$ (formally model independent)

$$\widehat{F}(\lambda x) = \frac{1}{\lambda}\left[\sum_{n=0}^{\infty} c_n f_n(x)\right]^2 \qquad \text{with} \qquad \int \mathrm{d}k\, \widehat{F}(k) = \sum_{n=0}^{\infty} c_n^2 = 1$$

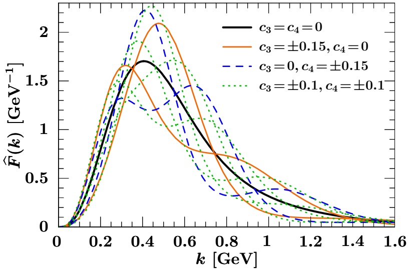
When fitting $\widehat{F}(k)$ from data

- Truncate at $n \leq N$, with N as required by precision of data
- Experimental uncertainties reflected in basis coefficients c_n
- Size of truncation error controlled by $1 - \sum_{n=0}^{N} c_n^2$

$\Rightarrow$ Systematic and realistic SF uncertainties

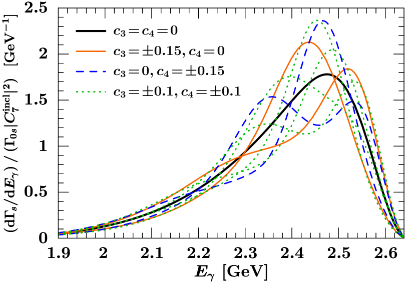
Incorporating Moment Constraints

$F^{1S\mathrm{i}}(k)$ with fixed m_b^{1S}, λ_1^{i}



$\Rightarrow$

$\mathrm{d}\Gamma_s/\mathrm{d}E_\gamma$ with fixed m_b^{1S}, λ_1^{i}



m_b, λ_1 dependence of $\mathrm{d}\Gamma$ enters via moments of $\widehat{F}(k)$

$$\int \mathrm{d}k\, k\, F^{1S\mathrm{i}}(k) = m_B - m_b^{1S} \qquad \int \mathrm{d}k\, k^2\, F^{1S\mathrm{i}}(k) = -\frac{\lambda_1^{\mathrm{i}}}{3} + (m_B - m_b^{1S})^2$$

- Can consistently include previous constraints on m_b^{1S}, λ_1^{i} (from $B \to X_c \ell\nu$, sum rules, etc.) via constraints on coefficients c_n
- Cleanly separates m_b and SF dependence

SIMBA

[Bernlochner, Lacker, Ligeti, Stewart, FT, Tackmann: work in progress]

SIMBA: Towards a Global Fit

Expanding $\widehat{F}(k)$ we get

$$\frac{\mathrm{d}\Gamma_s}{\mathrm{d}E_\gamma} = \sum_{n,m} \underbrace{\mathcal{N} c_n\, c_m}_{\text{fit}} \underbrace{\int \mathrm{d}k\, (m_B - p_X^+) \widehat{W}_{\text{pert}}(m_b, p_X^+, k)\, f_n(k)\, f_m(k)}_{\text{compute in advance}}$$

$$M_i(m_b^{1S}, \lambda_1^{\mathrm{i}}) = \sum_{n,m} \overbrace{c_n\, c_m} \overbrace{\int \mathrm{d}k\, k^i\, f_n(k)\, f_m(k)}$$

Combined fit to all measured (binned) spectra $\mathrm{d}\Gamma$ and moment constraints M_i

- Simultaneously determines normalization $\mathcal{N} \sim \Gamma_{0s}|C_7^{\text{incl}}|^2$ and c_n
- All perturbative information included through $\widehat{W}_{\text{pert}}$
- Consistently combines existing constraints on m_b, λ_1 with shape information from $B \to X_s\gamma$

$\Rightarrow$ SF and m_b uncertainties are determined by data, including correlations

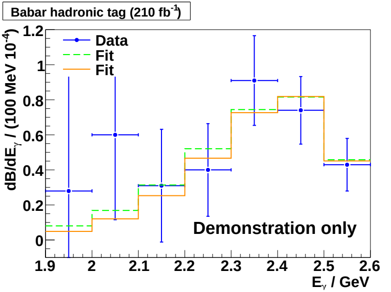
SIMBA: Some Preliminary Results

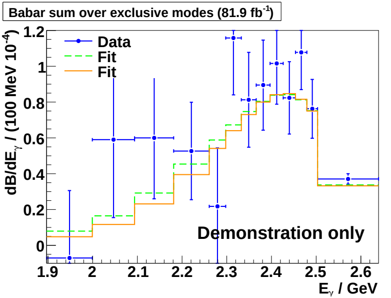
Do combined fit to

- Babar hadronic tag
- Babar sum over exclusive modes
- Belle $B \to X_c \ell \nu$ fit (including correlations)

$$m_b^{1S} = (4.72 \pm 0.12)\ \mathrm{GeV}$$
$$\lambda_1 = (-0.31 \pm 0.09)\ \mathrm{GeV}^2$$

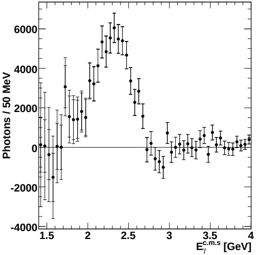
	w/o m_b, λ_1	with m_b, λ_1
χ^2/ndf	$8.7/18$	$9.1/19$
$\delta\mathcal{N}$	19%	13%
δc_1	45%	10%
δc_2	$-$	86%



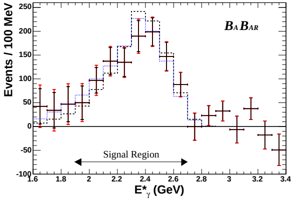


SIMBA: To Dos and Missing Pieces

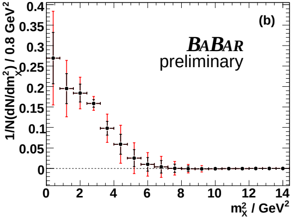
Belle fully inclusive



Babar leptonic tag



Babar $B \to X_u \ell \nu$



Add more experimental data

- Belle fully inclusive spectrum (strong correlations ...)
- Babar leptonic tag (not efficiency corrected, will be updated to full dataset)
- Spectra from $B \to X_u \ell \nu$

Missing theory pieces

- Subleading theory corrections
- Theory uncertainties & correlations

Conclusions

$\mathcal{B}(B \to X_s\gamma)$ provides stringent tests of SM/constraint on New Physics

- Calculation of $\mathcal{B}(B \to X_s\gamma)$ extremely advanced
- Measurements are most precise for large E_γ
- For a full description of spectrum need to combine different “optimal” theory descriptions

Want to combine all available information into global fit

- Simultaneously determine normalization of $B \to X_s\gamma$ (new physics sensitive) and nonperturbative parameters
- Let data decide about nonperturbative uncertainties
- Fit works, adding more data ...

Knowing the shape function is important in its own right

- Important input for other inclusive measurements ($|V_{ub}|$, $B \to X_s\ell^+\ell^-$)
- Contributes to experimental uncertainties via MC signal model