Inclusive semileptonic B decays

Paolo Gambino
Università di Torino

Our world is full of tensions

Outline

- Inclusive semileptonic decays, moments, fits, V_{cb}
- Do not use the word *precision* in vain
  how reliable are the mass determinations?
- Better not to forget unitarity
  a remark on heavy quark sum rules
- The inclusive V_{ub} determination
- Love your fellow theorists like yourself
  comparison of various approaches
- Conclusions

Inclusive semileptonic B decays: basic features

- ***Simple idea:*** inclusive decay do not depend on final state, factorize long distance dynamics of the meson. OPE allows to express it in terms of matrix elements of local operators

- The Wilson coefficients are perturbative, matrix elements of local ops parameterize non-pert physics: ***double series in α_s, Λ/m_b***

- Lowest order: decay of a free *b*, linear Λ/m_b absent. Depends on $m_{b,c}$, 2 parameters at $O(1/m_b^2)$, 2 more at $O(1/m_b^3)$...

$$\mu_\pi^2(\mu) = \frac{1}{2M_B}\left\langle B\left|\bar{b}\left(i\vec{D}\right)^2 b\right|B\right\rangle_\mu \qquad \mu_G^2(\mu) = \frac{1}{2M_B}\left\langle B\left|\bar{b}\frac{i}{2}\sigma_{\mu\nu}G^{\mu\nu}b\right|B\right\rangle_\mu$$

The total s.l. width in the OPE

$$\Gamma[\bar{B} \to X_c e \bar{\nu}] = \frac{G_F^2 m_b^5}{192\pi^3}|V_{cb}|^2 g(r)\left[1 + \frac{\alpha_s}{\pi} p_c^{(1)}(r,\mu) + \frac{\alpha_s^2}{\pi^2} p_c^{(2)}(r,\mu) - \frac{\mu_\pi^2}{2m_b^2} + \left(\frac{1}{2} - \frac{2(1-r)^4}{g(r)}\right)\frac{\mu_G^2 - \frac{\rho_{LS}^3 + \rho_D^3}{m_b}}{m_b^2} + \left(8\ln r - \frac{10r^4}{3} + \frac{32r^3}{3} - 8r^2 - \frac{32r}{3} + \frac{34}{3}\right)\frac{\rho_D^3}{g(r)\, m_b^3}\right]$$

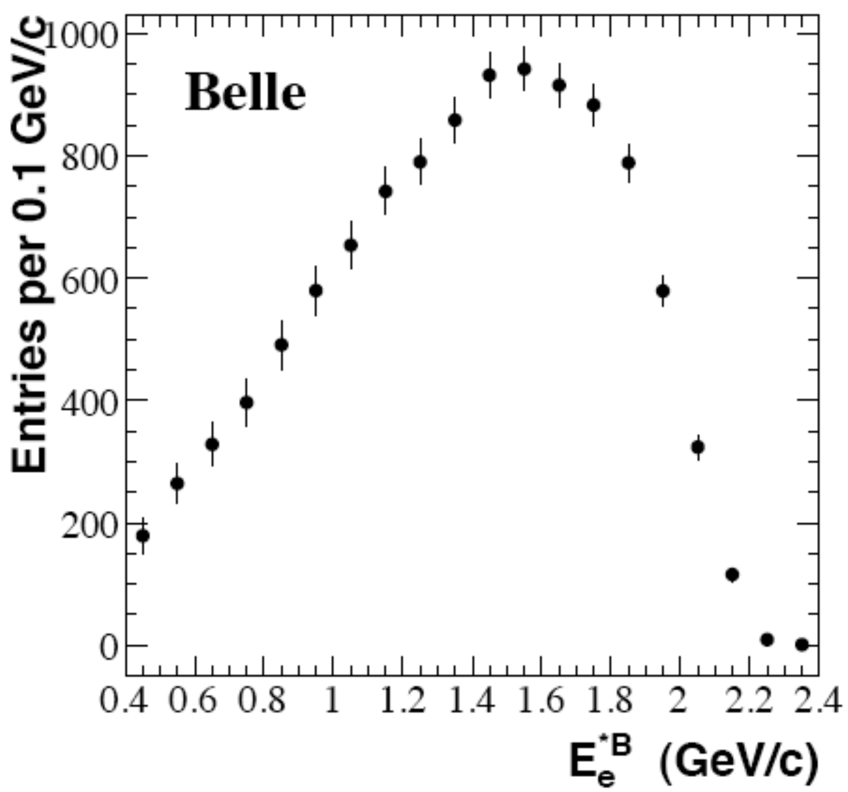
$$+ O\left(\alpha_s \frac{\mu_{\pi,G}^2}{m_b^2}\right) \quad + O\left(\frac{1}{m_b^4}\right)$$

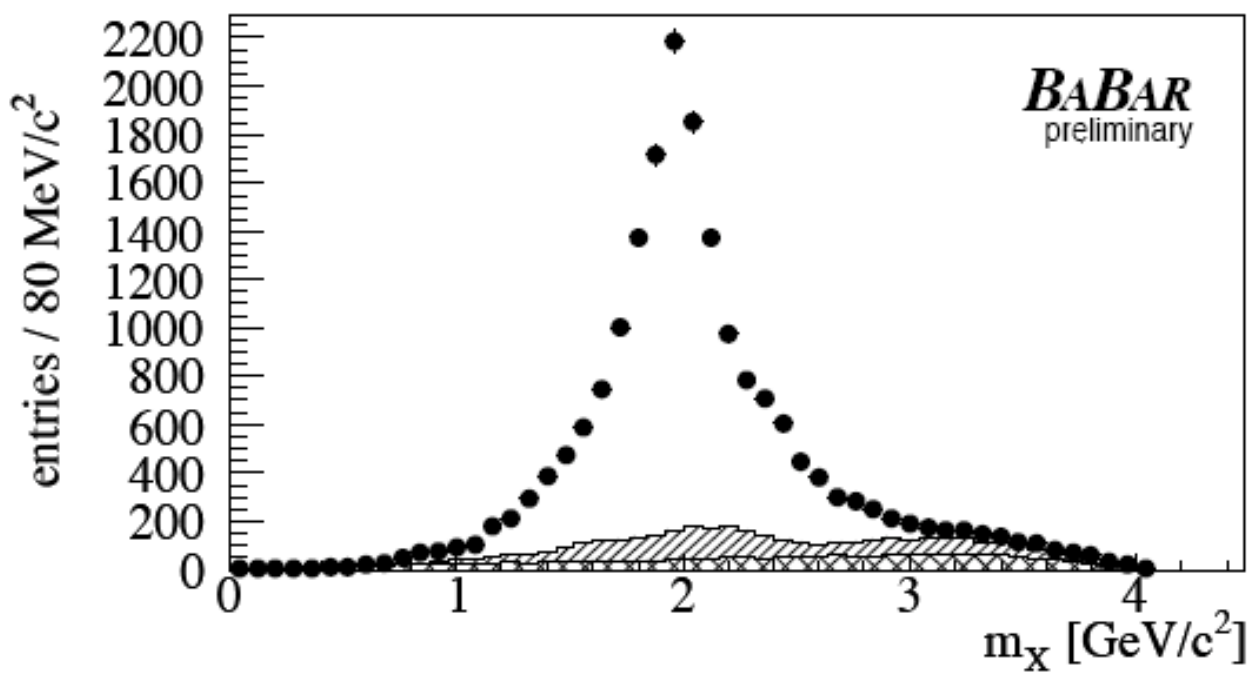
$$r = \frac{m_c^2}{m_b^2}$$

OPE valid for inclusive enough measurements, away from perturbative singularities ➠ moments

At present the implementations for moments include $O(\alpha_s^2\beta_0, 1/m_b^3)$ terms

Fitting OPE parameters to the moments





Total ***rate*** *gives $|V_{cb}|$, global* ***shape*** *parameters (moments of the distributions) tell us about B structure, m_b and m_c*

OPE parameters describe universal properties of the B meson and of the quarks

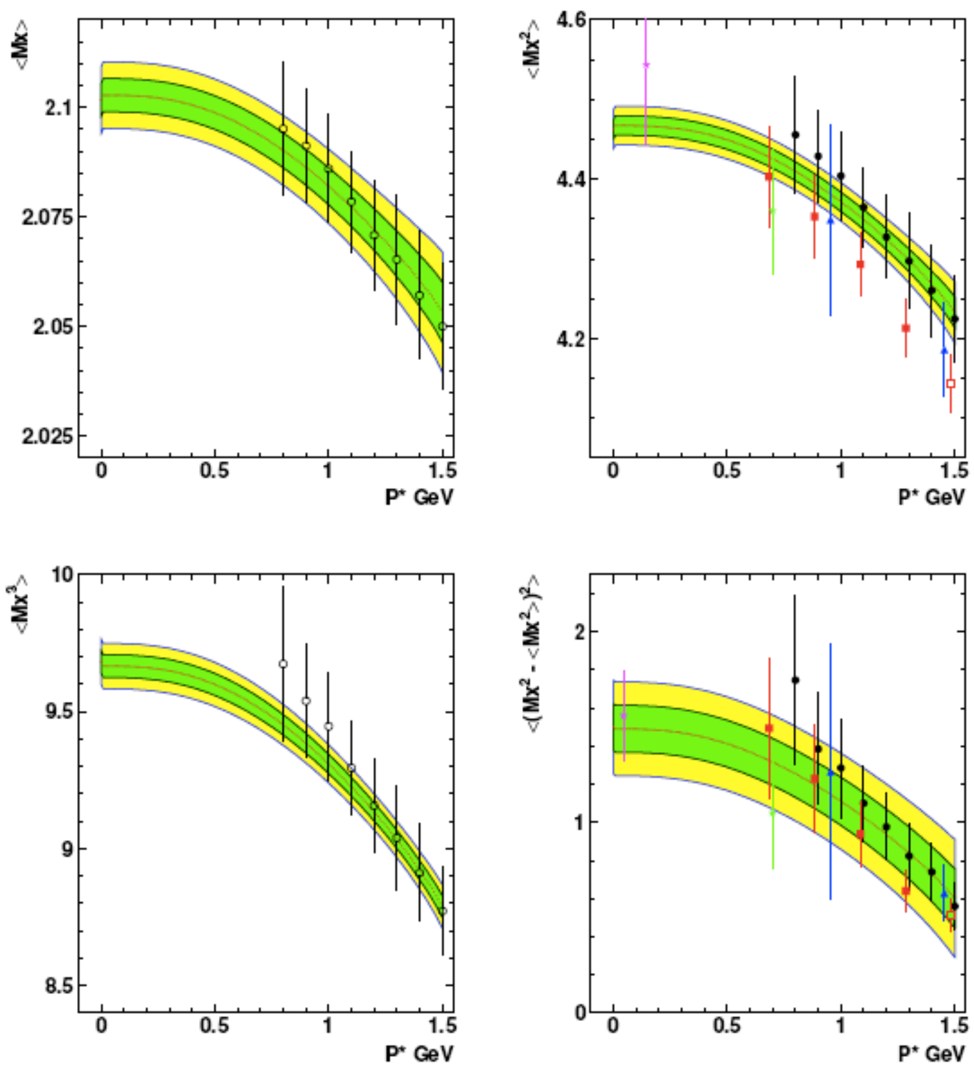
Global fit (kinetic scheme)

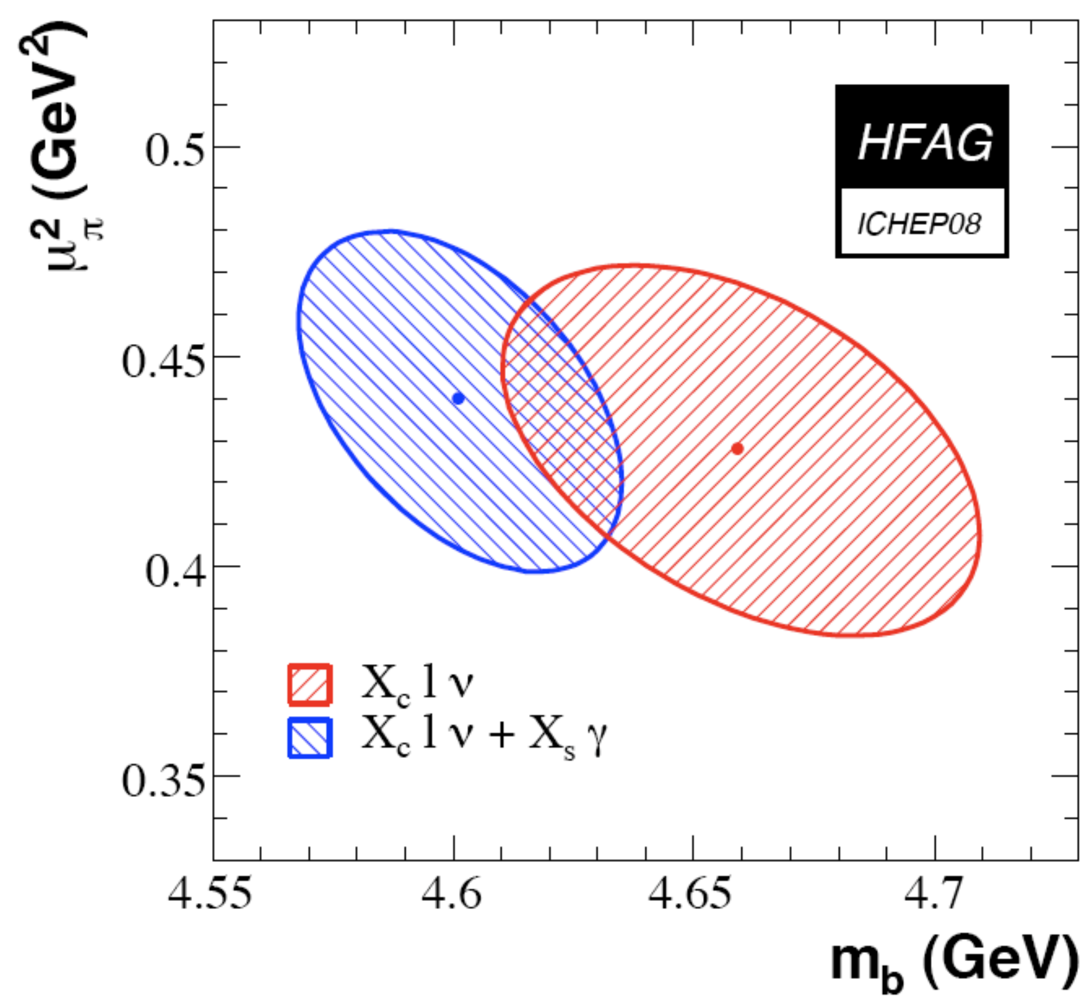
Inputs	$\lvert V_{cb}\rvert\ 10^3$	m_b^{kin}	χ^2/ndf
b→c & b→sγ	41.67(44)(58)	4.601(34)	29.7/57
b→c only	41.48(48)(58)	4.659(49)	24.1/46

Based on PG, Uraltsev, Benson et al

In the kinetic scheme *the contributions of gluons with energy below $\mu\approx 1$GeV are absorbed in the OPE parameters*

Here *scheme* means also a number of different assumptions inclusion of different data,and a recipe for theory errors



