Form Factors and Exclusive Decays

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Part I: Bloody Form Factors

Part I: Ever-so-difficult Form Factors

Form Factors of $B \to V(1^-)$ Transitions

$$\langle V(p)|\bar{q}\gamma_\mu(1-\gamma_5)b|B(p_B)\rangle = -ie^*_\mu(m_B+m_V)A_1(q^2) + i(p_B+p)_\mu(e^*q)\,\frac{A_2(q^2)}{m_B+m_V}$$

$$+\, iq_\mu(e^*q)\,\frac{2m_V}{q^2}\,\left(A_3(q^2)-A_0(q^2)\right) + \epsilon_{\mu\nu\rho\sigma}\epsilon^{*\nu}p_B^\rho p^\sigma\,\frac{2V(q^2)}{m_B+m_V}$$

$$\text{with } A_3(q^2) = \frac{m_B+m_V}{2m_V}\,A_1(q^2) - \frac{m_B-m_V}{2m_V}\,A_2(q^2) \;\text{ and }\; A_0(0) = A_3(0);$$

$$\langle V(p)|\bar{q}\sigma_{\mu\nu}q^\nu(1+\gamma_5)b|B(p_B)\rangle = i\epsilon_{\mu\nu\rho\sigma}\epsilon^{*\nu}p_B^\rho p^\sigma\,2T_1(q^2) + T_2(q^2)\left\{e^*_\mu(m_B^2-m_V^2)\right.$$

$$\left. -\,(e^*q)\,(p_B+p)_\mu\right\} + T_3(q^2)(e^*q)\left\{q_\mu - \frac{q^2}{m_B^2-m_V^2}\,(p_B+p)_\mu\right\}$$

A_0 is also the form factor of the pseudoscalar current:

$$\langle V|\partial_\mu A^\mu|B\rangle = (m_b+m_q)\langle V|\bar{q}i\gamma_5 b|B\rangle = 2m_V(e^*q)A_0(q^2).$$

Calculational Tools

- 8 independent form factors
- needed over a large kinematical range/momentum transfer: $0 \leq q^2 \leq 20\,\mathrm{GeV}^2$
- intrinsically non-perturbative quantities
- your options:
  - lattice: difficult for small q^2 & vectors, i.e. unstable particles (most recent calculation: $T_1^{B\to K^*}(0)$ by Becirevic et al., hep-ph/0611295)
  - QCD sum rules on the light-cone: good for small q^2, i.e. large final-state energies, unsuitable at very large q^2 (non-convergence of twist-expansion)
  - perturbative QCD: in connection with non-leptonic decays (e.g. Ali et al. hep-ph/0703162)
  - quark models etc.: no comment

Light-Cone Sum Rules in a Nutshell

light-cone/twist expansion:

correlation function $i\int d^4y e^{-ipy}\langle \bar{K}^*(p)|T\mathcal{O}(0)j_B^\dagger(y)|0\rangle$

$$\sim \sum_n T_H^{(n)} \otimes \phi^{(n)}.$$

- $\mathcal{O}$: weak current/effective operator
- $T_H^{(n)}$: perturbative scattering kernels
- $\phi^{(n)}$: non-perturbative parton distribution functions
- n: twist: 2, 3, 4, ...

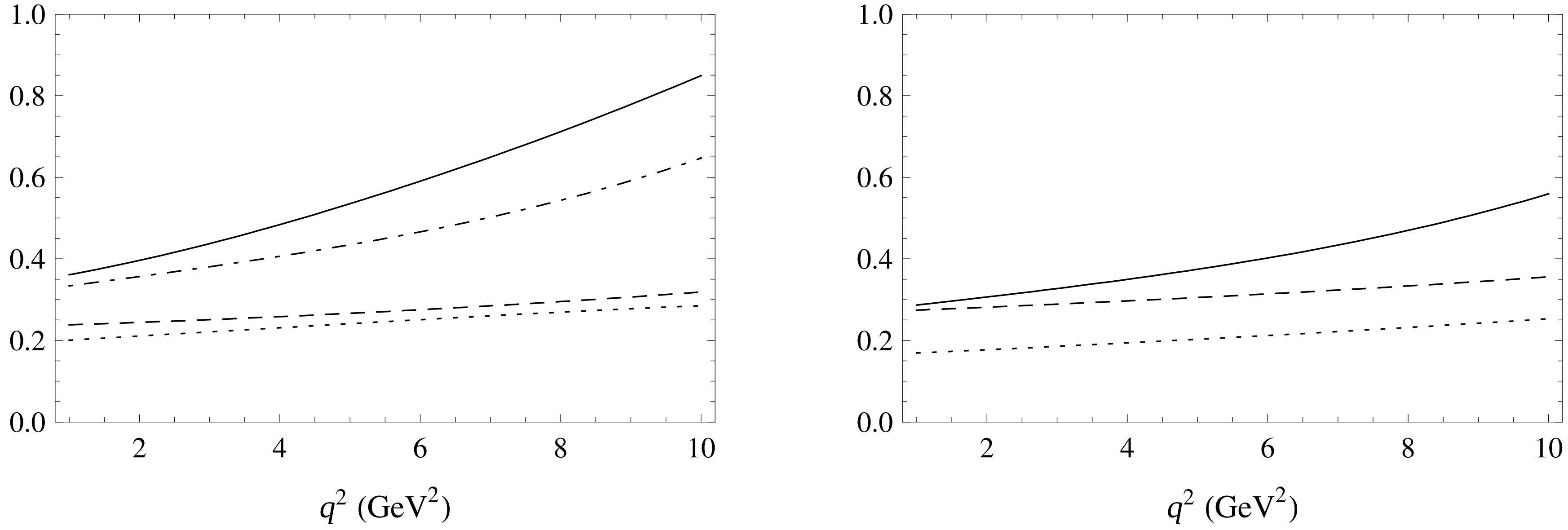
Saturate correlation function by hadronic states, isolate ground state (B meson), get final sum rule: (see talk by Khodjamirian)

$$\frac{m_B^2 f_B}{m_b} T_1(0) e^{-m_B^2/M^2} \equiv m_b \int_{u_0}^1 du\, e^{-m_b^2/(uM^2)}$$

$$\times \left[f_{K^*}^\perp R_{\text{twist 2}}(u) + f_{K^*}^\parallel \frac{m_{K^*}}{m_b} R_{\text{twist 3}}(u) + f_{K^*}^\perp \left(\frac{m_{K^*}}{m_b}\right)^2 R_{\text{twist 4}}(u) + O\left(\frac{m_{K^*}^3}{m_b^3}\right)\right]$$

