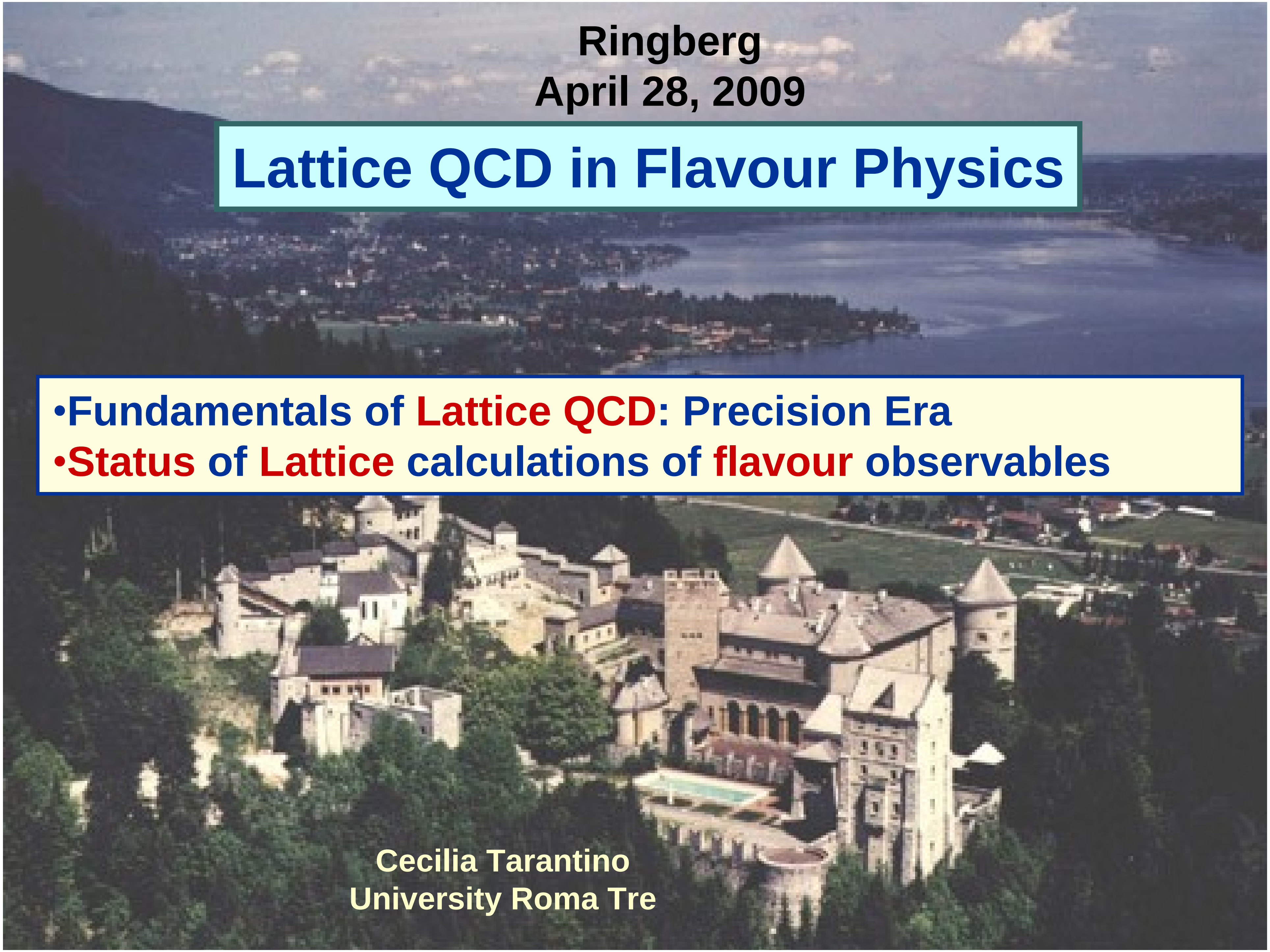


Ringberg
April 28, 2009

Lattice QCD in Flavour Physics

- Fundamentals of Lattice QCD: Precision Era
- Status of Lattice calculations of flavour observables

Cecilia Tarantino
University Roma Tre

Some fundamentals of Lattice QCD

Lattice QCD:

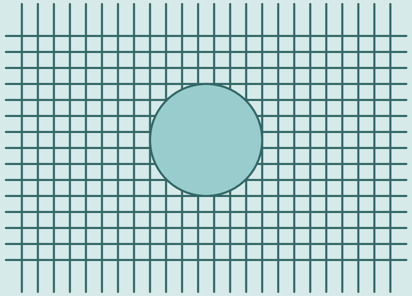
- ✓ **non-perturbative approach**
  (path-integral method)
- ✓ **only the QCD parameters**
- ✓ **theory regularization**
- ✗ **discrete space and finite volume**

Path Integral Method:

Green functions ≡ derivatives of the generator functional

$$Z(J_\mu,\eta,\bar{\eta}) = \int \delta A\,\delta\bar{q}\,\delta q\;\; e^{-S(A,q,\bar{q}) + \int J_\mu A_\mu + \int \bar{\eta} q + \int \bar{q}\eta}$$

To formally define the integrals, one considers
a discrete LATTICE of finite volume:
infinite-dimensional integrals → ordinary multiple integrals



$$\left\langle O(A,q,\bar{q})\right\rangle = \int \delta A\,\delta\bar{q}\,\delta q\;\; O(A,q,\bar{q})\;\; e^{-S(A,q,\bar{q})}$$

few configurations contribute ☺

$$\cong \bar{O} = \frac{1}{N}\sum_{i=1}^{N} O\left(\{C\}_i\right)$$

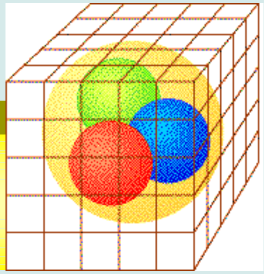
generated by a Monte Carlo

In the era of precision Flavour Physics

ε_K	$(2.280 \pm 0.013)\ 10^{-3}$	0.6%
Δm_d	$(0.507 \pm 0.005)\ \mathrm{ps}^{-1}$	1%
Δm_s	$(17.77 \pm 0.12)\ \mathrm{ps}^{-1}$	0.7%
Sin2β	0.672 ± 0.024	3%
$\lvert V_{us}\rvert\, f_+(0)$	0.21664 ± 0.00048	0.2%
…	…	…

we are also entering the era of

Precision LATTICE QCD



Unquenched calculations with relatively **low quark masses** are now being performed by **several groups** using **different approaches** (lattice action, renormalization, …).

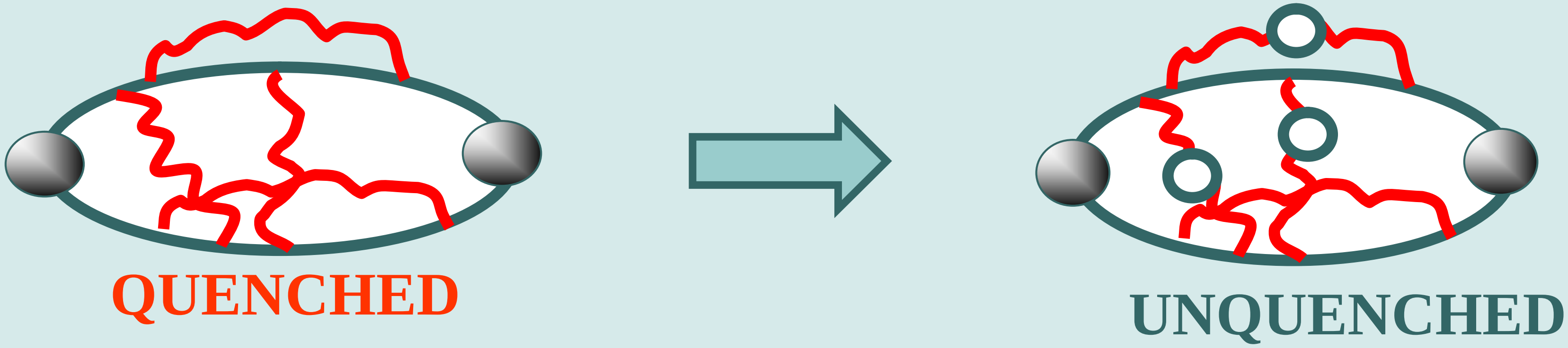
Crucial when aiming at a percent precision.