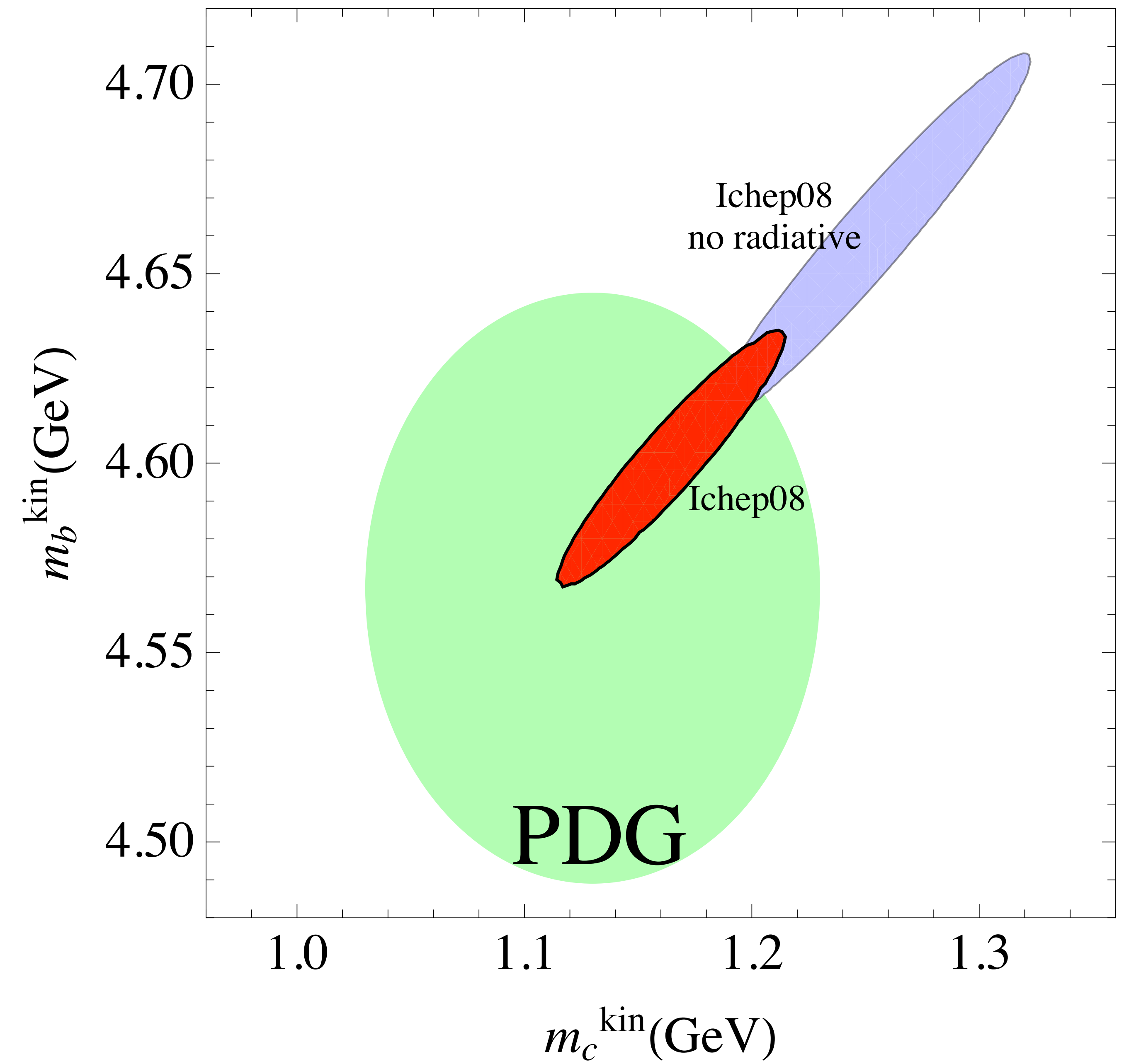


Fits & Quark Masses

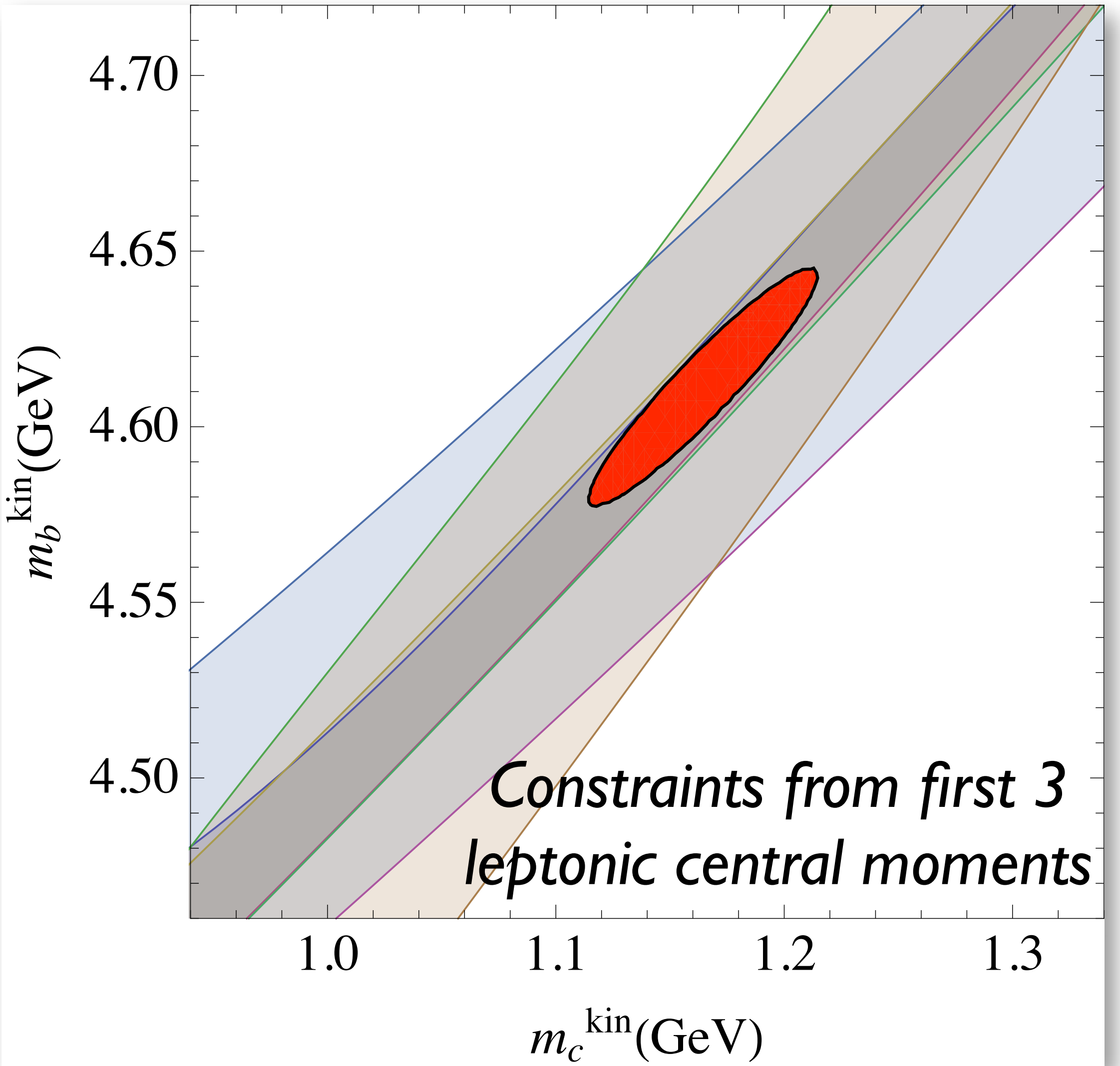
- *Assumes duality but it self-consistently checks it*
- *Very close result for $|V_{cb}|$ in 1S scheme* *(Bauer et al)*

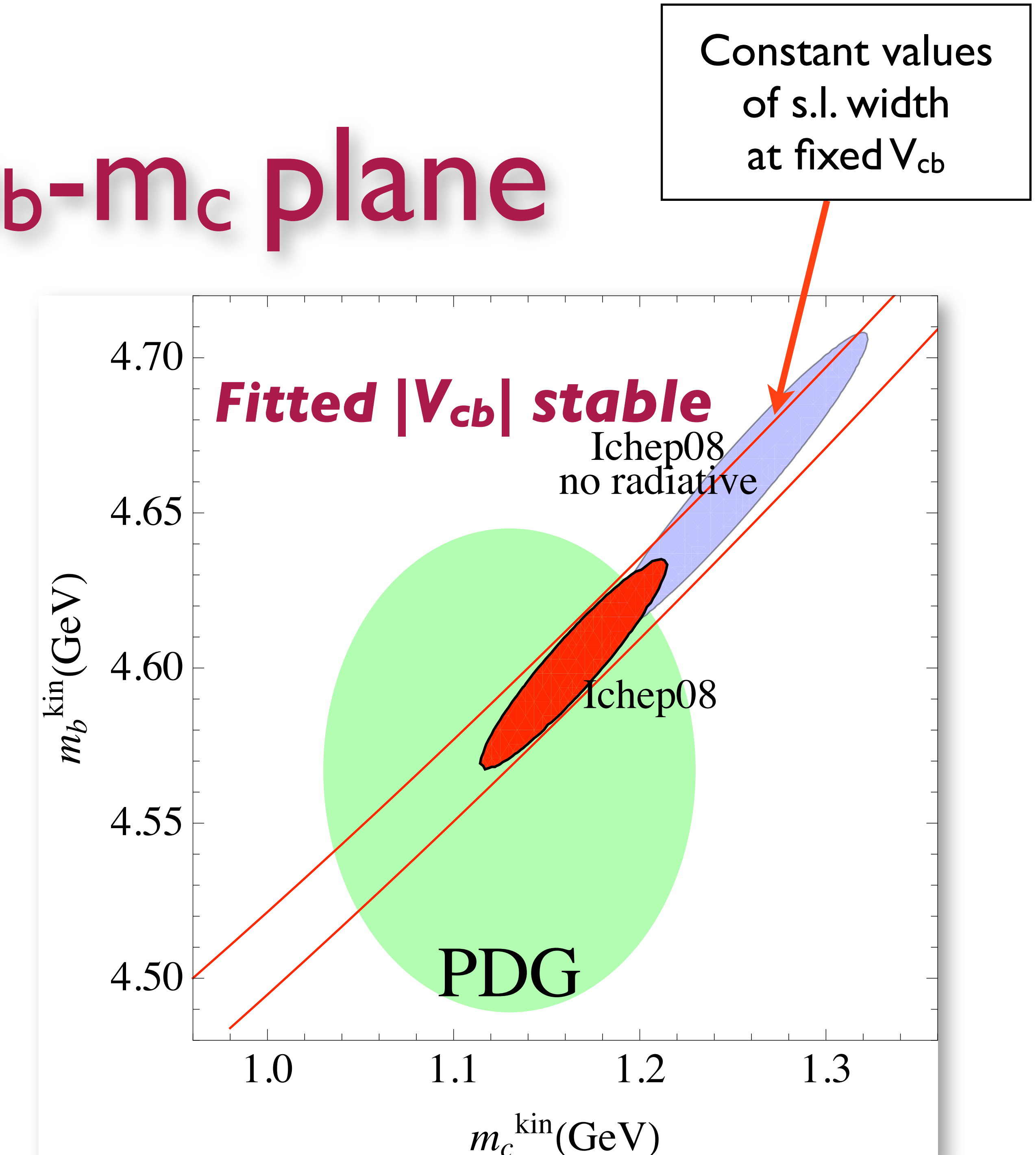
{
- *Higher order power corr. under control* *Mannel et al*
- *New pert $O(\alpha_s^2)$ ⇒* ***-0.5% in $|V_{cb}|$***
  Melnikov, Czarnecki, Pak
- *Part of $O(\alpha_s/m_b^2)$* *Becher et al*

- *New calculations give generally small contributions, will be included*
- *In the global HFAG fit the $B\to X_s\gamma$ moments* ***change significantly $m_{b,c}$*** *determination. Without radiative moments the masses are too high!*



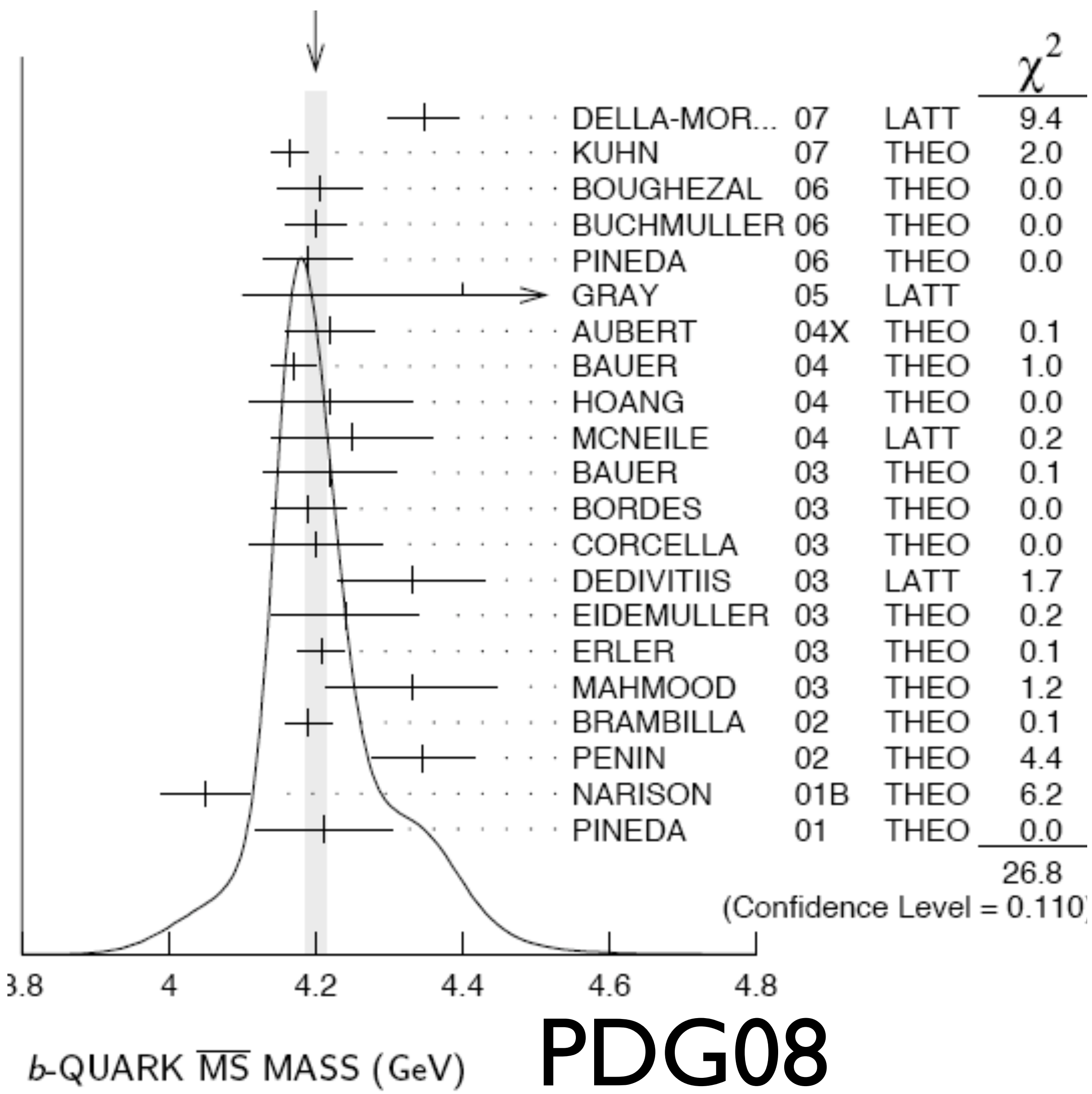
A strip in the m_b-m_c plane



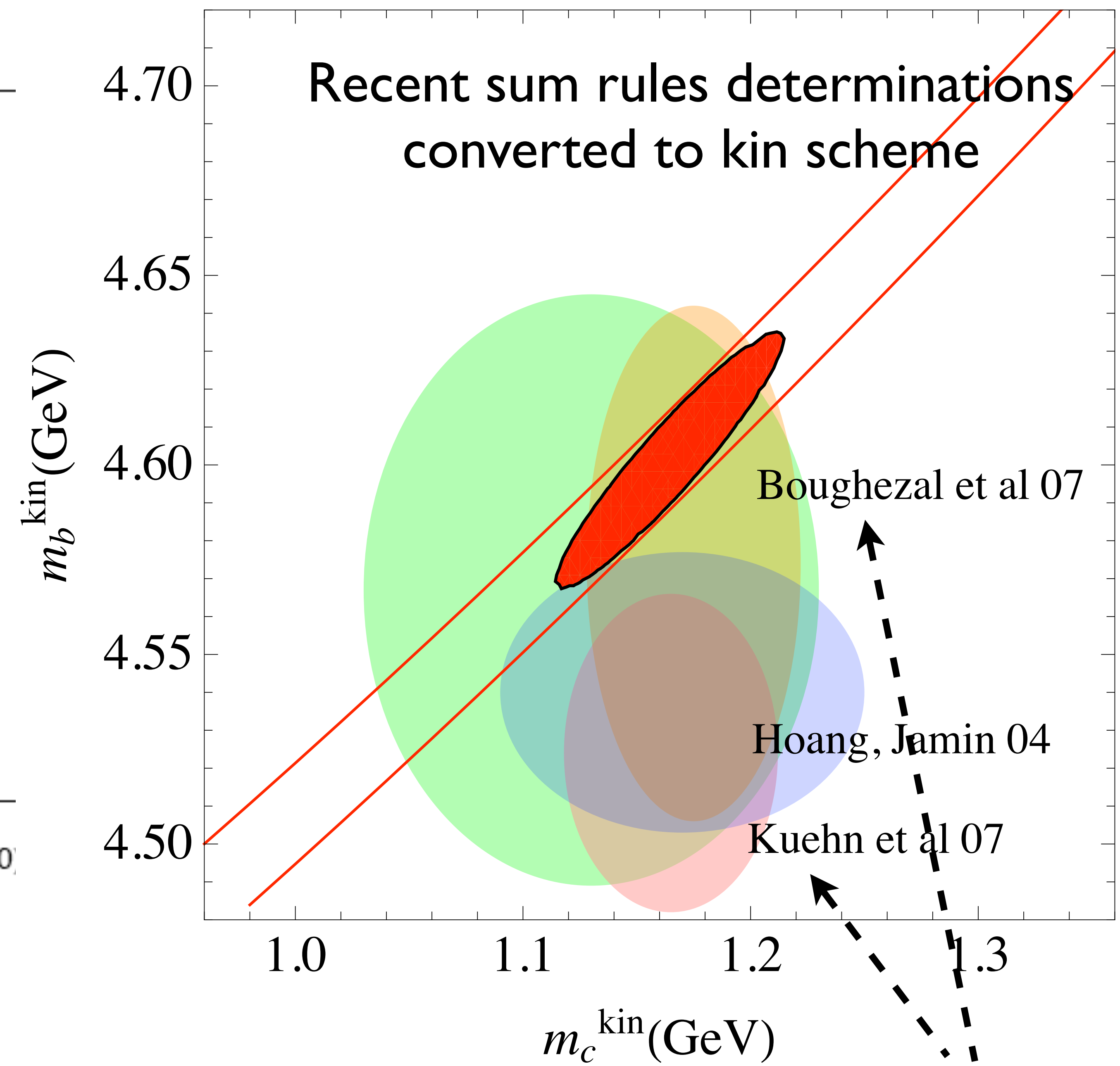


- *Semileptonic moments identify a strip in (m_b,m_c) plane along which the minimum is **shallow**.*
- *V_{ub} inclusive studies require m_b, μ_π, etc with correlations*
- *m_b,m_c and OPE parameters necessary for BR($B \to X_s \gamma$) etc*

Mass determinations



PDG08

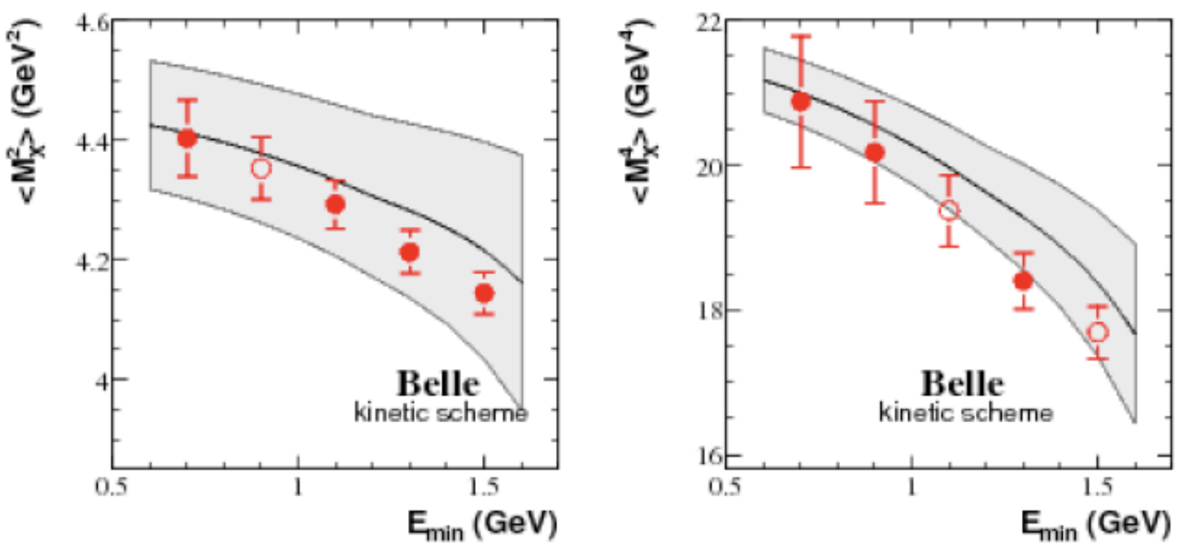


neglecting correlations between $m_{c,b}$ (α_s)

How reliable are mass determinations?

