$b \to s\ell^+\ell^-$ in the high q^2 region at two-loops

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in collaboration with
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Institute for Theoretical Physics
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Outline

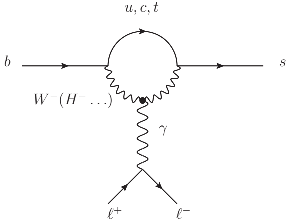
Framework and status of the calculation

NNLL calculation in the high q^2 region

Numerical issues

Some features about $b \to s\ell^+\ell^-$

- Induced by flavour changing neutral current
  $\Rightarrow$ loop-induced in the SM and sensitive to new physics



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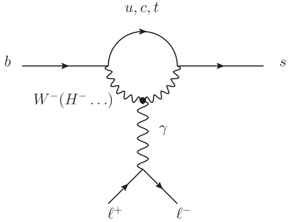

- Three body decay
  $\Rightarrow$ many kinematic observables can be measured like invariant mass spectrum of $\ell^+\ell^-$ and forward-backward asymmetry

How to treat the decay mode theoretically

- Theoretically clean predictions are possible by operator product expansion (OPE), which approximates full decay rate by the partonic decay rate:

$$\Gamma(B \to X_s \ell^+ \ell^-) = \Gamma(b \to X_s \ell^+ \ell^-) + \mathcal{O}(\frac{\Lambda^2_{\mathrm{QCD}}}{m_b^2})$$

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- Break down of OPE for dilepton invariant mass squared q^2 at
  - $c\bar{c}$ resonances
    $\Rightarrow$ Limitation of theoretical predictions to
      Low q^2: $1\text{GeV}^2 < q^2 < 6\text{GeV}^2$
      High q^2: $q^2 > 14.4\text{GeV}^2$ (Topic of the present talk)

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  - the endpoint m_b^2

    For $\int_{q_0^2}^{m_b^2} dq^2 \Gamma(B \to X_s \ell^+ \ell^-)$ effective expansion in

    $\Lambda_{\rm QCD}/(m_b - \sqrt{q_0^2})$ (Bauer, Ligeti, Luke '00, Neubert '00)

    Normalizing by $\int_{q_0^2}^{m_b^2} dq^2 \Gamma(B \to X_u \ell \nu)$ reduces the effect of

    $1/m_b^3$ corrections (Ligeti, Tackmann '07)

Effective Hamiltonian

- Decay amplitude is given by matrix elements of an effective Hamiltonian:
  $$\langle s\ell^+\ell^-|\mathcal{H}_{\text{eff}}|b\rangle = \sum_i C_i \langle s\ell^+\ell^-|\mathcal{O}_i|b\rangle$$
  with

$$
\begin{array}{lcl lcl}
\mathcal{O}_1 & = & (\bar{s}_L\gamma_\mu T^a c_L)(\bar{c}_L\gamma^\mu T^a b_L) & \mathcal{O}_2 & = & (\bar{s}_L\gamma_\mu c_L)(\bar{c}_L\gamma^\mu b_L) \\
\mathcal{O}_3 & = & (\bar{s}_L\gamma_\mu b_L)\sum_q(\bar{q}\gamma^\mu q) & \mathcal{O}_4 & = & (\bar{s}_L\gamma_\mu T^a b_L)\sum_q(\bar{q}\gamma^\mu T^a q) \\
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\mathcal{O}_7 & = & \frac{e}{g_s^2} m_b(\bar{s}_L\sigma^{\mu\nu} b_R)F_{\mu\nu} & \mathcal{O}_8 & = & \frac{1}{g_s} m_b(\bar{s}_L\sigma^{\mu\nu} T^a b_R)G^a_{\mu\nu} \\
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- Wilson coefficients C_i contain physics of the order m_t and M_W and resum large logarithms $\ln(m_b/M_W)$:
  LL: $(\alpha_s \ln \frac{m_b}{M_W})^n$, NLL: $\alpha_s(\alpha_s \ln \frac{m_b}{M_W})^n$, NNLL: $\alpha_s^2(\alpha_s \ln \frac{m_b}{M_W})^n$

