RG Improvement of the Higgs Production Cross Section

Thomas Becher

Fermilab

Ringberg workshop, April 26 - May 1, 2009

V. Ahrens, TB, M. Neubert and L. Yang

arXiv:0808.3008 and arXiv:0809.4283

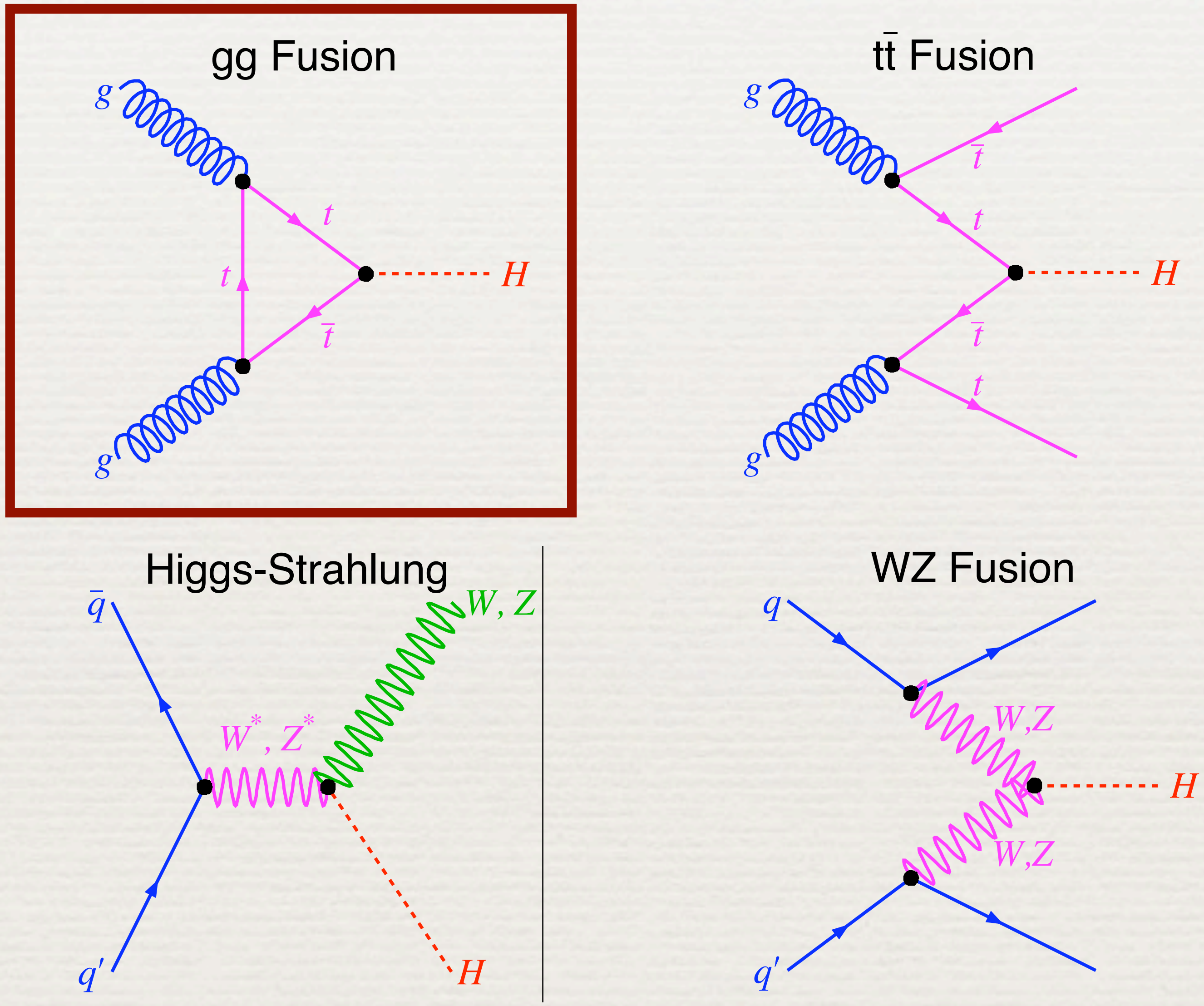
Higgs production $pp \rightarrow H+X$

- For good reasons, Higgs production is one of the best studied processes theoretically:
  - NNLO accuracy for total cross section Harlander and Kilgore '02, Anastasiou and Melnikov '02, Ravindran, Smith and van Nerven '03 as well as the differential decay to $\gamma\gamma$ Anastasiou, Melnikov and Petriello '05; Catani and Grazzini '07 and $W^+W^- \rightarrow \ell^+\nu\ell^-\bar{\nu}$ Anastasiou, Dissertori and Stöckli '07; Grazzini '08
- Large perturbative corrections. Leading order predictions off by more than a factor of two.

To resum or not to resum, that is the question.

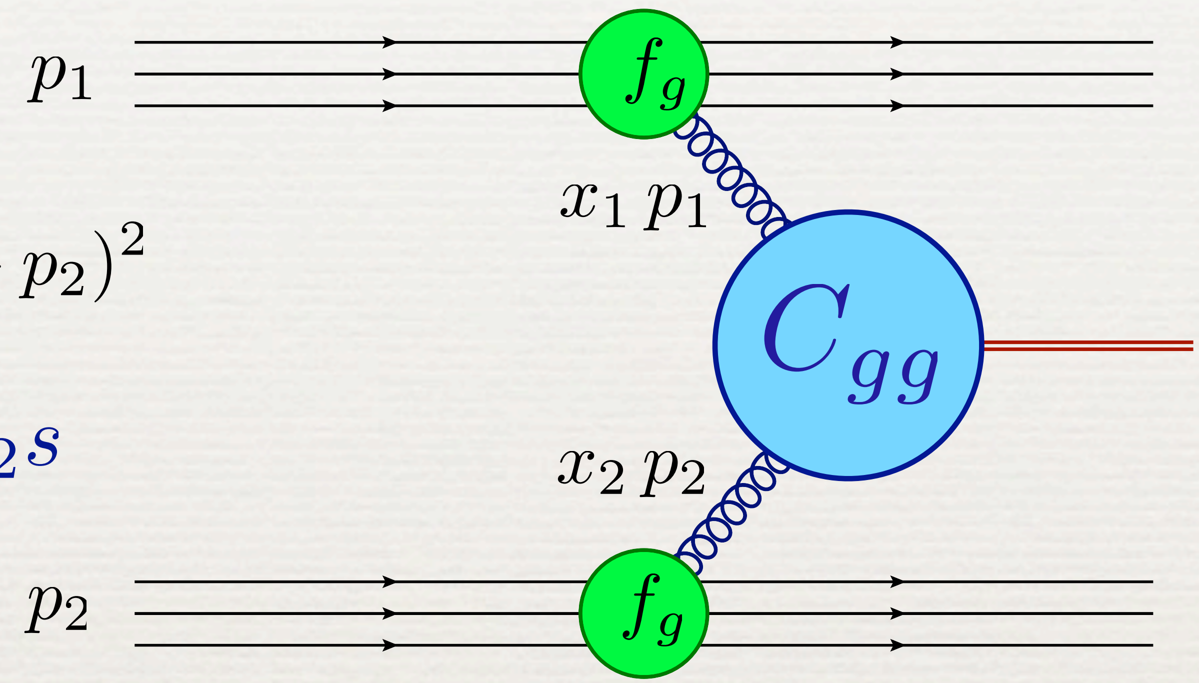
- Large perturbative corrections...
  - Soft-gluon resummation developed in '86 by Sterman. NNLL result known Catani et al. '03.
  - Can use RG evolution in SCET Manohar '03 (for DIS); Ji, Idilbi '06 (for Higgs), analytic expression for resummed kernels in momentum space TB, Neubert '06
- ... but are there large logarithms?
  - Inclusive production cross section, plenty of phase space for hard radiation.
  - We find: no large scale hierarchy, but large corrections associated with analytic continuation from space-like to time-like kinematics.

Higgs production mechanisms



- Gluon fusion via a top quark loop is an order of magnitude larger than the other production mechanisms

Factorization



Hadronic $s = (p_1 + p_2)^2$

Partonic $\hat{s} = x_1 x_2 s$

- Convolution of perturbative hard-scattering kernel C_{ij} with parton luminosity ff_{ij}

$$\sigma = \sigma_0 \sum_{ij} C_{ij} \otimes ff_{ij} \quad \text{with} \quad ff_{ij} = f_i \otimes f_j$$

- At the partonic threshold $z = M_H^2/\hat{s} \to 1$. In the convolution $z = M_H^2/s \ldots 1$

Hard-scattering kernels

- Known to NNLO for $m_H \ll 2m_t$. At NLO

Harlander and Kilgore '02, Anastasiou and Melnikov '02,
Ravindran, Smith and van Nerven '03

singular

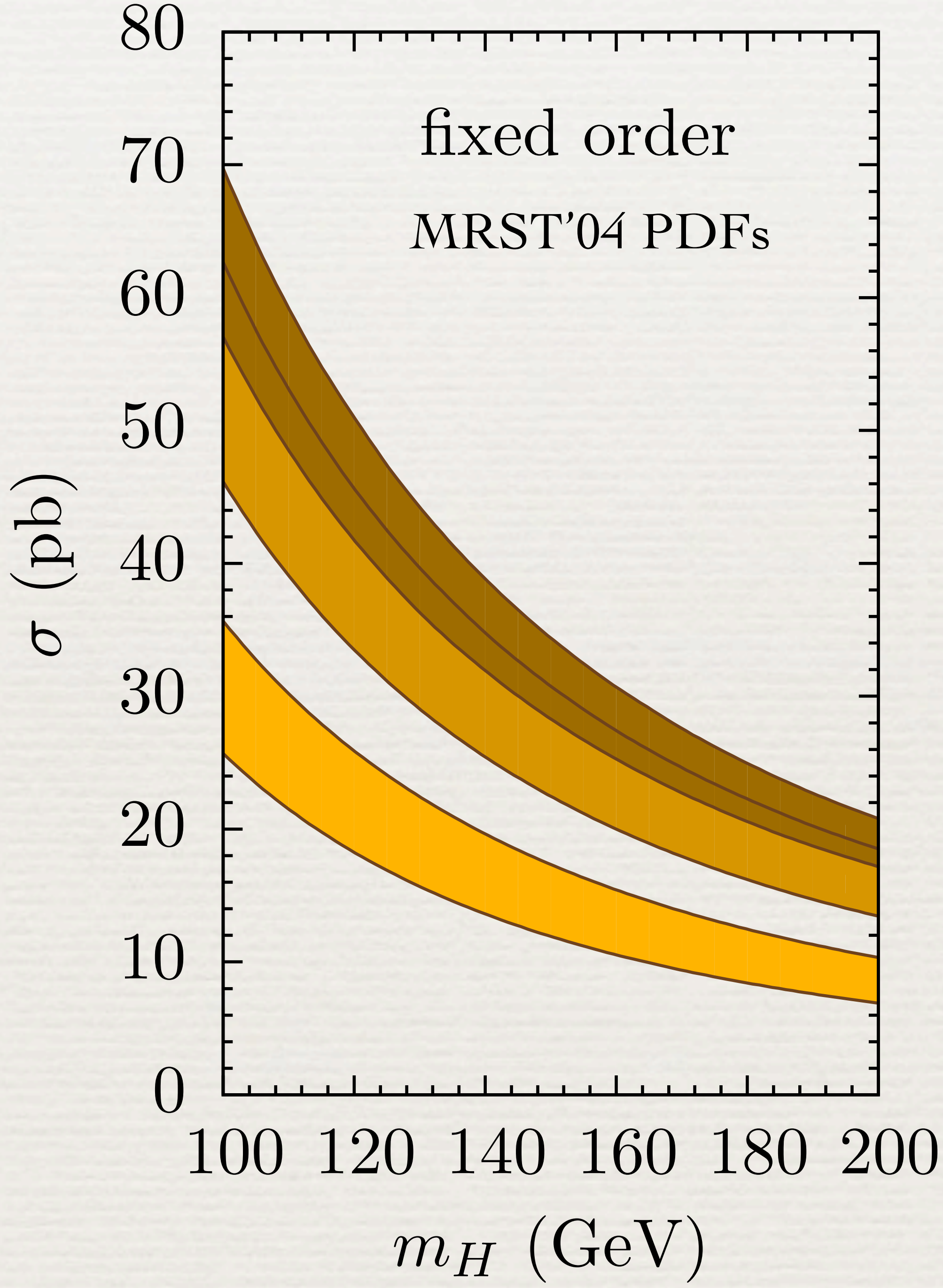
$$C_{gg}(z,\mu_f) = \delta(1-z) + \frac{\alpha_s}{\pi}\left\{\delta(1-z)\left(\frac{11}{2}+2\pi^2\right) + 6\left[\frac{1}{1-z}\ln\frac{m_H^2(1-z)^2}{\mu_f^2 z}\right]_+\right\}$$