In collaboration with C. Schwanda

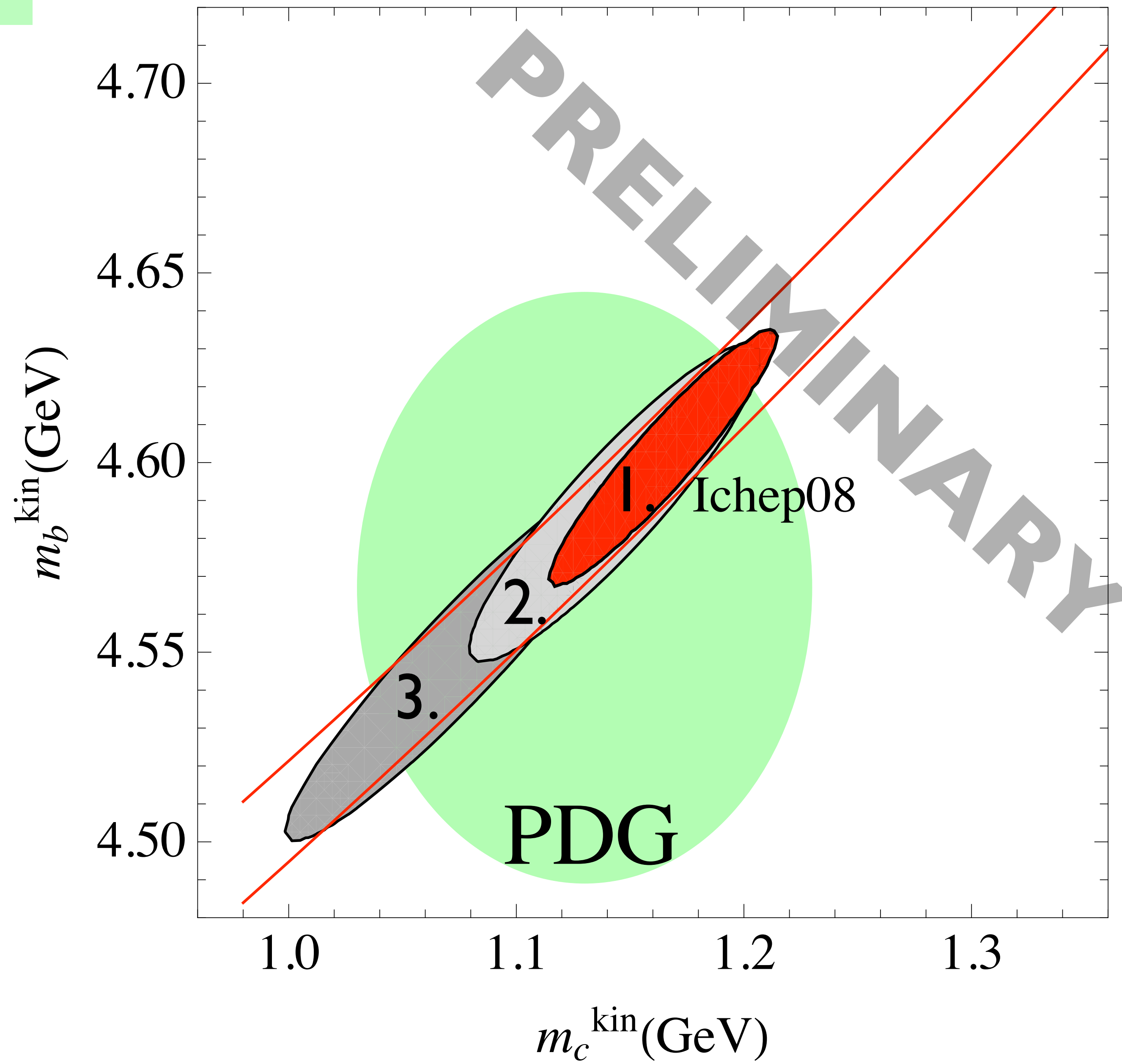
1. Theoretical correlations



Correlations between theory errors of moments with different cuts difficult to estimate. Examples:

1. 100% correlations
2. corr. computed from low-order
3. experimental correlations

always assume different central moments uncorrelated



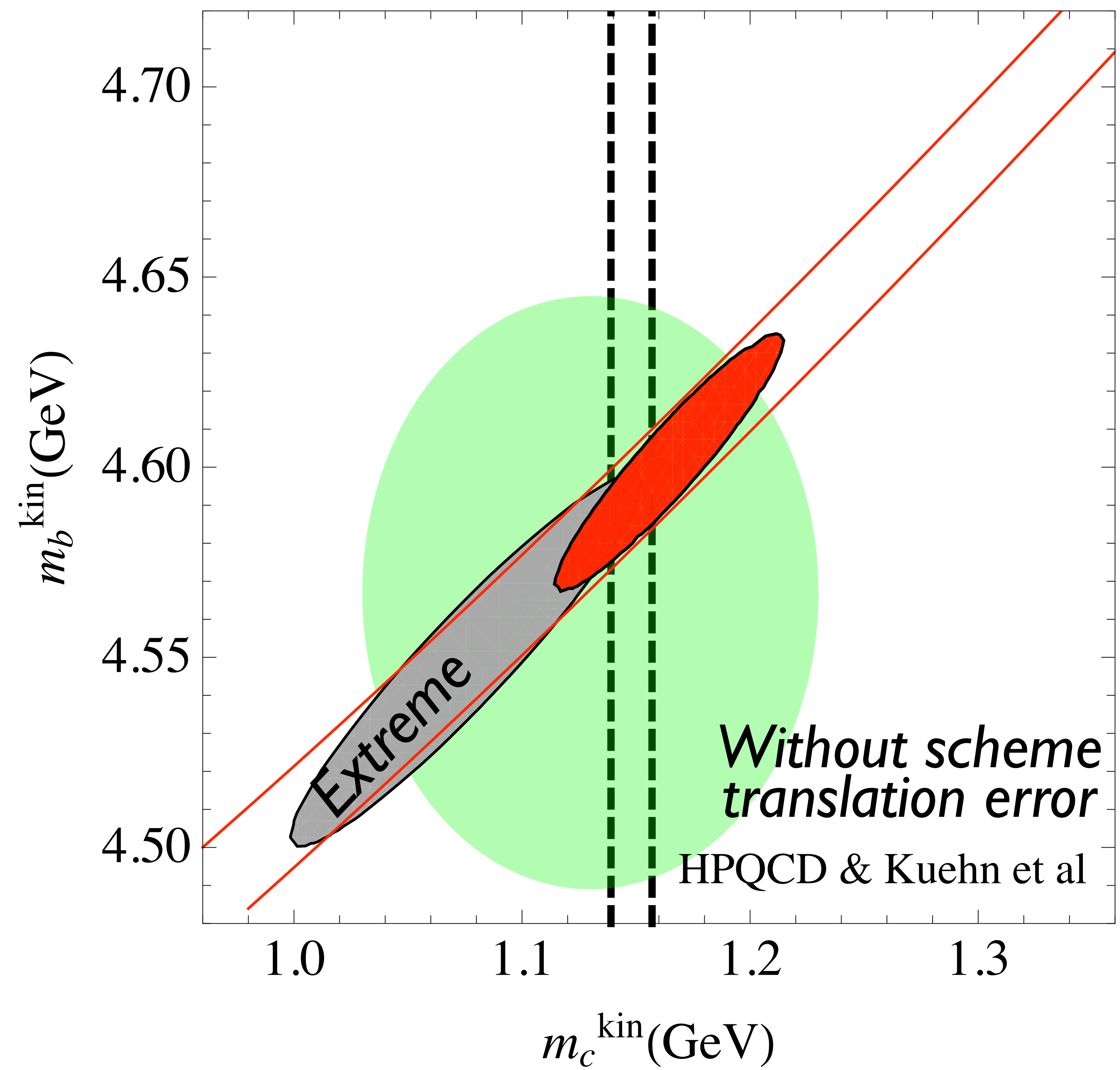
2. How important are radiative moments?
3. Can we include other constraints?

OPE fails for bsγ, but only at $O(\alpha_s)$ with operators $\neq O_7$. Unlikely to be relevant for normalized moments, but it must be studied

At the moment the role of radiative moments in the fits is almost ***identical*** *to using PDG07 bound $m_b(m_b)$=4.20(7)GeV*

Though bsγ are important as independent checks, the inclusion of additional constraints is in principle very useful. But which ones?

Fits to m_c(2-3GeV) coming soon: no scheme translation error

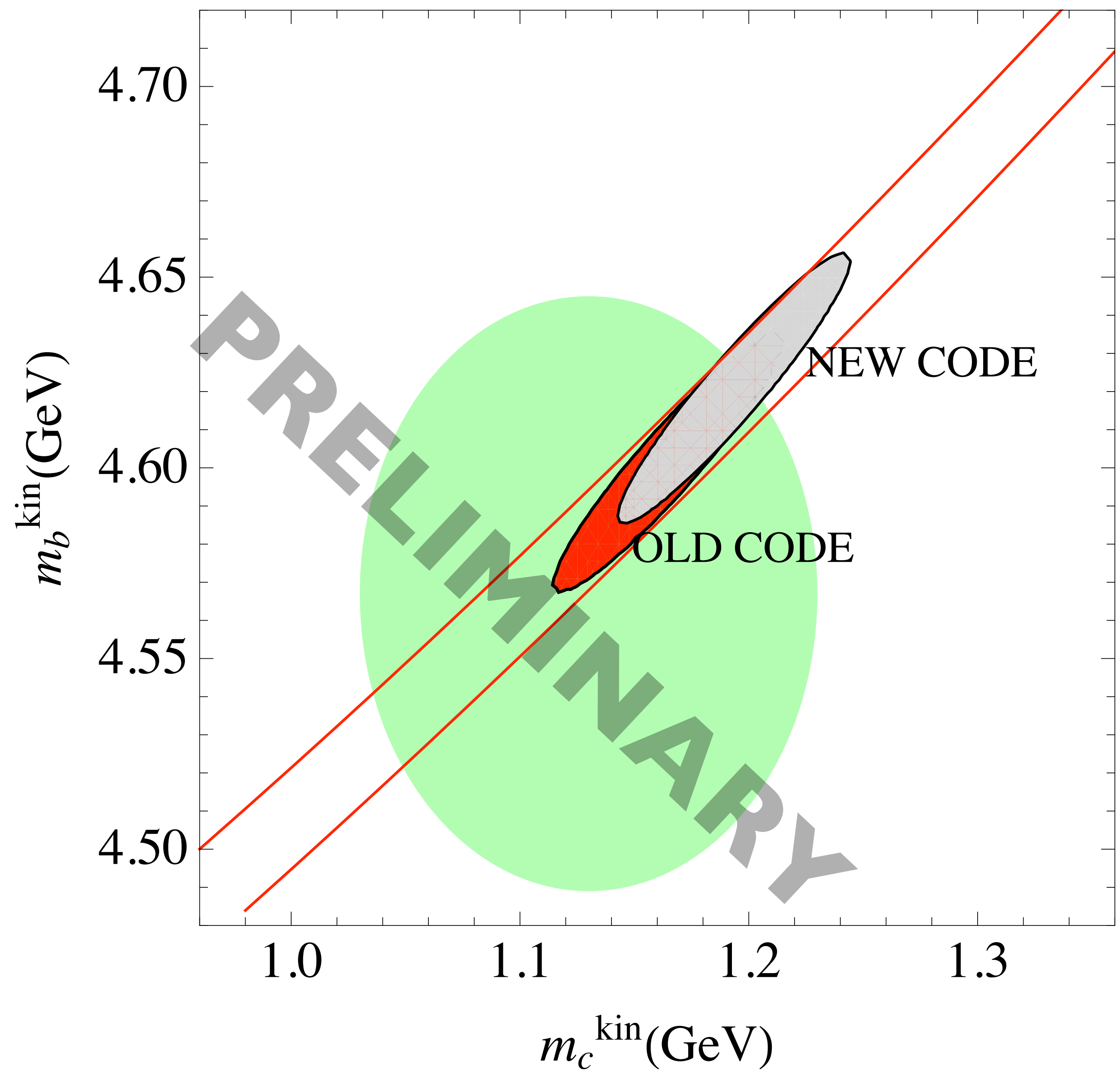


4. Fitting properly radiative moments

Since the bsγ moments are measured with a relatively high cut on E_γ, a purely local OPE is insufficient.

SF can be implemented at NLO + BLM (Benson et al) but depends on m_b, μ^2_π, ...

For the first time the fit is performed with the full parameter dependence.



Exclusive decays: $B \to D^* l \nu$

At zero recoil, where rate vanishes, the ff is

$$\mathcal{F}(1) = \eta_A (1 + \delta_{1/m^2})$$

Recent progress in the measurement of slopes and shape parameters Despite extrapolation, exp error ~2%

Main problem is normalization F(1): ***requires non-perturbative methods***

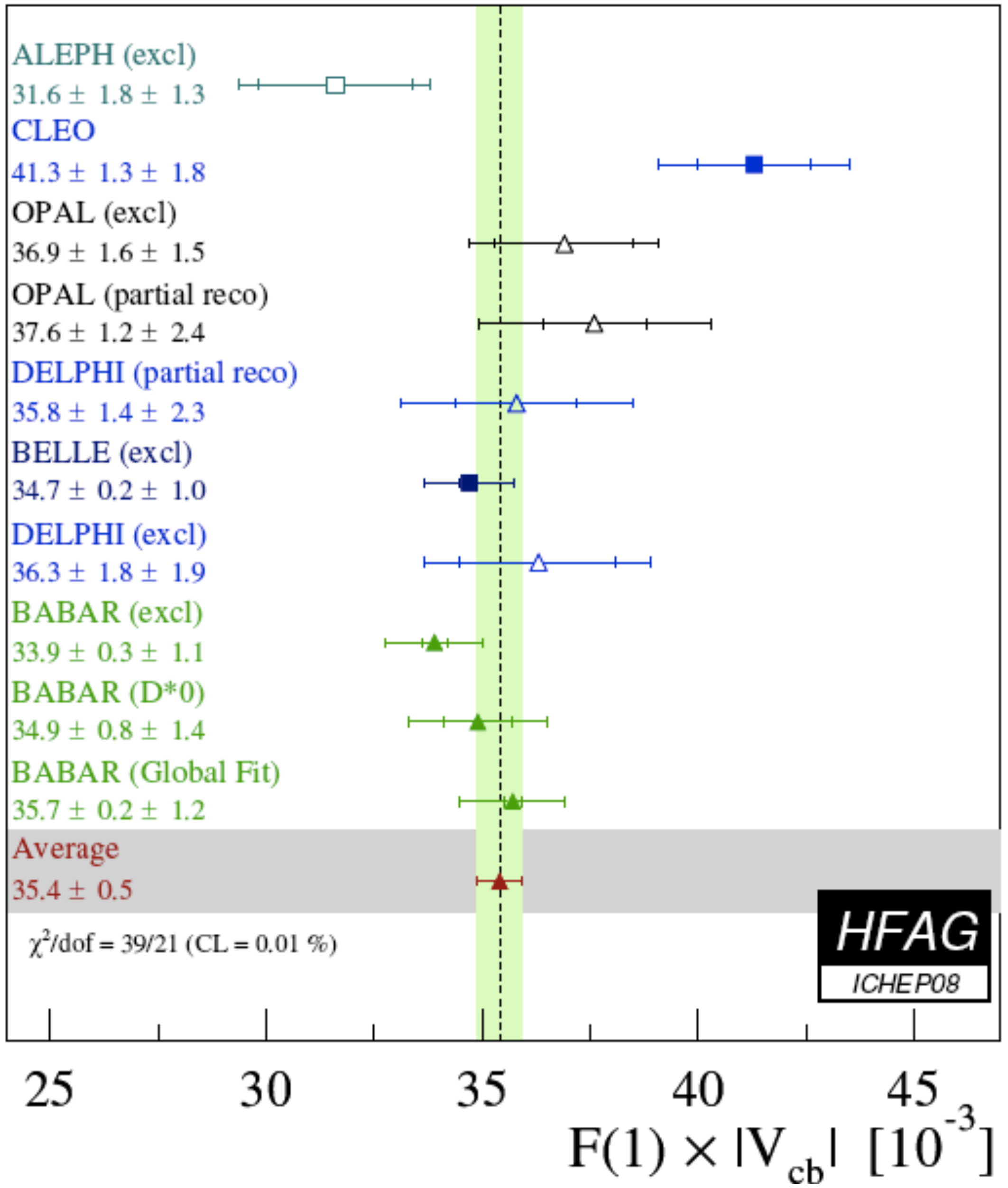
New and **only** unquenched Lattice QCD:

F(1) =0.921(24) Laiho et al 2008, HQET, double ratio

$$|V_{cb}|=38.2(0.5)(1.1)\times 10^{-3}$$

~2.4σ from inclusive determination which would imply F(1)=0.857(13)

$B \to D l \nu$ gives consistent but much less precise results



Lattice promising alternative: step scaling, w dependence, only quenched de Divitiis et al

Heavy Quark Sum Rules for B→D*lν

Heavy Quark Sum rules provide a (unitarity) bound on F(1):

$$F_{D^*}^2 + \underbrace{\sum_{f \neq D^*} |F_{B\to f}|^2}_{>0} = \xi_A^{\rm pert} - \frac{\mu_G^2}{3m_c^2} - \underbrace{\frac{\mu_\pi^2 - \mu_G^2}{4}}_{>0}\left(\frac{1}{m_c^2} + \frac{1}{m_b^2} + \frac{2}{3m_c m_b}\right) - \Delta_{\frac{1}{m_Q^3}} + \Delta_{\frac{1}{m_Q^4}} + \dots$$

Numerically (preliminary)

$$\sqrt{\xi_A^{\rm pert}} \simeq 0.96 \qquad -\Delta_{\frac{1}{m_Q^2}} - \Delta_{\frac{1}{m_Q^3}} = -0.14(2) \qquad F_{D^*} \lesssim 0.90$$

The quark masses and B meson expectation values measured in inclusive decays strongly disfavor F(1)>0.9

$$\sum_{f \neq D^*} |F_{B\to f}|^2 = \chi \cdot \left(\Delta_{\frac{1}{m_Q^2}} + \Delta_{\frac{1}{m_Q^3}} + \dots\right) \qquad F_{D^*} \simeq \sqrt{\xi_A^{\rm pert}} - \frac{1}{2}(1+\chi)\Delta$$

No reason to expect χ=0, typically χ>0.5 or F(1)<0.87 Uraltsev, Mannel, PG...