“PRECISION” LATTICE QCD: WHY NOW

1) Increasing of computational power

(TeraFlops machines)

→ Unquenched simulations



2) Algorithmic improvements:

→ Light quark masses in the ChPT regime

A comment on the results presented below, taken into account in estimating averages

V. Lubicz, C.T. 0807.4605

- The quenched approximation introduces a systematic uncertainty which depends on the observable and is difficult to estimate

HOWEVER

- Unquenched simulations are more "expensive" and some unquenched calculations have not reached yet the same accuracy in controlling other systematics as (cheaper) quenched calculations
(e.g. continuum extrapolation, non-perturbative renormalization)

- For some observables only few unquenched determinations are available

We will mainly present unquenched (N_f=2, 2+1) results, but without *forgetting* accurate quenched calculations !

Importance and Success of Lattice QCD in Flavour Physics

$|V_{us}| \equiv \lambda$
(CKM parameter)

- V_{us} and the "1st row" unitarity test
- The Unitarity Triangle Analysis (UTA)

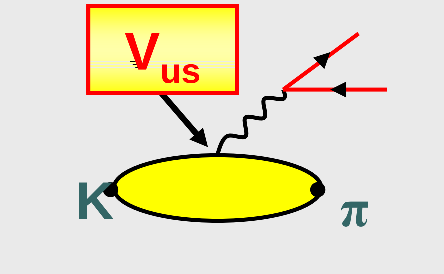
1st row: the most stringent unitarity test

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1$$

Source:	Nuclear β- dec.	Kl3,Kl2	b→u semil.
Abs. error:	$4 \cdot 10^{-4}$	$5 \cdot 10^{-4}$	$\sim 10^{-6}$

V_{us} from Kl3 decays

$$\Gamma_{K\to\pi l\nu} = C_K^2 \frac{G_F^2 m_K^5}{192\pi^2}\, I\, S_{EW}[1+\Delta_{SU(2)}+2\Delta_{EM}] \times \boxed{|V_{us}|^2 |f_+^{K\pi}(0)|^2}$$



Ademollo-Gatto: $f_+(0) = 1 - O(m_s - m_u)^2 \leftarrow$ O(1%). But represents the largest theoret. uncertainty

ChPT

$$f_+(0) = 1 + f_2 + f_4 + O(p^8)$$

Vector Current Conservation

$f_2 = -0.023$
Independent of L_i
(Ademollo-Gatto)

THE LARGEST UNCERTAINTY

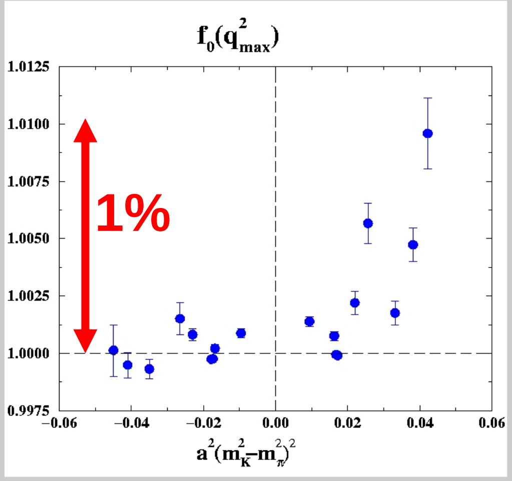
Old standard estimate:

Leutwyler, Roos (1984)
(QUARK MODEL)
$f_4 = -0.016 \pm 0.008$

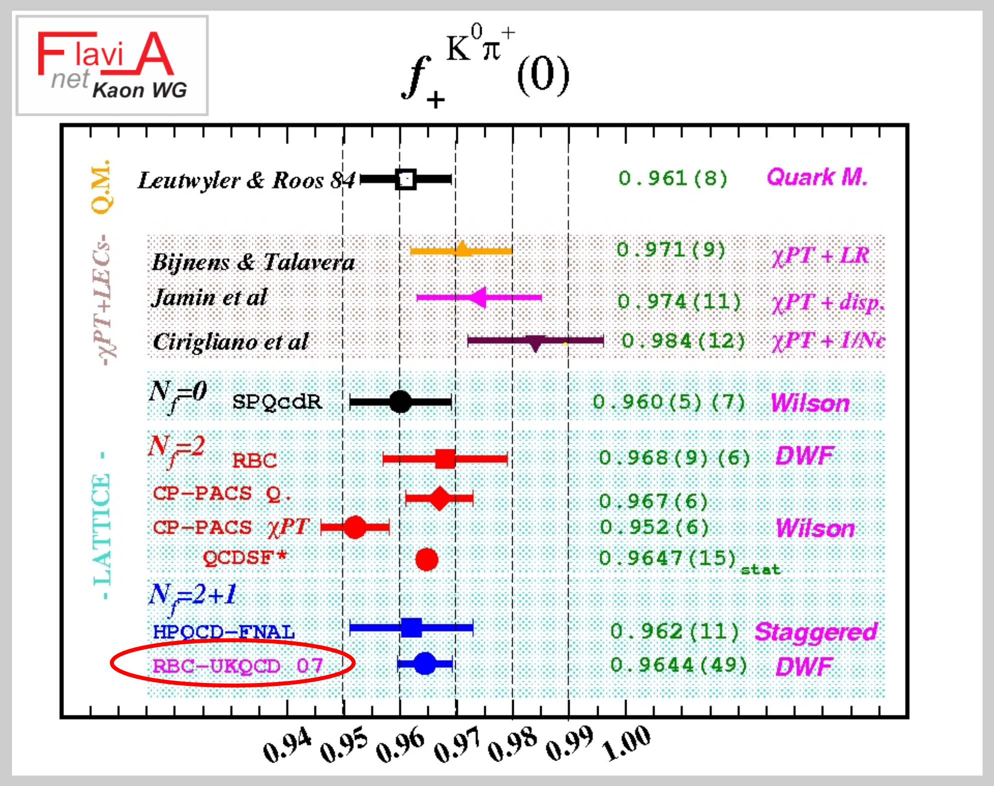
Lattice QCD

THE O(1%) PRECISION CAN BE REACHED

D.Becirevic, G.Isidori, V.Lubicz, G.Martinelli, F.Mescia, S.Simula, C.T., G.Villadoro. [NPB 705,339,2005]



The basic ingredient is a **double ratio** of correlation functions [FNAL for B→D,D*]



- **Good agreement** between Nf=2 and 2+1 calculations and the first quenched result

- The **error on** $\Delta f = f_+(0) - 1 - f_2$ quoted in the original calculation was 50%

- A **new precise** Nf=2+1 calculation by RBC/UKQCD

- **Analytical (model dependent) results slightly higher** than Lattice QCD

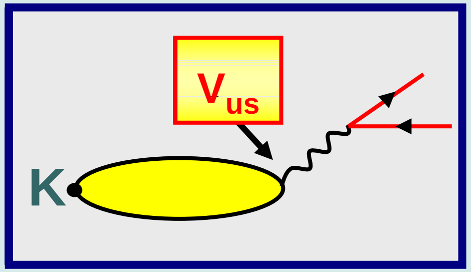
$f_+(0)=0.964(5) \longrightarrow |V_{us}|=0.2246(12)$

