Flavor Phyics in the Littlest Higgs model with T-parity

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T. Goto, Y. Okada and Y. Yamamoto, Phys. Lett. B **670** (2009) 378 [arXiv:0809.4753] $+\alpha$.

Introduction

Little Higgs models atract interests in recent years.

- Possible solution to *little hiearchy problem*.
  - ▷ NP scale $\gtrsim O$(TeV) vs. Higgs boson mass $\sim$ 100GeV.
- Basic idea: Higgs bosons are assumed to be pseudo Nambu-Goldstone bosons of a spontaneous global symmetry breaking.
- One loop correction to the Higgs boson mass is suppressed by *collective symmetry breaking*.

Littlest Higgs model (Arkani-Hamed *et al.*, 2002)

- An implementation based on $SU(5)/SO(5)$ nonlinear sigma model.
- Electroweak precision constraints
  $\Rightarrow SU(5) \to SO(5)$ SSB scale $\gtrsim$ 4TeV (Csaki *et al.*).
  - ▷ Little hierarchy problem revives.
- Discrete Z_2 symmetry "T-parity" helps (Cheng & Low, 2003).

$\Rightarrow$ LHT is phenomenologically viable.

Introduction: flavor physics in LHT

LHT provides new particles (heavy gauge bosons and quarks/leptons) and new flavor mixings.

$\Rightarrow$ Various signals in flavor observables expected and studied.

- Blanke *et al.* (2006–2007), Hubisz *et al.* (2006), Choudhury *et al.* (2007), ...

We have recalculated flavor changing amplitudes in LHT.

- A missing piece found, that cancels previously reported UV divergence.
- Applied to $K \to \pi \nu \overline{\nu}$ and LFVs.

Contents in the following:

- The model: LHT,
- Flavor changing Z penguin amplitude,
- Numerical results on $K \to \pi \nu \overline{\nu}$ and μLFVs.

Littlest Higgs model: $SU(5)/SO(5)$ nonlinear sigma model

- $SU(5)$ global symmetry is broken down to $SO(5)$ by a VEV of Σ ($\mathbf{15}$ of $SU(5)$) with a symmetry breaking scale f ~TeV:

$$\langle\Sigma\rangle = \Sigma_0 = \left(\begin{array}{cc|c|cc} 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ \hline 0 & 0 & 1 & 0 & 0 \\ \hline 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \end{array}\right).$$

- $SU(5)/SO(5)$: $24 - 10 = 14$ Nambu-Goldstone bosons.

$$\Sigma = \xi\Sigma_0\xi^T, \qquad \xi = \exp(i\Pi/f),$$

$$\Pi = \frac{1}{2}\left(\begin{array}{cc|c|cc} -\omega^0 - \frac{\eta}{\sqrt{5}} & -\sqrt{2}\omega^+ & -i\sqrt{2}\pi^+ & -i2\phi^{++} & -i\sqrt{2}\phi^+ \\ -\sqrt{2}\omega^- & \omega^0 - \frac{\eta}{\sqrt{5}} & h + i\pi^0 & -i\sqrt{2}\phi^+ & \sqrt{2}(\phi^P - i\phi^0) \\ \hline i\sqrt{2}\pi^- & h - i\pi^0 & \frac{4}{\sqrt{5}}\eta & -i\sqrt{2}\pi^+ & h + i\pi^0 \\ \hline i2\phi^{--} & i\sqrt{2}\phi^- & i\sqrt{2}\pi^- & -\omega^0 - \frac{\eta}{\sqrt{5}} & -\sqrt{2}\omega^- \\ i\sqrt{2}\phi^- & \sqrt{2}(\phi^P + i\phi^0) & h - i\pi^0 & -\sqrt{2}\omega^+ & \omega^0 - \frac{\eta}{\sqrt{5}} \end{array}\right)$$

LHT: gauge symmetries

$$\begin{pmatrix} \boldsymbol{SU(2)_1} & & \\ \hline & & \\ \hline & & \boldsymbol{SU(2)_2} \end{pmatrix}$$

global:	$SU(5)$	$\overset{f}{\Rightarrow}$	$SO(5)$
	$\cup$		$\cup$
gauged:	$[SU(2)\times U(1)]_1 \times [SU(2)\times U(1)]_2$	$\overset{f}{\Rightarrow}$	$SU(2)\times U(1)$ SM electroweak

T-parity: $[SU(2)\times U(1)]_1 \overset{\mathrm{T}}{\longleftrightarrow} [SU(2)\times U(1)]_2 \Rightarrow (g,g')_1 = (g,g')_2$.

Gauge bosons

$$\left.\begin{matrix} [W_1^a,\, B_1] \\ [W_2^a,\, B_2] \end{matrix}\right\} \Rightarrow \left\{\begin{matrix} W_L^\pm,\, Z_L,\, A_L & \text{“light” SM gauge bosons (T-even)} \\ W_H^\pm,\, Z_H,\, A_H & \text{“heavy” } O(f) \text{ masses (T-odd)} \end{matrix}\right.$$

(pseudo) Nambu-Goldstone bosons

- $\pi^{\pm,0}$, h $\rightarrow$ SM Higgs doublet ($\pi^{\pm,0}$ eaten by $W_L^\pm$, Z_L), T-even.
- $\omega^{\pm,0}$, η $\rightarrow$ eaten by $W_H^\pm$, Z_H, A_H, T-odd.
- $\phi^{\pm\pm,\pm,0,P}$ $\rightarrow$ physical, T-odd (effects on flavor obs. suppressed).

LHT: fermion sector

Fermions are doubled for T-parity.

