QCD Sum Rule Predictions for Leptonic and Semileptonic B and D Decays

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Content

- Status of the decay constants f_B and $f_{D_{(s)}}$:
  - QCD sum rules and bounds
- Light-cone sum rules for $B \to \pi$ form factor and $|V_{ub}|$
- new calculation of $D \to \pi$ and $D \to K$ form factors:
  - decreasing the theory error in $|V_{cd}|$ and $|V_{cs}|$
  - predicting the form factor shape

B & $D_{(s)}$ decay constants

- leptonic channels: $B^- \to l^- \nu_l$, $\quad (l = \tau)$

$$\Gamma(B^- \to l^- \bar{\nu}_l) = \frac{G_F^2 |V_{ub}|^2}{8\pi} m_l^2 m_B \left(1 - \frac{m_l}{m_B}\right)^3 f_B^2 \,,$$

  in charm sector: $D^+_{(s)} \to l^+ \nu_l$, sensitive to $|V_{cd(s)}|$, need $f_{D_{(s)}}$
- hypothetical new physics effects ($H^\pm$, SUSY, ..)
- f_{B_d} (f_{B_s}) determine the rare decay $B_{(s)} \to l^+ l^-$
- current precision:

  $BR(B^- \to \tau\bar{\nu}) = (1.65 \pm 0.38 \pm 0.38) \times 10^{-4}$ [Belle '08]
  $\pm 4\%$ projected at SuperB (for 75 ab^{-1})

  $BR(D_s^+ \to \mu^+\bar{\nu}) = (5.65 \pm 0.45 \pm 0.17) \times 10^{-3}$ [CLEO '09]
  $BR(D^+ \to \mu^+\bar{\nu}) = (3.82 \pm 0.32 \pm 0.09) \times 10^{-4}$ [CLEO '08]
- need decay constants with comparable accuracy

QCD calculation of decay constants

- lattice QCD: see the talk by C. Tarantino
- "non-lattice methods" $\Rightarrow$ QCD sum rules:
  Correlator of quark currents = hadronic sum
  $\Uparrow$
  {dispersion relation $\oplus$ unitarty}
- approximate analytic calculation of the correlator,
  based on operator-product expansion (OPE)

QCD sum rule for f_B

- Correlator of two $j_5 = m_b \bar{b} i\gamma_5 u$ currents, $\langle 0|j_5|B\rangle = m_B^2 f_B$

$$\int d^4x\, e^{iqx} \langle 0|T\{j_5(x) j_5^\dagger(0)\}|0\rangle = \frac{\langle 0|j_5|B\rangle\langle B|j_5^\dagger|0\rangle}{m_B^2 - q^2} + \sum_h \frac{\langle 0|j|h\rangle\langle h|j^\dagger|0\rangle}{m_h^2 - q^2}$$

$$q^2 \ll m_b^2 \quad \Downarrow \quad x \to 0$$

$$\boxed{\sum_{d=0,3,4,..} C_d(q^2, m_b, \alpha_s)\langle 0|O_d(0)|0\rangle} \quad \Leftarrow \text{OPE}$$

$d \neq 0$, $\langle 0|O_d|0\rangle$- vacuum condensates

$O_3 = \bar{q}q$, $O_4 = G_{\mu\nu}G^{\mu\nu}$, ...

- input: quark masses and QCD parameters

OPE Input

- use the correlators where the hadronic sum is known from experiment: determine quark masses and/or condensates
- quarkonium sum rules:

  $j\mu = \bar{b}\gamma_\mu b \Rightarrow \{\Upsilon(1S), \Upsilon(2S), ..\} \Rightarrow m_b$
  $j\mu = \bar{c}\gamma_\mu c \Rightarrow \{J/\psi(1S), \psi(2S), ..\} \Rightarrow m_c, \langle GG \rangle$

  the talk by J. Kühn
- strange quark $J^P = 0^-$ or $J^P = 0^+$ currents
  $\Rightarrow \{K, ...\}$ or $\{K\pi, ...\} \Rightarrow m_s \oplus$ ChPT $\Rightarrow m_{u,d}, \langle \bar{q}q \rangle$
  [A.K., K. Chetyrkin (2005), M.Jamin, Oller, A.Pich (2006)]