Flavianet Kaon WG

V_{us}/V_{ud} from Kμ2/πμ2 decays

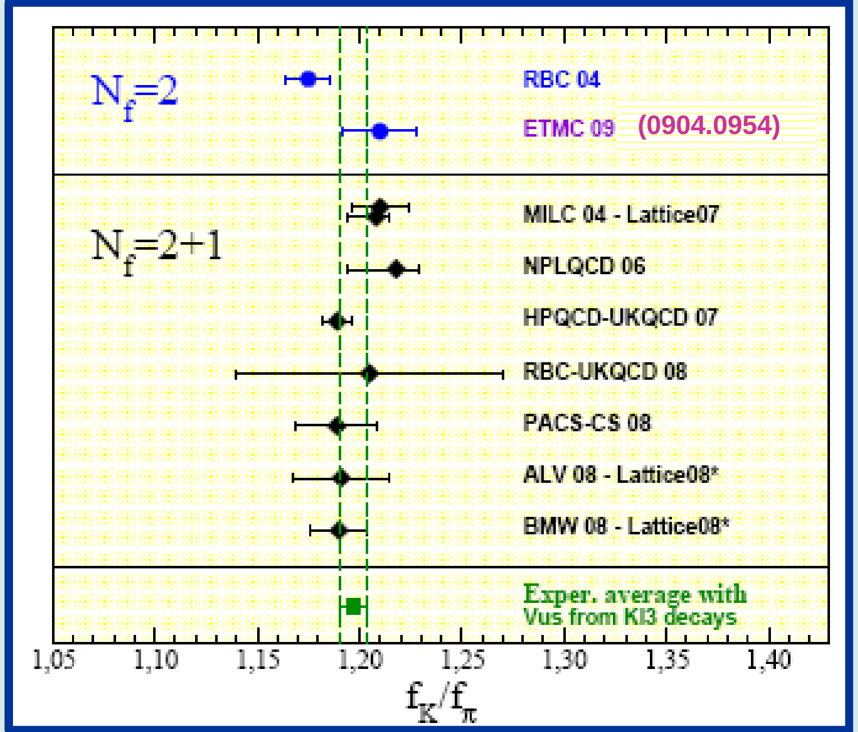
$$\frac{\Gamma(K \to \mu\bar{\nu}_\mu(\gamma))}{\Gamma(\pi \to \mu\bar{\nu}_\mu(\gamma))} = \frac{|V_{us}|^2}{|V_{ud}|^2}\left(\frac{f_K}{f_\pi}\right)^2 \frac{m_K(1-\frac{m_\mu^2}{m_K^2})}{m_\pi(1-\frac{m_\mu^2}{m_\pi^2})} \times 0.9930(35)$$

[Marciano 04]



The lattice determination of f_K/f_π, together with the experimental measurement of the leptonic decay Br's, and with $|V_{ud}|$ from nucleon beta decays, allows to extract $|V_{us}|$

f_K/f_π: LATTICE SUMMARY



Flavianet Kaon WG

f_K/f_π=1.189(7)

$|V_{us}|$=0.2261(15)

Kl3 and Kl2

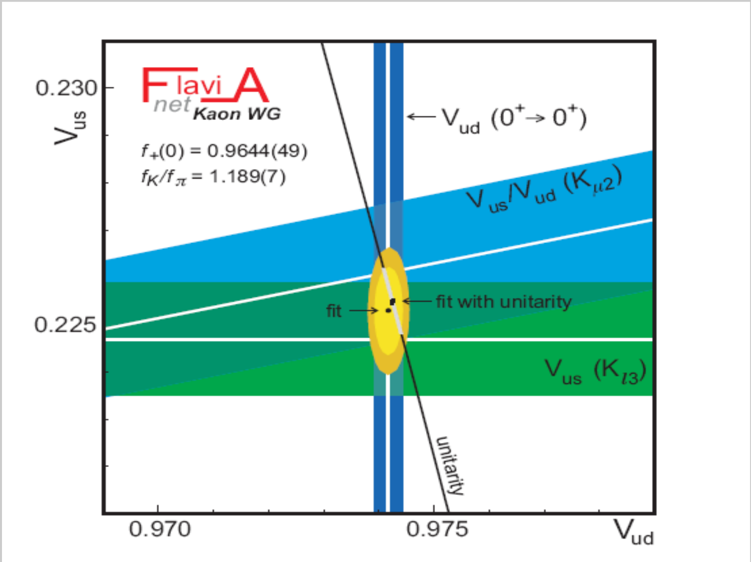
determinations of V_{us}

are in good agreement

First row

unitarity test

works well



Status of the **UTA** within the **Standard Model (SM)**

M. Ciuchini *et al.*, hep-ph/0012308
M. Bona *et al.* [UTfit Collaboration], hep-ph/0501199

The experimental constraints:

$$\varepsilon_K, \Delta m_d, \left|\frac{\Delta m_s}{\Delta m_d}\right|, \left|\frac{V_{ub}}{V_{cb}}\right|$$

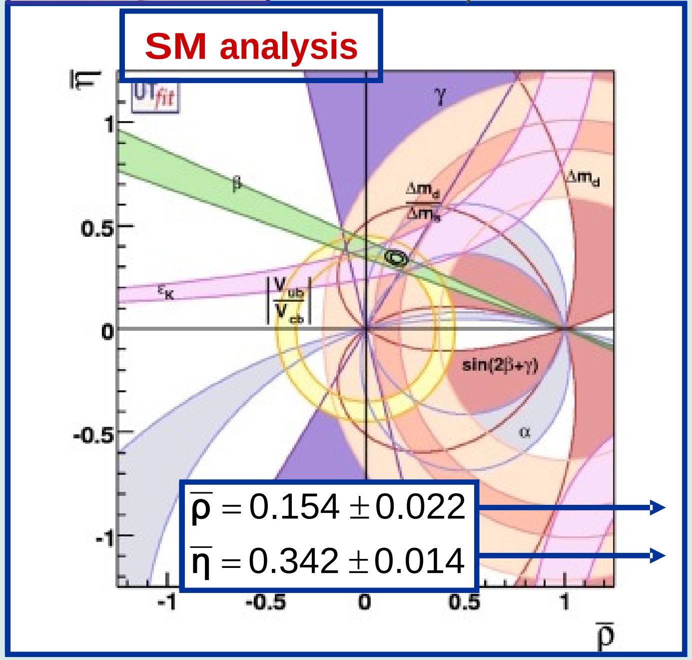
⇨ relying on theoretical calculations of hadronic matrix elements

$$\sin 2\beta, \cos 2\beta, \alpha, \gamma(\;2\beta+\gamma)$$

⇨ independent from theoretical calculations of hadronic parameters

overconstrain the CKM parameters consistently

www.utfit.org
C.T., 0902.3431



$\bar\rho = 0.154 \pm 0.022$ → ~14%

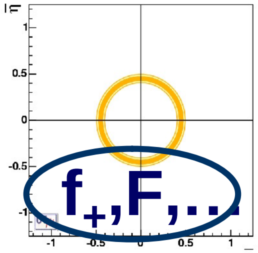
$\bar\eta = 0.342 \pm 0.014$ → ~ 4%

The UTA has established that the CKM matrix is the dominant source of flavour mixing and CP violation