Left-handed

	$SU(2)_1$	$SU(2)_2$	Y_1	Y_2	$SU(2)_L$	Y	T-parity
$q_1^i = \begin{pmatrix} u_1^i \\ d_1^i \end{pmatrix}$	$\mathbf{2}$	$\mathbf{1}$	$\frac{1}{30}$	$\frac{4}{30}$	$\mathbf{2}$	$\frac{1}{6}$	$q_1^i \leftrightarrow -q_2^i$
$q_2^i = \begin{pmatrix} u_2^i \\ d_2^i \end{pmatrix}$	$\mathbf{1}$	$\mathbf{2}$	$\frac{4}{30}$	$\frac{1}{30}$	$\mathbf{2}$	$\frac{1}{6}$	

Right-handed

	$SU(2)_1$	$SU(2)_2$	Y_1	Y_2	$SU(2)_L$	Y	T-parity
u_R^i	$\mathbf{1}$	$\mathbf{1}$	$\frac{1}{3}$	$\frac{1}{3}$	$\mathbf{1}$	$\frac{2}{3}$	$+$
d_R^i	$\mathbf{1}$	$\mathbf{1}$	$-\frac{1}{6}$	$-\frac{1}{6}$	$\mathbf{1}$	$-\frac{1}{3}$	$+$
$q_{HR}^i = \begin{pmatrix} u_{HR}^i \\ d_{HR}^i \end{pmatrix}$	nonlinear				$\mathbf{2}$	$\frac{1}{6}$	$-$

- Singlet top partners $T_\pm$ are also introduced to cancel one-loop top Yukawa contribution to SM Higgs boson mass (LH mechanism).
- $u \Rightarrow \nu$ and $d \Rightarrow e$ (with $U(1)$ charge adjustments) for leptons.

$SU(5)$ embedding for Yukawa and gauge interactions

Building blocks of gauge invariant Lagrangian: $SU(5)$ "containers".

$$\Psi_1(\bar{\mathbf{5}}) = \begin{pmatrix} -i\sigma^2 q_1 \\ 0 \\ 0 \end{pmatrix}, \quad \Psi_2(\mathbf{5}) = \begin{pmatrix} 0 \\ 0 \\ -i\sigma^2 q_2 \end{pmatrix}, \quad \Psi_R = \begin{pmatrix} * \\ * \\ -i\sigma^2 q_{HR} \end{pmatrix}.$$

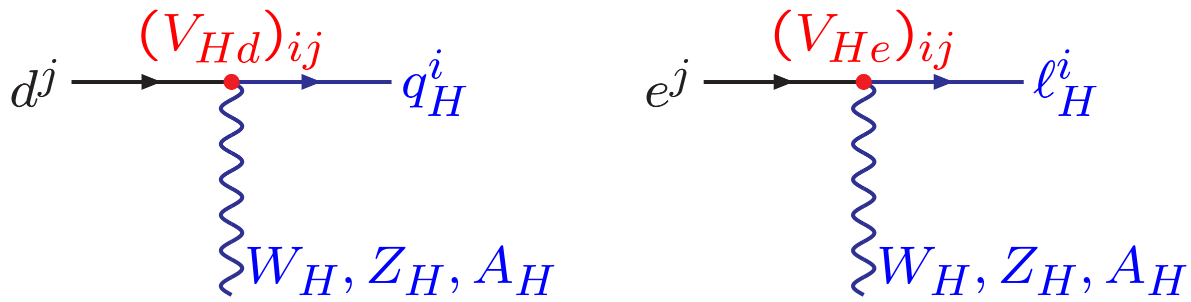
- $\Psi_{1,2}$: linear representations of $SU(5)$.
- Ψ_R: $\mathbf{5}$ of $SO(5)$, nonlinear representations of $SU(5)$.
  - ▷ $\Psi'_R = \xi \Psi_R$ transforms as $\mathbf{5}$ of $SU(5)$.

Yukawa coupling → $O(f)$ masses of T-odd fermions.

$$\mathcal{L}_H = -\kappa^{ij} f \left[\overline{\Psi}_2^i \Psi'^j_R - \overline{\Psi}_1^i \widetilde{\Psi}'^j_R \right] + \text{H.c.}, \qquad \widetilde{\Psi}'_R\text{: T-conjugate of } \Psi'_R.$$

κ^{ij} makes mismatch between light and heavy fermion mass bases.

⇒ new flavor mixing occurs in couplings with T-odd gauge bosons.



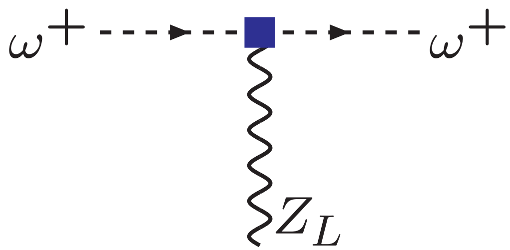
$O(\frac{v^2}{f^2})$ corrections in Z_L coupling

Vacuum configuration after EWSB: $h \to h + v$ ($v = 246\,\text{GeV}$).

$$\langle\Sigma\rangle_v = \xi_v \Sigma_0 \xi_v^T, \qquad \xi_v = 1 + O(\frac{v}{f}).$$

Expansion around $\langle\Sigma\rangle_v$ leads to $O(\frac{v^2}{f^2})$ corrections in gauge couplings of the pNG bosons and q_{HR}.

$$\mathcal{L} = \frac{f^2}{8}\,\mathrm{tr}\left[(\mathcal{D}^\mu\Sigma^\dagger)(\mathcal{D}_\mu\Sigma)\right] \Rightarrow$$