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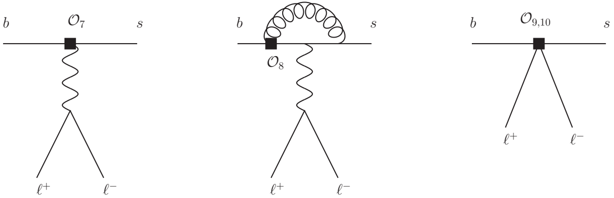
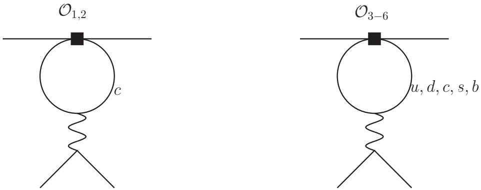
- Note extra factor $1/g_s^2$ in $\mathcal{O}_9$
$\Rightarrow$ Counting for the matrix elements: LL $\sim \alpha_s^{-1}$, NLL $\sim \alpha_s^0$, NNLL $\sim \alpha_s^1$,

Typical diagrams

- Two-quark operators



- Four-quark operators



$\Rightarrow$ lead to $c\bar{c}$ resonances that spoil OPE

Status of the calculation

- Wilson Coefficients up to NNLL

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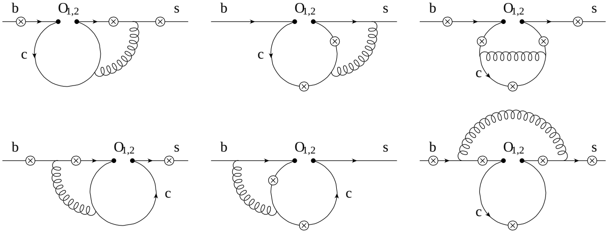
  - NNLL of $\langle \mathcal{O}_1 \rangle$ and $\langle \mathcal{O}_2 \rangle$
    - Low q^2: Expansion in m_c/m_b and q^2/m_b^2 Asatrian, Asatryan, Greub, Walker '01 '02 '02
    - High q^2:
      Numerically Ghinculov, Hurth, Isidori, Yao '04
      Analytically in an expansion in m_c/m_b Greub, V.P., Schüpbach '08

NNLL calculation in the high q^2 region

- Diagrams occurring at NNLL



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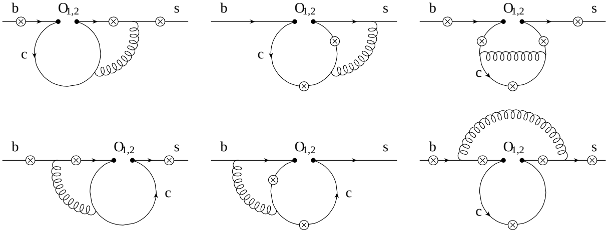

- Two ratios of scales: q^2/m_b^2 and m_c/m_b
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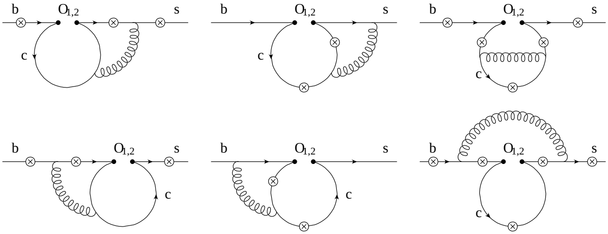

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  High q^2 region $\Rightarrow$ We keep $q^2 = \mathcal{O}(m_b^2)$ and expand in m_c/m_b
- Due to slow convergence we need powers up to $(m_c/m_b)^{20}$ to obtain an error less than 1%

Evaluation of two-loops Feynman integrals

- Reduction of tensor integrals to scalar integrals via Passarino-Veltman

$$\int d^d k_1 d^d k_2 \frac{[k_1^{\mu_1} \ldots k_1^{\mu_m}][k_2^{\nu_1} \ldots k_2^{\nu_n}]}{\prod D_i(k_1, k_2, p_{\text{extern}})} =$$
$$p_{\text{ext.}}^{\mu_1} \ldots p_{\text{ext.}}^{\nu_n} S_1 + g^{\mu_1,\mu_2} p_{\text{ext.}}^{\mu_3} \ldots p_{\text{ext.}}^{\nu_n} S_2 + \ldots$$