THE UTA CONSTRAINTS

Relying on **LATTICE** calculations

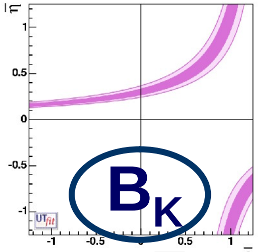
$|V_{ub}/V_{cb}|$



$f_+,F,\ldots$

$b\to u/b\to c$

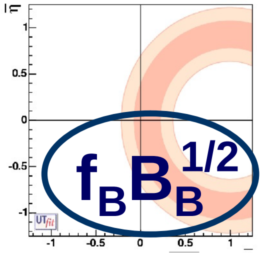
ε_K



B_K

$K^0-\bar{K}^0$

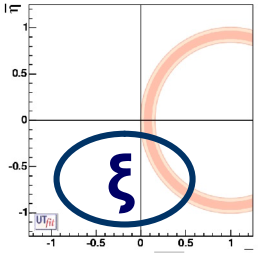
Δm_d



$f_B B_B^{1/2}$

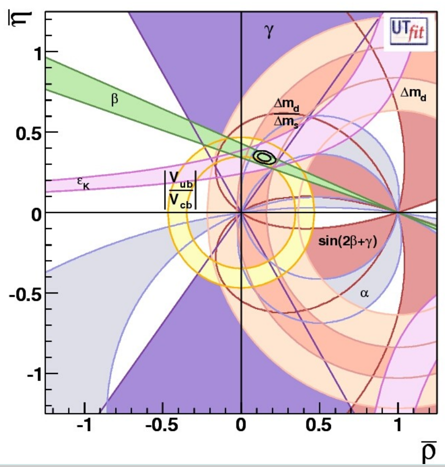
$B_d-\bar{B}_d$

$\Delta m_d/\Delta m_s$



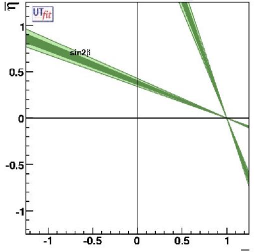
ξ

$B_s-\bar{B}_s$



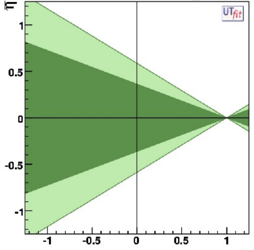
UT-ANGLES

$\sin 2\beta$



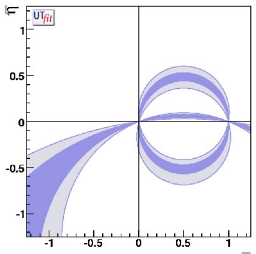
$B\to J/\Psi K^0$

$\cos 2\beta$



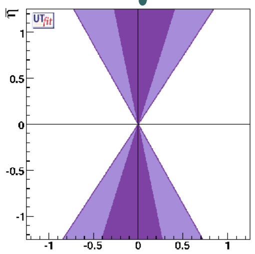
$B\to J/\Psi K^{*0}$

α



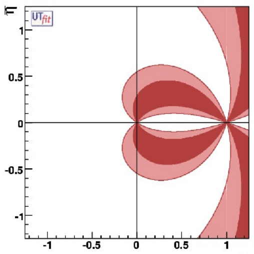
$B\to\pi\pi,\rho\rho$

γ



$B\to D^{(*)}K$

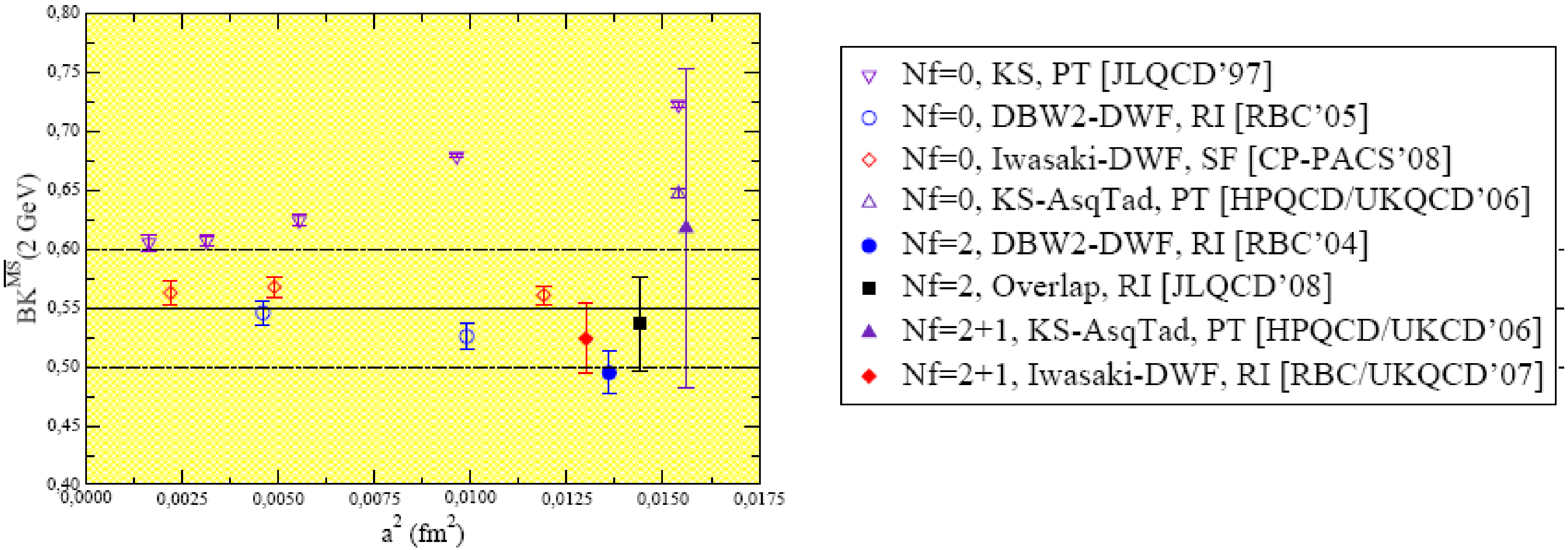
$\sin(2\beta+\gamma)$



$B\to D^{(*)}\pi,D\rho$

The Kaon bag-parameter: B_K

$$\langle \bar{K}^0 | Q(\mu) | K^0 \rangle = \frac{8}{3} f_K^2 m_K^2 B_K(\mu)$$



- •Recent unquenched results (so far at fixed lattice spacing)
- •Quenched calculations: O(a)-improved, multiplicative renormal., continuum limit
- •No evidence of quenching
- •Discretization effects could similarly affect quenched and unquenched results

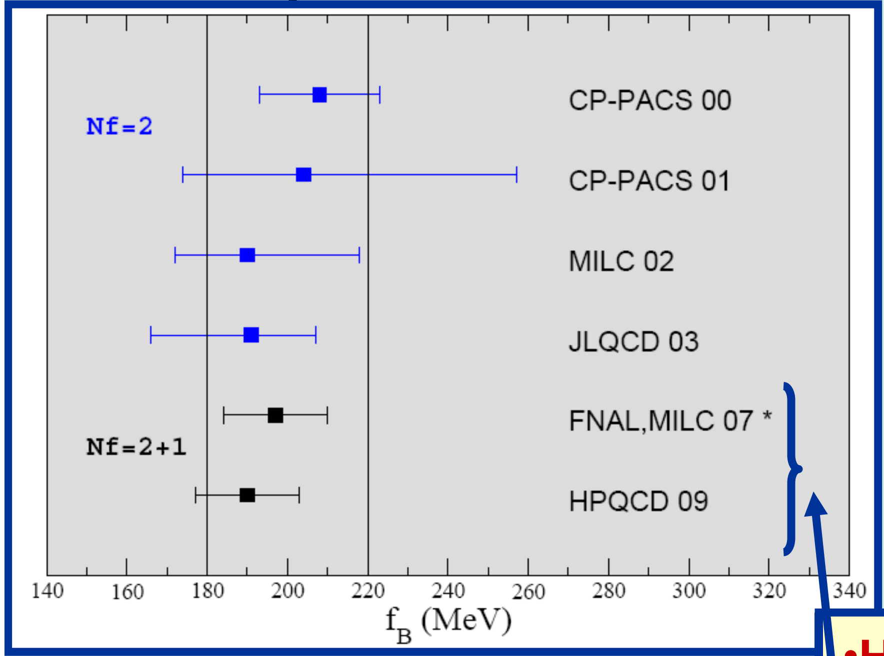
$\hat{B}_K = 0.75 \pm 0.07 \quad (B_K^{\overline{MS}}(2\text{GeV}) = 0.55 \pm 0.05)$