$$+\frac{\alpha_s}{\pi}\left\{6\left(\frac{1}{z}-2+z-z^2\right)\ln\frac{m_H^2(1-z)^2}{\mu_f^2 z} - \frac{11}{2}\frac{(1-z)^3}{z}\right\}$$

regular

- other partonic channels are regular
- in the following, we will analyze the singular terms and resum certain contributions to all orders
- will keep regular terms in fixed-order perturbation theory

Higher-order corrections



- Corrections are large: 70% NLO, 30% NNLO. [130% and 80% if PDFs and α_s are held fixed].
  - $\alpha_s(m_H) \approx 0.1$
- C_{gg} contains singular terms, these give 90% of NLO and 94% of NNLO correction
- contribution of C_{qg} and C_{qq} is small -1% and -8% of the NLO correction.

Effective Theory Analysis

Effective theory analysis

- Separate contributions associated with different scales, turning a multi-scale problems into a series of single-scale problems
- Evaluate each contribution at its natural scale, leading to improved perturbative behavior
- Use renormalization group to evolve contributions to an arbitrary factorization scale, thereby exponentiating (resumming) large corrections

When this is done consistently, large K-factors should never arise, since no large perturbative corrections should be left unexponentiated!

Scale hierarchy

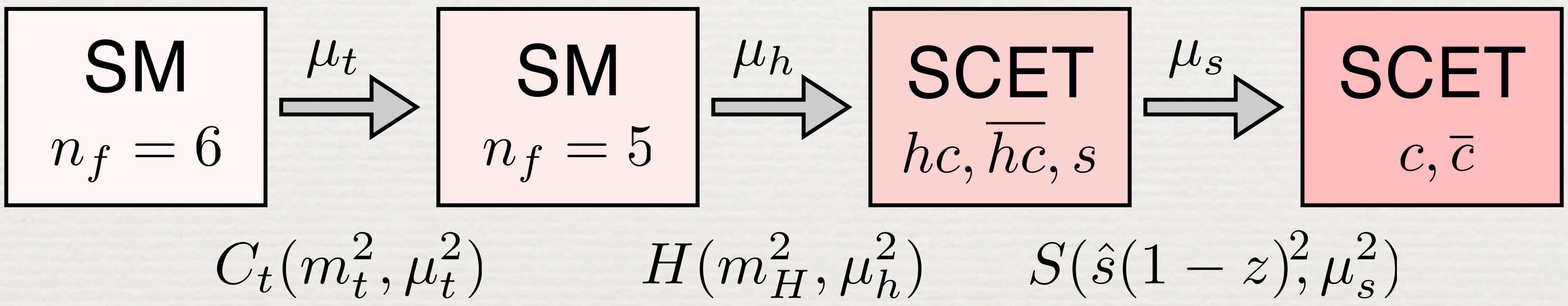
- We will analyze the Higgs cross section assuming the scale hierarchy [$z = M_H^2/\hat{s}$]

$$2m_t \gg m_H \sim \sqrt{\hat{s}} \gg \sqrt{\hat{s}}(1-z) \gg \Lambda_{\mathrm{QCD}}$$

- Expand to leading power in scale ratios
  - Expand kernels C_{ij} around partonic threshold z=1, keep only singular terms.
  - Singular terms give most of the cross section, but z is integrated over.
    - Will check later if $z \approx 1$ is fulfilled.

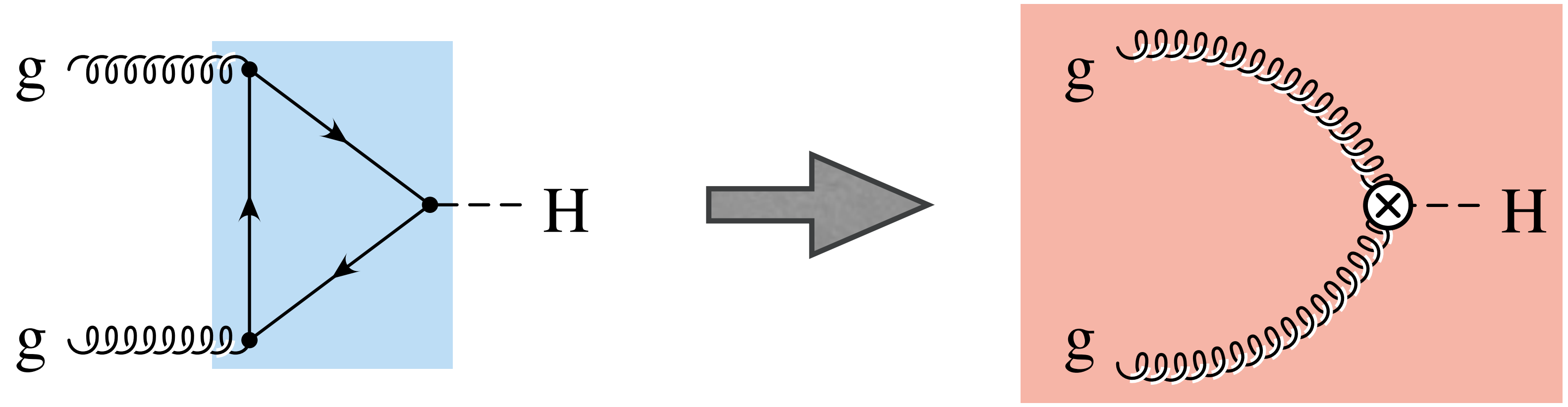
Sequence of EFTs

- Treating one scale at a time leads to a sequence of effective theories



- Effects associated with each scale are absorbed into Wilson coefficient.
- Solve RG equation to evolve from higher to lower scales.

First step: integrate out the top



$$\mathcal{L}_{\text{eff}} = C_t(m_t^2, \mu^2)\,\frac{H}{v}\,\frac{\alpha_s(\mu^2)}{12\pi}\,G_{\mu\nu,a}\,G_a^{\mu\nu}$$

- For $m_H \ll 2m_t$ we can integrate out the top quark, i.e. replace the SM by an effective theory with $n_f = 5$.
- Calculations in EFT are much simpler. One loop and one scale less.
    - NNLO results only available in EFT.

Matching and RG evolution

$$C_t(m_t^2,\mu) = 1 + \frac{11}{4}\frac{\alpha_s}{\pi} + \left(\frac{\alpha_s}{4\pi}\right)^2 \left[\frac{2777}{18} - 19\ln\frac{m_t^2}{\mu^2} + n_f\left(-\frac{67}{6} - \frac{16}{3}\ln\frac{m_t^2}{\mu^2}\right)\right] + \dots$$
$$\approx 1 + 0.09 + 0.007 + \dots \quad \text{for } \mu = m_t$$

NNNLO: Schröder and Steinhauser; Chetyrkin, Kühn and Sturm '05

- Natural scale choice $\mu \approx m_t$, small corr's.
- Wilson coefficient $C_t(m_t,\mu)$ fulfills RG equation

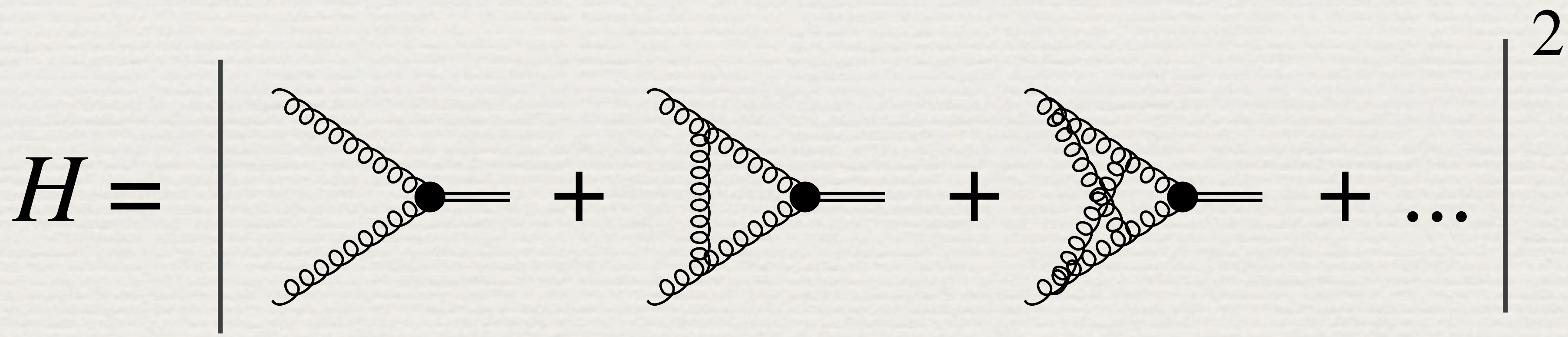
$$\frac{d}{d\ln\mu} C_t(m_t^2,\mu^2) = \gamma^t(\alpha_s)\, C_t(m_t^2,\mu^2)\,, \qquad \text{with} \quad \gamma^t(\alpha_s) = \alpha_s^2 \frac{d}{d\alpha_s}\frac{\beta(\alpha_s)}{\alpha_s^2}$$

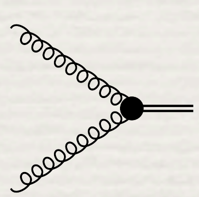
- Solution

$$C_t(m_t^2,\mu_f^2) = \frac{\beta\big(\alpha_s(\mu_f^2)\big)/\alpha_s^2(\mu_f^2)}{\beta\big(\alpha_s(\mu_t^2)\big)/\alpha_s^2(\mu_t^2)}\, C_t(m_t^2,\mu_t^2)$$