Use of QCD sum rule

- correlator calculated including $d \leq 6$ (power suppressed) and $O(\alpha_s^2)$ terms
- $\sum_h$ =quark-hadron duality approximation $\Rightarrow$ m_B, $\langle 0|j|h\rangle$
- "systematic" uncertainty minimized by Borel transform., fixing the hadron mass from SR
- currently: $\sim$ 10% error
  no way to get a better accuracy in future
- the spectral density is positive definite
  $\Rightarrow$ upper bounds independent from duality

OPE bounds for f_{D_s} and f_D

[A.K., hep/ph-0812.3747]

the hadronic matrix element:

$$(m_c + m_s)\langle 0|\bar{s} i\gamma_5 c|D_s\rangle = f_{D_s} m_{D_s}^2$$

- Correlation function of two charmed-strange currents:

$$j_5(x) = (m_c + m_s)\bar{s}(x) i\gamma_5 c(x)$$

$$\Pi(q^2) = i \int d^4x e^{iqx} \langle 0 \mid T\{j(x) j^\dagger(0)\} \mid 0\rangle$$
$$= \frac{f_{D_s}^2 m_{D_s}^4}{m_{D_s}^2 - q^2} + \sum_{h=D^*K,...} \frac{\langle 0 \mid j|h\rangle\langle h|j^\dagger \mid 0\rangle}{m_h^2 - q^2}$$

$s \to d$, $D_s \to D$,

Deriving the bound

- calculate $\Pi(q^2)$ and apply Borel transformation:

$$\Pi(M^2) = \sum_{n=0,1,2} \int\limits_{(m_c+m_d)^2}^{\infty} ds \left(\frac{\alpha_s}{\pi}\right)^n \rho^{(n)}(s) e^{-s/M^2}$$
$$+ \sum_{n=0,1} \left(\frac{\alpha_s}{\pi}\right)^n \Pi^{(n)}_{\langle \bar{q}q \rangle}(M^2) + \sum_{d=4,5,6} \Pi_d(M^2)\,. \qquad (1)$$

- equate to the hadronic sum and use the positivity of it:

$f_D^2 m_D^4 e^{-m_D^2/M^2} + ... = \Pi(M^2; m_c, m_s, \alpha_s, \text{cond.}, \ \mu,)$

- the same OPE as QCD SR , with no duality assumption involved

$\Rightarrow \ f_D < \sqrt{\Pi(M^2)/(m_D^4 e^{-m_D^2/M^2})}$

$M > 1.0$ GeV^2 and $\mu > 1.5$ GeV, OPE convergence

- bound for f_B is not constraining

B and $D_{(s)}$ decay constants [in MeV]

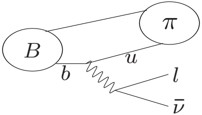
a sample of recent results:

method	f_B	f_D	f_{D_s}
exp. $\oplus$ CKM	$(242 \pm 28)\frac{3.99\times10^{-4}}{V_{ub}}$ *[Belle '08]*	$205.8 \pm 8.5 \pm 2.5$ *[CLEO'08], $V_{cd} = V_{us}$*	$259.5 \pm 6.6 \pm 3.1$ *[CLEO'09], $V_{cs} = V_{ud}$*
lattice	190 ± 13 *[HPQCD,'09]*	207 ± 4 *[HPQCD,UKQCD '08]*	241 ± 3 *[HPQCD,UKQCD '08]*
QCD SR	210 ± 19 *[Jamin-Lange '01]*	-	
	206 ± 20 *[Penin-Steinhauser'01]*	195 ± 20 *[Penin-Steinhauser'01]*	
		203 ± 20 *[Narison '02]*	235 ± 24 *[Narison '02]*
OPE bound	-	<230	<270

- f_{B_s}/f_B, SR in agreement with lattice QCD
- still some tension of exp. vs lattice f_{D_s} (and vs the bound)
- already some tension in f_B ?