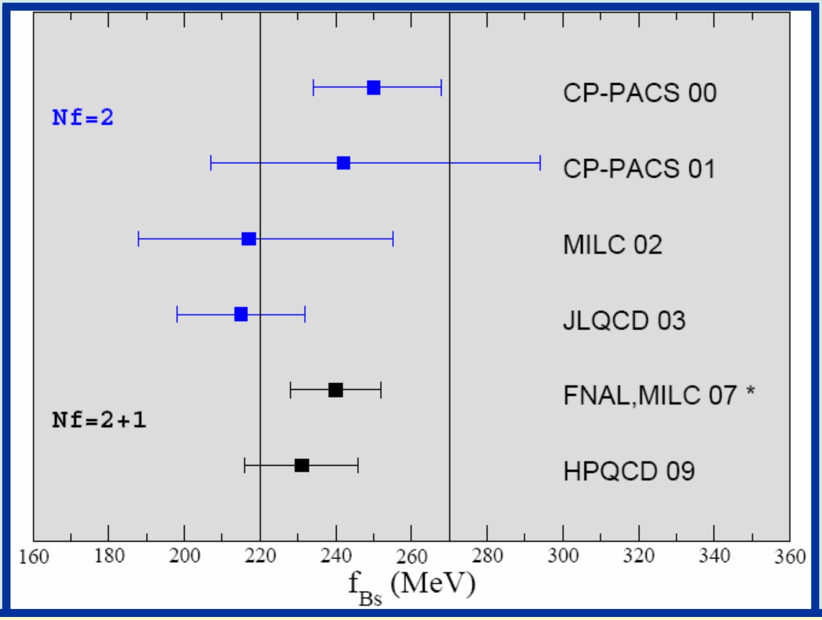
V. Lubicz, C.T. 0807.4605

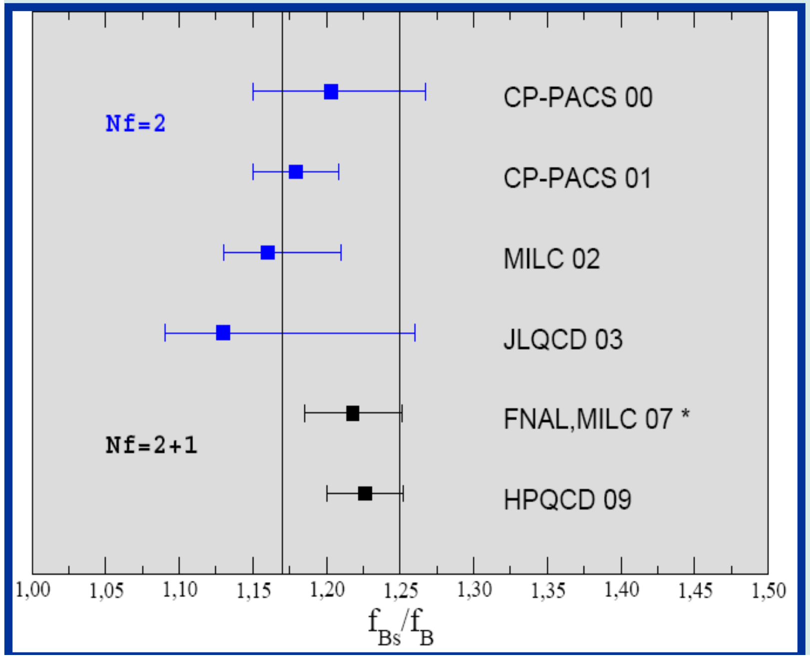
Unquenched simulations at several Lattice spacings are in progress !!

The $B_{d,s}$ decay constants: f_B , f_{Bs}

Inputs for $\Delta m_{d/s}$ and $B \to \tau\nu$







- **Heavy quark treated with two different approaches (NRQCD, FNAL)**
- **Importance for f_B of light quark masses ($< m_s/2$)**
- **Importance of continuum limit**

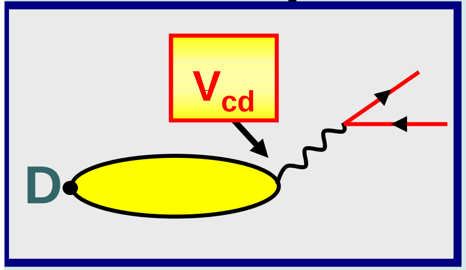
V. Lubicz, C.T. 0807.4605

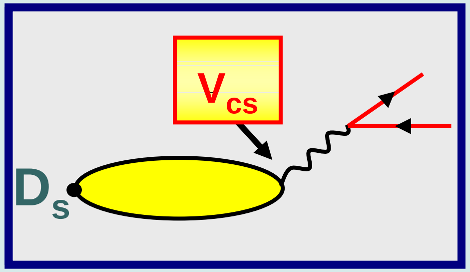
$$f_{Bs} = 245 \pm 25 \text{ MeV}$$

$$f_B = 200 \pm 20 \text{ MeV}$$

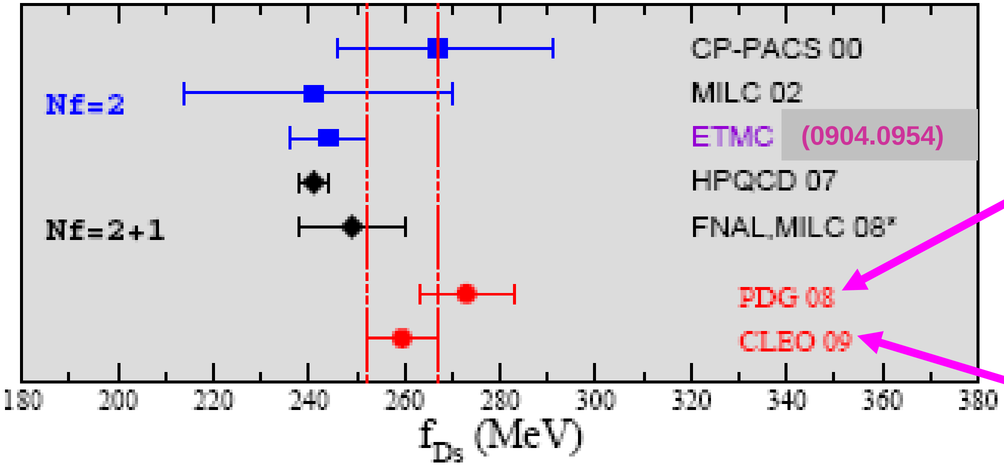
$$f_{Bs}/f_B = 1.21 \pm 0.04$$

The D and D_s decay constants: f_D , f_{Ds}



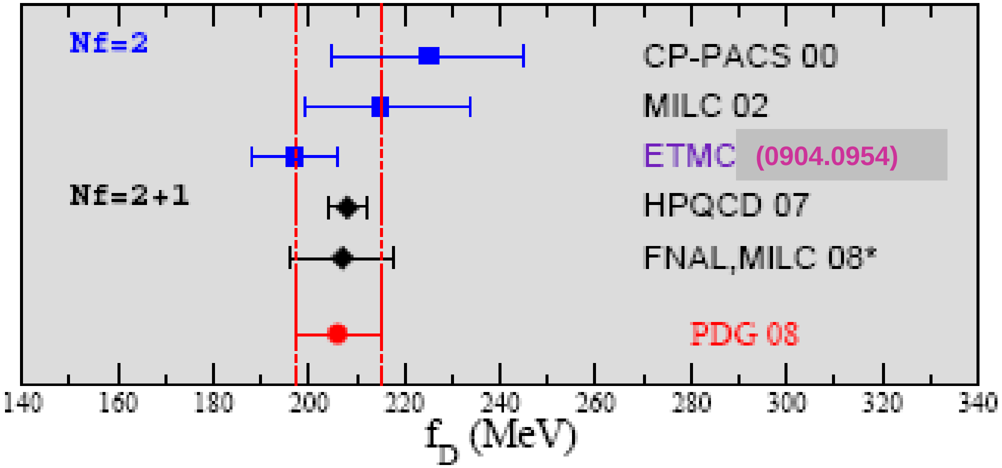


The Br's of th $D_{(s)}$ leptonic decays have been accurately measured at CLEO-c



In **2008**, the experimental measurements and the lattice results for f_{Ds} were ~ **2-3 σ** away

The new **CLEO-c** measurement **(0901.1216)** went towards lattice results



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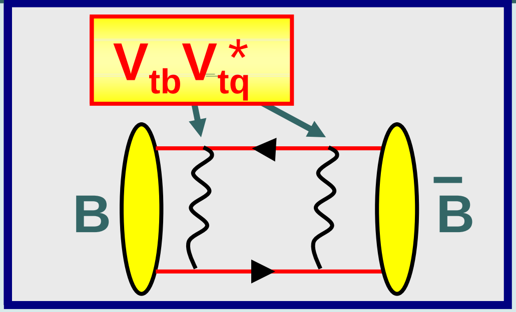
f_{Ds}= 250 ± 15 MeV