Second step: hard contributions H

- Separate the contributions of the hard scale $\hat{s}$ and the soft scale $\hat{s}(1-z)^2$
  - Set z=1, then only hard scale remains. These are diagrams w/o gluon emission:

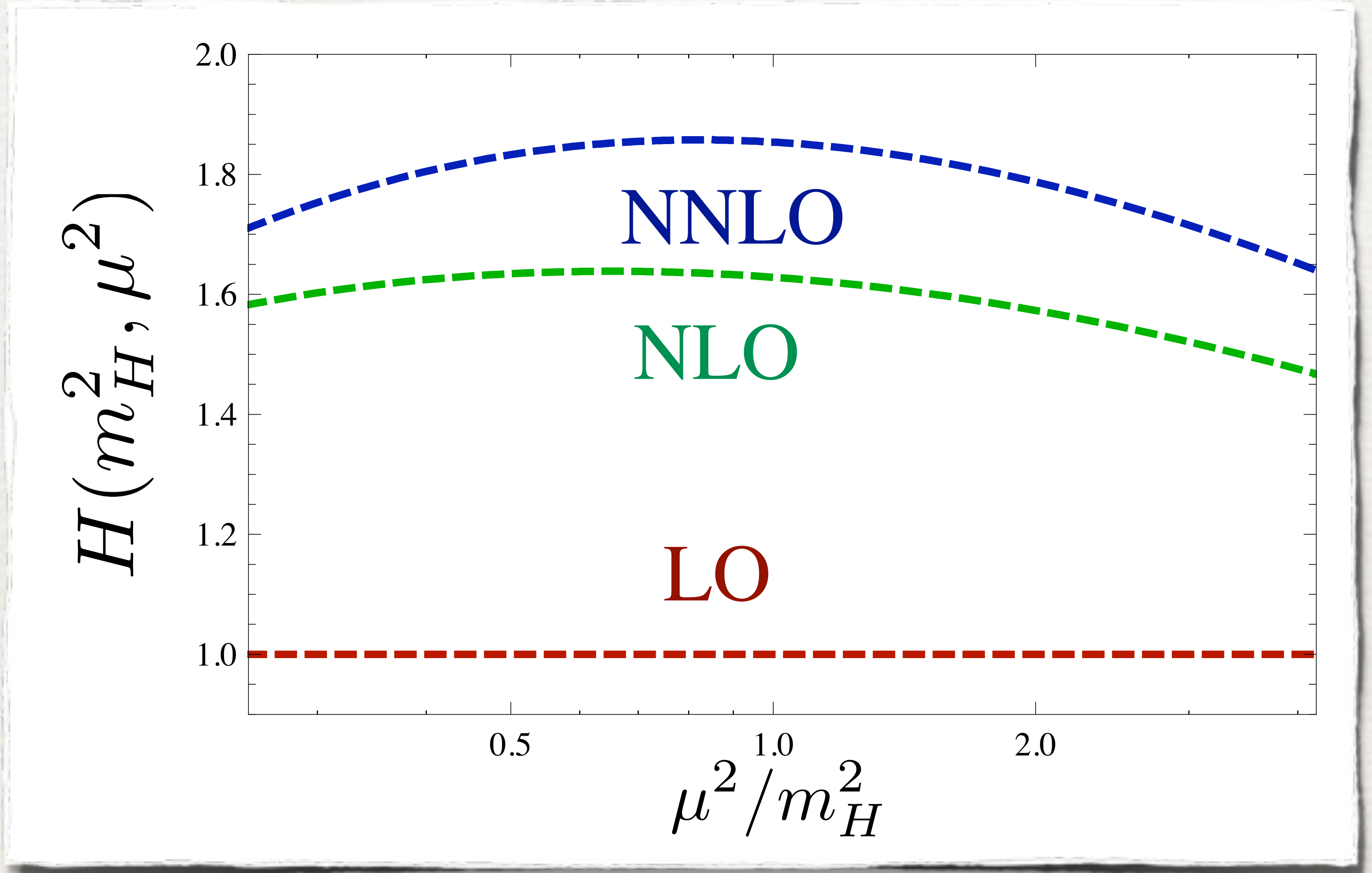




$$= C_t(m_t^2, \mu^2)\, \frac{H}{v}\, \frac{\alpha_s(\mu^2)}{12\pi}\, G_{\mu\nu,a}\, G_a^{\mu\nu}$$

  - H is the on-shell gluon form factor squared.

Choice of the hard scale



- Hard function is scale dependent.
- Corrections are large for any μ^2 !?!

Scalar form factor

- Hard function $H(m_H^2, \mu^2) = \left|C_S(-m_H^2 - i\epsilon, \mu^2)\right|^2$
- Scalar form factor

$$C_S(Q^2, \mu^2) = 1 + \sum_{n=1}^{\infty} c_n(L) \left(\frac{\alpha_s(\mu^2)}{4\pi}\right)^n, \quad L = \ln(Q^2/\mu^2)$$

$$c_1(L) = C_A \left(-L^2 + \frac{\pi^2}{6}\right)$$

Sudakov double logarithm

- Perturbative expansions

- space-like $C_S(Q^2, Q^2) = 1 + 0.393\, \alpha_s(Q^2) - 0.152\, \alpha_s^2(Q^2) + \ldots$

  time-like $C_S(-q^2, q^2) = 1 + 2.75\, \alpha_s(q^2) + (4.84 + 2.07i)\, \alpha_s^2(q^2)$

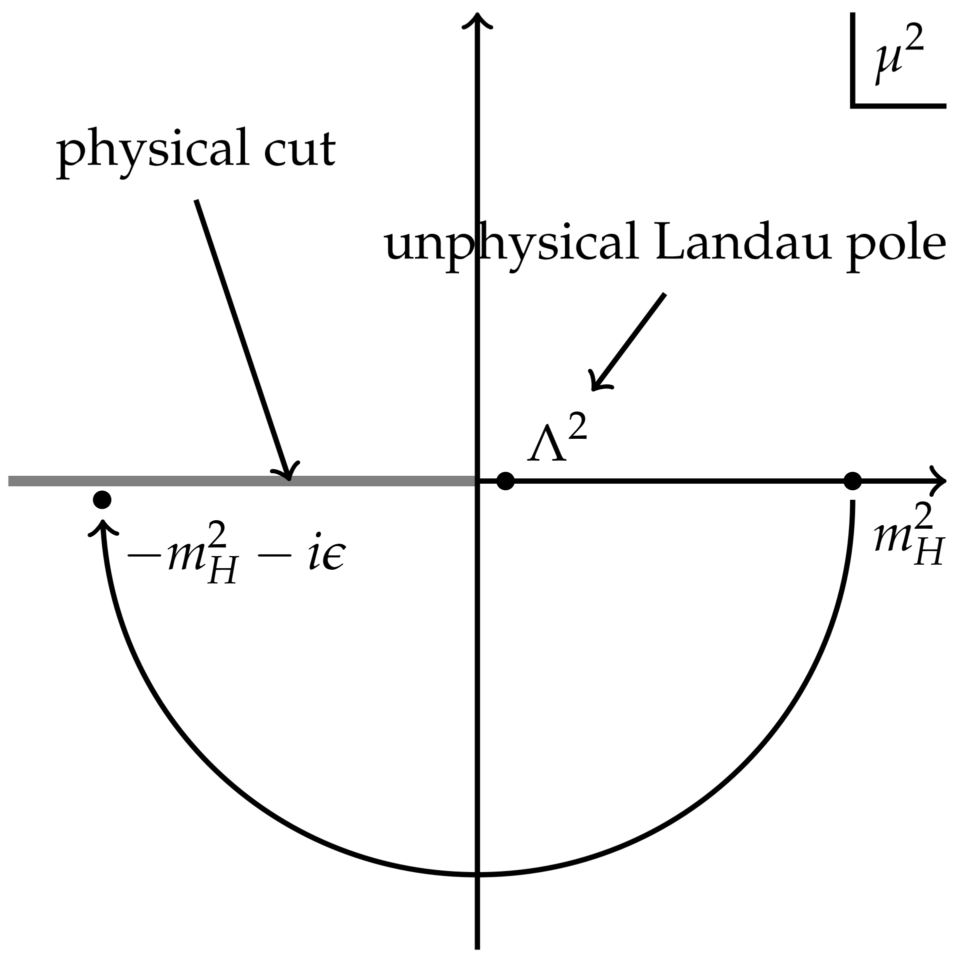
Solution

- Reason: $L \to \ln q^2/\mu^2 - i\pi$ and double log's give rise to π^2 terms. Parisi '80
  - Being related to Sudakov logs, they can be resummed. Magnea and Sterman and '90
- We can avoid the π^2 terms by choosing a time-like value $\mu^2 = -q^2$

$$C_S(-q^2, -q^2) = 1 + 0.393\,\alpha_s(-q^2) - 0.152\,\alpha_s^2(-q^2) + \ldots$$

  - same expansion coefficients as $C_S(Q^2, Q^2)$
- Note: RG-evolution defines $\alpha_s(\mu^2)$ for *any* μ^2