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- Reduction of scalar integrals to a set of simpler master integrals via integration by parts identities

$$0 = \int d^d k \, p^\mu \frac{\partial}{\partial k^\mu} f(k)$$

$\Rightarrow \mathcal{O}(20)$ master integrals

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- Evaluation of master integrals in expansion in m_c/m_b

Power expansion of Feynman integrals

- Expansion of Feynman integrals in powers of $z = m_c^2/m_b^2$ by solving a set of differential equations in z

$$\frac{d}{dz} I_\alpha = \sum_\beta h_{\alpha\beta} I_\beta + g_\alpha$$

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- Ansatz: Expansion of I_α in powers of z and $\ln z$

$$I_\alpha = \sum_{i,j,k} I_{\alpha,i}^{(j,k)} \epsilon^i z^j \ln^k z$$

Additionally expand $h_{\alpha\beta}$ and g_α in z:

$$h_{\alpha\beta} = \sum_{ij} h_{\alpha,i}^{(j)} \epsilon^i z^j \quad \text{and} \quad g_\alpha = \sum_{i,j,k} g_{\alpha,i}^{(j,k)} \epsilon^i z^j \ln^k z$$

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- Set of algebraic equations

$$0 = (j+1) I_{\alpha,i}^{(j+1,k)} + (k+1) I_{\alpha,i}^{(j+1,k+1)} - \sum_\beta \sum_{i'} \sum_{j'} h_{\alpha\beta,i'}^{(j')} I_{\beta,i-i'}^{(j-j',k)} - g_{\alpha,i}^{(j,k)}$$

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- This formalism also allows for numerical evaluation of the coefficients in the expansion $\Rightarrow$ additional cross-check.

A short description of this formalism

- Feynman parametrization:

$$I(z) \sim \int_0^1 d^{n-1}x \, \frac{1}{(z f_1(\vec{x}) + f_2(\vec{x}))^{n-d/2}}$$

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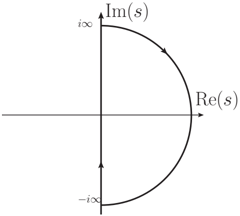
- Close integration contour to the right half



$\Rightarrow$ Summing up residua on the positive real axis leads to power expansion in z

$\Rightarrow$ $\ln(z)$ terms originate from terms like z^ϵ/ϵ

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- Location of the poles can be read off

$$s_{jN} = \frac{1 + N + A_j - B_j\epsilon}{C_j} \quad N \in \mathbb{N}_0$$

- We have $I(z) \sim \int_{-i\infty}^{i\infty} ds\, z^s \int_0^1 d^{n-1}x\, F(\vec{x}, s)$
  - Position of the poles in s give possible powers in z
  - We need information about the analytic structure of $\int_0^1 d^{n-1}x\, F(\vec{x}, s)$ without explicit evaluation of the integral
- Sector decomposition provides this property
  - Make sure that divergences in s come from integration over small x
  - Integral can be decomposed into terms like

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$\Rightarrow$ Ansatz

$$I(z) = \sum_{i,j,k \in S} I_i^{(j,k)} \epsilon^i z^j \ln^k z$$

where the set of indices S is known

Numerical convergence of the power expansion

- Decomposition of matrix elements

$$\langle s\ell^+\ell^-|\mathcal{O}_i|b\rangle_{\text{2-loops}} = -\left(\frac{\alpha_s}{4\pi}\right)^2\left[F_i^{(7)}\langle\mathcal{O}_7\rangle_{\text{tree}} + F_i^{(9)}\langle\mathcal{O}_9\rangle_{\text{tree}}\right]$$

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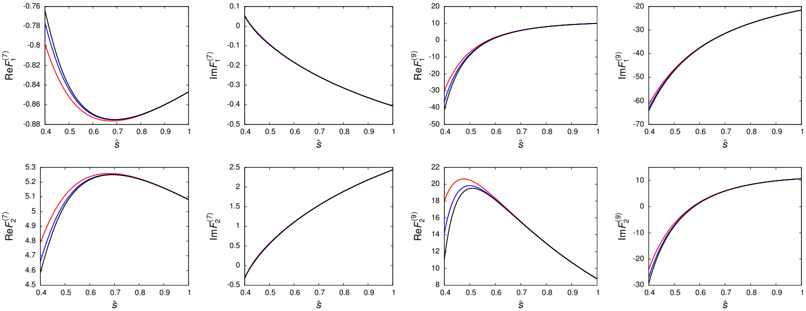