The total $B \to X_u l\nu$ width in the OPE

$$\Gamma[\bar B \to X_u e\bar\nu] = \frac{G_F^2 m_b^5}{192\pi^3}|V_{ub}|^2 \Bigg[1 + \frac{\alpha_s}{\pi} p_u^{(1)}(\mu) + \frac{\alpha_s^2}{\pi^2} p_u^{(2)}(r,\mu) - \frac{\mu_\pi^2}{2m_b^2} - \frac{3\mu_G^2}{2m_b^2}$$
$$+ \left(\frac{77}{6} + 8\ln\frac{\mu_{\rm WA}^2}{m_b^2}\right)\frac{\rho_D^3}{m_b^3} + \frac{3\rho_{LS}^3}{2m_b^3} + \frac{32\pi^2}{m_b^3} B_{\rm WA}(\mu_{\rm WA})\Bigg]$$
$$+ O\left(\alpha_s \frac{\mu_{\pi,G}^2}{m_b^2}\right) + O\left(\frac{1}{m_b^4}\right)$$

Weak Annihilation

Yes, life would be MUCH easier with the total width...

The problems with cuts

$|V_{ub}|$ from total BR($b\to ul\nu$) like incl $|V_{cb}|$ but we need kinematic cuts to avoid the ~100x larger $b\to cl\nu$ background:

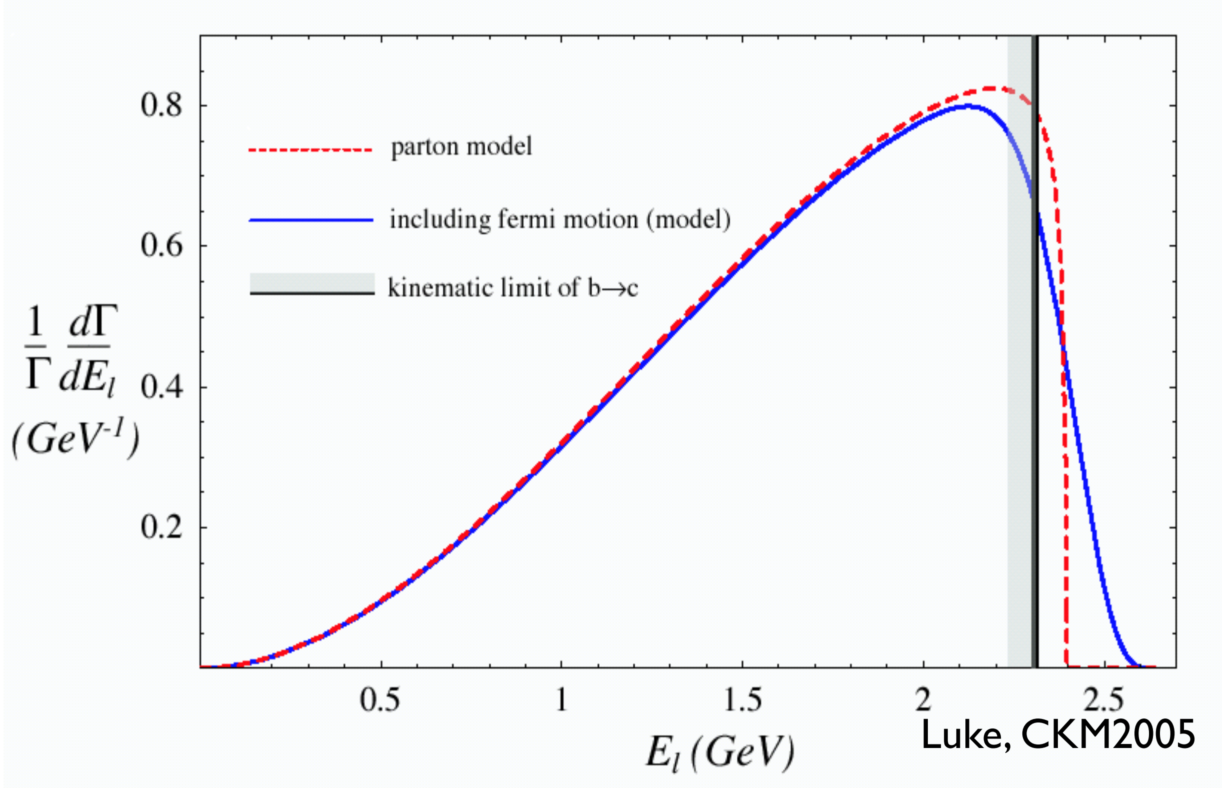
$m_X < M_D$ $\quad E_l > (M_B^2 - M_D^2)/2M_B$ $\quad q^2 > (M_B - M_D)^2$ ...

or combined (m_X, q^2) cuts

The cuts destroy convergence of the OPE that works so well in $b\to c$.

OPE expected to work only away from pert singularities

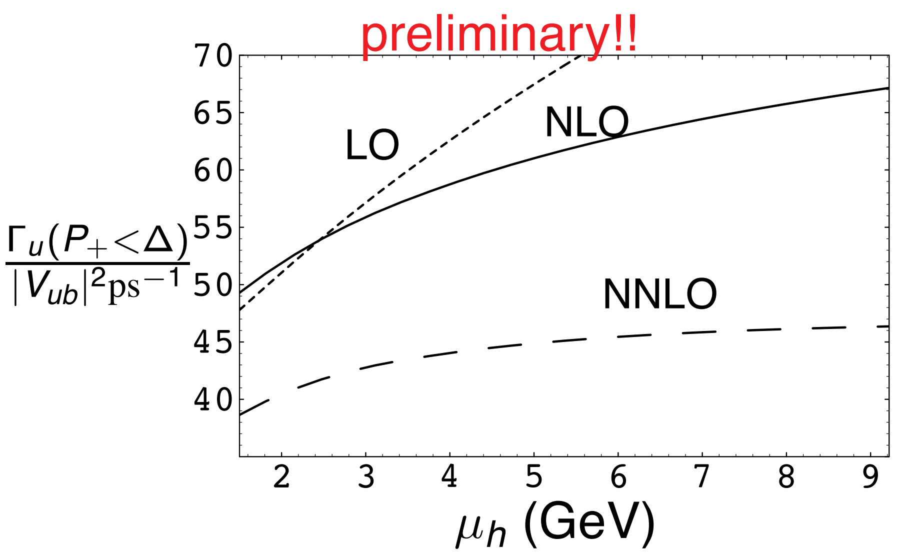
Rate becomes sensitive to "local" b-quark wave function properties like Fermi motion Dominant non-pert contributions can be resummed into a **SHAPE FUNCTION** f(k+)



Luke, CKM2005

Perturbative calculations

Partial rate for $P_+ < \Delta = M_D^2/M_B$



- NNLO result is smaller and less dependent on μ_h than NLO
- would lead to higher $|V_{ub}|$ compared to NLO (preliminary)

Some of the shift is due to different S at LO, NLO, NNLO

Ben Pecjak, ICHEP08

Complete $O(\alpha_s)$ implemented by all groups De Fazio-Neubert

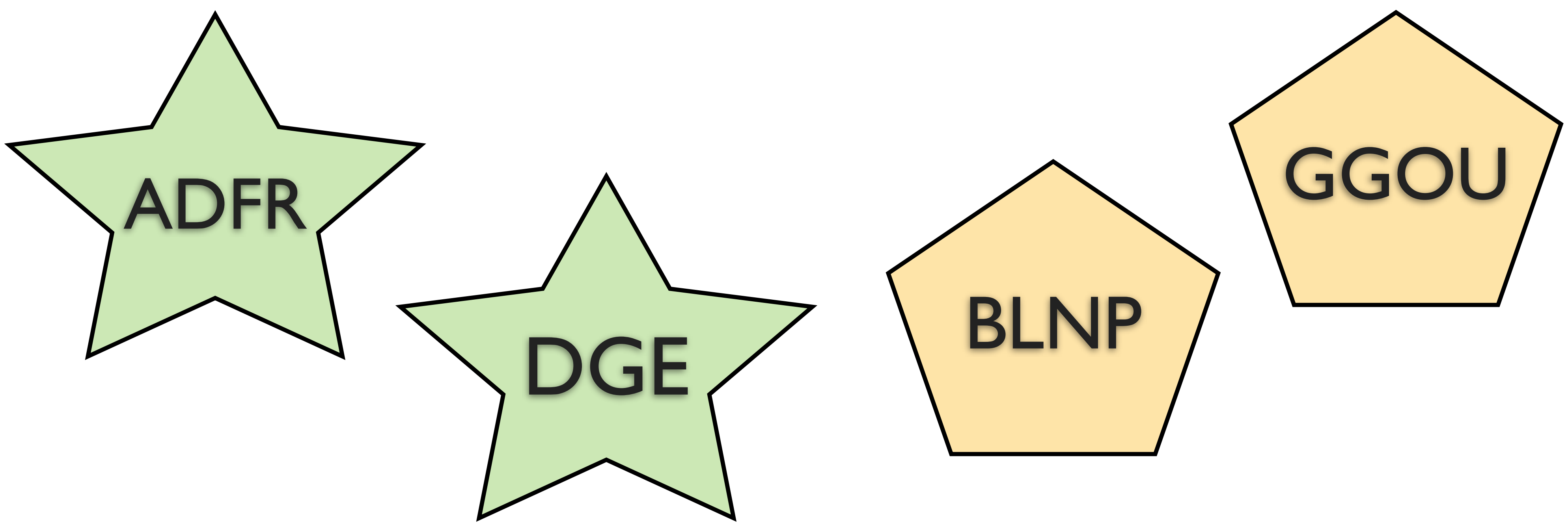
Complete running coupling NNLO $O(\alpha_s^2\beta_0)$ Gardi,Ridolfi, PG

in GGOU & DGE lead to -5% & +2%, resp. in $|V_{ub}|$

2008
Asatrian,Greub,Pecjak
Bonciani,Ferroglia,
Beneke,Huber, Li
G. Bell
in SCET-HQET
corresponds to fixed order $O(\alpha_s^2)$ in the SF region

How to access the SF?

Prediction *based* on resummed pQCD	OPE constraints + parameterization



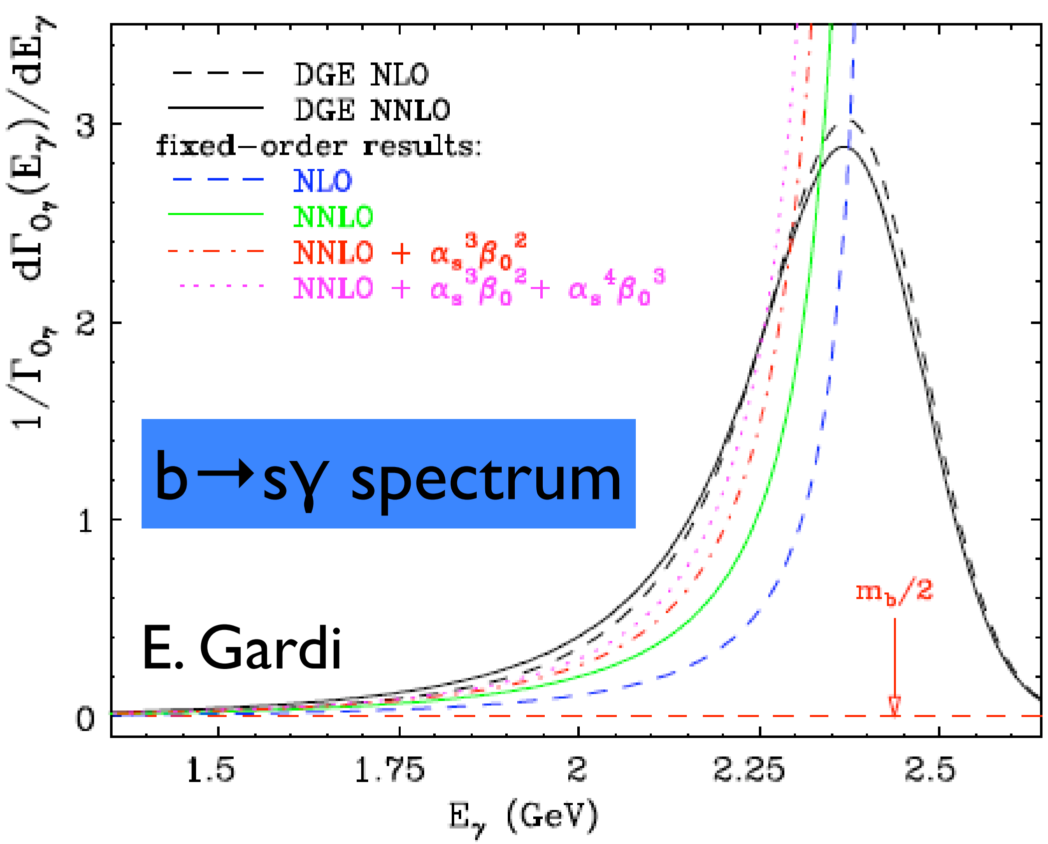
SF from perturbation theory

Resummed perturbation theory is qualitatively different: Support properties; stability! *(E. Gardi)*

b *quark* SF emerges from resummed pQCD but needs an IR prescription and power corrections for b →B

Dress Gluon Exponentiation (DGE) by Gardi et al employs renormalon resummation to define Fermi motion. Power corrections can be partly accomodated.

Aglietti et al (ADFR) use Analytic Coupling in the IR, a model



The SF in the OPE

Local OPE has also threshold singularities and SF can be equivalently introduced resumming dominant singularities *Bigi et al, Neubert*

Fermi motion can be parameterized within the OPE like PDFs in DIS. At leading order in m_b only a single universal function of one parameter enters (SF).

Unlike resumed pQCD, ***the OPE does not predict the SF****, only its first few moments. One then* ***needs an ansatz for its functional form****.*

$$\int dk_+ \; k_+^n \; F_i(k_+, q^2) = \quad \text{local OPE prediction} \Leftarrow \text{moments fits}$$

Two very different implementations:
PG,Giordano,Ossola,Uraltsev (GGOU)
Bosch,Lampe,Neubert,Paz (BLNP)

The SF in GGOU

Leading SF resums leading twist effects, $m_b \to \infty$
universal, q^2 indep

Finite m_b distribution functions
include all $1/m_b$ effects, *non-universal*
no need for subleading SFs

$$F(k_+) \longrightarrow F_i(k_+, q^2, \mu)$$

Structure function (i = 1,2,3)
q^2 dependence
cutoff dependence (gluons with $E_g<\mu$)

$$\frac{d^3\Gamma}{dq^2\,dq_0\,dE_\ell} = \frac{G_F^2|V_{ub}|^2}{8\pi^3}\left\{q^2W_1 - \left[2E_\ell^2 - 2q_0E_\ell + \frac{q^2}{2}\right]W_2 + q^2(2E_\ell - q_0)W_3\right\}$$

$$W_i(q_0, q^2) = m_b^{n_i}(\mu)\int dk_+\; F_i(k_+, q^2, \mu)\; W_i^{pert}\left[q_0 - \frac{k_+}{2}\left(1 - \frac{q^2}{m_b M_B}\right), q^2, \mu\right]$$

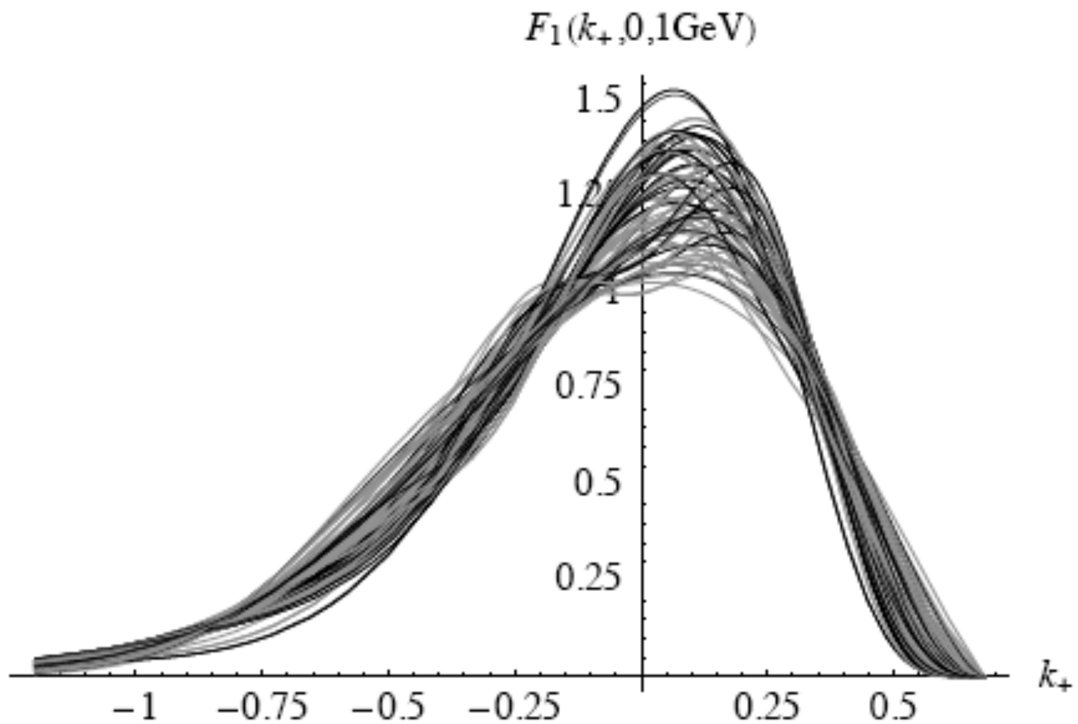
This factorization formula perturbatively defines the distribution functions