Status of Light-Cone Sum Rules

- latest results for $B \to \rho, \omega$ in Ball/Zwicky 04, $q^2_{\rm max} \approx 15\,{\rm GeV}^2$
- latest results for $B \to K^*$ in Altmannshofer et al. 0811.1214:



- include twist 2 ($O(\alpha_s)$), 3 ($O(\alpha_s)$) and 4 ($O(1)$)
- new strategy for symmetries/asymmetries in 0811.1214:
  Only ratios of FFs needed: constrain overall normalisation (f_B) from $B \to K^*\gamma \rightsquigarrow$ small errors $\sim O(10\%)$
- otherwise: error $\sim O(15-20\%)$ (Ball/Zwicky 04)

The Heavy Quark Limit

In the HQL the number of form factors reduces from 8 to 2:

$$\xi_\perp(q^2) = \frac{m_B}{m_B + m_{K^*}} V(q^2),$$
$$\xi_\parallel(q^2) = \frac{m_B + m_{K^*}}{2E} A_1(q^2) - \frac{m_B - m_{K^*}}{m_B} A_2(q^2).$$

In practice, the above (or similar) formulas (supplemented by hard perturbative corrections) are used to determine ξ from $A_{1,2}, V$.

Questions:

1. Do light-cone sum rules fulfill the HQL relations imposed by the reduction from 8 to 2 form factors?
2. What is the size of $1/m_b$ corrections induced in $\xi_{\perp,\parallel}$ by $A_{1,2}, V$?

Light-Cone Sum Rules in the HQL: $\xi_\parallel$

$$\frac{(m_B + m_{K^*})/(2E)\, A_1(q^2) - (m_B - m_{K^*})/m_B\, A_2(q^2)}{(m_{K^*}/E)\, A_0(q^2)} \quad = \quad 1 + O(\alpha_s)$$

Note that numerator appears to start at twist 2, but denominator is twist 3, hence twist 2 in numerator must cancel!

Twist-2 contributions to numerator:

$$\begin{aligned}
&(m_B + m_{K^*})/(2E)\, A_1 - (m_B - m_{K^*})/m_B\, A_2 \\
\sim \quad & \frac{m_b\, e^{m_B^2/M^2}}{m_B^2 f_B} \int_{u_0}^1 du\, e^{-\frac{m_b^2 - \bar{u} q^2}{u M^2}} f_{K^*}^\perp \, \frac{\phi_\perp(u)}{u} \, \frac{q^2(m_b^2 - q^2 - u(m_B^2 - q^2))}{2 m_B (m_B^2 - q^2) u} \\
\neq \quad & 0
\end{aligned}$$

Oops, looks like a problem?

Light-Cone Sum Rules in the HQL: $\xi_\parallel$

$$\frac{m_b\, e^{m_B^2/M^2}}{m_B^2 f_B}\int_{u_0}^1 du\, e^{-\frac{m_b^2-\bar{u}q^2}{uM^2}} f_{K^*}^\perp\, \frac{\phi_\perp(u)}{u}\, \frac{q^2(m_b^2-q^2-u(m_B^2-q^2))}{2m_B(m_B^2-q^2)u} \neq 0 \quad ?$$

Recall that LCSRs are evaluated near the minimum in M^2: $(\bar{u}=1-u)$

$$\frac{d}{dM^2}\int_{u_0}^1 du\; e^{\frac{m_B^2}{M^2}-\frac{m_b^2-\bar{u}q^2}{uM^2}}\,\rho(u)=0$$

$$\longleftrightarrow\quad \int_{u_0}^1 du\, e^{\frac{m_B^2}{M^2}-\frac{m_b^2-\bar{u}q^2}{uM^2}}\,\frac{um_B^2-m_b^2+\bar{u}q^2}{u}\,\rho(u)=0$$

Hence twist-2 contribution to $\xi_\parallel$ vanishes if LCSR is evaluated at minimum in M^2!

HQL relation between $A_{0,1,2}$ is fulfilled to twist-3 accuracy also for $O(\alpha_s)$ corrections – but is violated at twist 4 $\rightsquigarrow$ power-suppressed corrections!

Caveat Emptor – Numerical Values for $\xi_{\|}$

Don't combine $A_1(q^2)$ and $A_2(q^2)$ à la

$$\xi_{\|}(q^2) \quad = \quad \frac{m_B + m_{K^*}}{2E}\, A_1(q^2) - \frac{m_B - m_{K^*}}{m_B}\, A_2(q^2)$$

Altmannhofer et al. 0811.1214:

- result from LCSR for $\xi_{\|}(0)$: $\boxed{0.118 \pm 0.008}$
- result from combination of LCSRs for $A_1(0)$ and $A_2(0)$:
  $\boxed{0.265 - 0.158 = 0.107 \pm 0.054}$
- for comparison, Egede et al. 0807.2589 quote 0.16 ± 0.03

Light-Cone Sum Rules in the HQL: $\xi_\perp$

$$\frac{A_1(q^2)}{V(q^2)} = \frac{2Em_B}{(m_B+m_{K^*})^2} \qquad \text{(no hard corrections)}$$