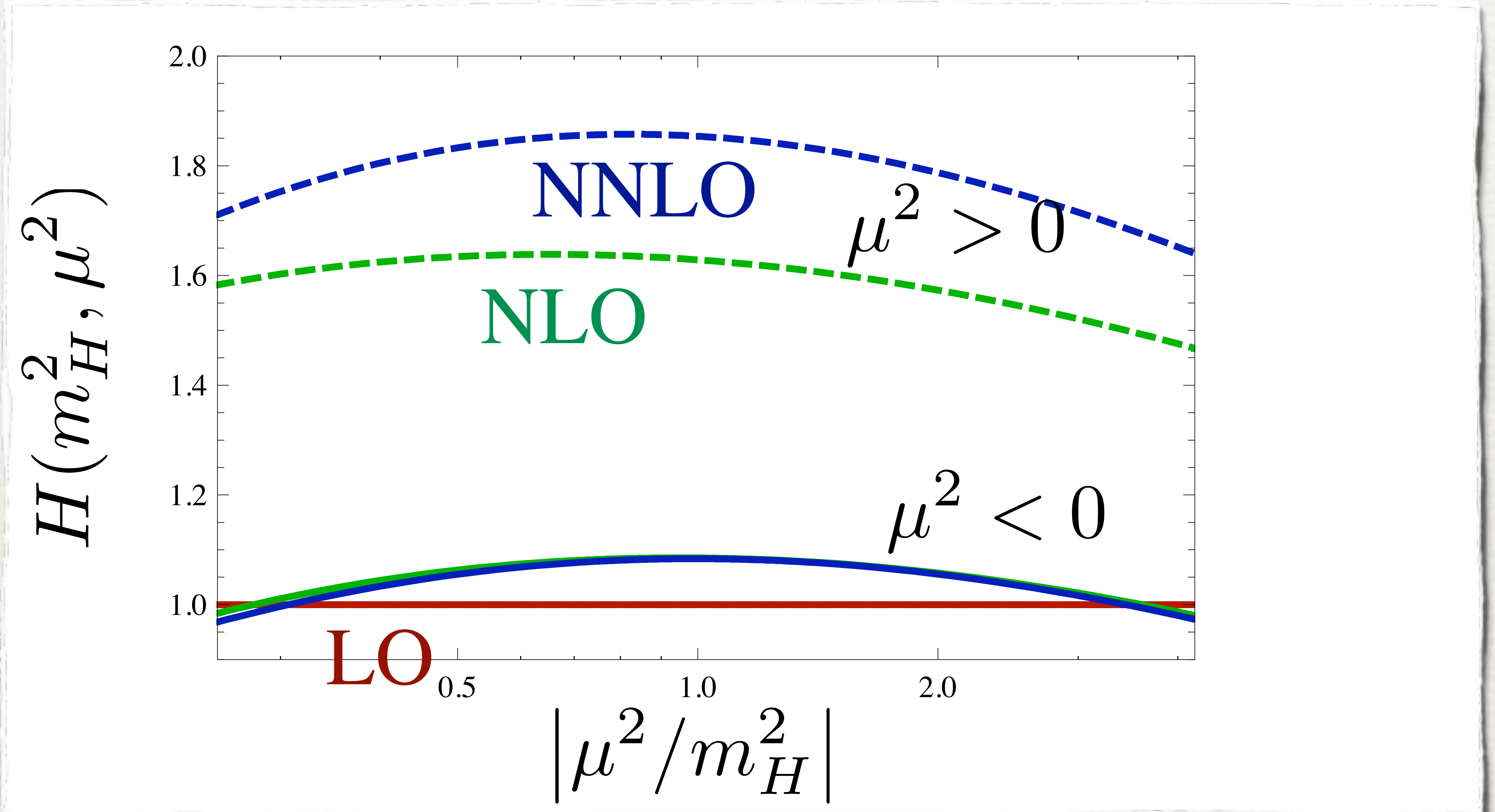
α_s in the complex μ^2 plane



$$\int_{\alpha_s(\mu^2)}^{\alpha_s(-\mu^2)} \frac{d\alpha}{\beta(\alpha)} = -\frac{i\pi}{2}$$

- Can avoid the Landau pole when going to negative μ^2 .
- Size of expansion parameters is similar. For m_H=120 GeV
  - $\alpha_s(m_H^2)$=0.112
  - $\alpha_s(-m_H^2+i\varepsilon)$=0.107+0.024$i$

Time-like vs. space-like μ²



- Convergence is much better for $\mu^2 < 0$
- Evaluate *H* for $\mu^2 < 0$ where convergence is good and use RG to evolve to arbitrary scale

RG evolution

- Hard function fulfills RG equation

$$\frac{d}{d\ln\mu}C_S(-m_H^2-i\epsilon,\mu^2)=\left[\Gamma^A_{\text{cusp}}(\alpha_s)\,\ln\frac{-m_H^2-i\epsilon}{\mu^2}+\gamma^S(\alpha_s)\right]C_S(-m_H^2-i\epsilon,\mu^2)$$

produces Sudakov double log's

- Exact solution

$$C_S(-m_H^2-i\epsilon,\mu_f^2)=\exp\left[2S(\mu_h^2,\mu_f^2)-a_\Gamma(\mu_h^2,\mu_f^2)\,\ln\frac{-m_H^2-i\epsilon}{\mu_h^2}-a_{\gamma^S}(\mu_h^2,\mu_f^2)\right]C_S(-m_H^2-i\epsilon,\mu_h^2)$$

  - with

$$S(\nu^2,\mu^2)=-\int\limits_{\alpha_s(\nu^2)}^{\alpha_s(\mu^2)}d\alpha\,\frac{\Gamma^A_{\text{cusp}}(\alpha)}{\beta(\alpha)}\int\limits_{\alpha_s(\nu^2)}^{\alpha}\frac{d\alpha'}{\beta(\alpha')}\,,\qquad a_\Gamma(\nu^2,\mu^2)=-\int\limits_{\alpha_s(\nu^2)}^{\alpha_s(\mu^2)}d\alpha\,\frac{\Gamma^A_{\text{cusp}}(\alpha)}{\beta(\alpha)}\,,$$

Approximate solution

- Neglect single log's and running of α_s

$$\frac{d}{d\ln\mu} C_S(-m_H^2,\mu^2) = C_A \frac{\alpha_s}{\pi} \ln \frac{-m_H^2 - i\epsilon}{\mu^2} C_S(-m_H^2,\mu^2)$$

- Solution

$$C_S(-m_H^2,\mu^2) = \exp\left(C_A \frac{\alpha_s}{4\pi} \ln^2 \frac{-m_H^2}{\mu^2} \right) \times C_S(-m_H^2,-m_H^2)$$

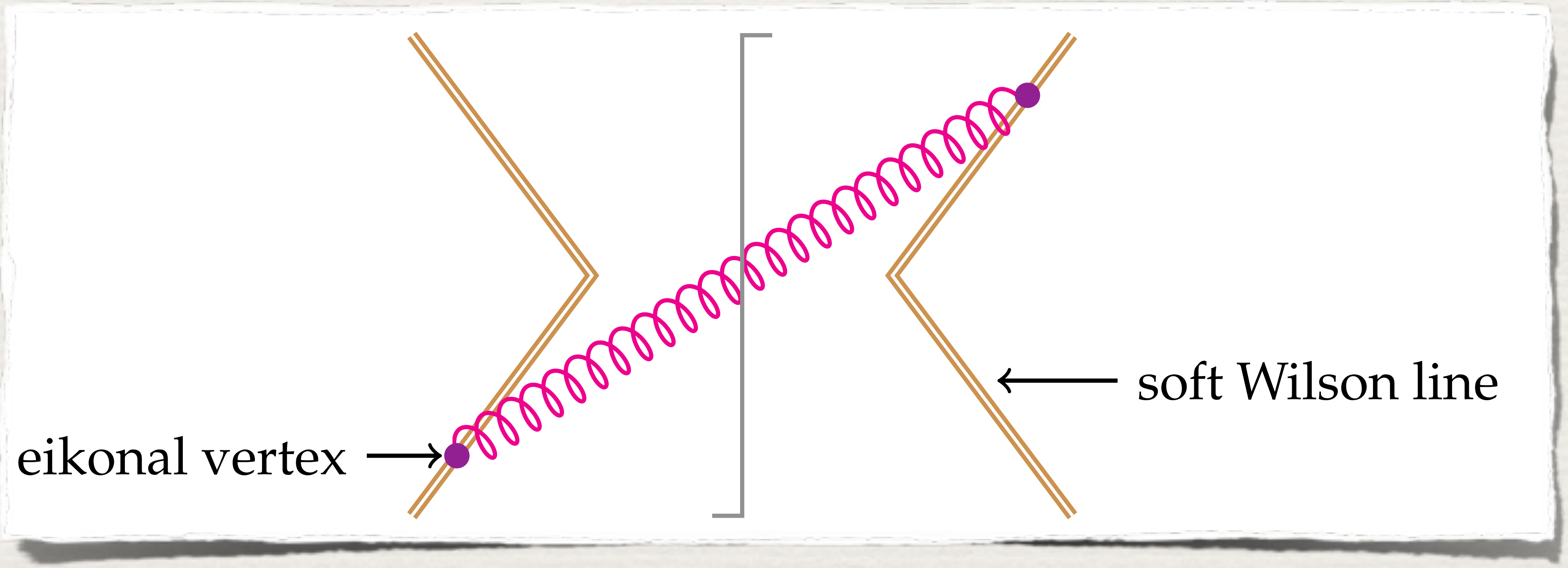
- Hard function

$$H(m_H^2,\mu^2 = +m_H^2) = \exp\left(C_A \frac{\alpha_s}{2\pi} \pi^2 \right) \times |C_S(-m_H^2,-m_H^2)|^2$$

$$\approx 1.7$$

Soft function S

- $S(\sqrt{\hat{s}}(1-z),\mu)$ is the vacuum expectation value of a Wilson loop constructed from soft gluon fields.

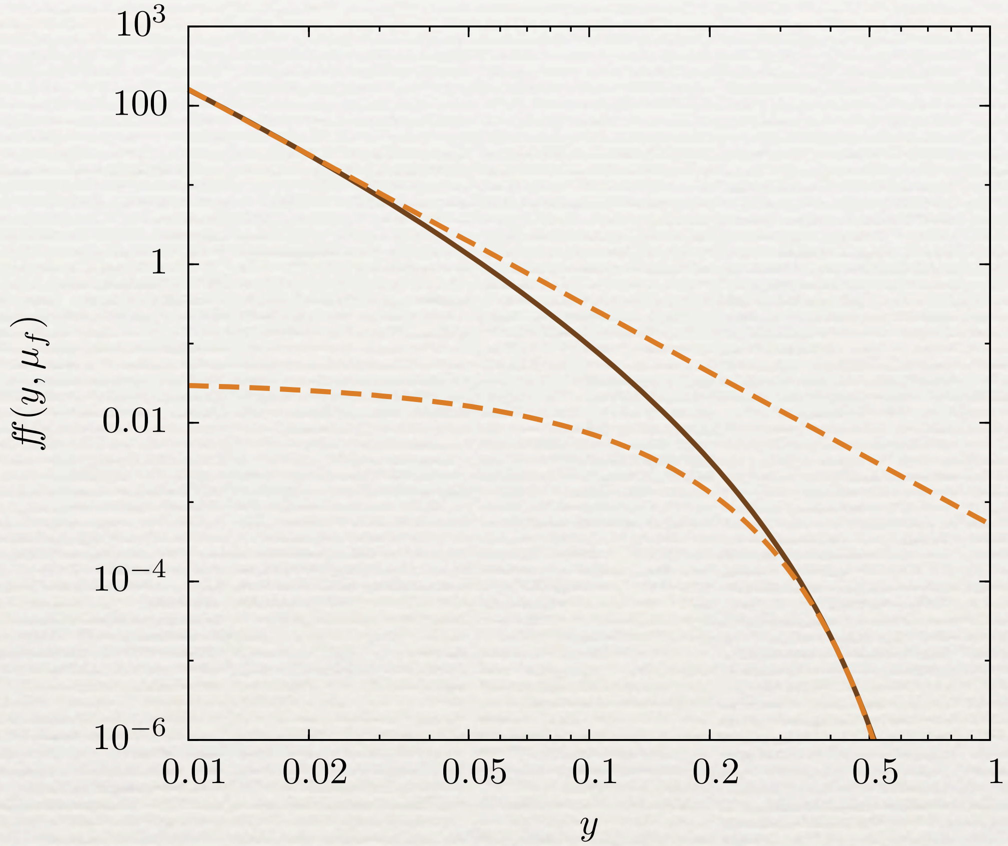


Soft function $S(\sqrt{\hat{s}}(1-z),\mu)$

- Could avoid large logarithms by choosing the scale $\mu = \sqrt{\hat{s}}(1-z)$
  - but z is integrated from z=τ...1, with $\tau = m_H^2/s$. Ill-defined convolution due to Landau-pole.
- Instead choose scale such that the convolution integral does not contain large log's.