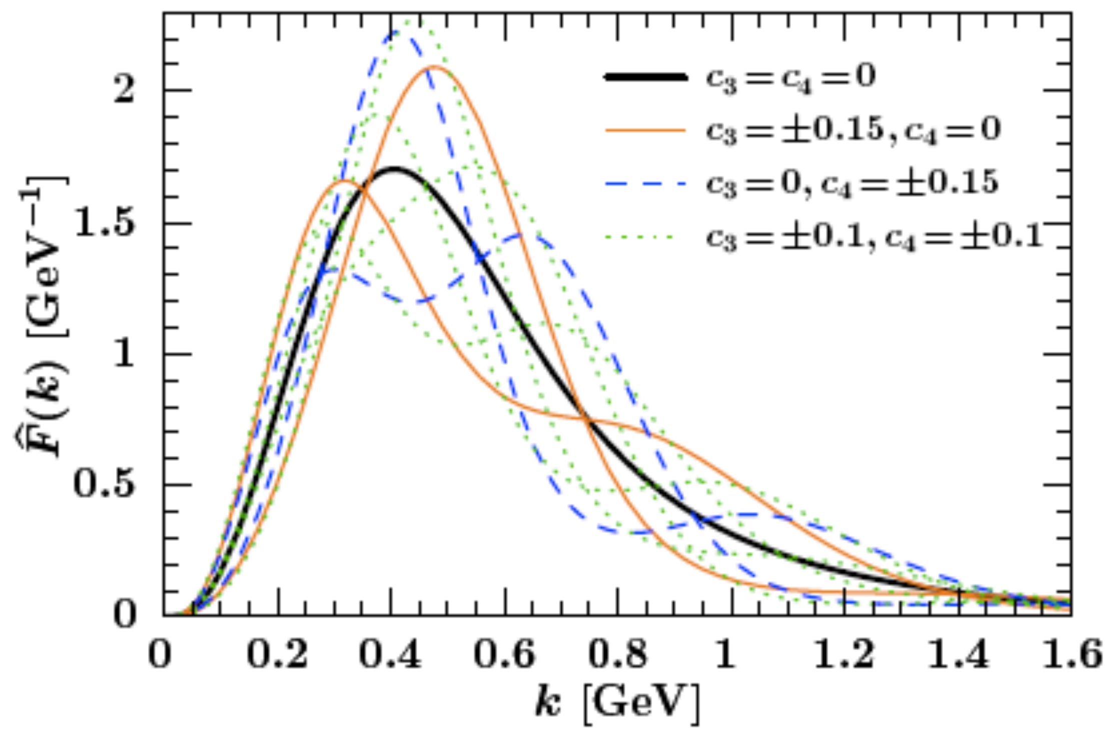
see also Benson, Bigi, Uraltsev for bsγ

$$\int dk_+\; k_+^n\; F_i(k_+, q^2) = \text{local OPE}$$

Importance of subleading effects

Functional forms





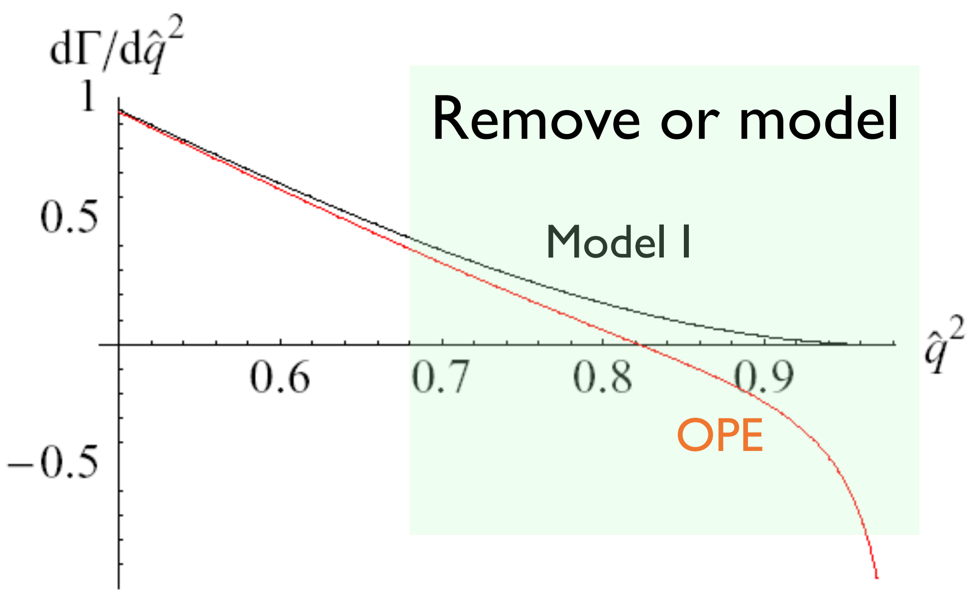
About 100 forms considered in GGOU, large variety, double max discarded. Small uncertainty (1-2%) on V_{ub}

Recent more systematic method by Ligeti et al. arXiv:0807.1926 Plot shows 9 SFs that satisfy all the first three moments

The high q^2 tail

At high q^2 higher dimensional operators are not suppressed leading to pathological features. Origin in the non-analytic square root

$$\frac{d\Gamma}{dq_0\, dq^2} \propto \sqrt{q_0^2 - q^2} \quad \Rightarrow \quad \frac{d\Gamma}{dq^2} \sim -\sum_{n=1}^{\infty} \frac{(-1)^n\, b_n(\hat{q}^2)}{(1-\hat{q}^2)^{n-2}} \left(\frac{\bar{\Lambda}}{m_b}\right)^n$$



In the integrated rate the $1/m_b^3$ singularity is removed by the WA operator: needs modelling for q^2 spectrum

$$\delta\Gamma \sim \left[C_{\rm WA}\, B_{\rm WA}(\mu_{\rm WA}) - \left(8\ln\frac{m_b^2}{\mu_{\rm WA}^2} - \frac{77}{6}\right)\frac{\rho_D^3}{m_b^3} + \mathcal{O}(\alpha_s)\right]$$

WA matrix element B_{WA} parameterizes global properties of the tail, affects V_{ub} depending on cuts, <u>tends to decrease V_{ub}</u> , may pollute all present determinations

Comparing the existing approaches at common m_b

(HFAG ichep08, CKM08)

m_b^{kin} =4.601(35)GeV ➟ $m_b(m_b)$=4.23GeV

μ_π^2=0.440(40)GeV^2

very strong dependence on m_b,

twice larger than in total rate

$|V_{ub}|$ from DGE

Gardi & Andersen

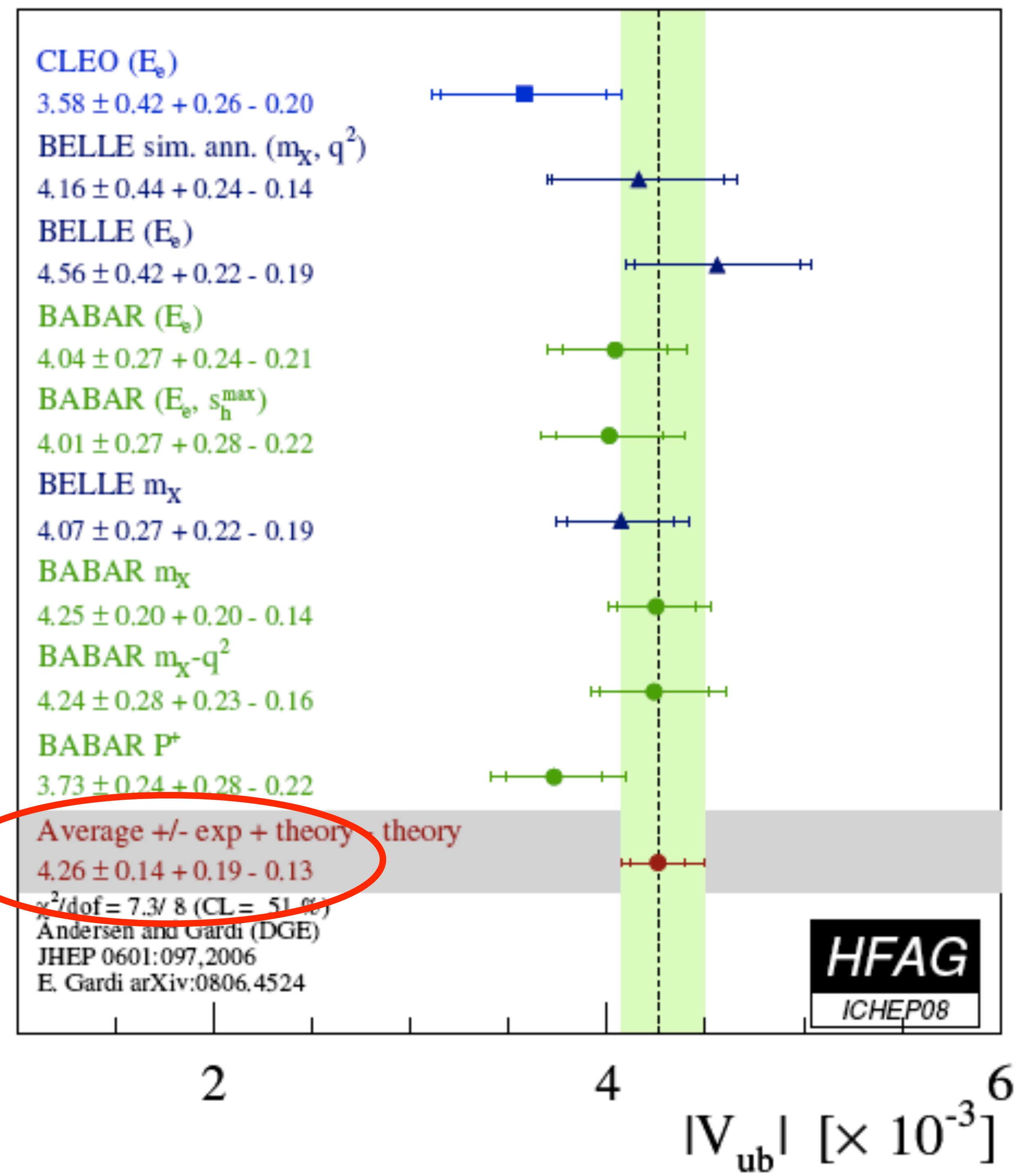
Main features of the spectra are reproduced ➠ $|V_{ub}|$ stable, small errors and good χ^2

NNLL and $O(\alpha_s^2\beta_0)$ implemented

Power corrections in the SF region are included here only in theor. err.
No subleading SF.
Matches to local OPE.

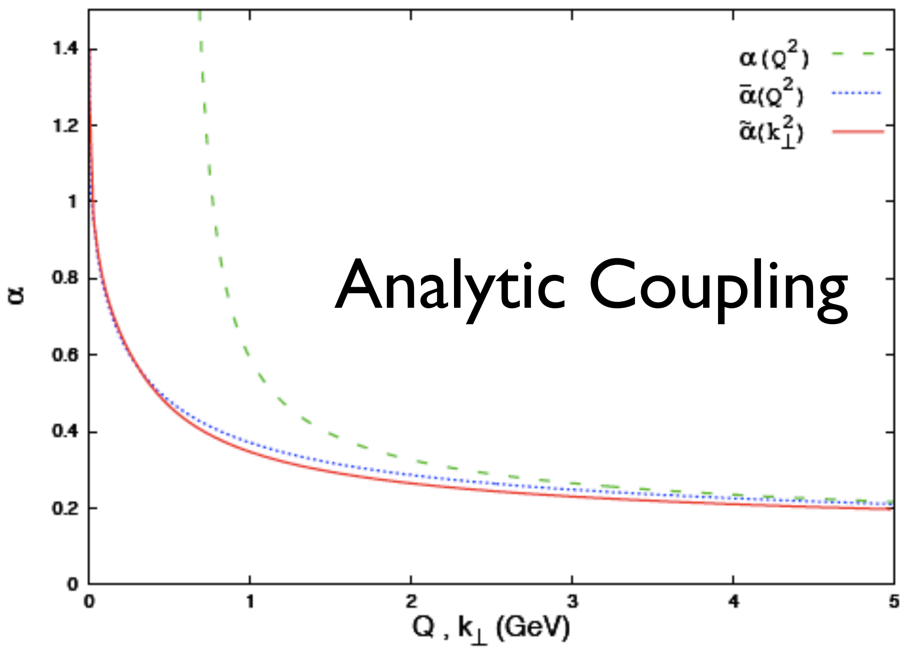
Only input other than α_s
$m_b(m_b)$=4.24(4) from global fit

5-6% total error, mostly m_b



$|V_{ub}|$ from ADFR

Aglietti, Di Lodovico, Ferrera, Ricciardi



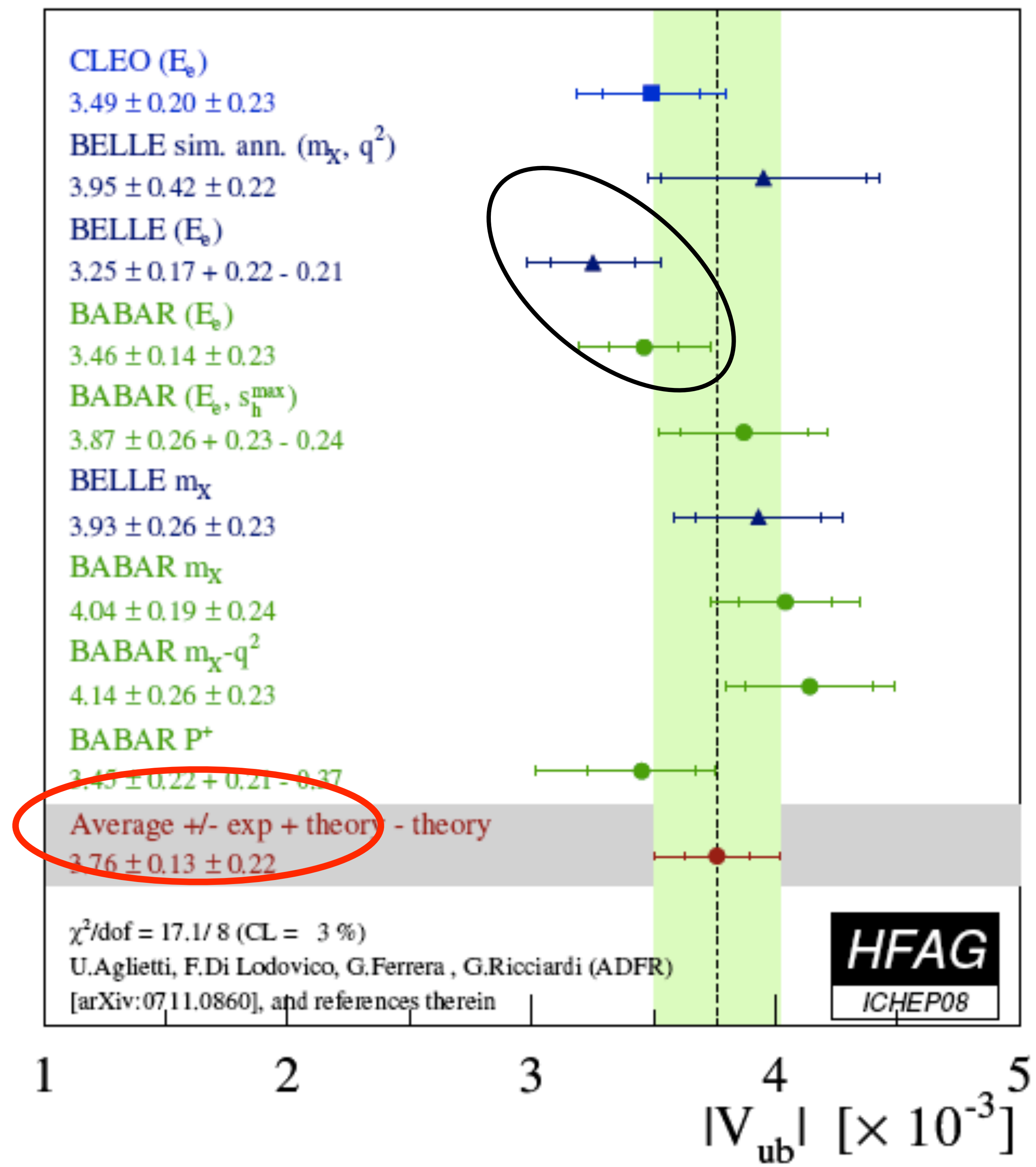
Worse consistency here.