Explicit LCSRs:

$$A_1(q^2) = \frac{m_b\, e^{m_B^2/M^2}}{m_B^2 f_B (m_B+m_{K^*})} \int_{u_0}^1 du\, e^{-\frac{m_b^2-\bar{u}q^2}{uM^2}} \frac{1}{u} \Big[f_{K^*}^\perp (m_b^2-q^2) \frac{\phi_\perp(u)}{2u} + f_{K^*}^{\|} m_b m_{K^*}\, g_v(u) + \dots \Big]$$

$$V(q^2) = \frac{m_B+m_{K^*}}{2} \frac{m_b\, e^{m_B^2/M^2}}{m_B^2 f_B} \int_{u_0}^1 du\, e^{-\frac{m_b^2-\bar{u}q^2}{uM^2}} \frac{1}{u} \Big[f_{K^*}^\perp \phi_\perp(u) - \frac{u}{2(m_b^2-q^2)} f_{K^*}^{\|} m_b m_{K^*} \frac{d}{du} g_a(u) + \dots \Big]$$

Twist-2 ratio: $$\frac{A_1(q^2)}{V(q^2)} = \frac{m_B^2-q^2}{(m_B+m_{K^*})^2} \; : \qquad \text{OK in HQL!}$$

Light-Cone Sum Rules in the HQL: $\xi_\perp$

$$\frac{A_1(q^2)}{V(q^2)} = \frac{2Em_B}{(m_B+m_{K^*})^2}$$ (no hard corrections)

Explicit LCSRs:

$$A_1(q^2) = \frac{m_b\, e^{m_B^2/M^2}}{m_B^2 f_B (m_B+m_{K^*})}\int_{u_0}^1 du\, e^{-\frac{m_b^2-\bar{u}q^2}{uM^2}}\frac{1}{u}\left[f_{K^*}^\perp (m_b^2-q^2)\frac{\phi_\perp(u)}{2u}\right.$$
$$\left. + f_{K^*}^\parallel m_b m_{K^*}\, g_v(u) + \dots\right]$$

$$V(q^2) = \frac{m_B+m_{K^*}}{2}\frac{m_b\, e^{m_B^2/M^2}}{m_B^2 f_B}\int_{u_0}^1 du\, e^{-\frac{m_b^2-\bar{u}q^2}{uM^2}}\frac{1}{u}\left[f_{K^*}^\perp \phi_\perp(u)\right.$$
$$\left. -\frac{u}{2(m_b^2-q^2)} f_{K^*}^\parallel m_b m_{K^*}\frac{d}{du}g_a(u) + \dots\right]$$

Numerically: $\boxed{\xi_\perp(0) = 0.266 \pm 0.032}$

$1/m_b$ Corrections

Ratios in the HQL: $\dfrac{A_1(q^2)}{V(q^2)} = \dfrac{2Em_B}{(m_B+m_{K^*})^2}$

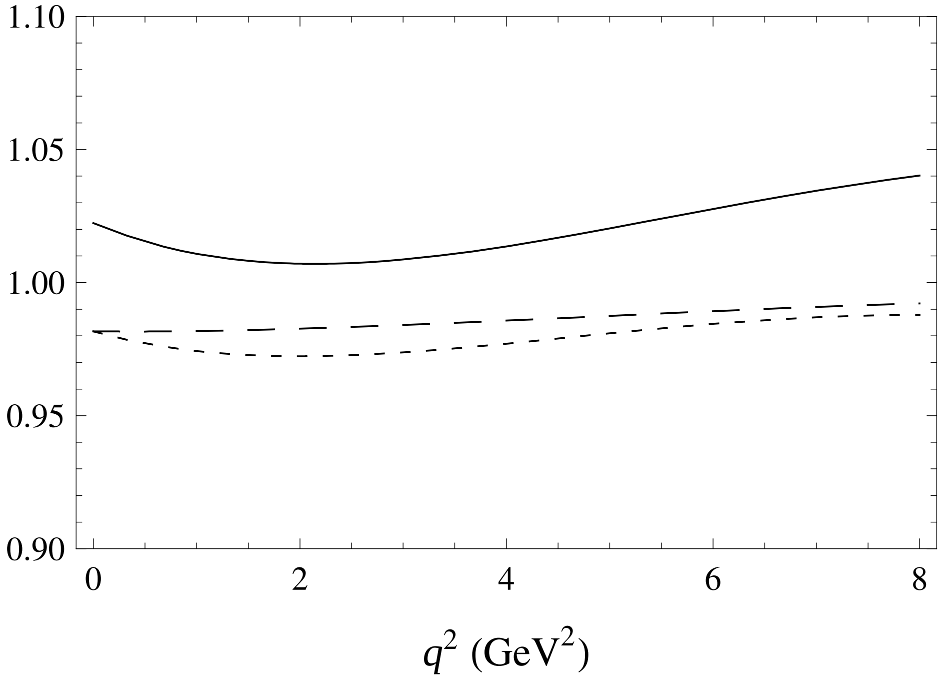
$$\frac{T_1(q^2)}{V(q^2)} = \frac{m_B}{m_B+m_{K^*}}\left(1+\frac{\alpha_s C_F}{4\pi}\left[\ln\frac{m_b^2}{\mu^2}-L\right]+\frac{\alpha_s C_F}{4\pi}\,\frac{m_B}{4E}\,\frac{\Delta F_\perp}{\xi_\perp(q^2)}\right),$$