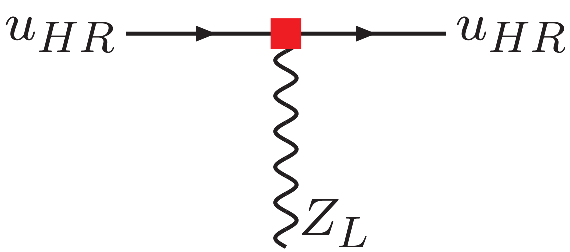


$$\propto \left(\cos^2\theta_W - \frac{v^2}{8f^2}\right).$$

[Blanke *et al.*, 2007]

$$\mathcal{L}_{\mathrm{kin}}(q_{HR}) = \frac{1}{2}\left[\overline{\Psi}'_R i\not{D}\Psi'_R + \overline{\widetilde{\Psi}}'_R i\not{D}\widetilde{\Psi}'_R\right], \qquad (\Psi'_R = \xi\Psi_R),$$

$$\Rightarrow$$



$$\propto \left(\frac{1}{2} - Q_u\sin^2\theta_W - \frac{v^2}{8f^2}\right).$$

Flavor changing processes in LHT

We studied:

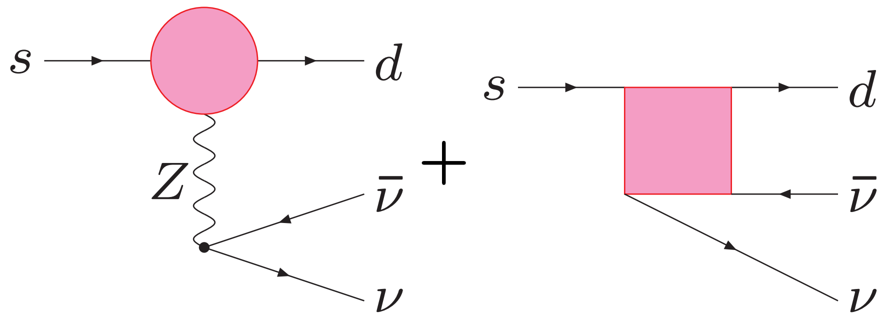
- $K \to \pi \, \nu \, \overline{\nu}$ [PLB670(2009)378],
- Muon LFVs ($\mu \to e \, \gamma$, $\mu \to e \, e \, e$, $\mu \to e$ conversion) [preliminary].

FCNC in LHT: $K \to \pi \nu \bar{\nu}$ ($s \to d \nu \bar{\nu}$)

Low energy effective Lagrangian:

$$\mathcal{L}^{\text{eff}}_{[d\nu]} = C^{ijlm}_{[d\nu]LL} (\overline{d}^i \gamma^\mu d^j_L)(\overline{\nu}^l \gamma_\mu \nu^m_L), \qquad i \neq j.$$

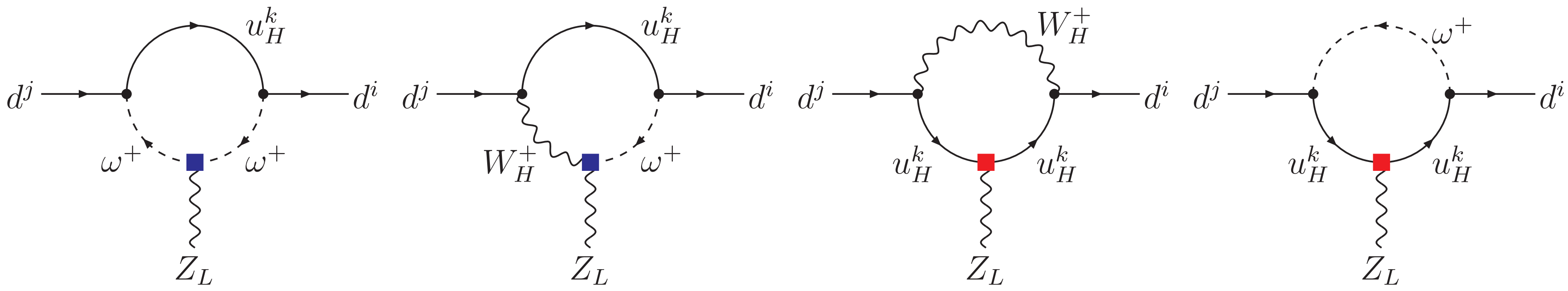
- Right-handed $d^j \to d^i$ current is suppressed in LHT (as in SM).
- Leading order = one loop (no tree level contributions).



- In 't Hooft–Feynman gauge, box terms are manifestly finite.
- T-even sector = SM $\oplus$ $SU(2)$ singlet vector-like top partner T_+.
  - ▷ $C^{ijkl}_{[d\nu]LL}$(T-even) $\propto \lambda_t^{(ij)} = (V^*_{\text{CKM}})_{ti}(V_{\text{CKM}})_{tj}$.

T-odd loops: Z penguins

- Zeroth order terms in $\frac{v^2}{f^2}$ expansion vanishes.
- Relevant diagrams for leading $O(\frac{v^2}{f^2})$ contributions:



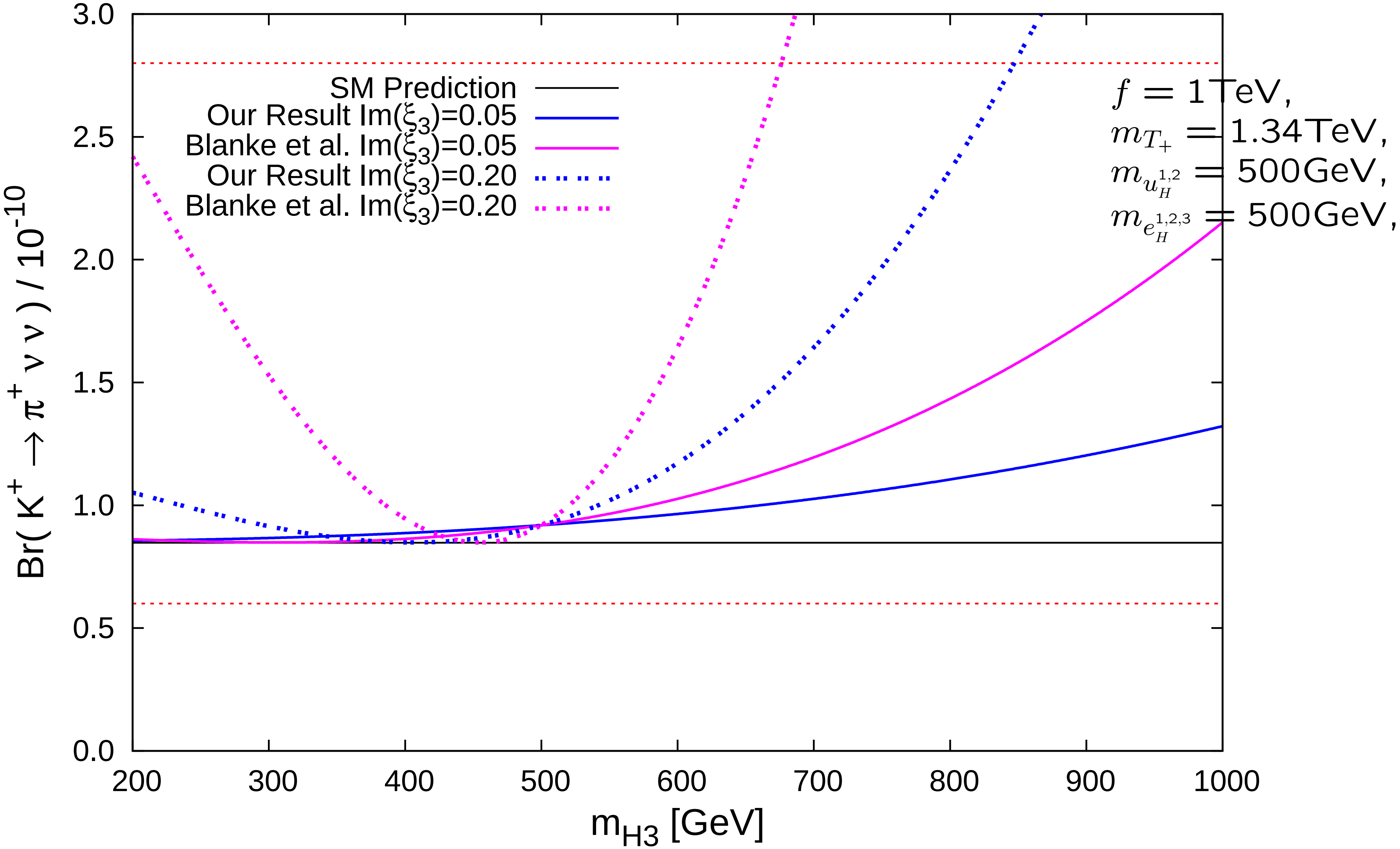
- UV divergences cancel in the sum $\Rightarrow$ finite amplitude obtained.

$$C^{ijlm}_{[d\nu]LL}(\text{T-odd}) = -\frac{g^4}{(4\pi)^2 m^2_{W_L}} \sum_{k,n=1}^{3} \lambda_k^{Hd(ij)} \lambda_n^{H\nu(lm)} \times \frac{v^2}{16 f^2}\left[z_k F_9(z_k) - G^{\square\text{odd}}_{[du]}(z_k, y_n, r)\right],$$

$$z_k = m^2_{q_H^k}/m^2_{W_H}, \quad y_n = m^2_{\ell_H^n}/m^2_{W_H}, \quad y = m^2_{A_H}/m^2_{Z_H}.$$

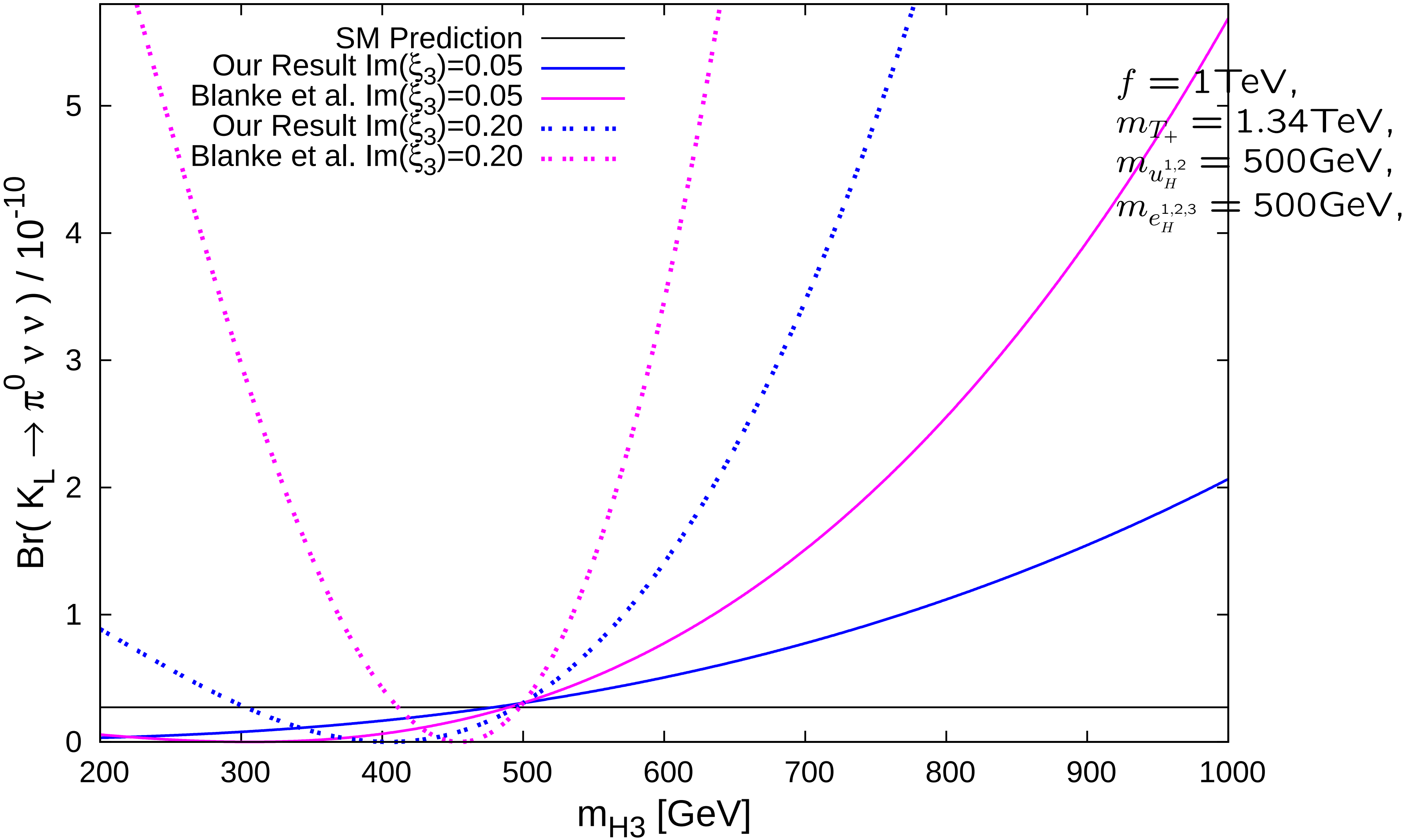
$$\lambda_k^{Hd(ij)} = (V^*_{Hd})_{ki}(V_{Hd})_{kj}, \quad \lambda_n^{H\nu(lm)} = (V^*_{H\nu})_{nl}(V_{H\nu})_{nm}.$$