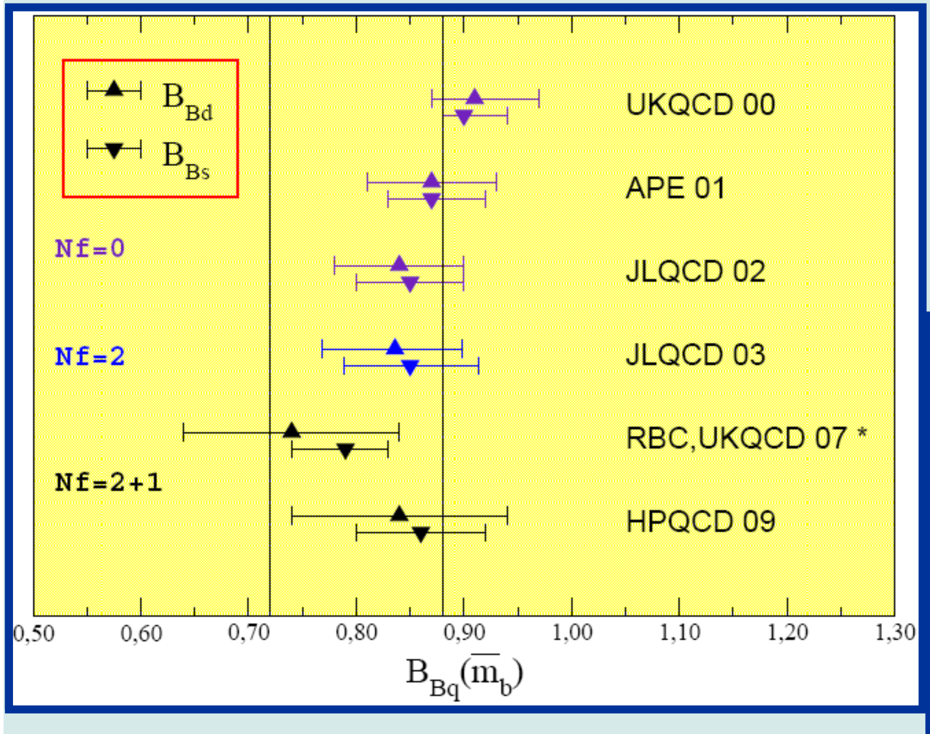
f_D= 212 ± 14 MeV

f_{Ds}/f_D = 1.18 ± 0.03

The $B_{d,s}$ bag-parameters: B_{Bd} , B_{Bs}



$$\langle \bar{B} | Q(\mu) | B \rangle = \frac{8}{3} m_B^2 f_B^2 B_B(\mu)$$



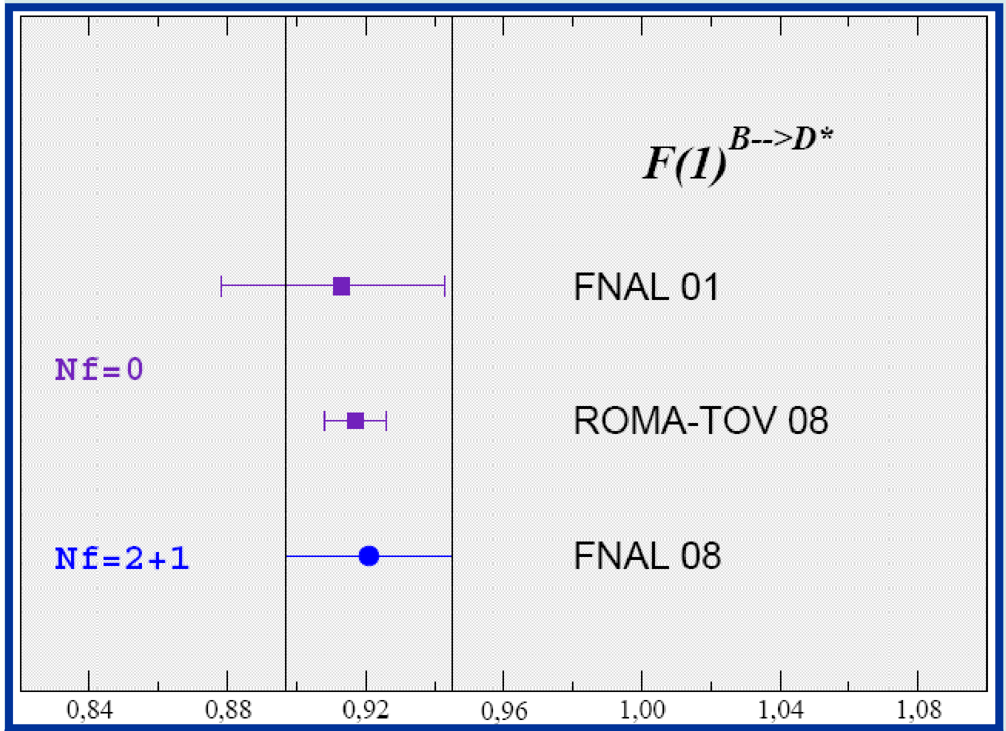
- $B_{Bs} \approx B_{Bd}$
- **Chiral logs** are **unimportant** in the B-parameters

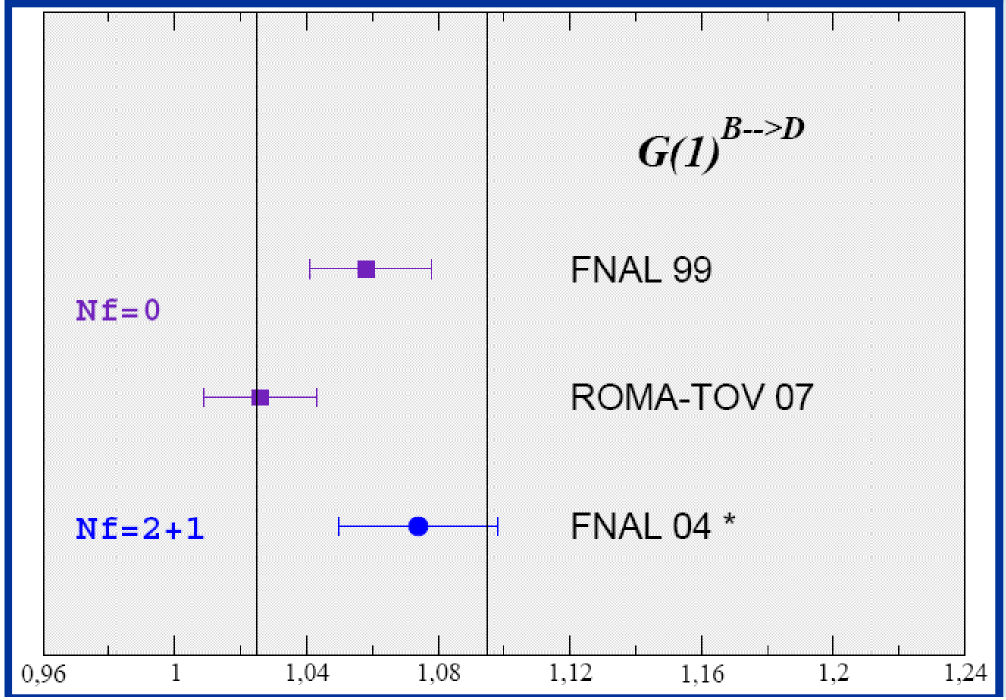
$$\hat{B}_{B_s} = \hat{B}_{B_d} = 1.22 \pm 0.12$$

V. Lubicz, C.T. 0807.4605

$$f_{B_s}\sqrt{\hat{B}_{B_s}} = 270 \pm 30\,\text{MeV} \;, \qquad f_B\sqrt{\hat{B}_{B_d}} = 225 \pm 25\,\text{MeV}$$

$$\xi = \frac{f_{B_s}\sqrt{\hat{B}_{B_s}}}{f_B\sqrt{\hat{B}_{B_d}}} = 1.21 \pm 0.04$$