$$\int_\tau^1 \frac{dz}{z} S(\sqrt{\hat{s}}(1-z),\mu)\, ff_{gg}(\tau/z)$$

Parton luminosity $\text{ff}_{gg}(y)$



- Steeply falling function. Convolution integral is dominated by $y \sim \tau$.
- $\text{ff}_{gg}(y) \sim y^{-a}$ for $y < 0.03$ with $a \approx 2.5$

  $\text{ff}_{gg}(y) \sim (1-y)^b$ for $y > 0.3$ with $b \approx 14.5$

Convolution

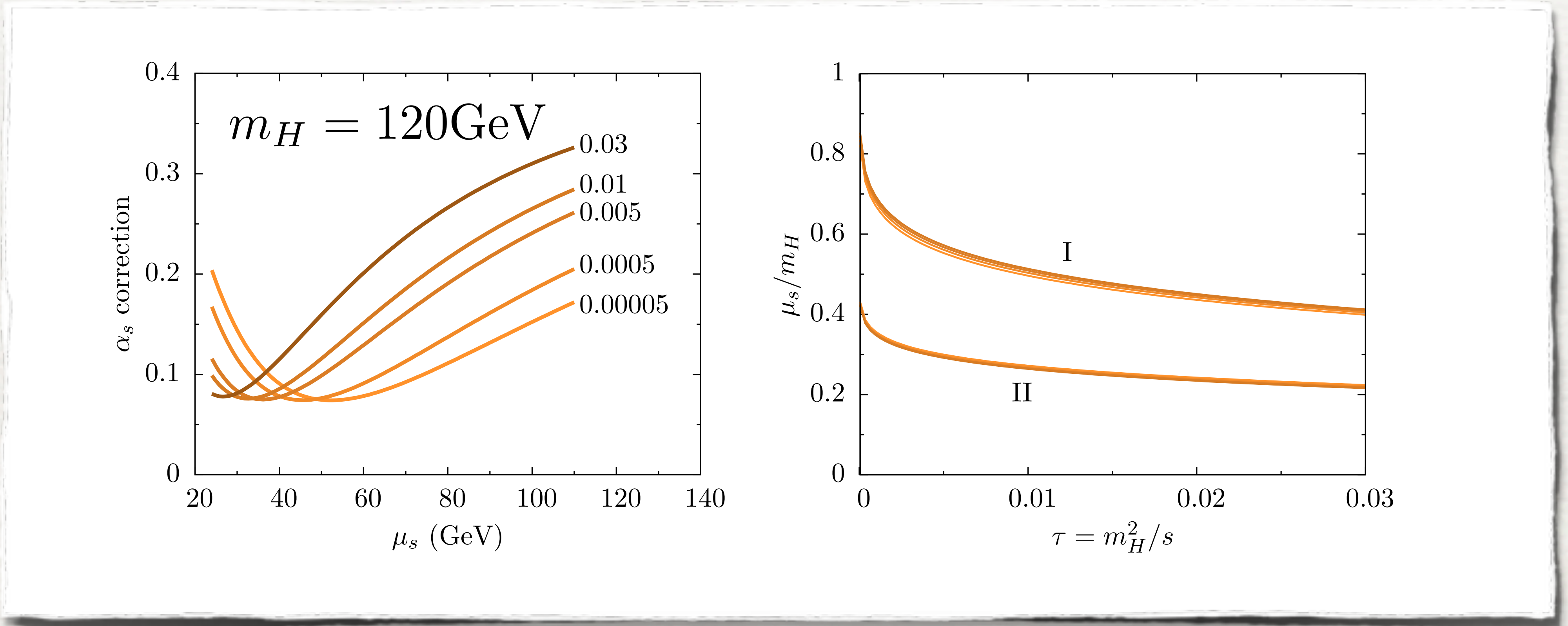
- Since $\tau = m_H^2/s < 0.03$ for $m_H <$ 350GeV at the Tevatron, $ff_{gg}(y, \mu_f) \propto y^{-a}$ and we can approximate

$$\int_\tau^1 \frac{dz}{z} S(\sqrt{\hat{s}}(1-z), \mu)\, ff_{gg}(\tau/z)$$

$$\approx ff_{gg}(\tau) \int_0^1 dz\, S(\sqrt{\hat{s}}(1-z), \mu)\, z^{a-1}$$

  - Since a-1=1.5 no strong enhancement of the threshold region.

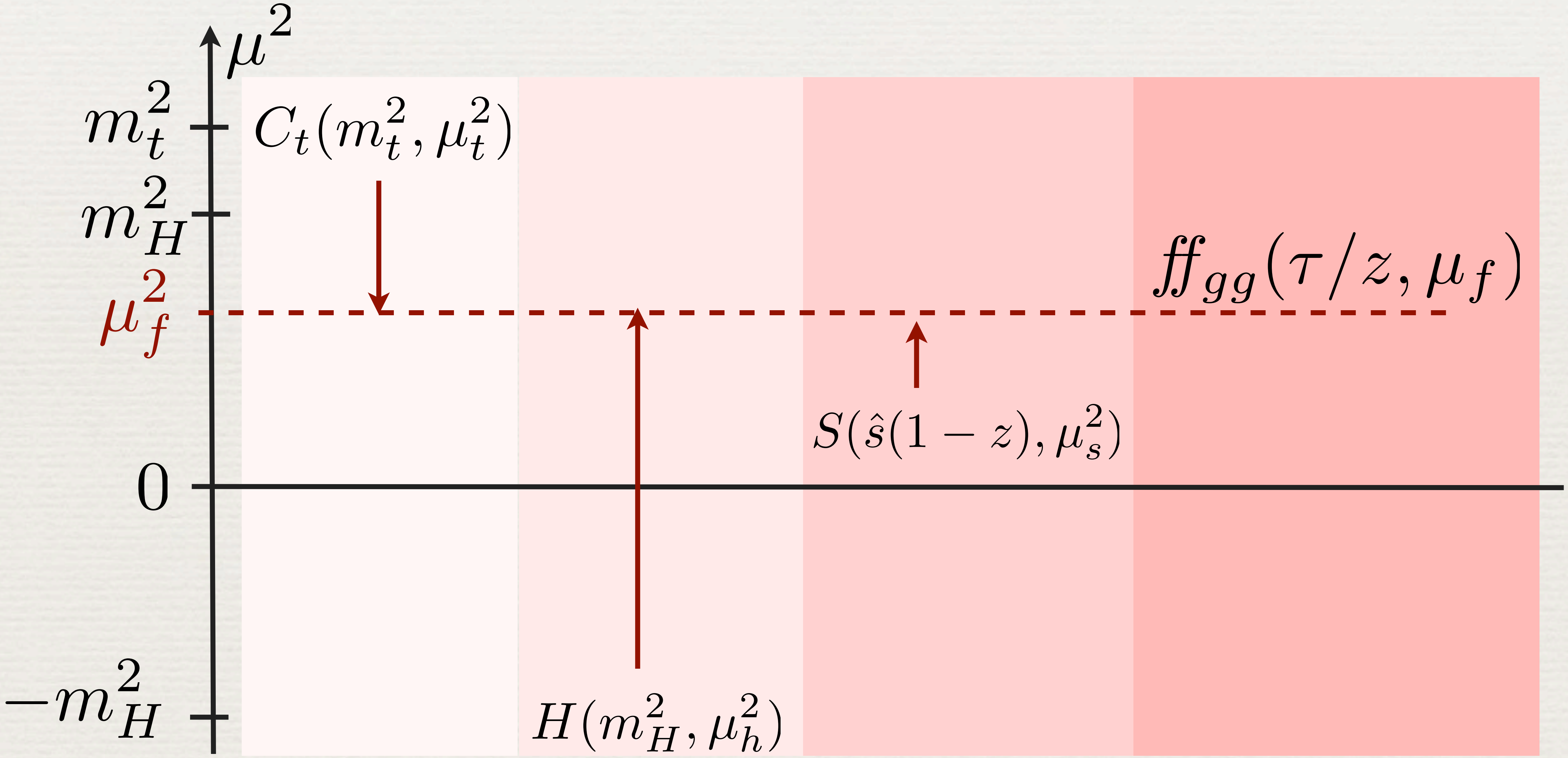
Choice of the soft scale



- Good perturbative behavior with $\mu_s \sim m_H/2$.
- No large logarithms
  - Soft-gluon resummation is a small effect

Summary

- Evaluate each part at its characteristic scale, evolve to common scale:



Resummed kernel

$$C(z, m_t, m_H, \mu_f) = \left[C_t(m_t^2, \mu_t^2)\right]^2 \left|C_S(-m_H^2 - i\epsilon, \mu_h^2)\right|^2 U(m_H, \mu_t, \mu_h, \mu_s, \mu_f)$$
$$\times \frac{z^{-\eta}}{(1-z)^{1-2\eta}} \, \widetilde{s}_{\text{Higgs}}\left(\ln \frac{m_H^2(1-z)^2}{\mu_s^2 z} + \partial_\eta, \mu_s^2\right) \frac{e^{-2\gamma_E \eta}}{\Gamma(2\eta)} \,,$$