$\mathbf{B}(K^+ \to \pi^+ \nu \bar{\nu}) \qquad \mathbf{B}(\exp) = (1.5^{+1.3}_{-0.9}) \times 10^{-10}$



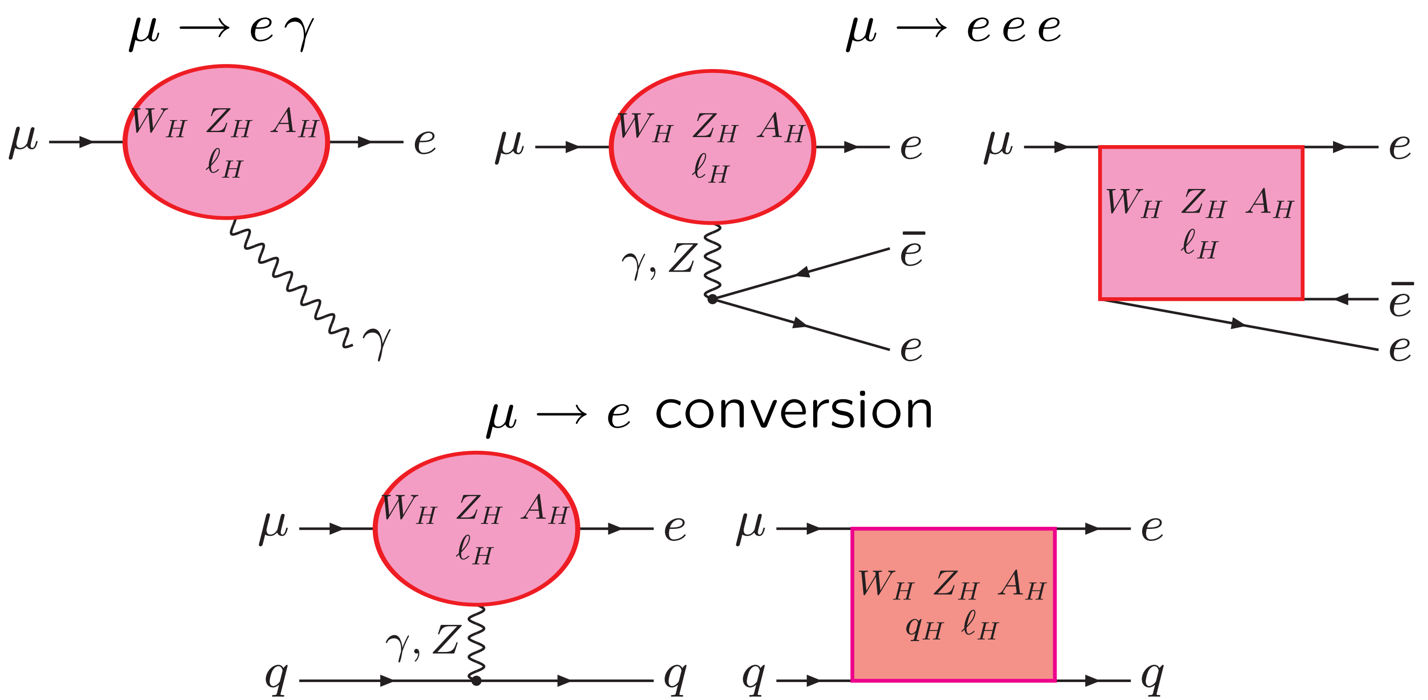
$\xi_3 = (V_{Hd})_{31}(V_{Hd})^*_{32}$. $\mathrm{Re}[(V_{Hd})^*_{31}(V_{Hd})_{32}] = 0$ (to suppress T-odd contrib. to ε_K).

$\mathbf{B}(K_L \to \pi^0 \nu \bar{\nu}) \qquad \mathbf{B}(\text{exp}) < 2.1 \times 10^{-7}$



$\xi_3 = (V_{Hd})_{31}(V_{Hd})^*_{32}$. $\text{Re}[(V_{Hd})^*_{31}(V_{Hd})_{32}] = 0$ (to suppress T-odd contrib. to ε_K).