NNLO resummation, NLO constants

Consider E_l cuts higher than 2.3GeV because their E_l apparently does not reproduce data (see later)

employs M_B in on-shell calculation of spectra: no renormalon cancellation, no convergence to OPE.
no model error

~7% total error, mostly m_c



$|V_{ub}|$ in BLNP

Bosch,Lange,Neubert,Paz

$$\tilde{W}_1^{(0)}(P_+, y) = U_y(\mu_h, \mu_i)\, H(y, \mu_h) \int_0^{P_+} d\hat{\omega}\, m_b J(y, m_b(P_+ - \hat{\omega}), \mu_i)\, \hat{S}(\hat{\omega}, \mu_i)$$

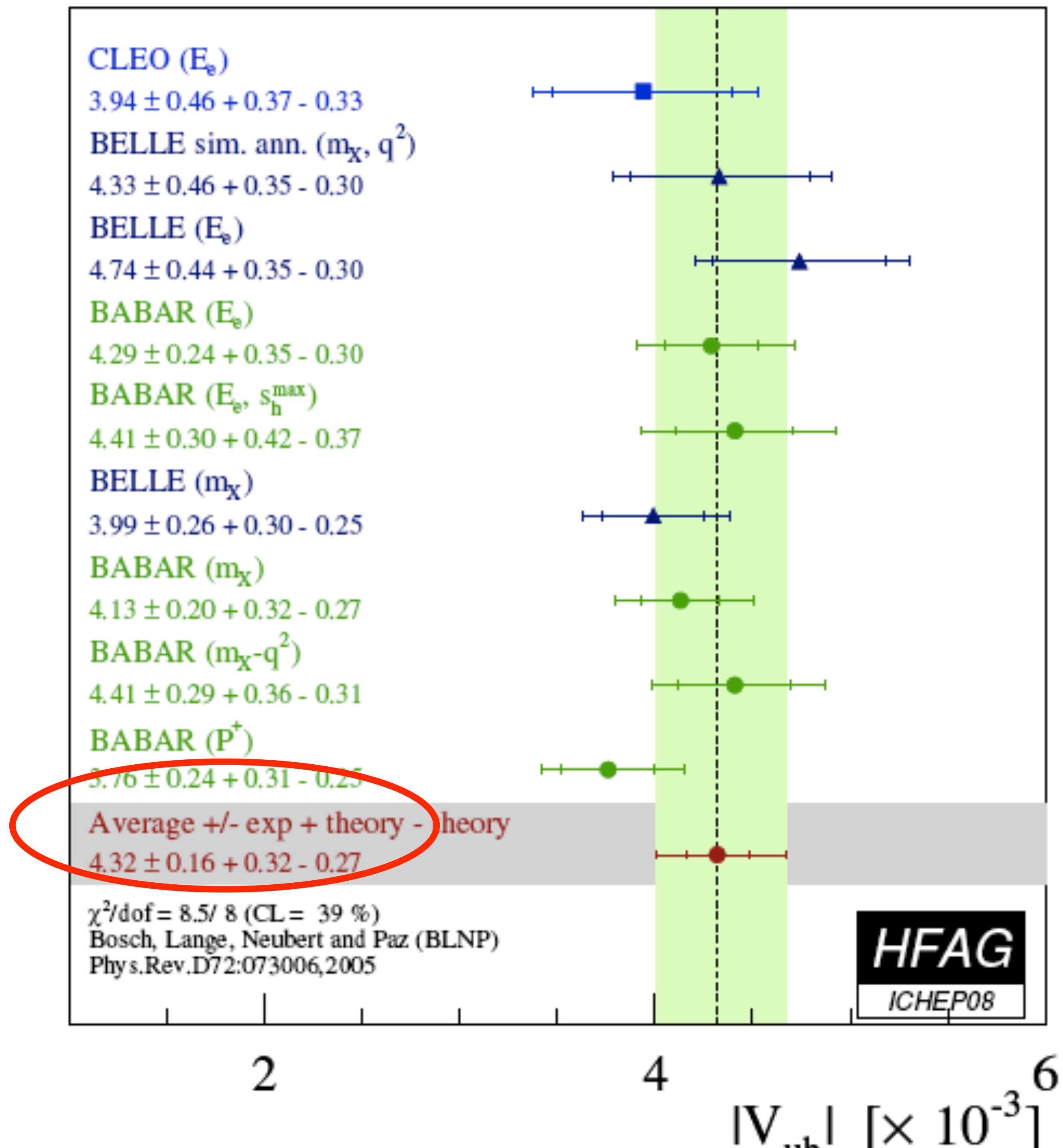
$$d\Gamma = HJ \otimes \hat{S} + \frac{1}{m_b} H_i' J_i' \otimes \hat{S}_i' + \dots$$

Good consistency. Uses elegant multiscale OPE that resums soft-collinear logs, but many largely unconstrained subleading SFs

NNLL resummation, only $O(\alpha_s, \Lambda^2/m_b^2)$ matching to OPE, 3 ffs for leading SF, extensive modelling of SSF.

m_b and μ_π^2 in SF scheme obtained from global fit in the kin scheme

~7-8% total error, main error HQE parameters



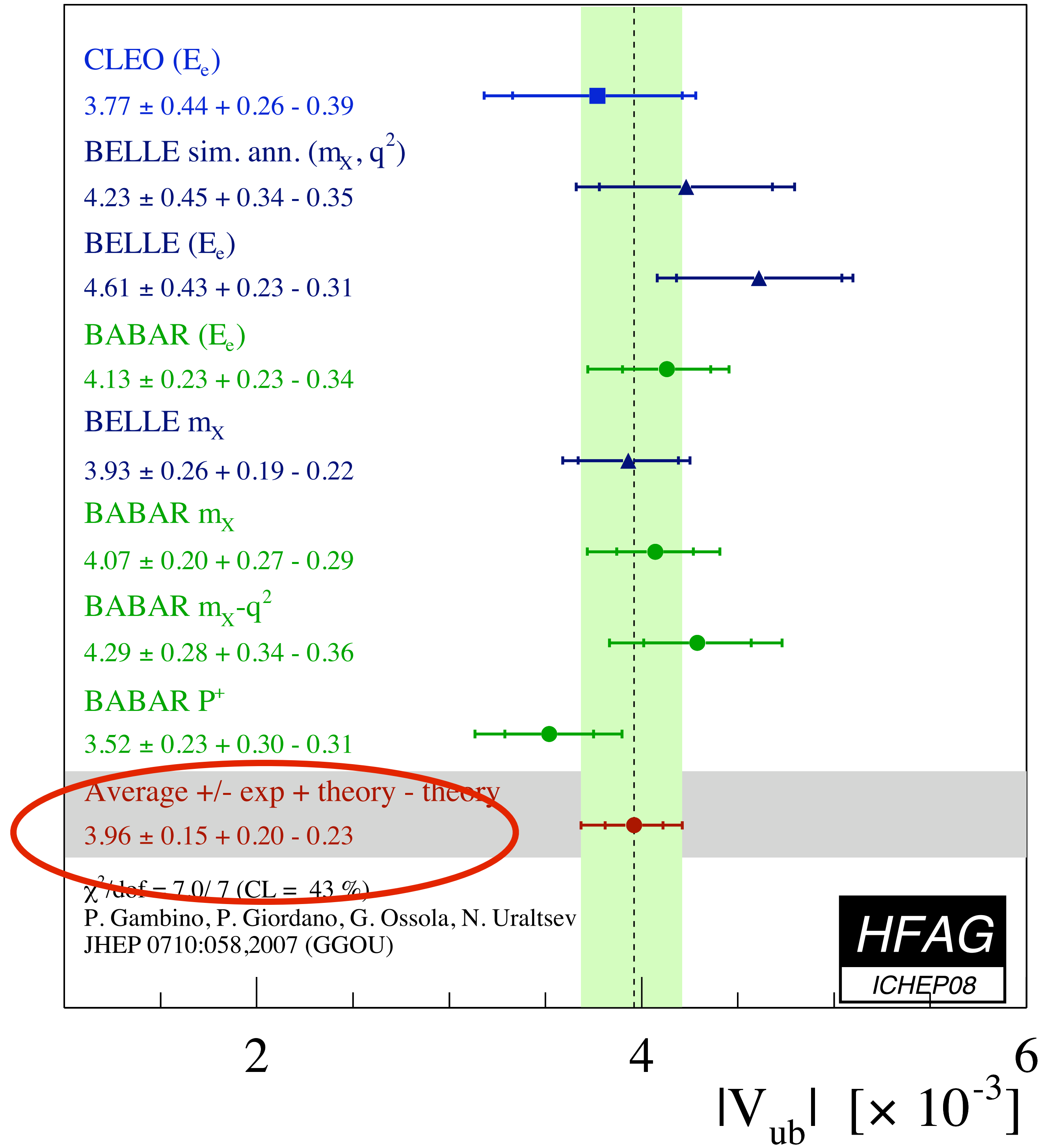
$|V_{ub}|$ in the kinetic scheme -GGOU

PG,Giordano,Ossola,Uraltsev

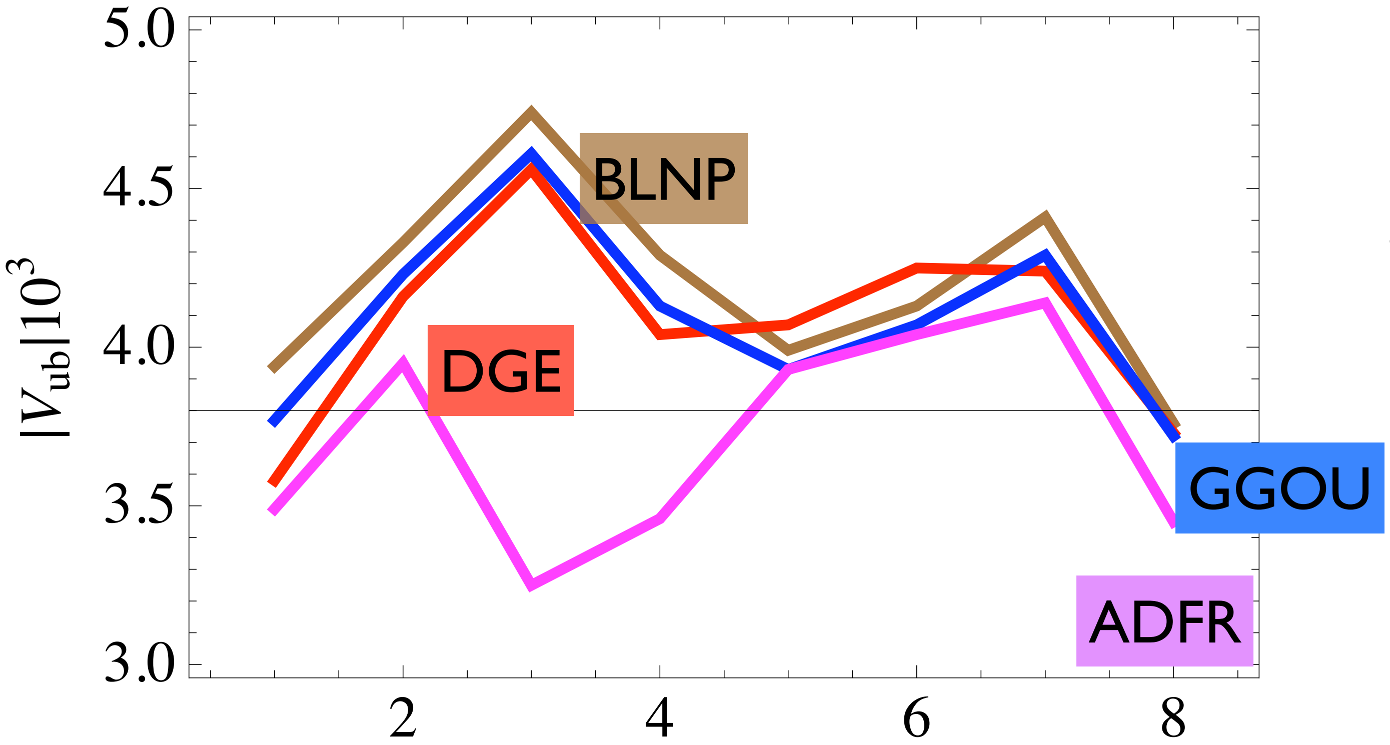
Good consistency & small th error.
OPE in a scheme with Wilsonian IR cutoff ~1GeV, all subleading $1/m_b$ and $O(\alpha_s^2\beta_0)$ terms consistently included,
careful treatment of high q^2 tail.