Heavy-light form factors

- semileptonic channels: $\overline{B}_d \to \pi^+ l \bar{\nu}_l$



$\Rightarrow |V_{ub}|$

$$d\Gamma(\overline{B}_d \to \pi^+ l\bar{\nu}_l)/dq^2 = \frac{G_F^2 |V_{ub}|^2}{24\pi^3} p_\pi^3 |f^+_{B\pi}(q^2)|^2$$

semileptonic region: $0 < q^2 < (m_B - m_\pi)^2 = q^2_{max} \simeq 26.4\ \text{GeV}^2$

- in charm sector: $D \to \pi(K) l \nu_l$, sensitive to $|V_{cd(s)}|$, need $f^+_{D\pi_{(K)}}(q^2)$
- accurately measured BR's and slopes , $BR(\bar{B}^0 \to \pi^- l \bar{\nu}) = (1.36 \pm 0.09) \times 10^{-4}$ [PDG '08]
- need form factors $f^+_{B\pi}$, $f^+_{D\pi(K)}$ with comparable accuracy
- $B \to P$ form factors also for $B \to PP$, $B \to Pll$, $(P = \pi, K)$

Light-cone sum rule (LCSR) for $B \to \pi$ form factor

(also the talk by Patricia Ball)

- the correlator of $j_5 = m_b \bar{b} i\gamma_5 d$ and $j_\mu^W = \bar{u} i\gamma_\mu b$

$$\int d^4x e^{iqx} \langle \pi(p)| T\{j_5(x) j_W(0)\}|0\rangle = \frac{\langle 0|j_5|B\rangle\langle\pi|j_W|B\rangle}{m_B^2-(p+q)^2} + \sum_h \frac{\langle 0|j_5|h\rangle\langle\pi|j_W|h}{m_h^2-(p+q)^2}$$

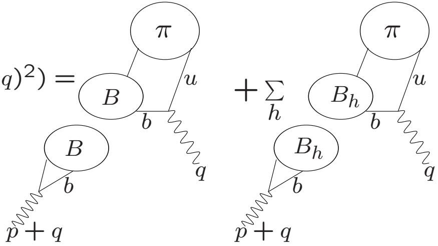
$$|q^2| \sim |(p+q)^2| \ll m_b^2 \Downarrow \quad x^2 \to 0$$

$$\boxed{\sum_{t=2,3,..} C_t(q^2,(p+q)^2,m_q,\alpha_s)\langle\pi(p)|O_t(x,0)|0\rangle} \Leftarrow \text{OPE}$$

- $\langle\pi(p)(|O_t(x,0)|0\rangle$ - pion light-cone distribution amplitudes
$O_2 \to \bar{u}(x)\gamma_\mu\gamma_5 d(0)$ twist 2, Gegenbauer moments

LCSR , outline of derivation

$F(q^2,(p+q)^2)$ - analytical resut of light-cone OPE for the correlator,
valid at $q^2,(p+q)^2 \ll m_b^2$

$F(q^2,(p+q)^2) =$ 

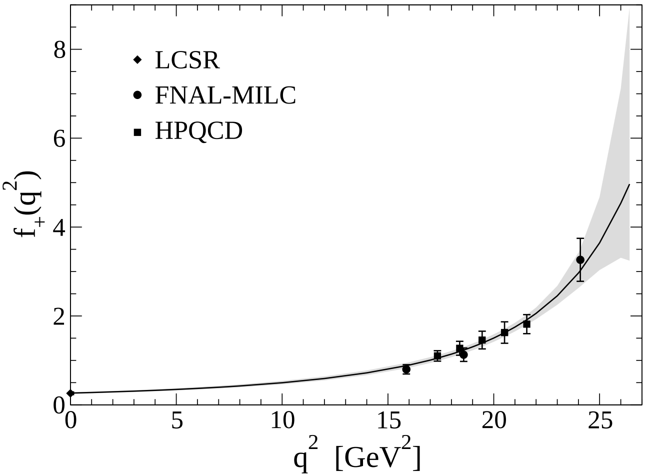
$f_B f^+_{B\pi}(q^2)$ $\qquad$ $\sum_{B_h} \rightarrow duality\ (s_0^B)$

- calculation at finite m_b
- Gegenbauer moments: pion form factor, 2-point sum rules
- accuracy achieved $t \leq 4$, $O(\alpha_s)$
- the form factor includes both "soft" and "hard scattering terms