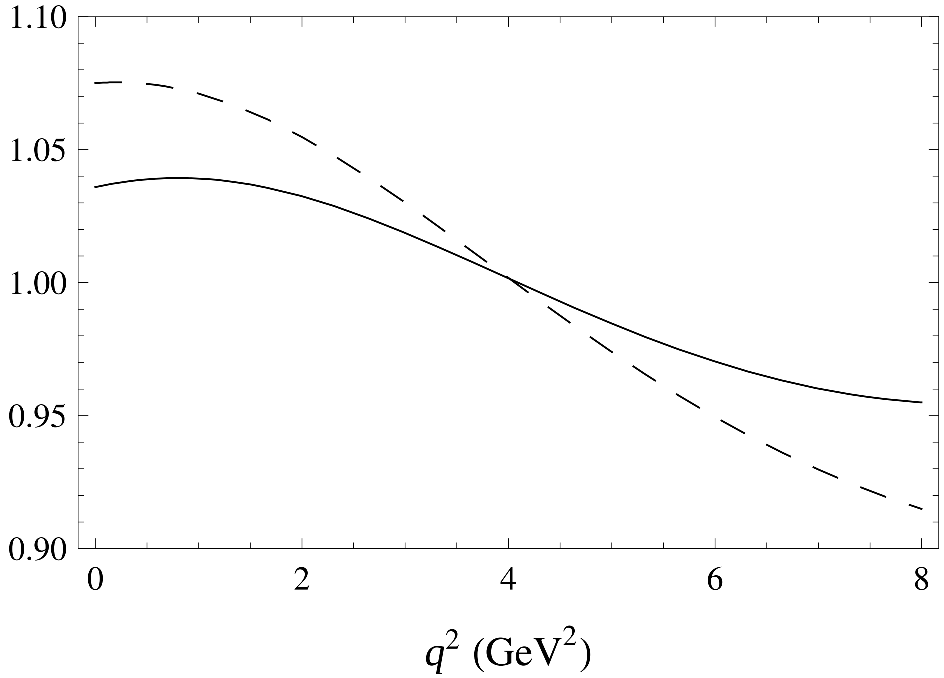
$$\frac{(m_B+m_{K^*})/(2E)\,A_1(q^2)-(m_B-m_{K^*})/m_B\,A_2(q^2)}{(m_{K^*}/E)\,A_0(q^2)} =$$

$$1+\frac{\alpha_s C_F}{4\pi}\left[-2+2L\right]-\frac{\alpha_s C_F}{4\pi}\,\frac{m_B(m_B-2E)}{(2E)^2}\,\frac{\Delta F_\parallel}{(E/m_{K^*})\,\xi_\parallel(q^2)},$$

$$\frac{(m_B/2E)\,T_2(q^2)-T_3(q^2)}{(m_{K^*}/E)A_0(q^2)} = 1+\frac{\alpha_s C_F}{4\pi}\left[\ln\frac{m_b^2}{\mu^2}-2+4L\right] -\frac{\alpha_s C_F}{4\pi}\left(\frac{m_B}{2E}\right)^2\frac{\Delta F_\parallel}{(E/m_{K^*})\,\xi_\parallel(q^2)},$$

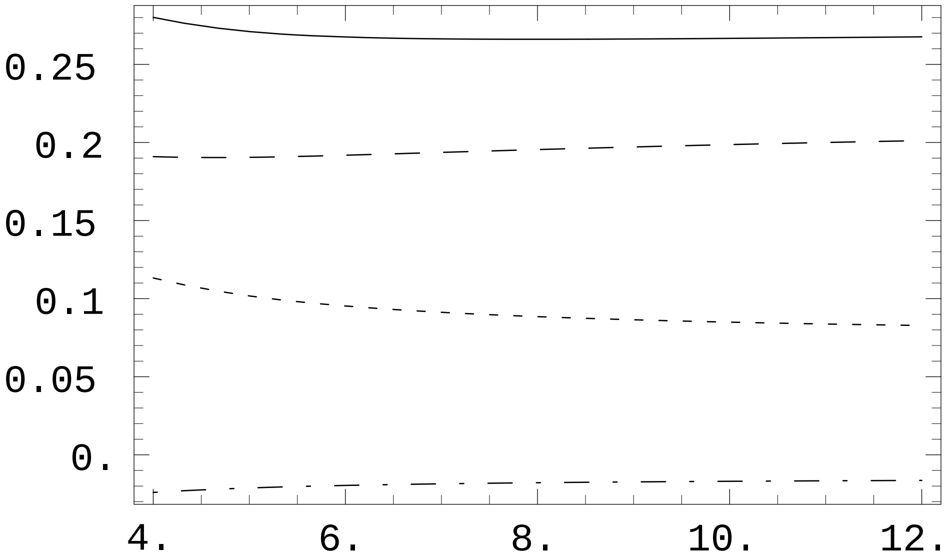
$1/m_b$ Corrections

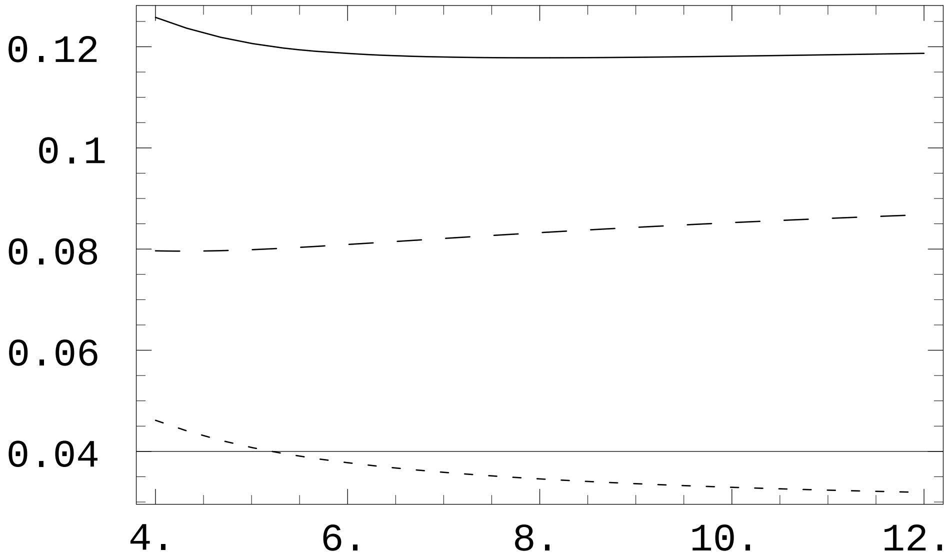




Ratios based on $\xi_\perp$ and on $\xi_\|$

$\xi_{\|,\perp}$ in the strict HQL





$\xi_\perp(0)$ and T2, T3, T4 contributions . . .