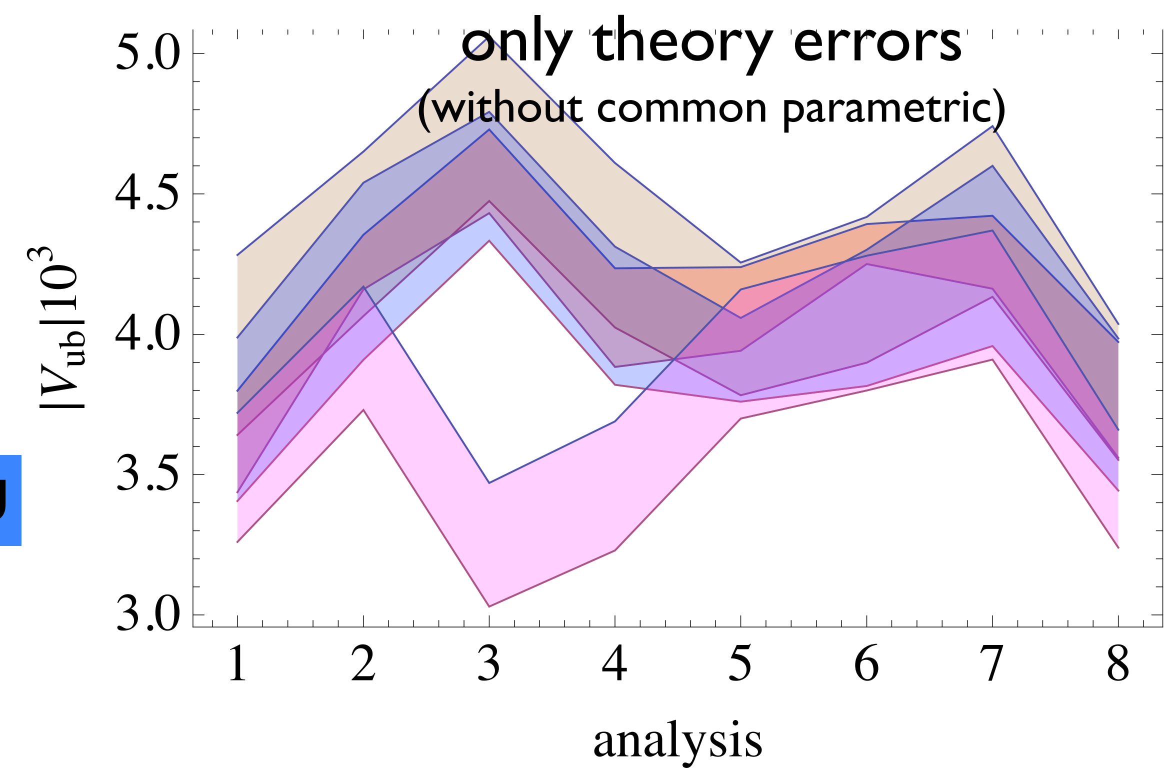
Inputs from global fit to the moments

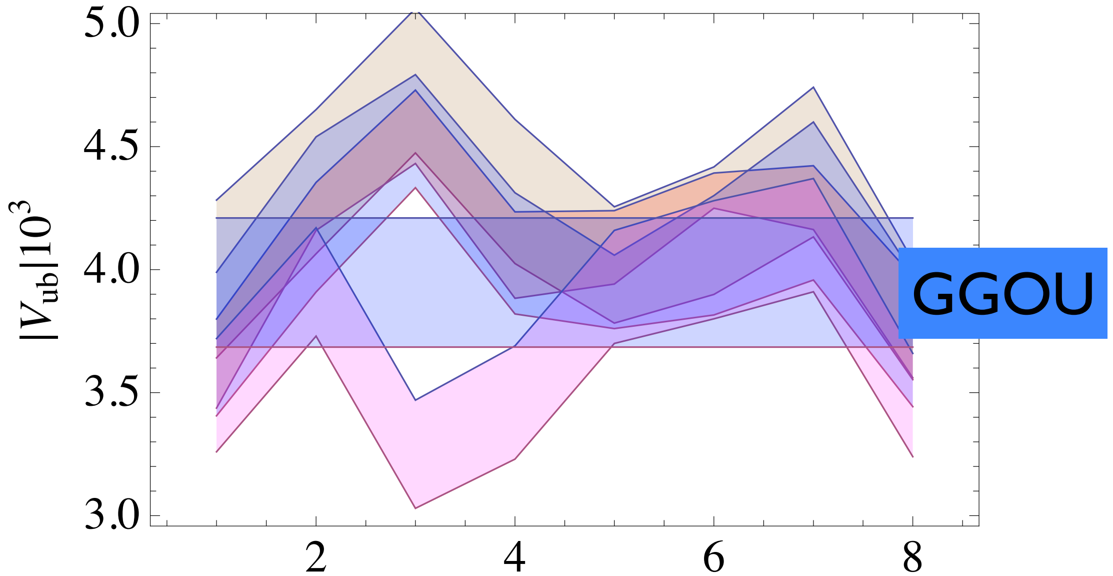
+6.3-7.0% total error



A global comparison



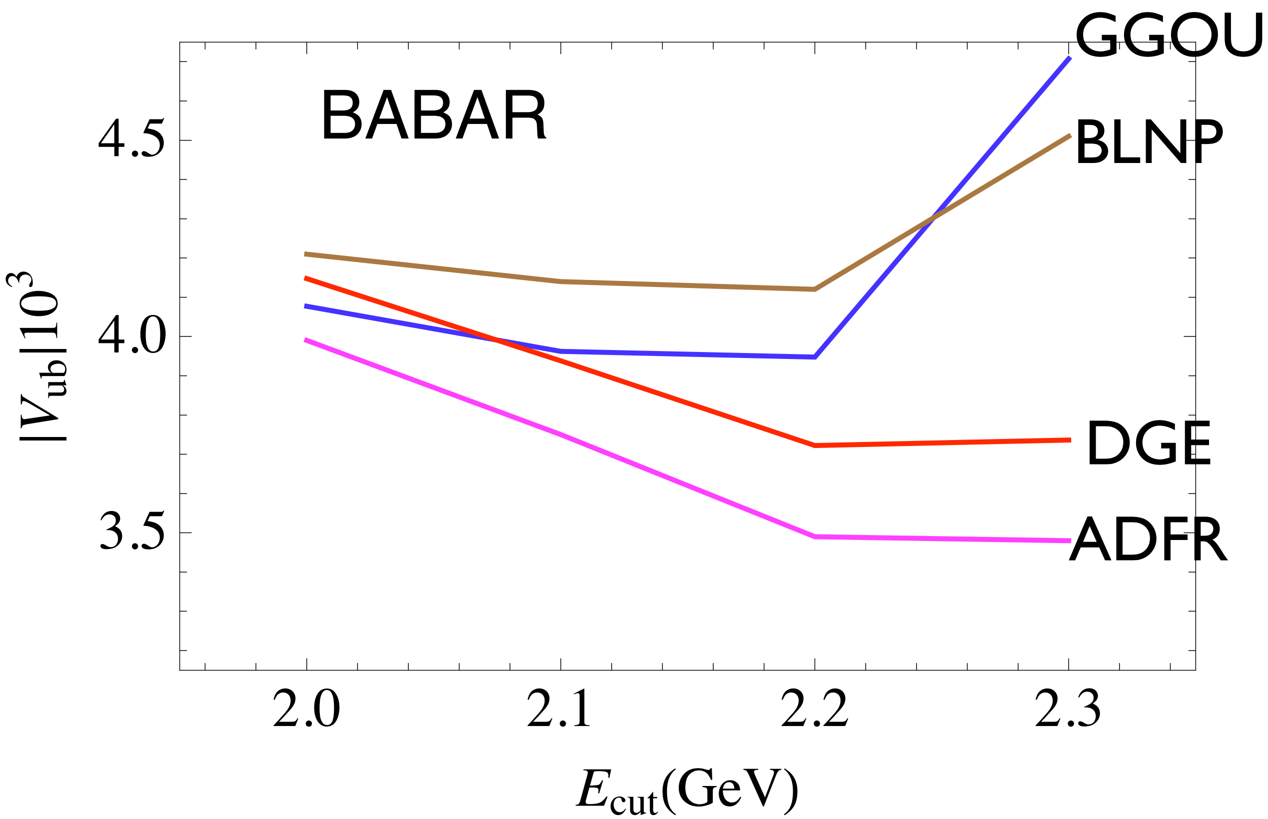


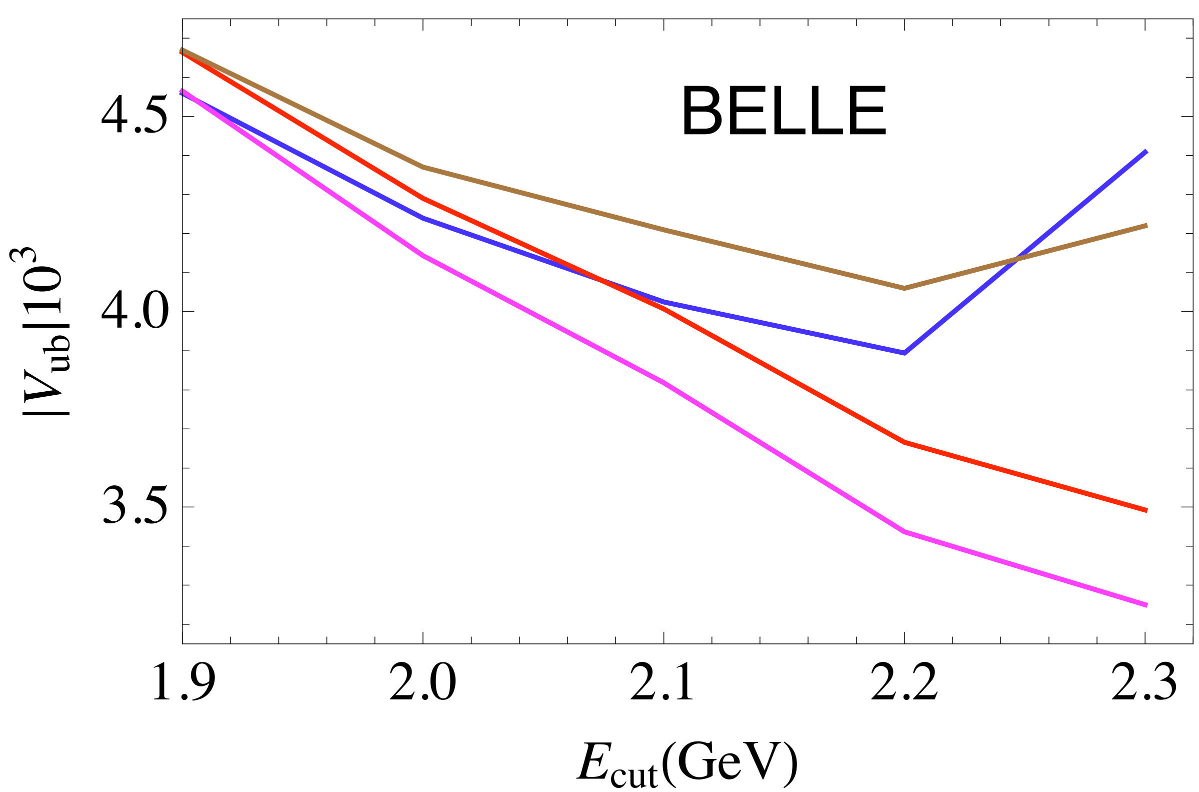


- ✳ common inputs (except ADFR)
- ✳ Overall good agreement with one exception
  SPREAD WITHIN TH ERRORS!
- ✳ Systematic offset of central values: normalization? to be investigated
- ✳ Very different methods, common systematics? WA, inputs, pert corrections

Why do central values differ up to 9-10%?

The lepton spectrum



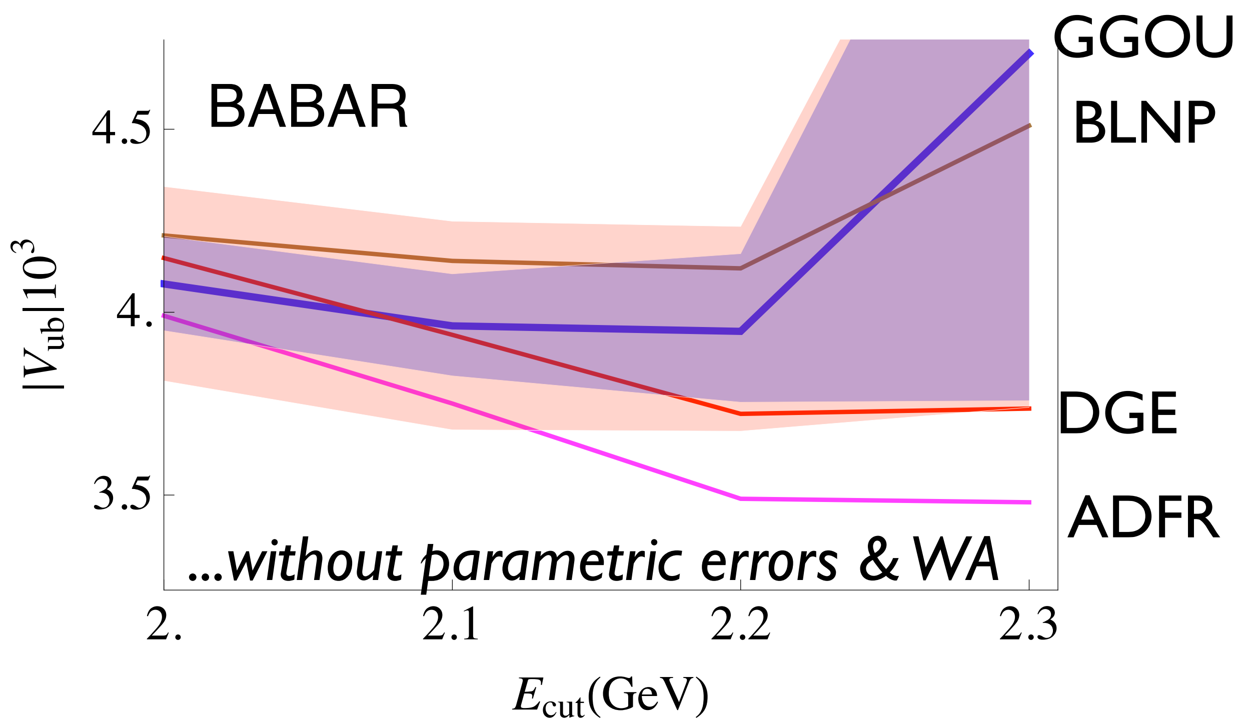


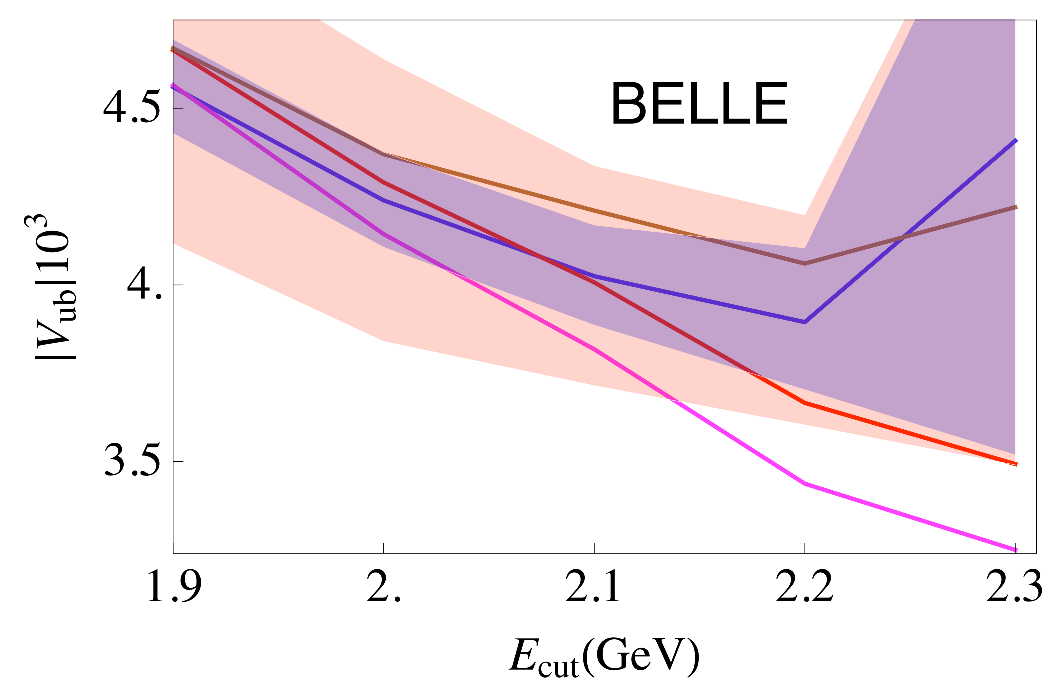
Babar *E_l determination* ***Belle*** *E_l determination*

Common inputs, m_b^{kin}=4.60GeV or $m_b(m_b)$=4.24GeV.

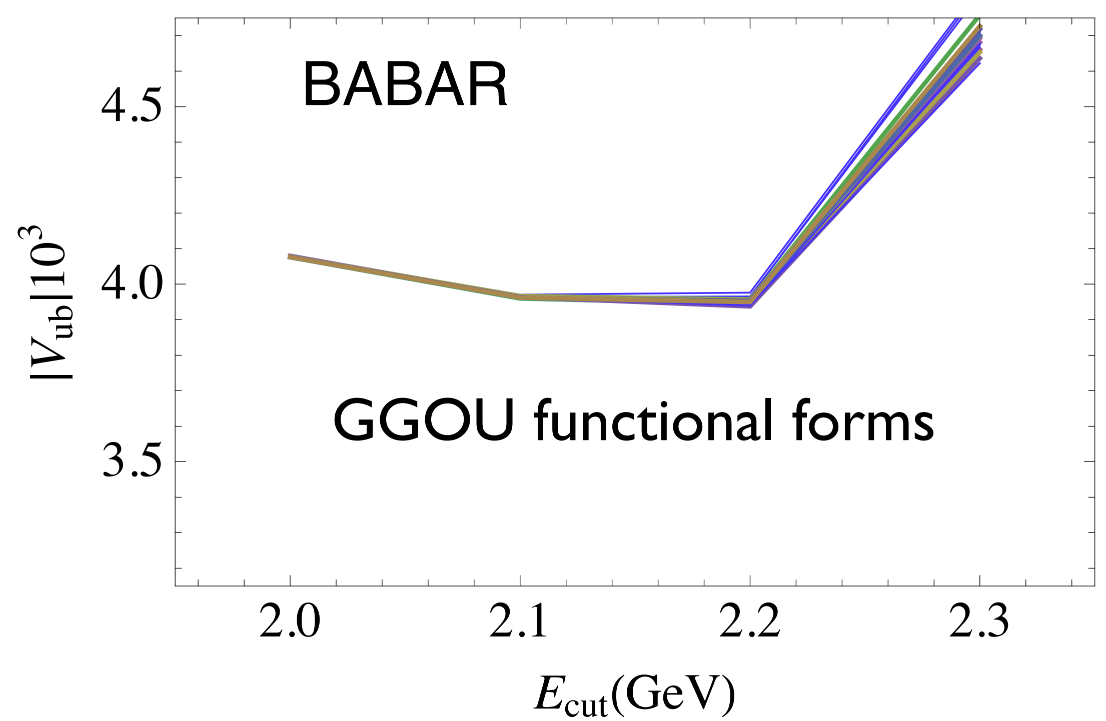
Exp analyses depend strongly on generator (inconsistent!!!!!)

The lepton spectrum

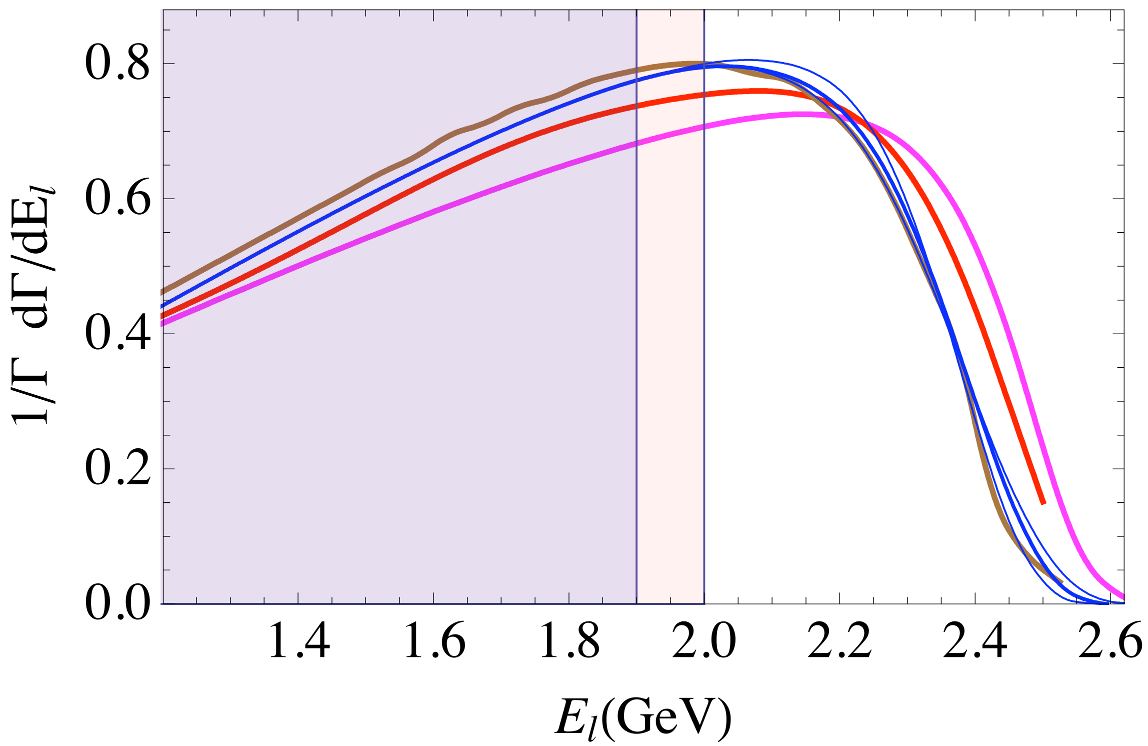




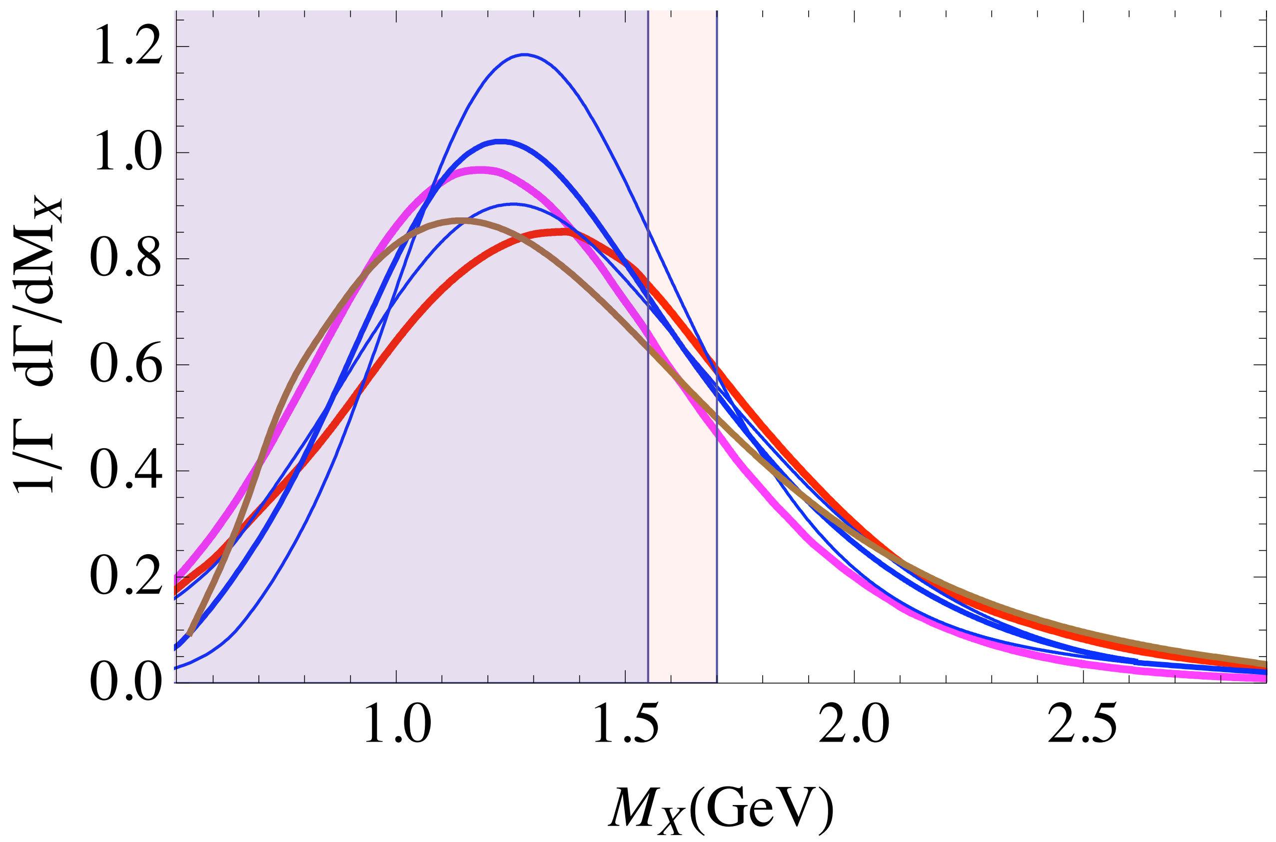
The spectrum does provide information: OPE based methods close to each other up to 2.2GeV, resummed methods show larger slope, only *seem* to behave in same way