V_{cb} from $B\to D/D^*\ l\ \nu$ decays

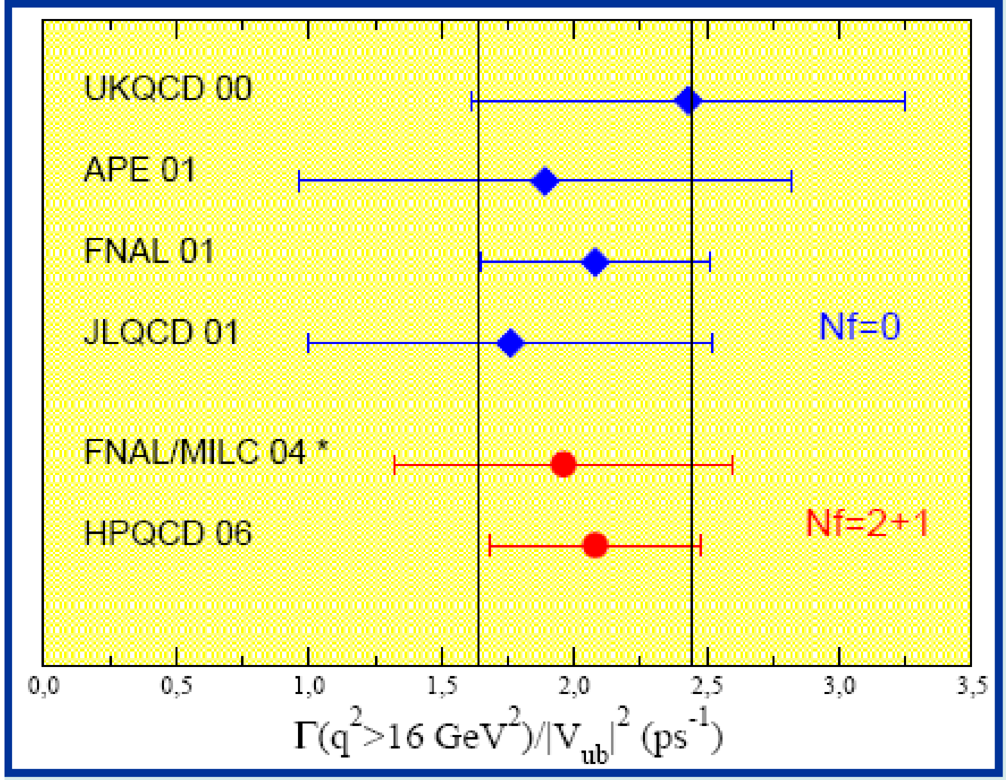
- **Form factors calculated by FNAL and ROMA-TOV Collaborations, with different techniques → in agreement**
- **Percent level accuracy achieved, as required to evaluate corrections to the static limit**
- **Privileged role of $B\to D^*\ l\ \nu$ due to the better experimental accuracy**

V. Lubicz, C.T. 0807.4605

$$\left|V_{cb}\right|^{B\to D^* l\nu}_{excl.} = (39.2 \pm 1.1)\cdot 10^{-3}$$

$$\left|V_{cb}\right|^{B\to D l\nu}_{excl.} = (39.9 \pm 4.4)\cdot 10^{-3}$$

V_{ub} from $B \rightarrow \pi\, l\, \nu$



- •Different techniques to treat the b quark (NRQCD,relQCD,FNAL)→ in good agreement

- •No evidence of quenching effects

V. Lubicz, C.T. 0807.4605

$$|V_{ub}|^{LQCD}_{excl.} = (35.4 \pm 4.0) \cdot 10^{-4}$$

In agreement with a new improved lattice study (FNAL/MILC08) which combines lattice and BaBar data for $f_+(q^2)$ in a model independent way:
$|V_{ub}|=(33.8\pm 3.6)\cdot 10^{-4}$ [0811.3640]

QCD Sum Rules provide well compatible results

(see talk by Patricia Ball)

Due to many experimental constraints, some hadronic quantities can be extracted from the (overcostraint) UTA and compared to Lattice calculations

UTfit

Extracting them as free parameters from the UTA:
(UTfit, update of hep-ph/0606167)

ε_K → $\hat{B}_K^{UT}=0.75\pm0.07$

Δm_d → $f_{Bs}\sqrt{\hat{B}_{Bs}}^{UT}=265\pm4$ MeV

$\Delta m_s/\Delta m_d$ → $\xi^{UT}=1.25\pm0.06$

Averaging recent accurate Lattice results:
(V.Lubicz, C.T., 0807.4605)

$\hat{B}_K^{LAT}=0.75\pm0.07$

$f_{Bs}\sqrt{\hat{B}_{Bs}}^{LAT}=270\pm30$ MeV

$\xi^{LAT}=1.21\pm0.04$

Remarkable agreement:

- **Additional evidence of the SM success in describing flavour physics**
- **Reliability of Lattice QCD**

Further improvements in Lattice calculations of B_K and ξ are looked forward to increase the UTA accuracy

New accurate studies are in progress!!


THANKS!

BACKUP

K^0- $\overline{K}^0$ mixing: the complete operator basis

$$\langle \bar{K}^0|O_1(\mu)|K^0\rangle = \frac{8}{3} M_K^2 f_K^2\, B_1^{sd}(\mu)\ ,$$

$$\langle \bar{K}^0|O_2(\mu)|K^0\rangle = -\frac{5}{3}\left(\frac{M_K}{m_s(\mu)+m_d(\mu)}\right)^2 M_K^2 f_K^2\, B_2^{sd}(\mu)\ ,$$

$$\langle \bar{K}^0|O_3(\mu)|K^0\rangle = \frac{1}{3}\left(\frac{M_K}{m_s(\mu)+m_d(\mu)}\right)^2 M_K^2 f_K^2\, B_3^{sd}(\mu)\ ,$$

$$\langle \bar{K}^0|O_4(\mu)|K^0\rangle = 2\left(\frac{M_K}{m_s(\mu)+m_d(\mu)}\right)^2 M_K^2 f_K^2\, B_4^{sd}(\mu)\ ,$$

$$\langle \bar{K}^0|O_5(\mu)|K^0\rangle = \frac{2}{3}\left(\frac{M_K}{m_s(\mu)+m_d(\mu)}\right)^2 M_K^2 f_K^2\, B_5^{sd}(\mu)\ ,$$

RI-MOM
(2 GeV)

All quenched

	B_1^{sd}	B_2^{sd}	B_3^{sd}	B_4^{sd}	B_5^{sd}
hep-lat/9920027	0.68(21)	0.67(7)	0.95(15)	1.00(9)	0.66(11)
hep-lat/0605016	0.56(6)	0.87(8)	1.41(16)	0.94(6)	0.62(8)
hep-lat/0610075	0.52(4)	0.54(2)	0.71(2)	0.70(1)	0.62(1)

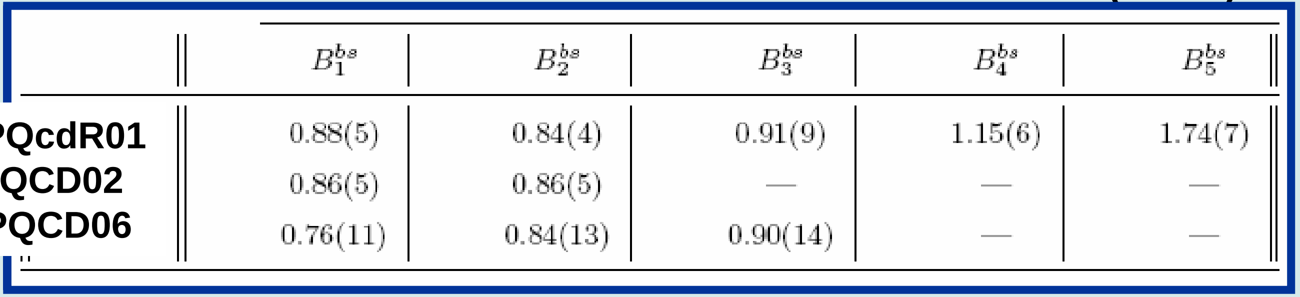
- **The differences among the three determinations are larger than quoted errors**
- **New studies are needed**