$\xi_\|(0)$ and T3, T4 contributions

Part II: Application to $B \to K^*\mu\mu$

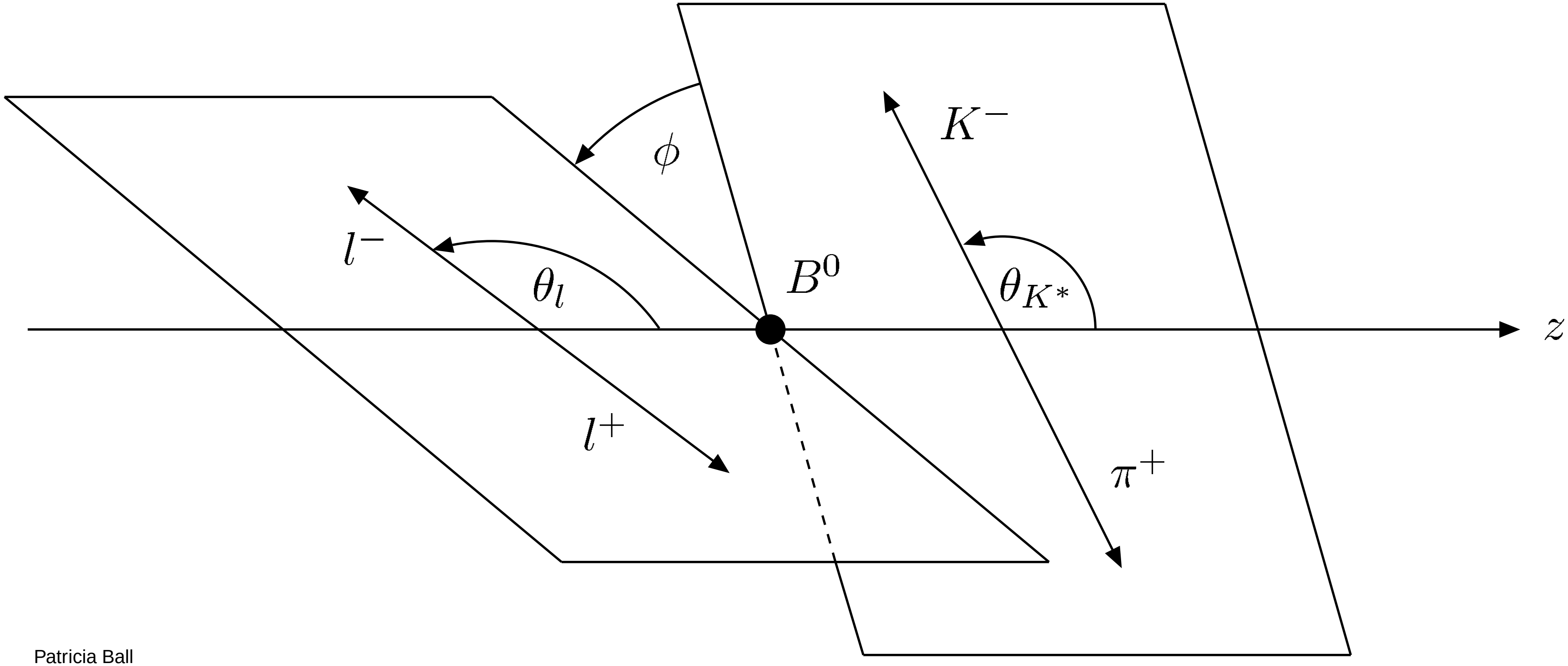
0811.1214

Kinematics

K^* decays to $K\pi$ ($\approx$100 %). Angular distribution of $K\pi$ indicative for polarisation of K^*.

Full angular spectrum:

$$\frac{d^4\Gamma}{dq^2\, d\cos\theta_l\, d\cos\theta_{K^*}, d\phi} = \frac{9}{32\pi} I(q^2, \theta_l, \theta_{K^*}, \phi)$$