The leptonic and M_X spectra



DGE GGOU BLNP ADFR



The leptonic spectrum is not sensitive to the SF except quite close to the endpoint. At 1.5GeV all methods should agree (it's pQCD after all)

- *Not all observables are equally* clean. eg high q^2 tail is sensitive to WA
- Need spectra: only way to test frameworks (see E_l spectrum).
- More inclusive measurements, less dependence on m_b
- Theory errors are partly parametric: m_b dependence is crucial

	Average $\lvert V_{ub}\rvert \times 10^3$
DGE	$4.26(14)_{ex}{}^{+19}_{-13}$
BLNP	$4.31(16)_{ex}{}^{+32}_{-27}$
GGOU	$3.96(15)_{ex}{}^{+20}_{-23}$

2.1, 1.9, 1.3σ from $B\to\pi l\nu$ (MILC-FNAL)

3.1, 2.4, 1.5σ from UTFit (because of sin2β)

NEW preliminary Belle Multivariate analysis only E_l>1GeV

$$|V_{ub}| = (4.45 \pm 0.26^{+0.13}_{-0.22}) \times 10^{-3}$$

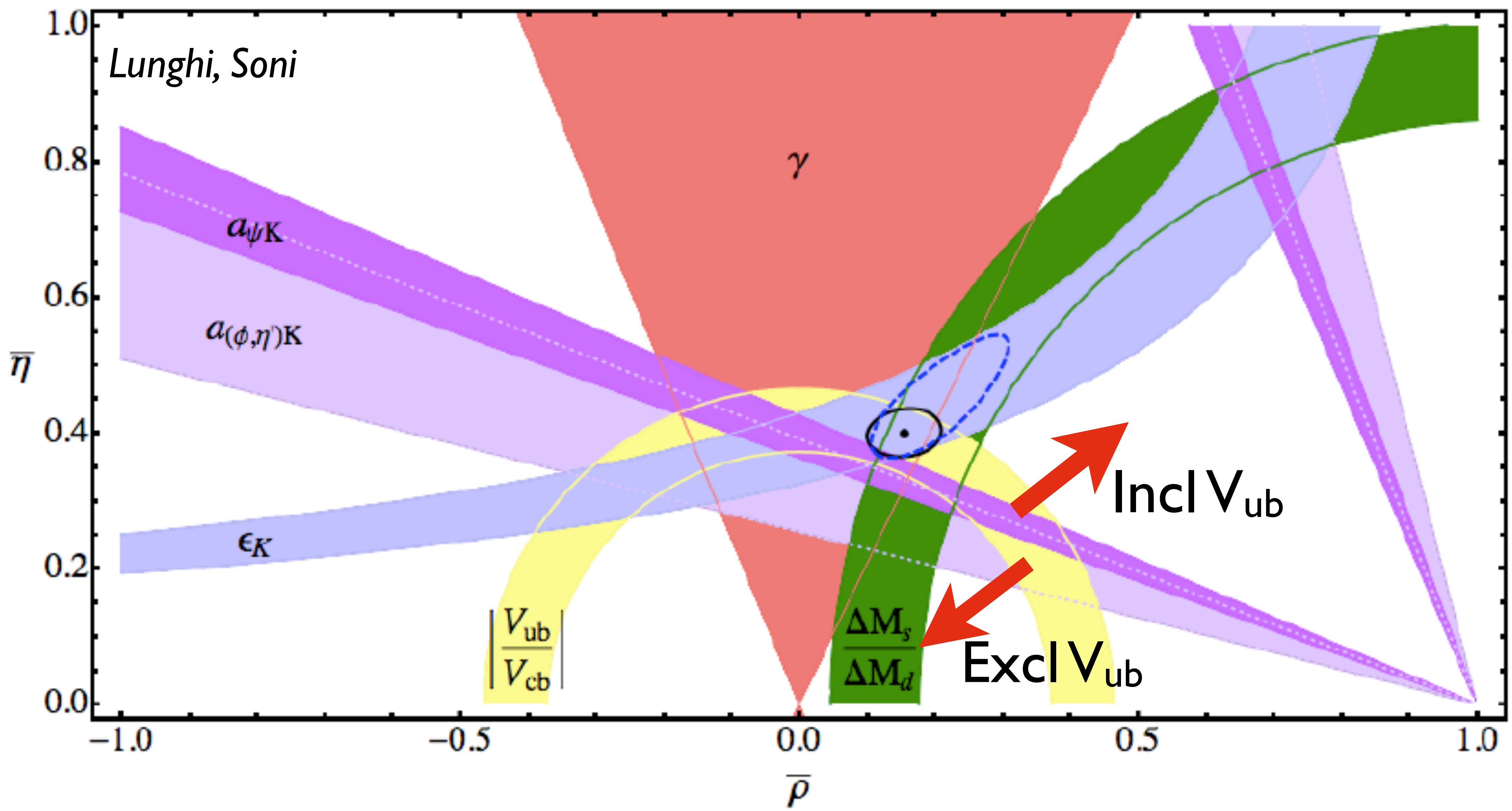
GGOU

2.1σ from excl, 2.5σ from UTFit probably a bit less after fit upgrade

This includes about 90% of the rate really <u>*inclusive*</u> *measurement, no need for SF. Only crucial input m_b needs to be confirmed!*

NEW PHYSICS?

eg LR models Chen,Nam



Recent lattice results for B_K and previously neglected contributions lead to 15% smaller ε_K, in ~1.8-2σ conflict with exp sin2β. *Buras,Guadagnoli*

Perhaps sin2β is simply too low...

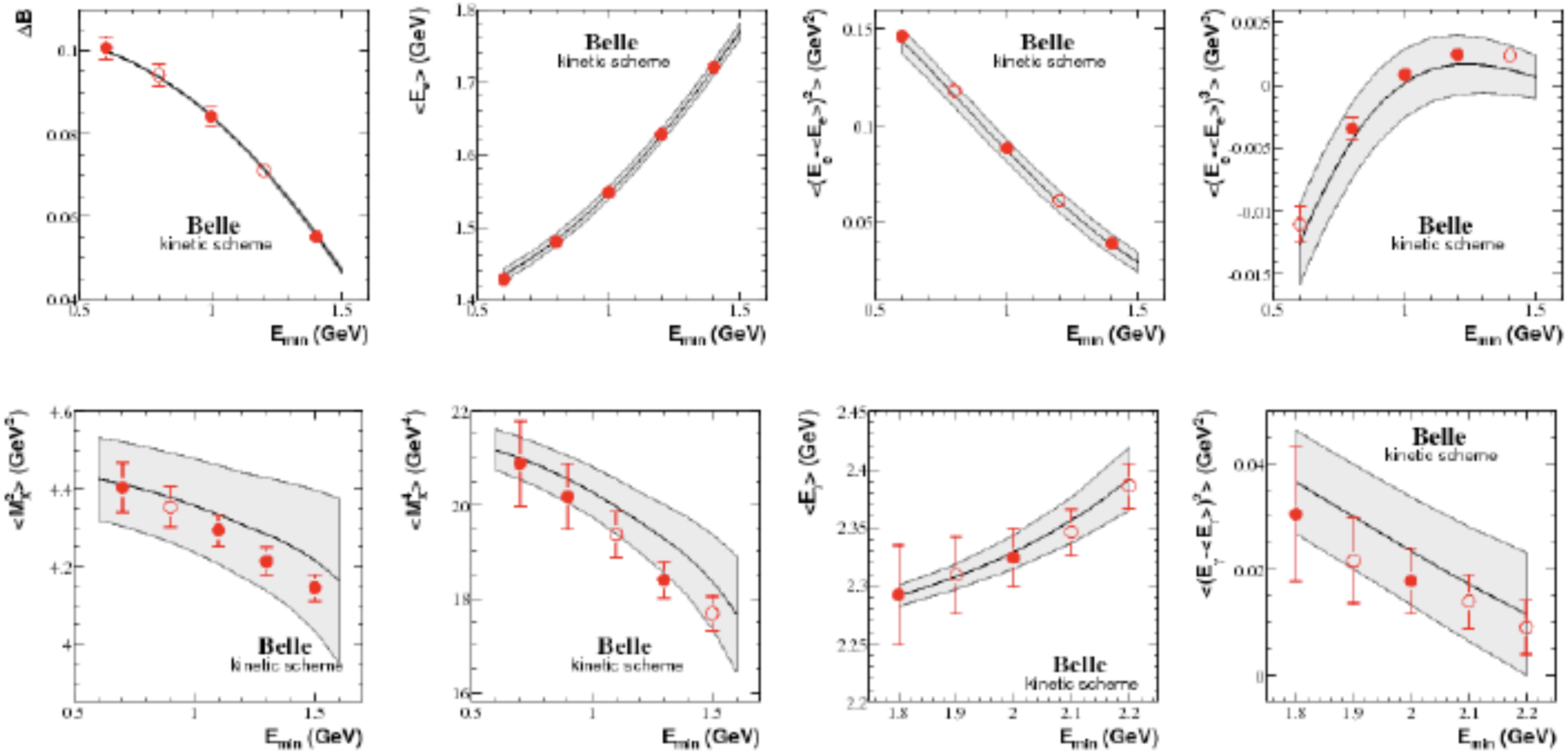
too early to say

Conclusions

- Inclusive $|V_{cb}|$ seems OK, good prospects for th error reduction
- m_b-m_c well determined by semileptonic fits, individual errors somewhat underestimated. Expect improvements by summer
- Latest FNAL result for $B\to D^*$ f.f. clashes with heavy quark sum rules
- Approaches to V_{ub} agree and seem consistent with data, need spectra and varying cuts
- No real problem between excl-incl V_{ub}, 1.5σ with UTfit, but new Belle multivariate result implies even larger V_{ub} with small th uncertainty

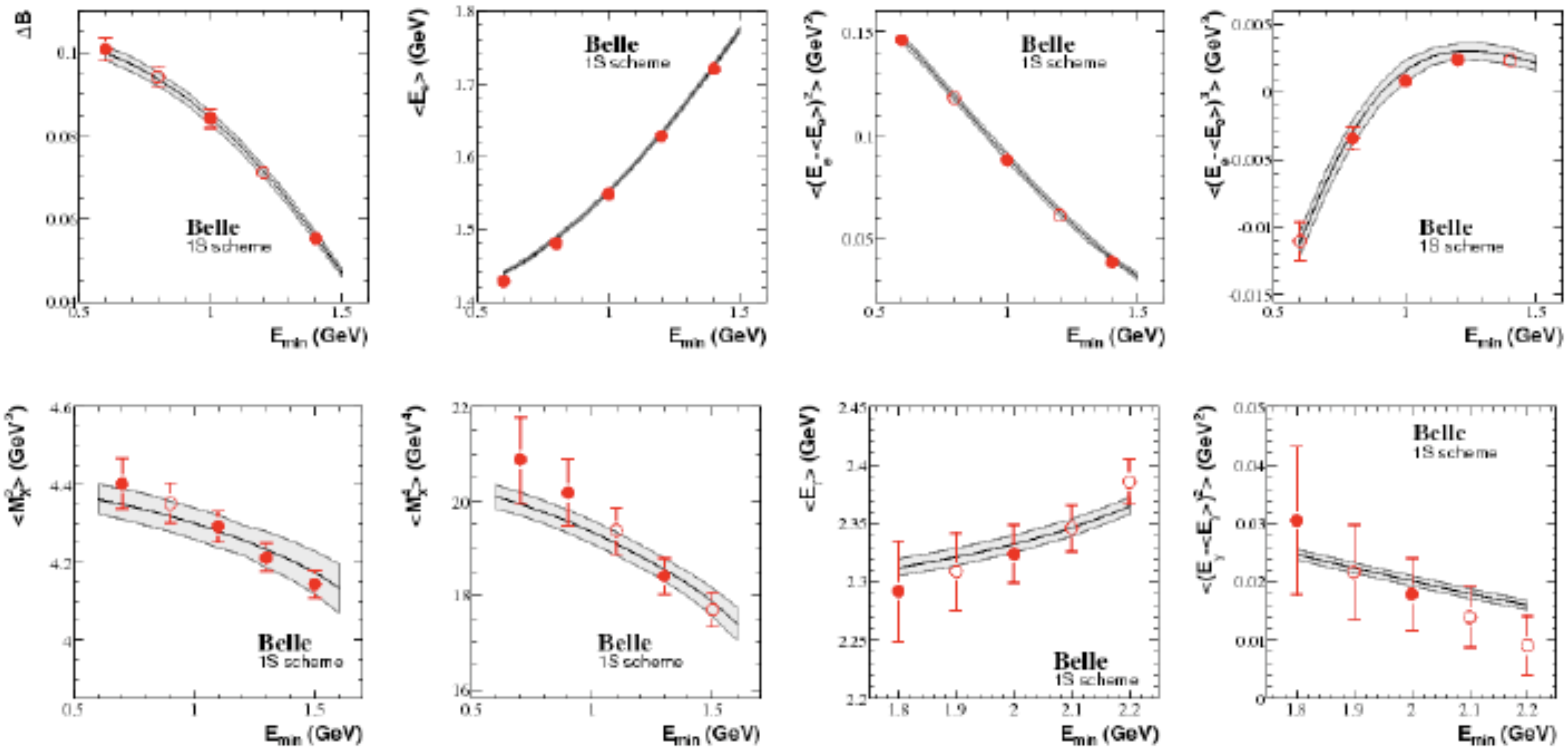
Back-up slides

Fit result in the kinetic scheme



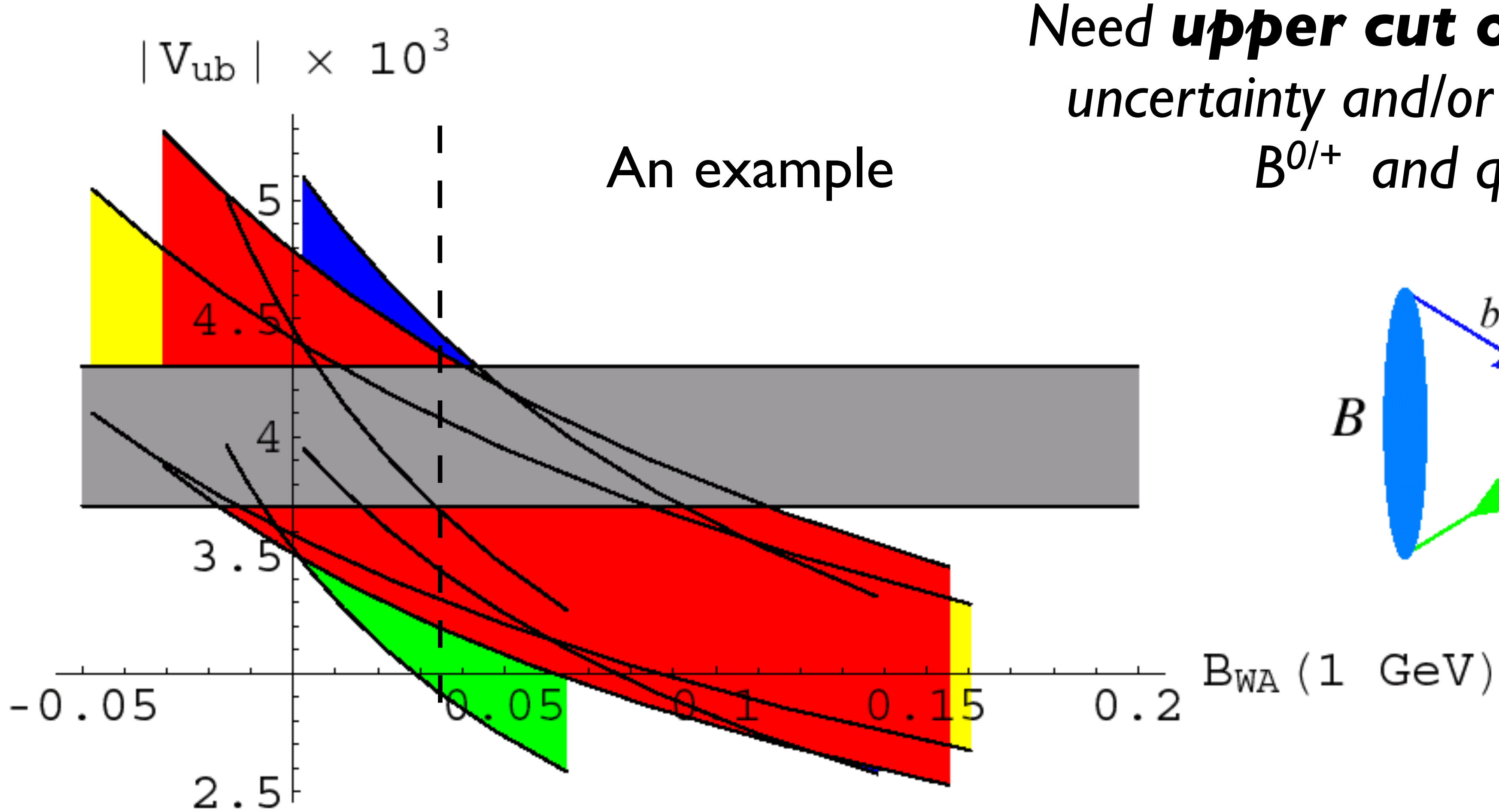
χ^2/ndf. = 4.7 / (25-7)

Fit result in the 1S scheme



χ^2/ndf. = 7.3 / (25-7)

Constraining Weak Annihilations



WA may pollute all present estimates, and tend to ***decrease*** *the extracted V_{ub}. Need* ***upper cut on q^2*** *to remove this uncertainty and/or constrain WA from $B^{0/+}$ and q^2 spectrum*