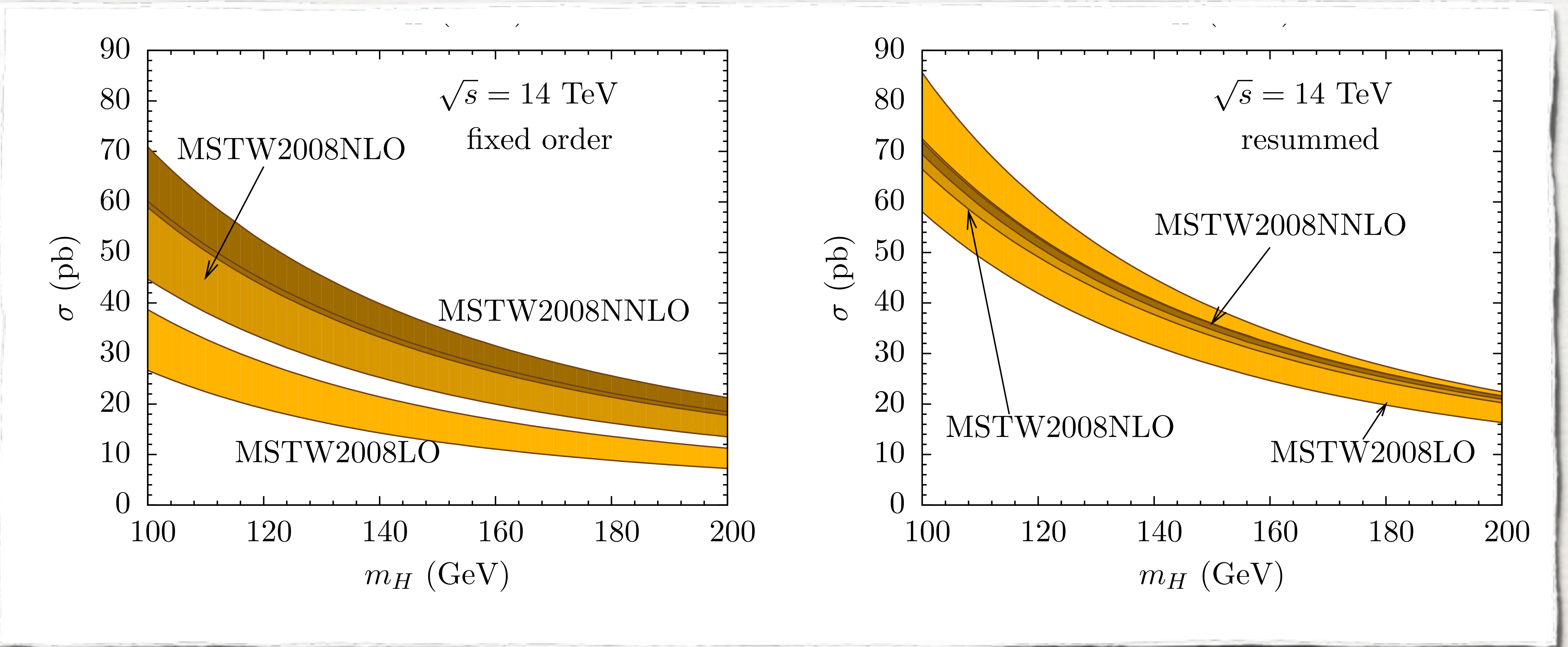
where

$$U(m_H, \mu_t, \mu_h, \mu_s, \mu_f) = \frac{\alpha_s^2(\mu_s^2)}{\alpha_s^2(\mu_f^2)} \left[\frac{\beta\big(\alpha_s(\mu_s^2)\big)/\alpha_s^2(\mu_s^2)}{\beta\big(\alpha_s(\mu_t^2)\big)/\alpha_s^2(\mu_t^2)}\right]^2 \left|\left(\frac{-m_H^2 - i\epsilon}{\mu_h^2}\right)^{-2a_\Gamma(\mu_h^2, \mu_s^2)}\right|$$
$$\times \left|\exp\left[4S(\mu_h^2, \mu_s^2) - 2a_{\gamma^S}(\mu_h^2, \mu_s^2) + 4a_{\gamma^B}(\mu_s^2, \mu_f^2)\right]\right| .$$

- Contribution of all scales separated, evolution factor U evolves from one scale to another.
- Have matching to 2 loops, evolution to 3-loop accuracy.

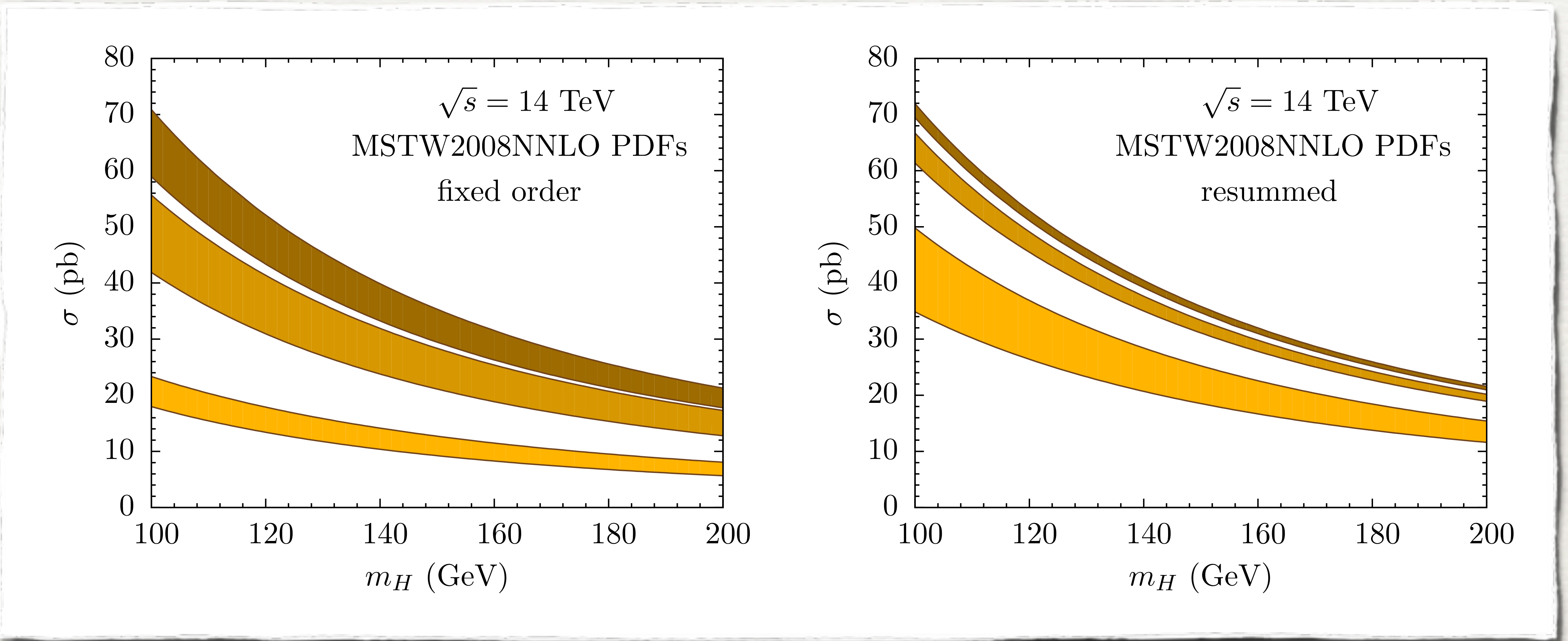
Phenomenological results

Cross section at the LHC



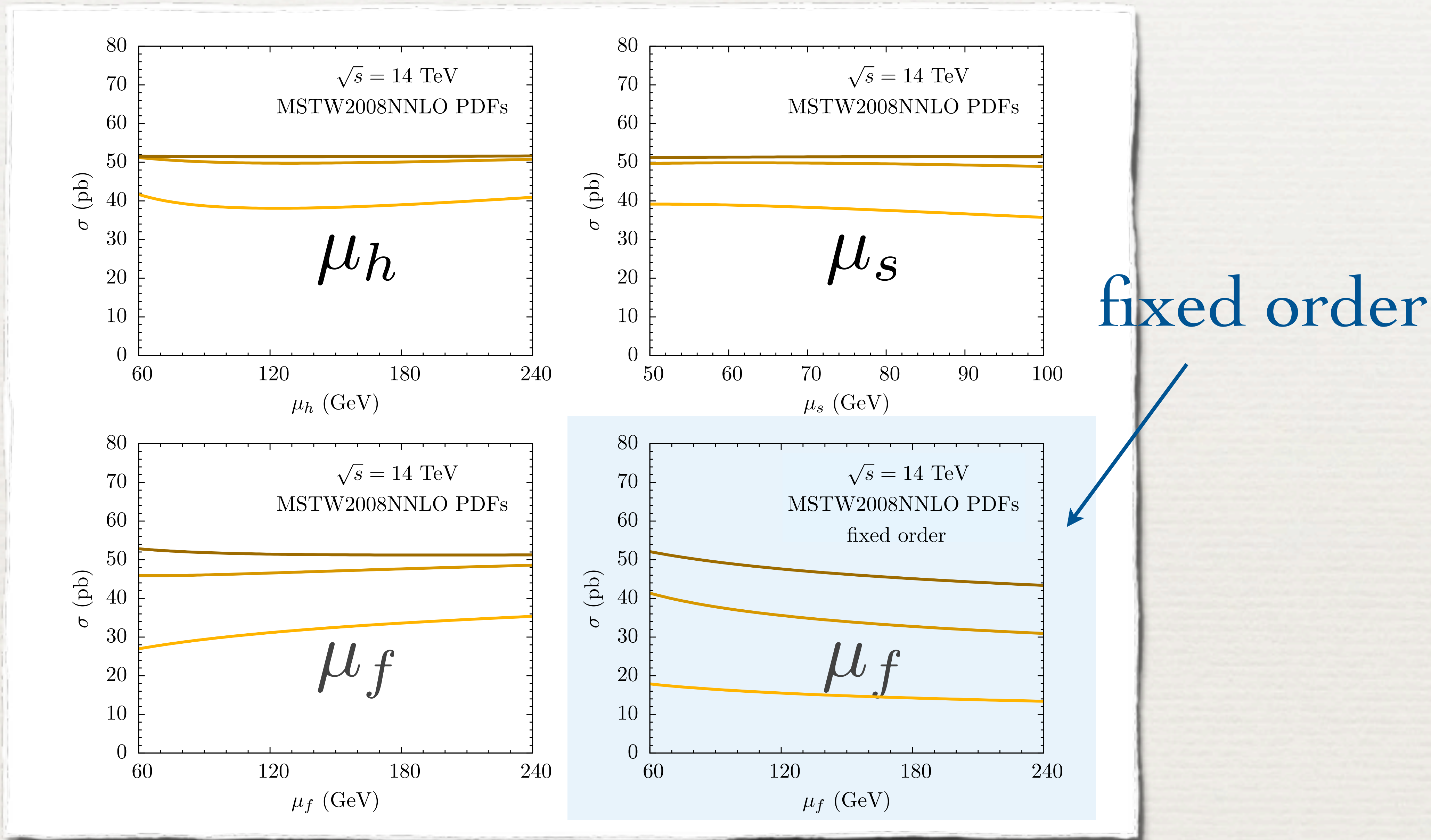
- Different MSTW PDFs at each order.
- Faster convergence, smaller scale dependence. K-factor close to 1. (Note: with '04 PDFs resummed result had K=1.3.)
- Note: for $\mu_f = m_{\rm H}/2$, fixed order result is close to resummed.

Cross section at the LHC



- Same plot, but using the same PDF everywhere.

Scale dependence for m_H=120GeV



- Negligible dependence on μ_t is not shown

Comparison with ordinary threshold resummation

	$\mu_h^2 > 0$		$\mu_h^2 < 0$	
	fixed order	threshold	π^2-enhanced	threshold + π^2
LO	$15.5^{+2.4+0.4}_{-2.1-0.5}$	$17.8^{+3.3+0.4}_{-2.7-0.6}$	$27.1^{+4.0+0.6}_{-3.8-0.8}$	$31.2^{+5.7+0.8}_{-4.8-1.0}$
NLO	$35.5^{+5.9+0.8}_{-4.6-1.1}$	$37.7^{+3.6+0.9}_{-1.2-1.2}$	$45.0^{+3.0+1.1}_{-3.3-1.4}$	$46.6^{+2.5+1.1}_{-1.1-1.5}$
NNLO	$47.6^{+4.5+1.1}_{-4.2-1.5}$	$48.5^{+2.5+1.2}_{-0.5-1.5}$	$51.4^{+1.7+1.2}_{-1.6-1.6}$	$51.4^{+1.4+1.2}_{-0.3-1.6}$

pdf uncertainty

scale uncertainty

cross section at LHC in pb for m_H =120 GeV

- additional uncertainty from α_s.
- threshold resummation only has a small effect.
- both resummations increase cross section.