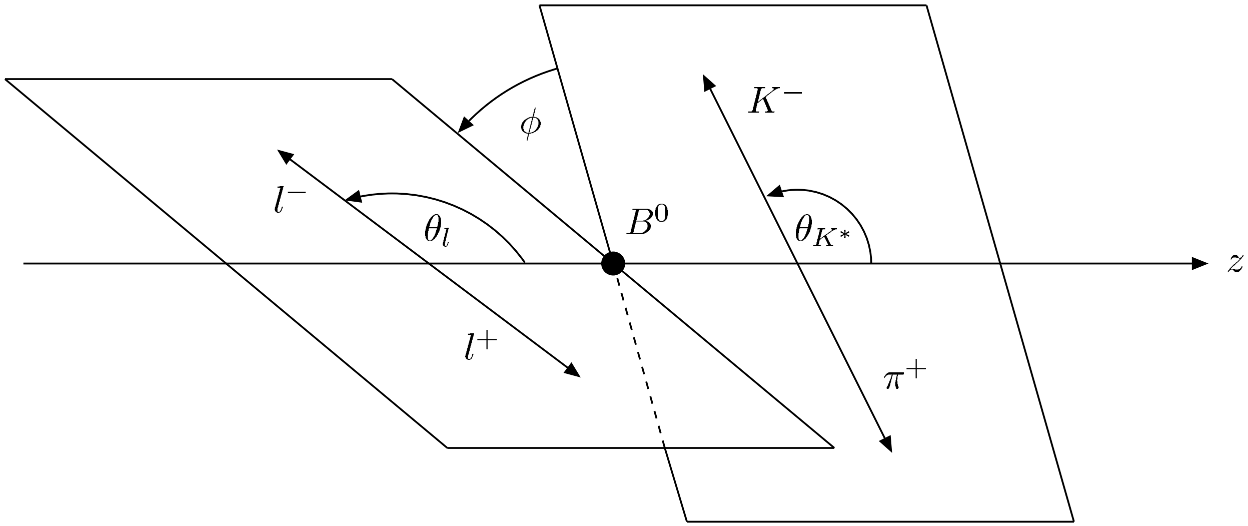


Kinematics

$$\frac{d^4\Gamma}{dq^2\, d\cos\theta_l\, d\cos\theta_{K^*},d\phi} = \frac{9}{32\pi} I(q^2,\theta_l,\theta_{K^*},\phi)$$

$$\begin{aligned} I(q^2,\theta_l,\theta_{K^*},\phi) &= I_1^s \sin^2\theta_{K^*} + I_1^c \cos^2\theta_{K^*} + (I_2^s \sin^2\theta_{K^*} + I_2^c \cos^2\theta_{K^*})\cos 2\theta_l \\ &+ I_3 \sin^2\theta_{K^*} \sin^2\theta_l \cos 2\phi + I_4 \sin 2\theta_{K^*} \sin 2\theta_l \cos\phi \\ &+ I_5 \sin 2\theta_{K^*} \sin\theta_l \cos\phi \\ &+ (I_6^s \sin^2\theta_{K^*} + I_6^c \cos^2\theta_{K^*})\cos\theta_l + I_7 \sin 2\theta_{K^*} \sin\theta_l \sin\phi \\ &+ I_8 \sin 2\theta_{K^*} \sin 2\theta_l \sin\phi + I_9 \sin^2\theta_{K^*} \sin^2\theta_l \sin 2\phi. \end{aligned}$$

I_j: experimental observables: angular correlations



$\rightarrow$ complete information on $B \rightarrow K^* \mu\mu$

I_j and Form Factors/Wilson Coefficients

Relation between observables I_j and $\langle K^* \mu\mu | \mathcal{H}_{\rm eff} | B\rangle$ given in terms of transversity amplitudes $A_{\parallel,\perp,0,t}$, e.g.

$$I_6^s = 2\beta_\ell \left[\text{Re}(A_\parallel^L A_\perp^{L\,*}) - (L \to R) \right],$$

with

$$A_{\perp L,R} = N\sqrt{2}\lambda^{1/2} \left[\left[(C_9^{\rm eff} + C_9^{\rm eff\prime}) \mp (C_{10} + C_{10}') \right] \frac{V(q^2)}{m_B + m_{K^*}} + \frac{2m_b}{q^2}(C_7^{\rm eff} + C_7^{\rm eff\prime}) T_1(q^2) \right]$$

$$A_{\parallel L,R} = -N\sqrt{2}(m_B^2 - m_{K^*}^2) \left[(C_9^{\rm eff} - C_9^{\rm eff\prime}) \mp (C_{10} - C_{10}') \right] \frac{A_1(q^2)}{m_B - m_{K^*}} + \frac{2m_b}{q^2}(C_7^{\rm eff} - C_7^{\rm eff\prime}) T_2(q^2) \Big]$$