μLFV: $\mu \to e\,\gamma$, $\mu \to e\,e\,e$ and $\mu \to e$ conversion

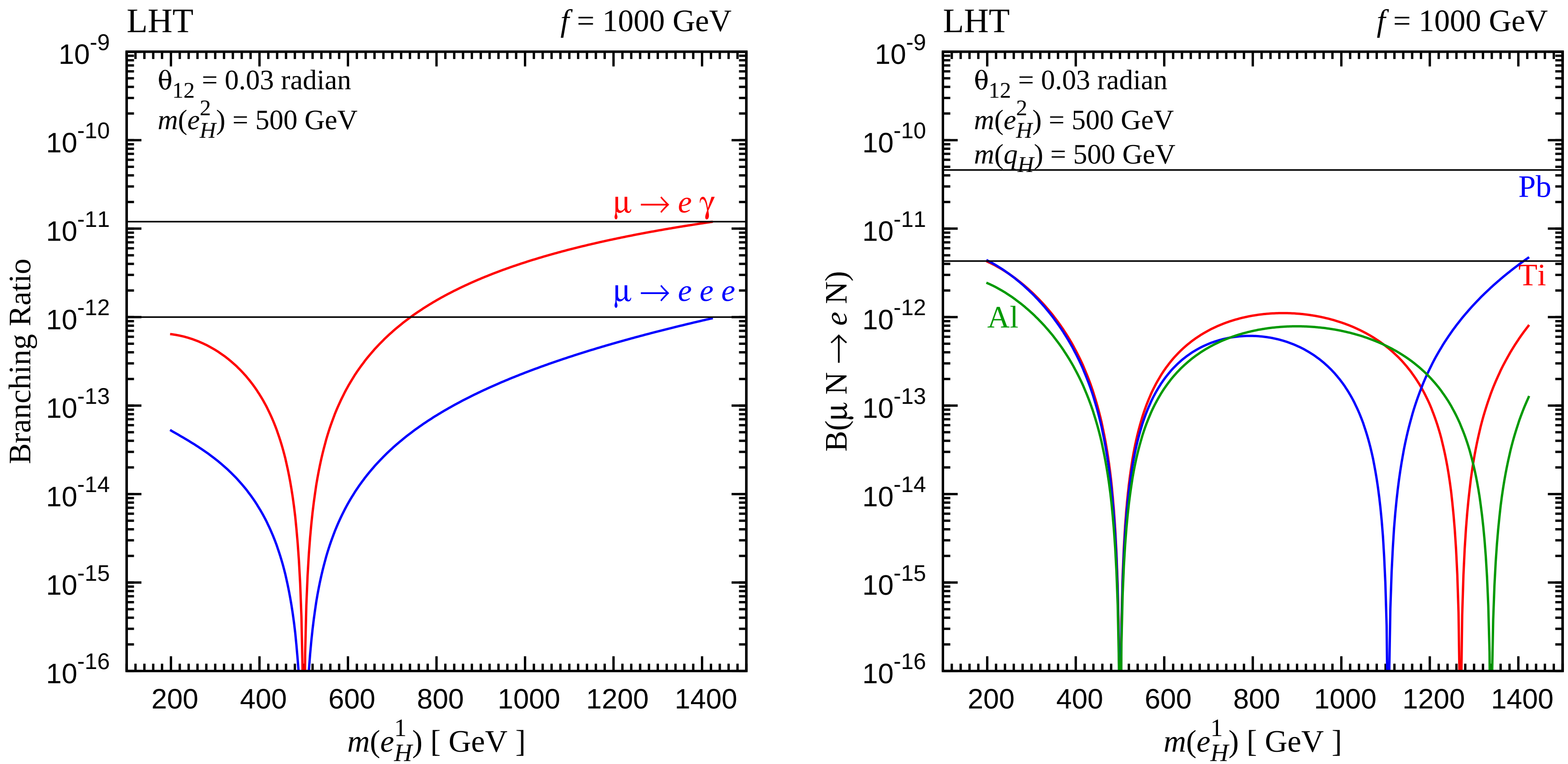


- T-odd loops only.
- Calculated in the same way as quark FCNC: $u \Leftrightarrow \nu$, $d \Leftrightarrow e$.

See also: del Águila, Illana & Jenkins, JHEP0901(2009)080;