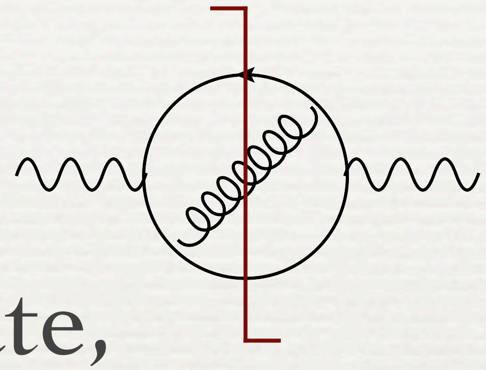
RG improvement for other time-like processes

$\mu^2<0$ for other processes

- Interesting to see the effect of choosing a time-like renormalization point for other processes, in particular
  - Higgs decay $H \to X$
  - $e^+e^- \to$ hadrons
  - τ-decays

  } no Sudakov double log's
  - Drell-Yan process $pp \to \gamma^*/Z + X \to l^+l^- + X$

Hadronic Higgs decay $H \to X$



- Analogous to e+e− → hadrons
- No Sudakov log's in inclusive decay rate, therefore no associated π^2 terms in the analytic continuation.
- Only effect is due to the running of α_s from $-\mu^2$ to $+\mu^2$, which is a small effect at high energies.
  - Equivalent to Contour Improved PT
  - π^2 only at NNLO
  - however large $\beta_0\alpha_s$ term at NLO, so this effect might be more important

Drell-Yan: pp $\rightarrow \gamma^*/Z$ +X $\rightarrow l^+l^-$ +X

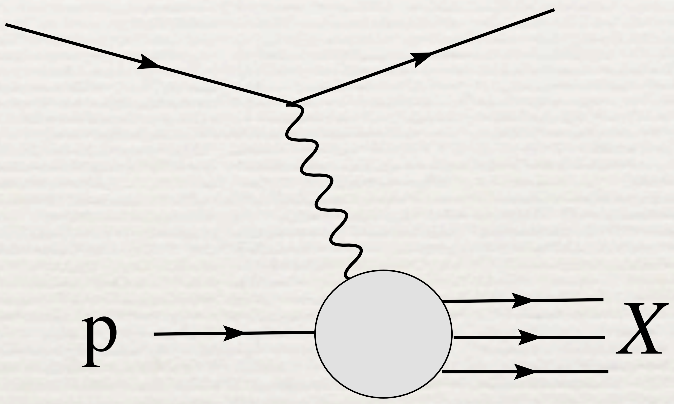
- Near the partonic threshold DY fulfills a factorization theorem similar to Higgs production
- Corresponding hard function is given by quark vector form factor

$$|C_V(-q^2, q^2)|^2 = 1 + 0.0845 + 0.0292 + \dots\,,$$
$$|C_V(-q^2, -q^2)|^2 = 1 - 0.1451 - 0.0012 + \dots\,.$$

  - effect is smaller by C_F/C_A

Drell-Yan vs. DIS

- While DY involves the time-like form factor, DIS involves the space-like form factor



$$\left|\frac{C_V(q^2,\mu^2)}{C_V(-q^2,\mu^2)}\right|^2 \approx \exp\left(\frac{C_F\alpha_s\pi}{2}\right)$$

- Parisi first pointed this out '80; Sterman and Magnea derived resummation formula '90.

Drell-Yan process

- For $\sqrt{q^2} = 8\text{GeV}, \sqrt{s} = 39\text{GeV}$:

	fixed order	threshold	threshold + π^2
LO	$0.299^{+0.051}_{-0.040}$	$0.436^{+0.062}_{-0.071}$	$0.700^{+0.091}_{-0.106}$
NLO	$0.449^{+0.051}_{-0.041}$	$0.493^{+0.011}_{-0.014}$	$0.559^{+0.014}_{-0.035}$
NNLO	$0.505^{+0.021}_{-0.025}$	$0.512^{+0.002}_{-0.004}$	$0.534^{+0.009}_{-0.006}$

- NNLO difference scales like $\alpha(q^2)^3$. For high values of the invariant mass of the lepton pair, the effect is small.

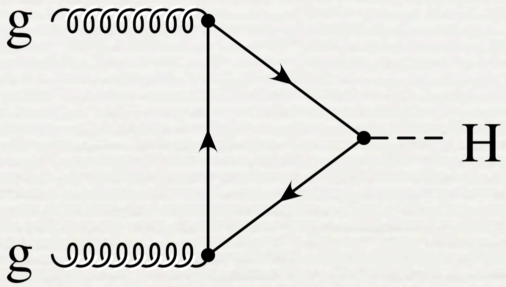
Summary

- Have performed an EFT analysis of Higgs production near the partonic threshold region
    - there are no numerically large Sudakov log's to be resummed, but large corrections arise in the analytic continuation of these log's from space-like to time-like kinematics
    - these can be avoided by evaluating the hard function *H* for $\mu^2<0$ and using the renormalization group to evolve to positive μ^2 values.
- RG-improved prediction has improved convergence and smaller scale dependence. At NNLO, for m_H=120GeV , the cross section is 8% larger at LHC (13% at the Tevatron) than the fixed order result.

extra slides

Fixed-order cross section

- The prefactor is



$$\sigma_0 = \frac{G_F}{\sqrt{2}} \frac{m_H^2\, \alpha_s^2(\mu_f^2)}{288\pi s} \left| \sum_q A(x_q) \right|^2 \quad \text{with } x_q \equiv 4m_q^2/m_H^2$$

- Asymptotic behavior of A

$$A(x_q) \to 1 \qquad \text{for} \quad x_q \to \infty \qquad \text{(heavy quark)}$$
$$A(x_q) \propto x_q \qquad \text{for} \quad x_q \to 0 \qquad \text{(light quark)}$$

  - Contribution of light quarks strongly suppressed. Negligible except for interference of top and bottom (a few % effect).

Time dependence of PDFs

- Shifts in Higgs cross section in updated PDF sets have turned out to be larger than the assigned uncertainties.
- NNLO cross section in pb Anastasiou, Boughezahl and Petriello '09

	MRST01	MRST04	MRST06	MRST08
Tevatron m_H=170 GeV	0.3833	0.3988	0.3943±5%	0.3444±10%
LHC,10 TeV m_H=120 GeV	28.9	29.9	32.6	35.4

  - Cross section σ(10 TeV) ≈ 0.6 σ(14 TeV).
  - Note: LHC numbers above do not include *b*-quark and EW

Three-loop form factor

Baikov, Chetyrkin, Smirnov, Steinhauser '09

- Third order coefficient is sizable
- Space-like:

$$C_S(Q^2, Q^2) = 1 + 0.393\alpha_s - 0.152\alpha_s^2 - 2.05\alpha_s^3$$

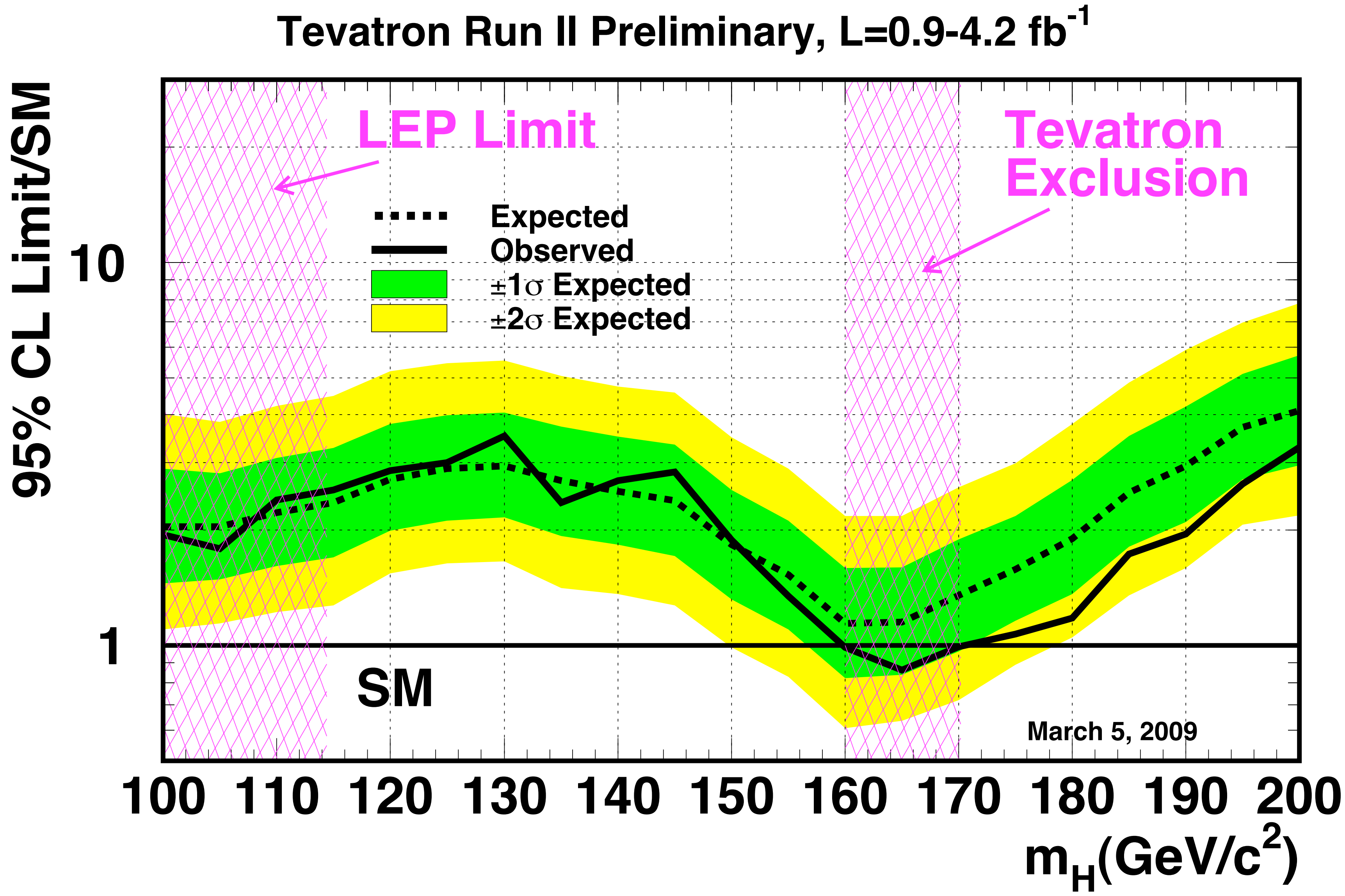
$$H(-q^2, -q^2) = 1 + 0.79\alpha_s - 0.15\alpha_s^2 - 4.22\alpha_s^3$$