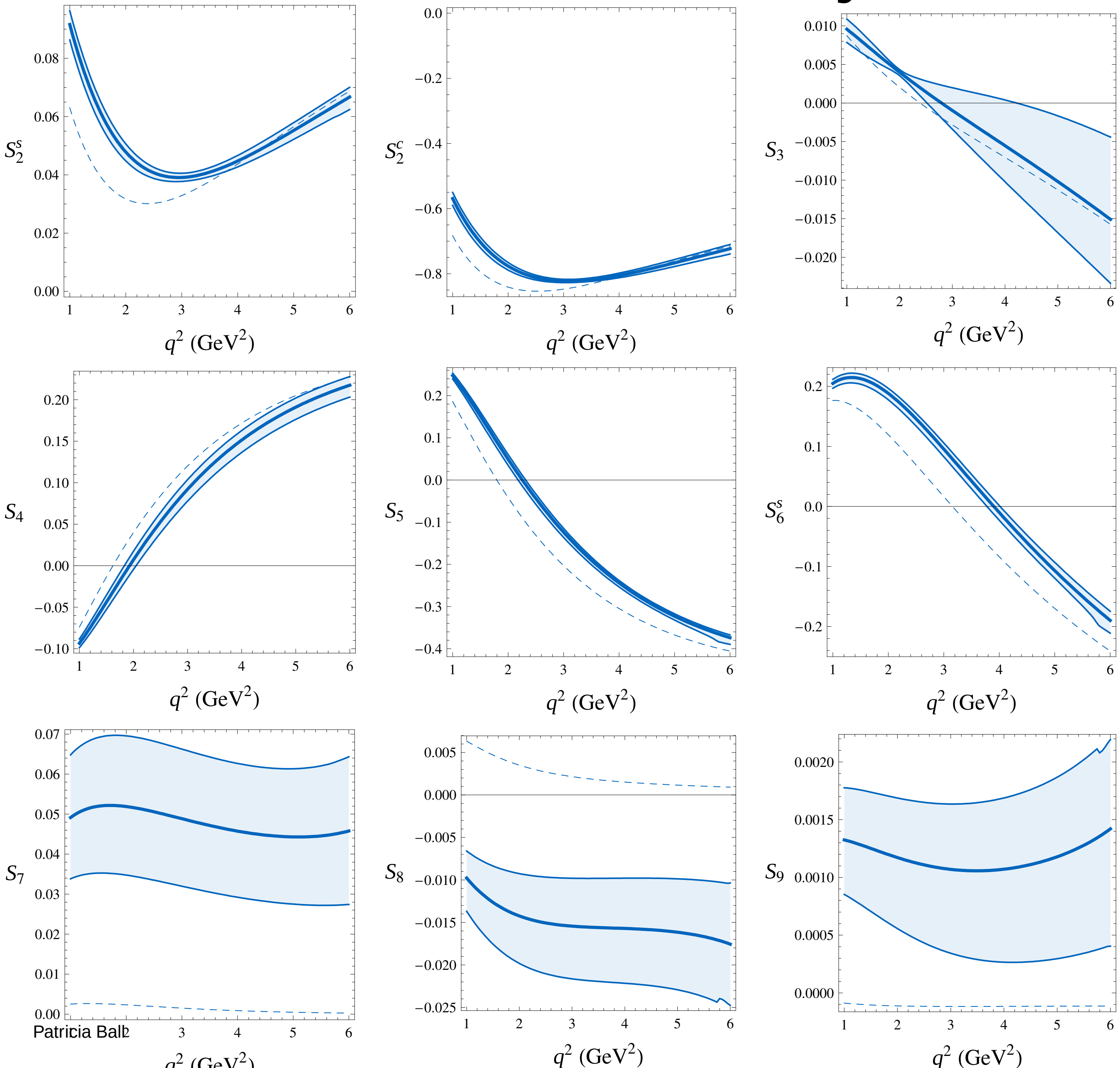
Observables

- order principle: behaviour under CP trafos: I_j from b ($\bar{B}^0$) decay, $\bar{I}_j$ from $\bar{b}$ (B^0) decay
  - CP-even (CP-averaged) $I_j + \bar{I}_j$, CP-odd $I_j - \bar{I}_j$
- normalize by $d(\Gamma + \bar{\Gamma})/dq^2$ (q^2: invariant dilepton mass)
- new standard observables: symmetries S_j and asymmetries A_j:

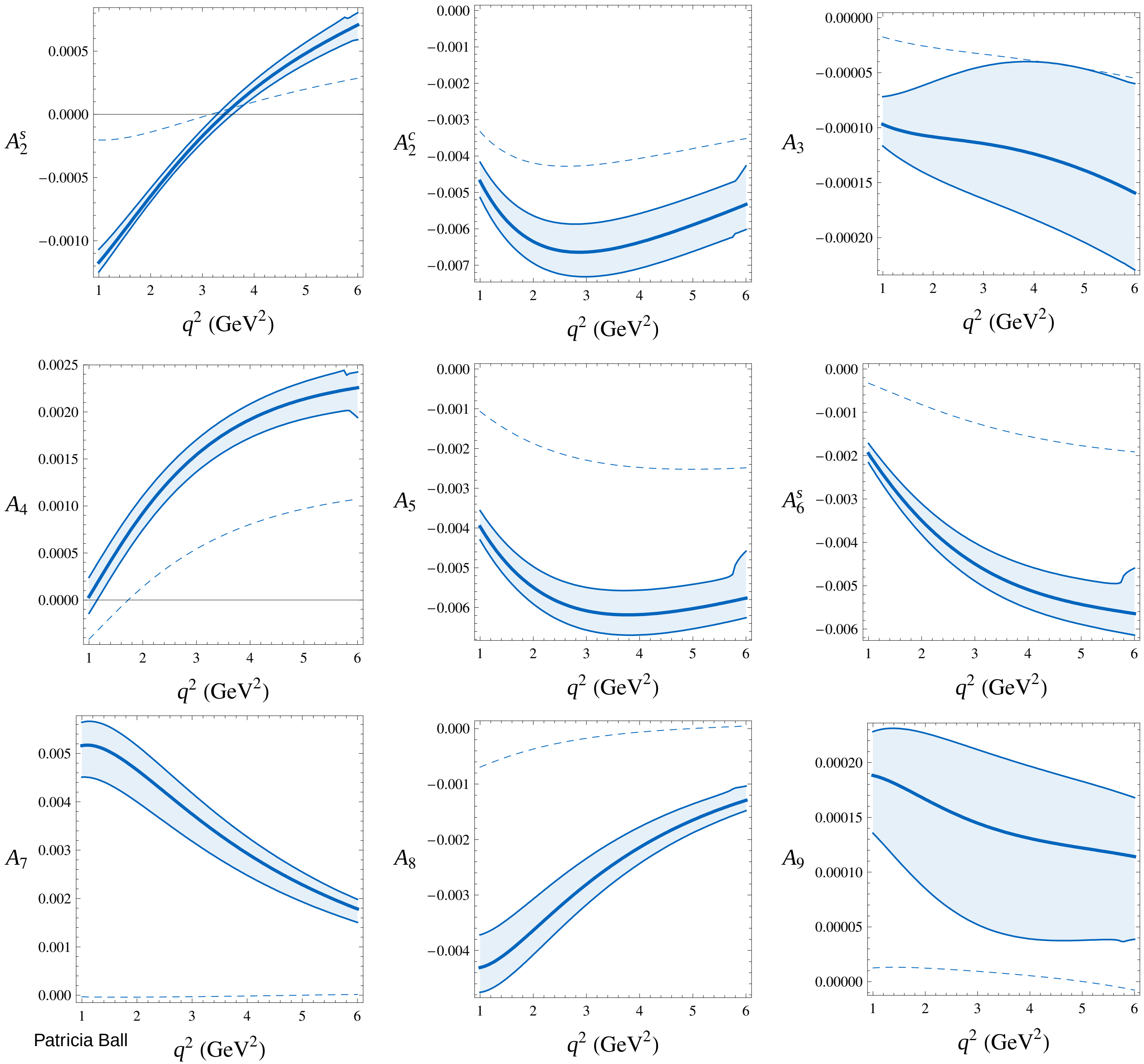
$$S_j = \frac{I_j + \bar{I}_j}{d(\Gamma + \bar{\Gamma})/dq^2}\,, \qquad A_j = \frac{I_j - \bar{I}_j}{d(\Gamma + \bar{\Gamma})/dq^2}$$

  Taking ratios reduces theory errors!
- advantage symmetries: increased statistics
- advantage asymmetries: sensitivity to new CP-violating phases induced by BSM (all A_j very close to 0 in SM)

Observables in the SM: Symmetries



Observables in the SM: Asymmetries



How to measure S_i and A_i?