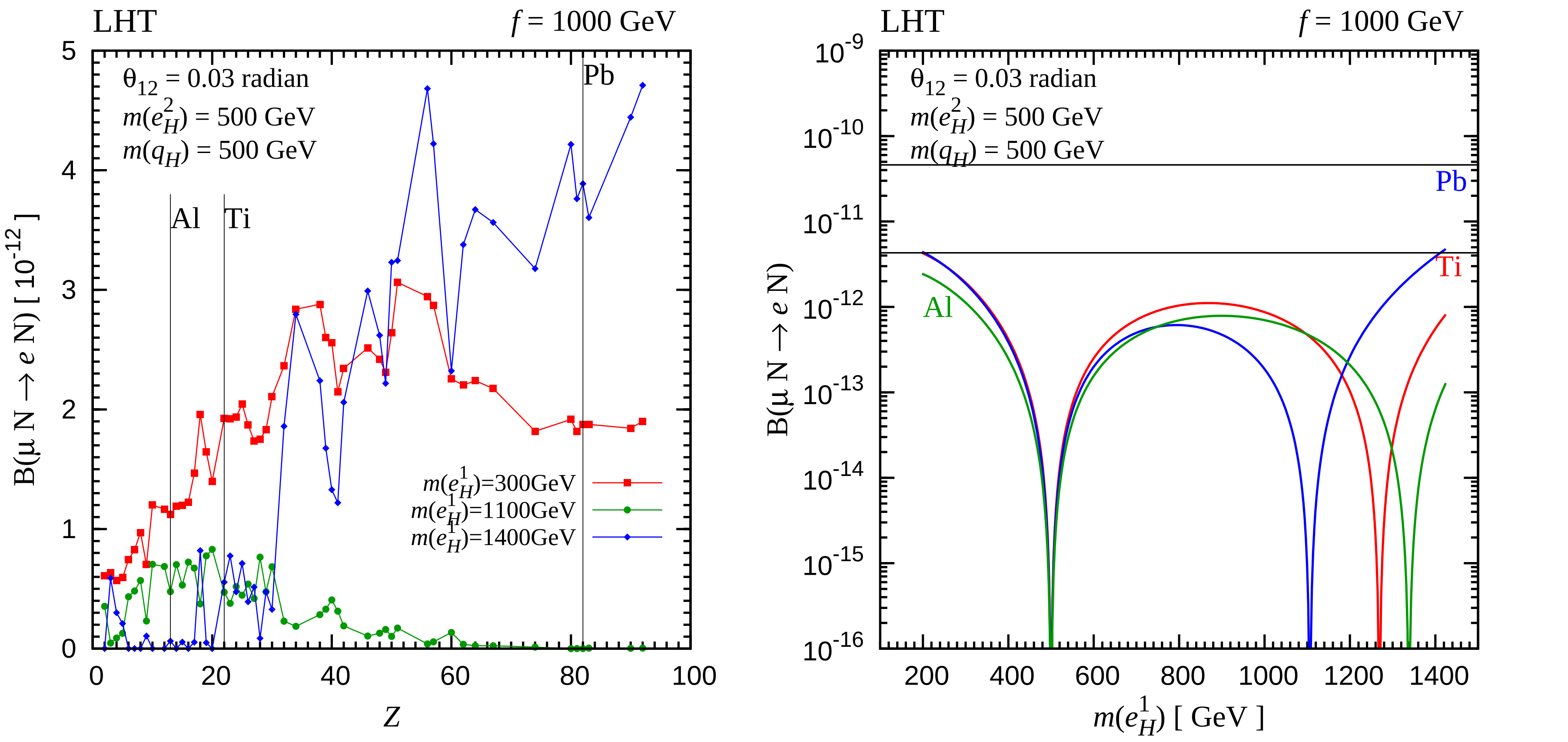
Blanke *et al.*, JHEP05(2007)013.

μLFV: $\mu \to e\,\gamma$, $\mu \to e\,e\,e$ and $\mu \to e$ conversion



- Assumptions: $\theta_{i3} = \delta_{ij} = 0$ in V_{He}; q_H degenerate in mass.
- Matrix elements for $\mu \to e$ conv. taken from Kitano, Koike and Okada, PRD66(2002)096002.

μLFV: $\mu \to e$ conversion



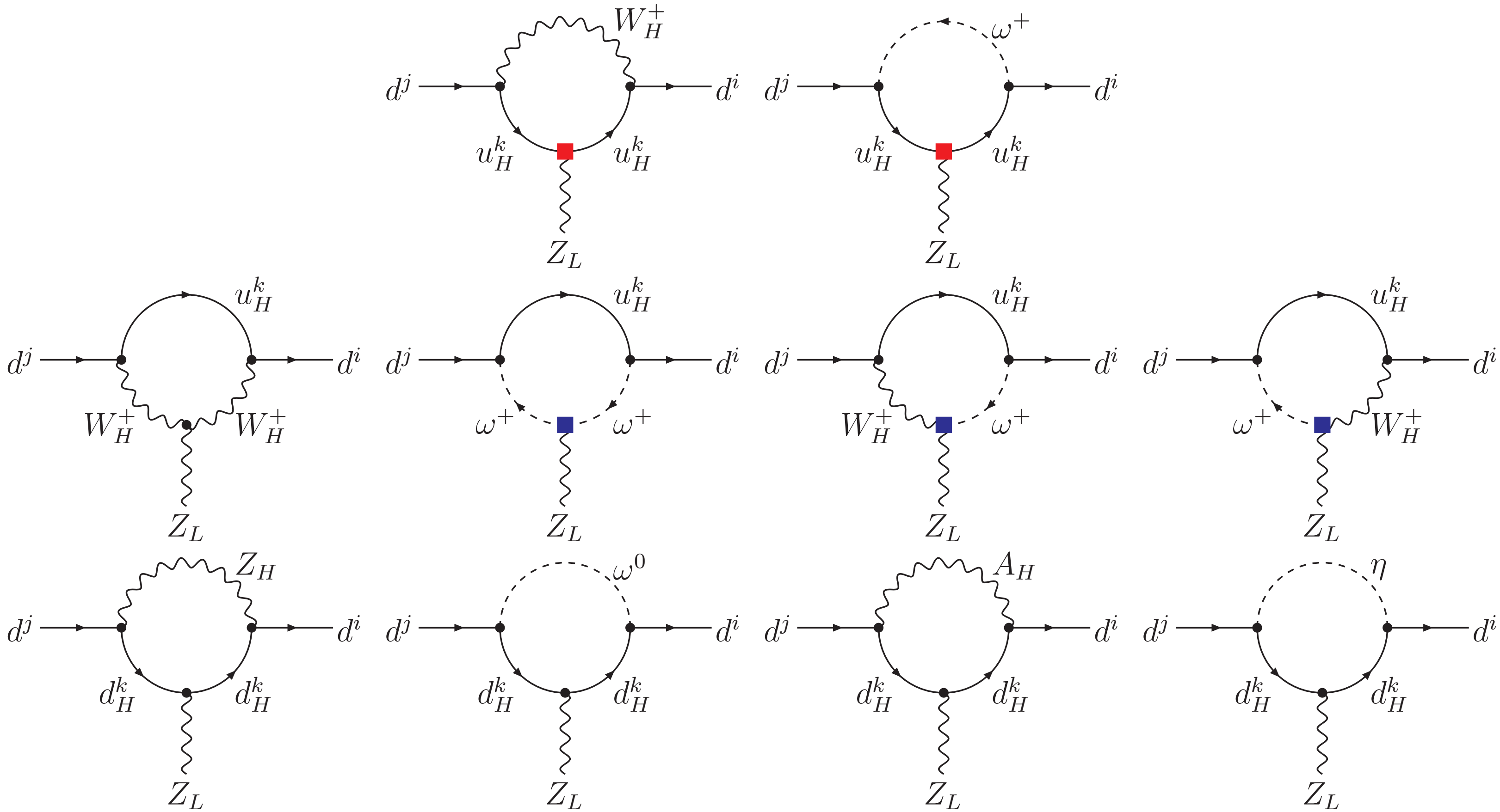
- B=conversion rate/capture rate depends on nucleus.
- Cancelation among amplitudes possible.
- Experiments aiming 10^{-16} planned (Mu2e@FNAL, COMET@J-PARC).