$B \to \pi$ form factor and $|V_{ub}|$

- the current status accumulated in a single figure:

from [Bourrely, Caprini, Lellouch, 0807.222 hep-ph]



- $q^2 = 0$: Light-cone sum rules (LCSR) , recent update

[G.Duplancic, A.K., B.Melic, Th.Mannel, N.Offen (2007)

$$f^+_{B\pi}(0) = 0.26^{+0.04}_{-0.03}$$

- large q^2 lattice *[FNAL-MILC, HPQCD]*:

$f^+(q^2)$ with errors at the level of $\pm 12\%$

- the curve: analyticity $\oplus$ "conformal mapping" $q^2 \to z$

$\Rightarrow$ series (in z) parameterization

- fitting theory $\oplus$ exp. shape dBR/dq^2 [BaBar, Belle]

to series parameterization

Recent $|V_{ub}|$ determinations from $B \to \pi l \nu_l$

[ref.]	$f^+_{B\pi}(q^2)$ calculation	$f^+_{B\pi}(q^2)$ input	$\lvert V_{ub}\rvert \times 10^3$
Okamoto et al. '05	lattice ($n_f = 3$)	-	3.78±0.25±0.52
HPQCD '06	lattice ($n_f = 3$)	-	3.55±0.25±0.50
Flynn et al '07	-	lattice $\oplus$ LCSR	$3.47 \pm 0.29 \pm 0.03$
Ball, Zwicky '04	LCSR	-	$3.5 \pm 0.4 \pm 0.1$
DKMMO '07	LCSR	-	$3.5 \pm 0.4 \pm 0.2 \pm 0.1$
Bourrely, Caprini, Lellouch '08	-	lattice$\oplus$ LCSR	3.54 ± 0.24

- $f^+_{B\pi}$ from "lattice " and "non-lattice" (LCSR) have comparable uncertainties,
- this will change , if lattice calculations achieve the claimed goal of $\sim 1 \div 2\%$ accuracy [App.A SuperB report '07]
not achievable with LCSR ...
- other "non-lattice" tools:
  - sum rules in effective theories (HQET,SCET)

    [LCSR in SCET F. De Fazio, Th. Feldmann T.Hurth '06
  - LCSR with B meson distribution amplitudes
    [A.K., N. Offen, Th. Mannel '06]
    - new application $B \to D^{(*)}$, talk by Ch. Klein

$D \to \pi, K$ form factors from LCSR

[Ch.Klein, A.K., Th. Mannel, N.Offen, paper in prepar.]

- replace $b \to c$ in the correlator /LCSR for $B \to \pi$, $\bar{u} \to \bar{d}(\bar{s}) \Longrightarrow$ LCSR for $D \to \pi(K)$
- input updates:
  - $\overline{MS}$ for c-quark mass, instead of pole mass,
  - $m_s(2GeV) = 98 \pm 16$ MeV, QCD SR with α_s^4 accuracy
  - Gegenbauer moments: $a_2^\pi = 0.16 \pm 0.01$ $a_4^\pi = 0.04 \pm 0.01$

  constrained from fitting the $B \to \pi$ shape to experiment,

  $a_1^K = 0.10 \pm 0.04$ from new determination

  [K.Chetyrkin, A.K., A.Pivovarov (2008)]
- using very accurate experimental number for f_D

Comparison of $D \to \pi, K$ form factors at $q^2 = 0$

Method	[Ref.]	$f^+_{D\pi}(0)$	$f^+_{DK}(0)$
Lattice QCD	[APE(2001)]	$0.57 \pm 0.06 \pm 0.02$	$0.66 \pm 0.04 \pm 0.01$
	[Aubin et al (2005)]	$0.64 \pm 0.03 \pm 0.06$	$0.73 \pm 0.03 \pm 0.07$
	[QCDSF(2009)]	$0.74 \pm 0.06 \pm 0.04$	$0.78 \pm 0.05 \pm 0.04$
LCSR	[A.K. et al. (2000)]	0.65 ± 0.11	$0.78^{+0.2}_{-0.15}$
	[P. Ball (2006)]	0.63 ± 0.11	0.75 ± 0.12
	this work (prel.)	$0.67^{+0.10}_{-0.07}$	$0.74^{+0.11}_{-0.08}$

theoretical uncertainties added in quadrature

$$\frac{f^+_{D\pi}(0)}{f^+_{DK}(0)} = 0.90 \pm 0.06 \,,$$

some uncertainties cancel, also agrees with [P.Ball (2006)]