Here $z = 0.1$, $\hat{s} = q^2/m_b^2$, red curve: up to $\mathcal{O}(z^6)$, blue curve: up to $\mathcal{O}(z^8)$, black curve: up to $\mathcal{O}(z^{10})$

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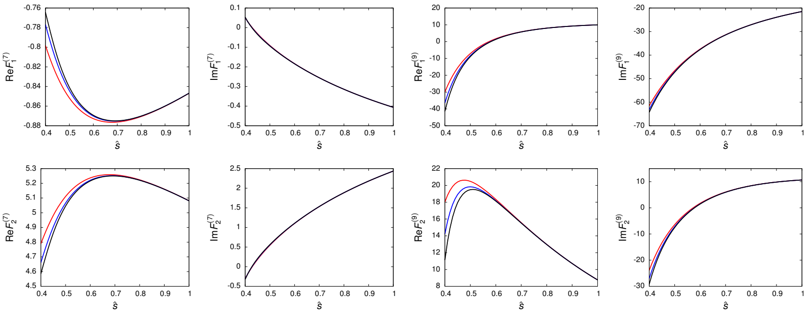

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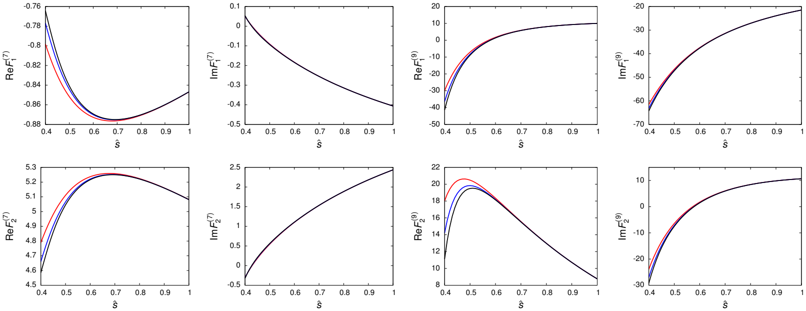

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- By comparison with numerical calculation of Ghinculov et al. we find deviation less than 1%

Numerical impact of $\langle \mathcal{O}_{1,2} \rangle_{\text{2-loops}}$ on the BRs

- Simple ratio with small dependence on $m_{b,pole}$:

$$R(\hat{s}) = \frac{1}{\Gamma(\bar{B} \to X_c e^- \bar{\nu}_e)} \frac{d\Gamma(\bar{B} \to X_s \ell^+ \ell^-)}{d\hat{s}}$$

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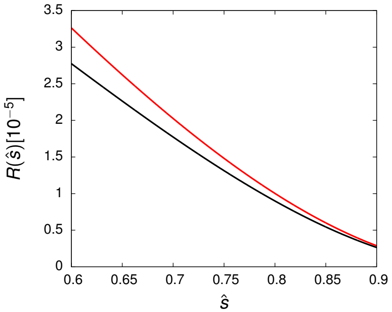
- Significant effect of 2-loops contribution on $R(\hat{s})$ of the order 10%



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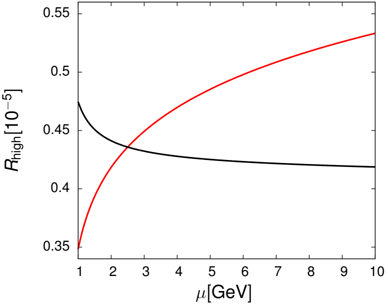
- Reduction of scale-dependence of $R_{\text{high}} = \int_{0.6}^{1} d\hat{s} R(\hat{s})$ to 2% $(2\text{GeV} \leq \mu \leq 10\text{GeV})$



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- Combining method of regions with differential equation techniques we obtained an expansion in m_c/m_b of the Feynman integrals
- This analytical result confirmed a former numerical calculation and is now completely published