V. Lubicz, C.T. 0807.4605

$$B_2^{sd} = 0.7(2)\quad ,\quad B_3^{sd} = 1.0(4)\quad ,\quad B_4^{sd} = 0.9(2)\quad ,\quad B_5^{sd} = 0.6(1)$$

B^0- $\overline{B}^0$ mixing: the complete operator basis

RI-MOM
(2 GeV)



	B_1^{bs}	B_2^{bs}	B_3^{bs}	B_4^{bs}	B_5^{bs}
SPQcdR01	0.88(5)	0.84(4)	0.91(9)	1.15(6)	1.74(7)
JLQCD02	0.86(5)	0.86(5)	—	—	—
unquenched→ **HPQCD06**	0.76(11)	0.84(13)	0.90(14)	—	—

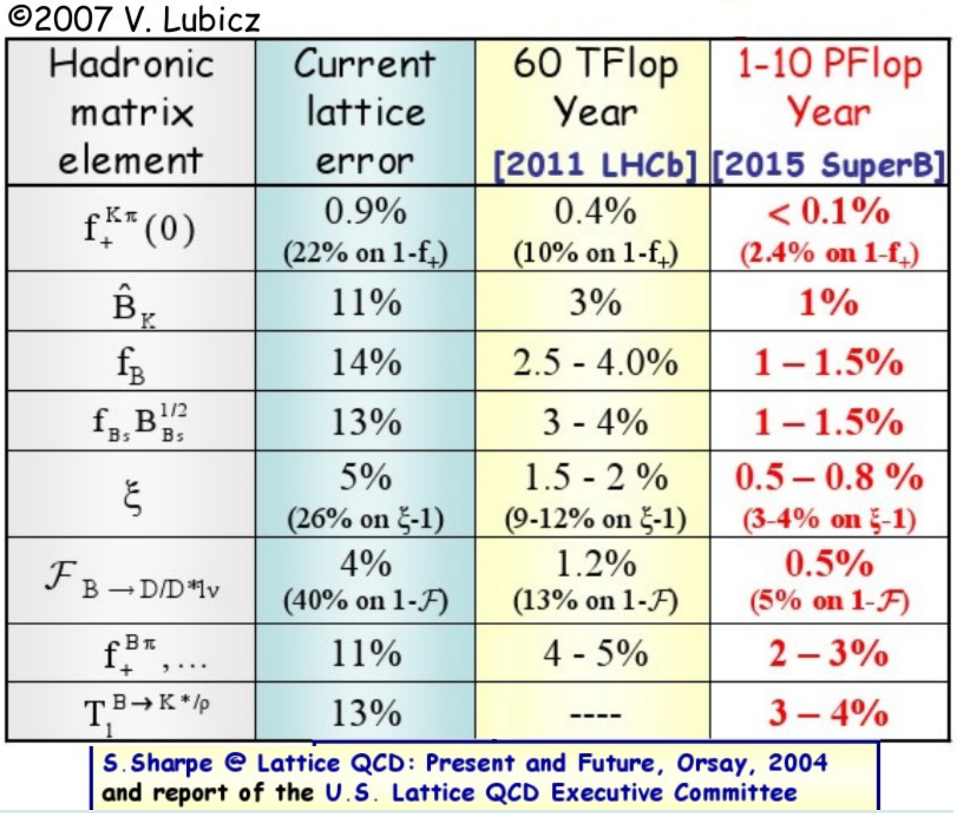
V. Lubicz, C.T. 0807.4605

$$B_2^{bq} = 0.85(10) \quad , \quad B_3^{bq} = 0.90(13) \quad , \quad B_4^{bq} = 1.15(13) \quad , \quad B_5^{bq} = 1.74(19)$$

- New unquenched studies are looked forward

Trying to predict the future...

V. Lubicz@SuperB workshop, 2007



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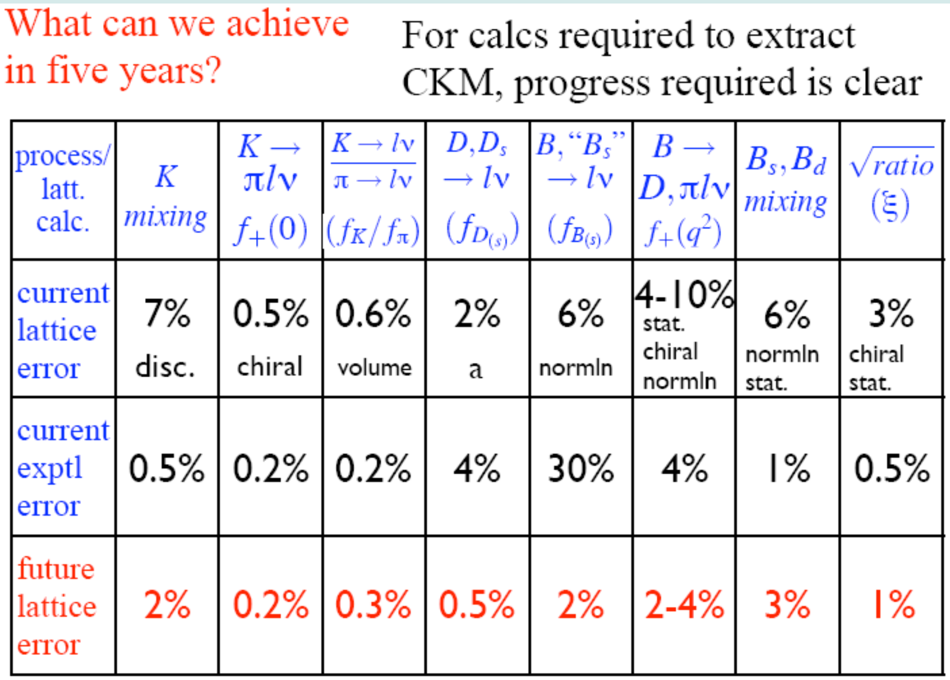
Hadronic matrix element	Current lattice error	60 TFlop Year [2011 LHCb]	1-10 PFlop Year [2015 SuperB]
$f_+^{K\pi}(0)$	0.9% (22% on 1-f_+)	0.4% (10% on 1-f_+)	< 0.1% (2.4% on 1-f_+)
$\hat{B}_K$	11%	3%	1%
f_B	14%	2.5 - 4.0%	1 – 1.5%
$f_{B_s}B_{B_s}^{1/2}$	13%	3 - 4%	1 – 1.5%
ξ	5% (26% on ξ-1)	1.5 - 2 % (9-12% on ξ-1)	0.5 – 0.8 % (3-4% on ξ-1)
$\mathcal{F}_{B\to D/D^*l\nu}$	4% (40% on 1-$\mathcal{F}$)	1.2% (13% on 1-$\mathcal{F}$)	0.5% (5% on 1-$\mathcal{F}$)
$f_+^{B\pi}, \dots$	11%	4 - 5%	2 – 3%
$T_1^{B\to K^*/\rho}$	13%	----	3 – 4%

S.Sharpe @ Lattice QCD: Present and Future, Orsay, 2004
and report of the U.S. Lattice QCD Executive Committee

C. Davies@SuperB workshop, 2009



What can we achieve in five years?

For calcs required to extract CKM, progress required is clear

process/ latt. calc.	K mixing	$K\to\pi l\nu$ $f_+(0)$	$\frac{K\to l\nu}{\pi\to l\nu}$ (f_K/f_π)	$D,D_s\to l\nu$ $(f_{D_{(s)}})$	$B,$"B_s"$\to l\nu$ $(f_{B_{(s)}})$	$B\to D,\pi l\nu$ $f_+(q^2)$	B_s,B_d mixing	$\sqrt{ratio}$ (ξ)
current lattice error	7% disc.	0.5% chiral	0.6% volume	2% a	6% normln	4-10% stat. chiral normln	6% normln stat.	3% chiral stat.
current exptl error	0.5%	0.2%	0.2%	4%	30%	4%	1%	0.5%
future lattice error	2%	0.2%	0.3%	0.5%	2%	2-4%	3%	1%