Determination of $|V_{cd}|$ and $|V_{cs}|$

- LCSR predicts the product, $[f_D f_{D\pi}(0)]_{LCSR} = 138^{+20}_{-13}$ MeV
- previously a two-point SR was used for f_D
  (adding own uncertainty, although some errors compensated)
- $CLEO_c$ data:

$$BR(D \to l\nu_l) \Rightarrow f_D|V_{cd}| = 46.5 \pm 2.0 \text{ MeV}$$

  (they quote f_D assuming $|V_{cd}| = |V_{us}|$)

$$dBR/dq^2(D \to \pi l\nu_l) \Rightarrow f_{D\pi}(0)|V_{cd}| = 0.143 \pm 0.005 \pm 0.002,$$

  (from the series parameterization fit of the q^2-bins)
- multiply two exp. numbers and divide by LCSR prediction:

$$|V_{cd}| = 0.219 \pm [0.005]_{exp1} \pm [0.004]_{exp2}{}^{+0.016}_{-0.010},$$

  the LCSR error is effectively halved !
  CLEO$\oplus$ lattice: $|V_{cd}| = 0.223 \pm 0.008 \pm 0.003 \pm 0.023$

- the same for $|V_{ub}|$, if $B \to l\nu_l \to f_B|V_{ub}|$ will have a smaller error
- from the predicted ratio of $D \to \pi$ and $D \to K$ form factors and CLEO data (f_D cancels):

$$\frac{|V_{cd}|}{|V_{cs}|} = 0.214 \pm [0.008]_{exp} \pm [0.002]_{exp} \pm 0.014\,,$$

$$|V_{cs}| = 1.03 \pm [0.08]_{ratio} \left[{}^{+0.08}_{-0.06}\right]_{V_{cd}}\,,$$

- compare with CLEO$\oplus$ lattice: $|V_{cs}| = 1.019 \pm 0.019 \pm 0.007 \pm 0.106$

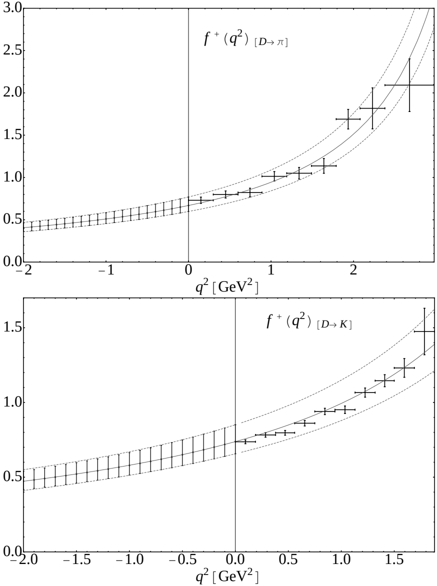
all results still preliminary, some correlations not accounted for

Predicting the shape of $D \to \pi, K$ form factors

- LCSR valid at $q^2 < m_c^2 - 2m_c\tau, \quad \tau \sim 1$ GeV scale
  $\Rightarrow$ only $q^2 \leq 0$ accessible
- the semileptonic region is larger:
  $(m_D - m_{\pi(K)})^2 = 2.98(1.88) GeV^2$
- new : calculating the form factor at negative q^2 and accessing the whole semileptonic region via analytic continuation (applyng BK parameterization of dispersion relation or z expansion)
-

$D \to \pi$ and $D \to K$ form factor shapes

(preliminary)



Outlook

- "non-lattice" decay constants and form factors :
  QCD sum rules /bounds and LCSR agree with and complement the lattice QCD results
- the level of accuracy in the correlators cannot be essentially improved, task for future: refine/adjust the input, calculate twist-5 terms in LCSR
- combining various exp. data may help,
  more accurate data on the shapes in semileptonic decay distributions
- quark-hadron duality approximation is the main concern
  more data on the hadron spectroscopy in $D_{(s)}$ and B channels : radial excitations