Conclusion

- FCNC and LFV processes in the Littlest Higgs model with T-parity are (re-)studied.
- One loop $d^i \to d^j$ and $e^i \to e^j$ amplitudes are UV finite at $O(\frac{v^2}{f^2})$.
- Branching fractions of $K \to \pi\,\nu\,\overline{\nu}$ decays may differ from SM predictions significantly.
- $\mu \to e\,\gamma$, $\mu \to e\,e\,e$ and $\mu \to e$ conversion can be measurable in near future experiments.

Backups

Z penguins: T-odd loops



Calculated in $\frac{v^2}{f^2}$ expansion.

- ■ and ■: $O(\frac{v^2}{f^2})$ corrections.
- zeroth order vanishes.

Loop functions

$$
\begin{aligned}
F_9(x) &= \frac{x-6}{2(x-1)} + \frac{(3x+2)\log x}{2(x-1)^2}, \\
G^{\square\text{odd}}_{[dd]}(x,y,r) &= G^{\square}_{[dd]}(x,y) - \frac{3}{4}\left[\mathbf{g}_{2[1]}(x,y) + \frac{r}{25}\mathbf{g}_{2[1]}(\frac{x}{r},\frac{y}{r}) + \frac{2r}{5}\mathbf{g}_2(x,y,r)\right], \\
G^{\square\text{odd}}_{[du]}(x,y,r) &= G^{\square}_{[du]}(x,y) + \frac{3}{4}\left[\mathbf{g}_{2[1]}(x,y) + \frac{r}{25}\mathbf{g}_{2[1]}(\frac{x}{r},\frac{y}{r}) - \frac{2r}{5}\mathbf{g}_2(x,y,r)\right]. \\
G^{\square}_{[dd]}(x,y) &= \left(1+\frac{xy}{4}\right)\mathbf{g}_{2[1]}(x,y) - 2xy\mathbf{g}_{1[1]}(x,y), \\
G^{\square}_{[du]}(x,y) &= \left(4+\frac{xy}{4}\right)\mathbf{g}_{2[1]}(x,y) - 2xy\mathbf{g}_{1[1]}(x,y), \\
\mathbf{g}_2(x,y,z) &= \frac{x^2\log x}{(x-1)(x-y)(x-z)} + (\text{cycl. perm.}), \\
\mathbf{g}_{2[1]}(x,y) &= \mathbf{g}_2(x,y,1).
\end{aligned}
$$

Result of T-even loops for $d^j \to d^i \nu \bar{\nu}$

$$
\begin{aligned}
C^{ijlm}_{[d\nu]LL}(\text{T-even}) &= -\frac{g^4}{(4\pi)^2 m_{W_L}^2}\delta^{lm}\left[\sum_{k=c,t}\lambda_k^{d(ij)} X_{\text{SM}}(x_k) + \lambda_t^{d(ij)}\bar{X}_{\text{even}}\right], \\
X_{\text{SM}}(x) &= \frac{x}{8}\left[\frac{(3x-6)\log x}{(x-1)^2} + \frac{x+2}{x-1}\right], \\
\bar{X}_{\text{even}} &= s_L^2\left[X_{\text{SM}}(x_{T_+}) - X_{\text{SM}}(x_t)\right] \\
&\quad + \frac{s_L^2 c_L^2}{4}\left[-\frac{x_t + x_{T_+}}{2} + \frac{x_t x_{T_+}(\log x_t - \log x_{T_+})}{x_t - x_{T_+}}\right].
\end{aligned}
$$

Numerical inputs

LHT has 20 new parameters:

- f, m_{T_+},
- $m_{u_H^k}$, $m_{e_H^k}$,
- 3 angles + 3 phases in V_{Hd}, 3 angles + 3 phases in $V_{H\nu}$.

In the $K \to \pi \nu \overline{\nu}$ plots, we fix

- $f = 1$TeV, $m_{T_+} = 1.34$TeV,
- $m_{u_H^{1,2}} = m_{e_H^{1,2,3}} = 500$GeV,
- $\mathrm{Re}[(V_{Hd})^*_{31}(V_{Hd})_{32}] = 0$ (to suppress T-odd contribution to ε_K).

$\Rightarrow$ free inputs: $m_{u_H^3}$ and $\mathrm{Im}[(V_{Hd})^*_{31}(V_{Hd})_{32}]$.

In the μLFV plots, we fix

- $f = 1$TeV, ($m_{T_+} = 1.34$TeV),
- $m_{e_H^{2,3}} = m_{q_H^{1,2,3}} = 500$GeV,
- $\theta_{23} = \theta_{13} = \delta_{ij} = 0$ for V_{He}.

$\Rightarrow$ free inputs: $m_{e_H^1}$ and θ_{12} in V_{He}.