- full angular spectrum
- or: suitable integrals
  - if observable is smooth in q^2:

$$S_i(q^2) \to \langle S_i \rangle \equiv \int_{1\,\mathrm{GeV}^2}^{6\,\mathrm{GeV}^2} dq^2\, S_i(q^2)$$

(QCD factorization only valid for small q^2: choose interval $1\,\mathrm{GeV}^2 \leq q^2 \leq 6\,\mathrm{GeV}^2$ below (theory) charm threshold $4m_c^2$)

  - for S_5, e.g., only need small number of bins in angles:

$$S_5 = -\frac{4}{3}\left[\int_{\pi/2}^{3\pi/2} - \int_0^{\pi/2} - \int_{3\pi/2}^{2\pi}\right] d\phi \left[\int_0^1 - \int_{-1}^0\right] \times d\cos\theta_K \frac{d^3(\Gamma-\bar{\Gamma})}{dq^2\, d\cos\theta_K d\phi} \Big/ \frac{d(\Gamma+\bar{\Gamma})}{dq^2}$$

Features to look out for

- zeros in S_i
  - most famous: zero in forward-backward asymmetry:

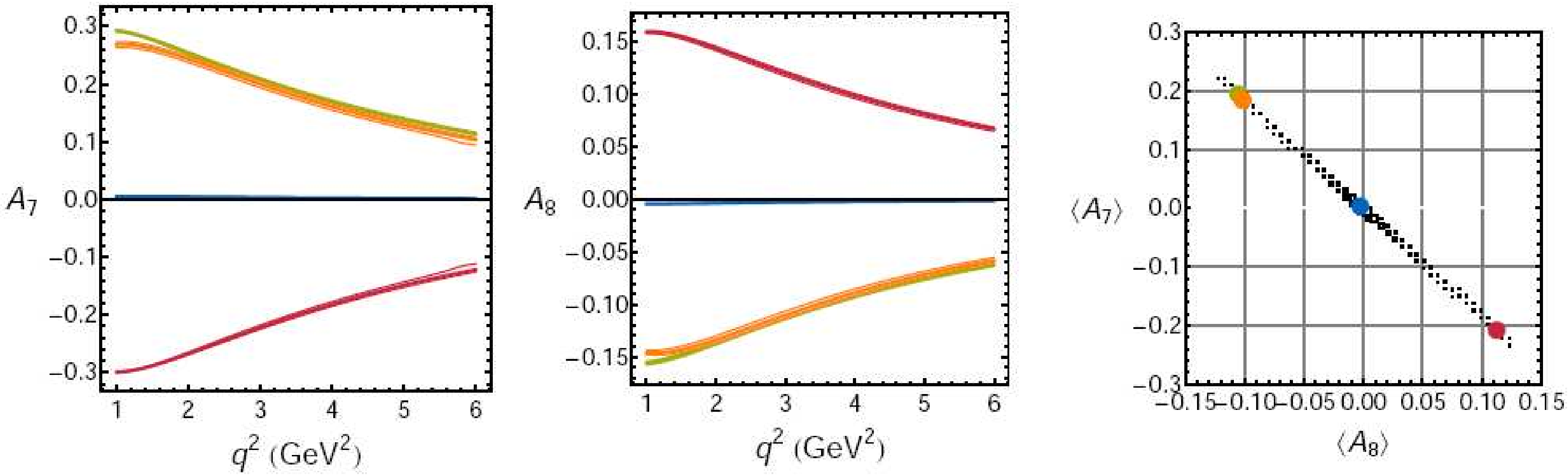
$$A_{\rm FB} = \frac{3}{8}\left(2S_6^s + S_6^c\right)$$

  - zeros also present (or, depending on NP, absent) in S_4, S_5
- large A_i
  - presence of new CP violating phases
- correlations between various observables

Features to look out for

Yesterday's talk by Altmannshofer:

CP Asymmetries in $B^0 \to K^{0*}(\to K^+\pi^-)\ell^+\ell^-$



- The CP asymmetries A_7 and A_8 are negligible small in the SM
- In the FBMSSM huge effects are possible and they are highly correlated
- Deviations from the correlation point clearly towards sizeable complex NP contributions to other Wilson coefficients than C_7

Summary

- form factors needed to predict exclusive B decays
- particularly relevant for the LHC: $B \to K^*$
- no new (=unquenched) lattice calculations for $B \to K^*$
- quite a few recent analyses based on QCD factorisation and FFs in the heavy quark limit ($\xi_{\perp,\|}$). Emphasis on observables with reduced dependence on FFs (e.g. zero of forward-backward asymmetry)
- Altmannshofer et al. 0811.124:
  - use full QCD form factors from light-cone sum rules
  - reduce uncertainty by predicting symmetries & asymmetries, sensitive only to ratios of form factors
  - include correlated erors
  - $\rightsquigarrow$ full angular analysis, 24 observables
  - only fully exploitable at LHCb upgrade?
  - and what about the elephant in the room – non-resonant $K\pi$ contributions?