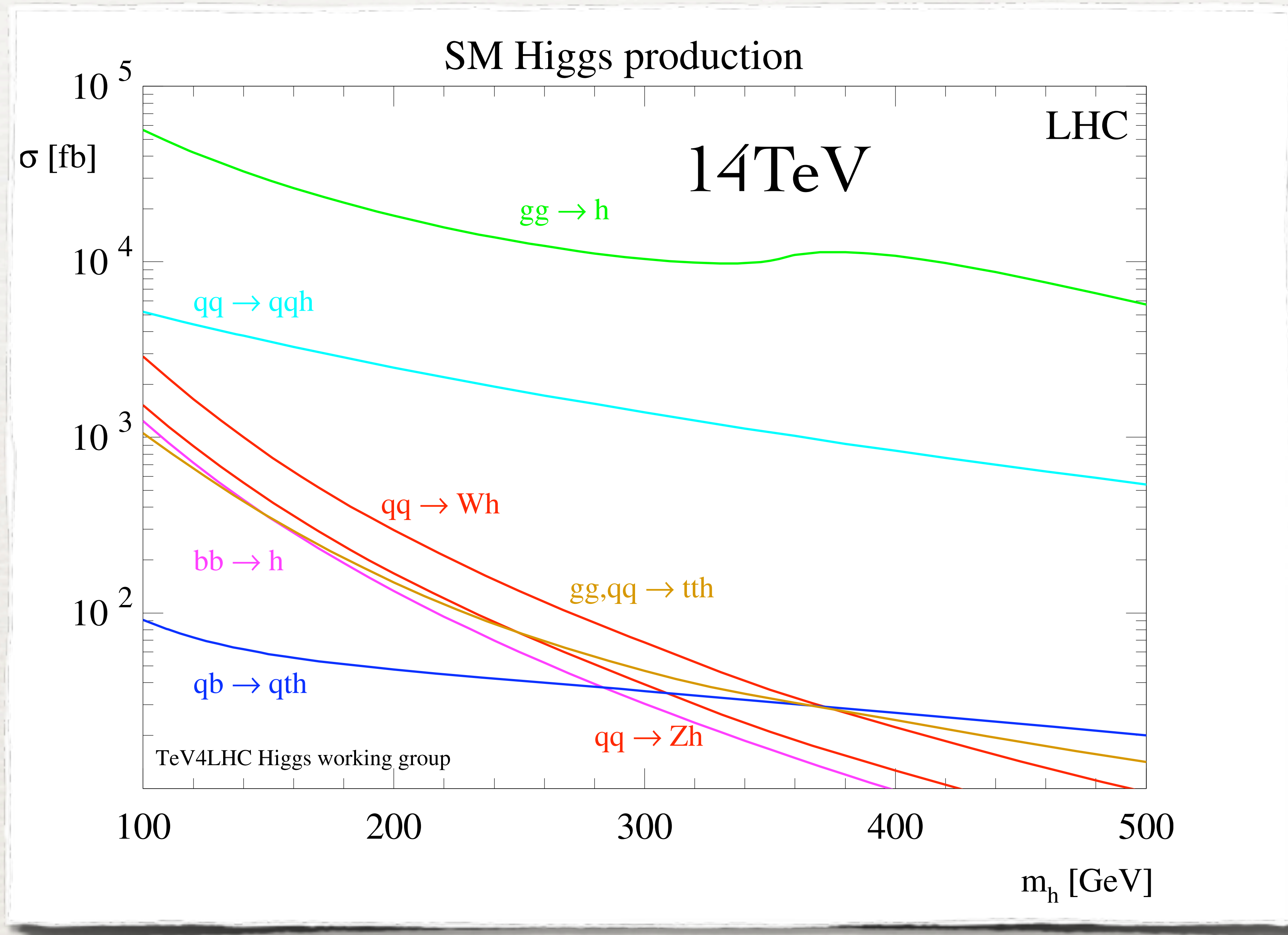
- Time-like

$$C_S(-q^2, q^2) = 1 + 2.75\,\alpha_s + (4.84 + 2.07i)\,\alpha_s^2 + (2.57 + 6.33i)\,\alpha_s^3$$

$$H(-q^2, -q^2) = 1 + 5.5\alpha_s + 17.2\alpha_s^2 + 31.8\alpha_s^3$$

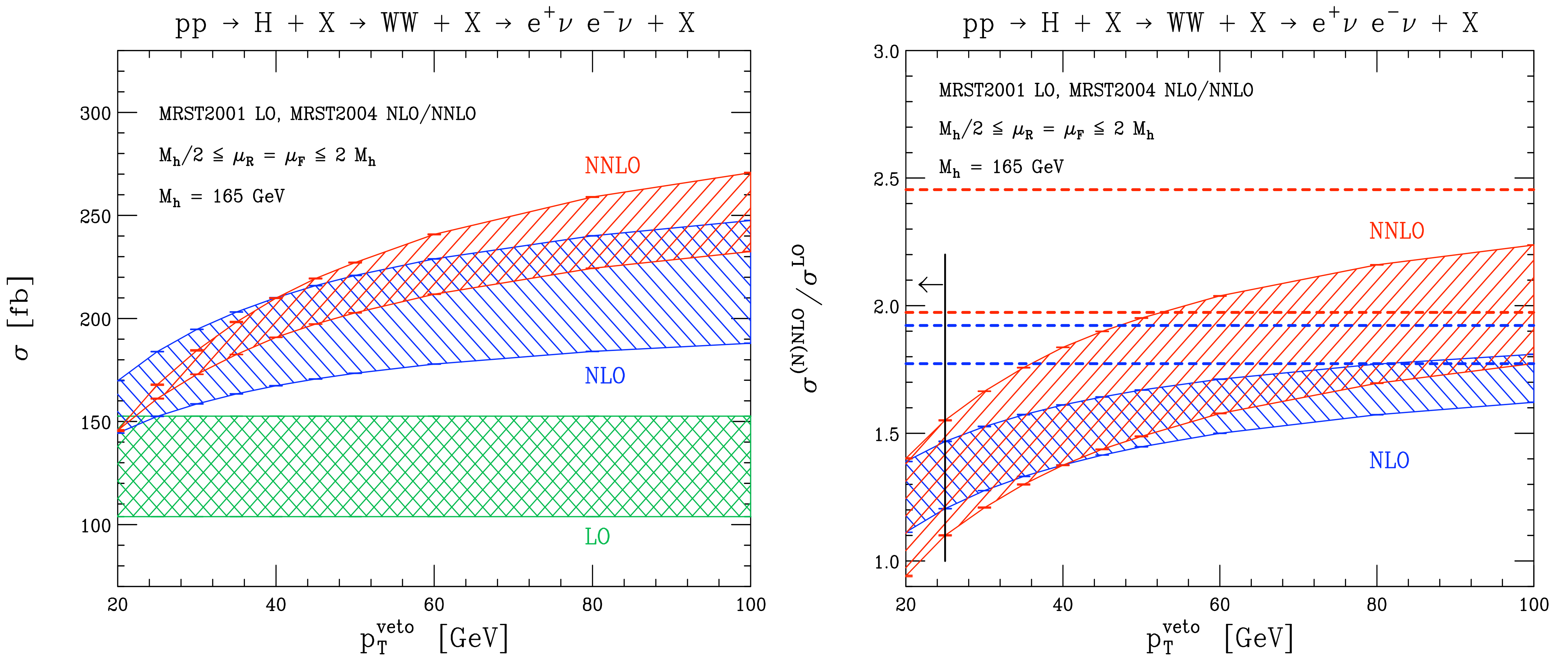
New Tevatron exclusion





- gluon fusion dominates
- note: production cross section is ~50% lower at 10TeV.

Jet veto

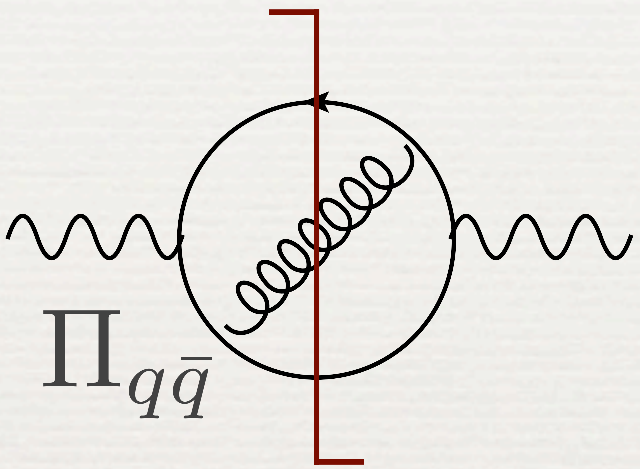


$$|\eta| < 2.5 \text{ and } p_{\rm T}^{\rm jet} > p_{\rm T}^{\rm veto}$$

Anastasiou, Dissertori, Stöckli '07

$e^+e^- \to$ hadrons

$$R(s) = \frac{\sigma(e^+e^- \to \text{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)} = N_c \left(\sum_q e_q^2\right) 4\pi\, \text{Im}\Pi_{q\bar{q}}(-s+i\epsilon)$$



- Standard expansion:

$$4\pi^2\, \text{Im}\Pi_{q\bar{q}}(-s+i\epsilon) = 1 + d_1 \frac{\alpha_s(s)}{4\pi} + d_2 \left(\frac{\alpha_s(s)}{4\pi}\right)^2 + \ldots .$$

- Expansion of $\Pi_{q\bar{q}}$ using time-like scale

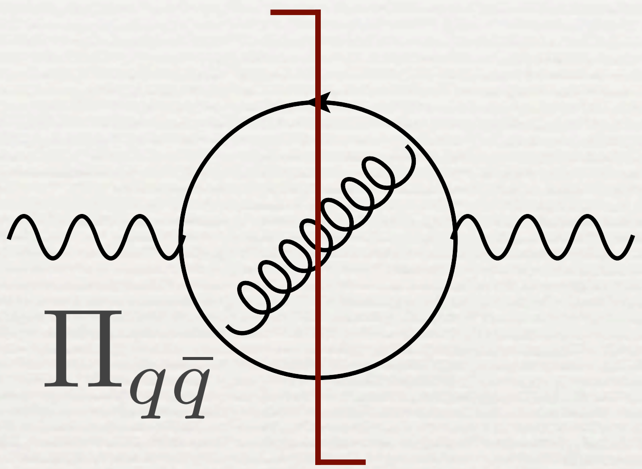
$$4\pi\, \Pi_{q\bar{q}}(-s+i\epsilon) = \frac{1}{\pi}\ln(-s+i\epsilon) - \frac{1}{\pi\beta_0}\left[d_1 \ln\alpha_s(-s+i\epsilon) + \left(d_2 - \frac{d_1\beta_1}{\beta_0}\right)\frac{\alpha_s(-s+i\epsilon)}{4\pi} + \ldots\right] + \text{const.}$$

  - same as contour improved PT

- *Inclusive* rate: no Sudakov log's, no associated π^2's. Only effect is running of the coupling. Numerically, small except for very low values as s (such as $s = m_\tau$).

$e^+e^- \to$ hadrons

- Time-like coupling in terms of $\alpha_s(\mu^2)$



$$\frac{\alpha_s(\mu^2)}{\alpha_s(-\mu^2)} = 1 - ia(\mu^2) + \frac{\beta_1}{\beta_0}\frac{\alpha_s(\mu^2)}{4\pi}\ln\left[1 - ia(\mu^2)\right] + \mathcal{O}(\alpha_s^2)$$

$$a(\mu^2) \equiv \beta_0\alpha_s(\mu^2)/4$$

- Eliminate time-like for space-like

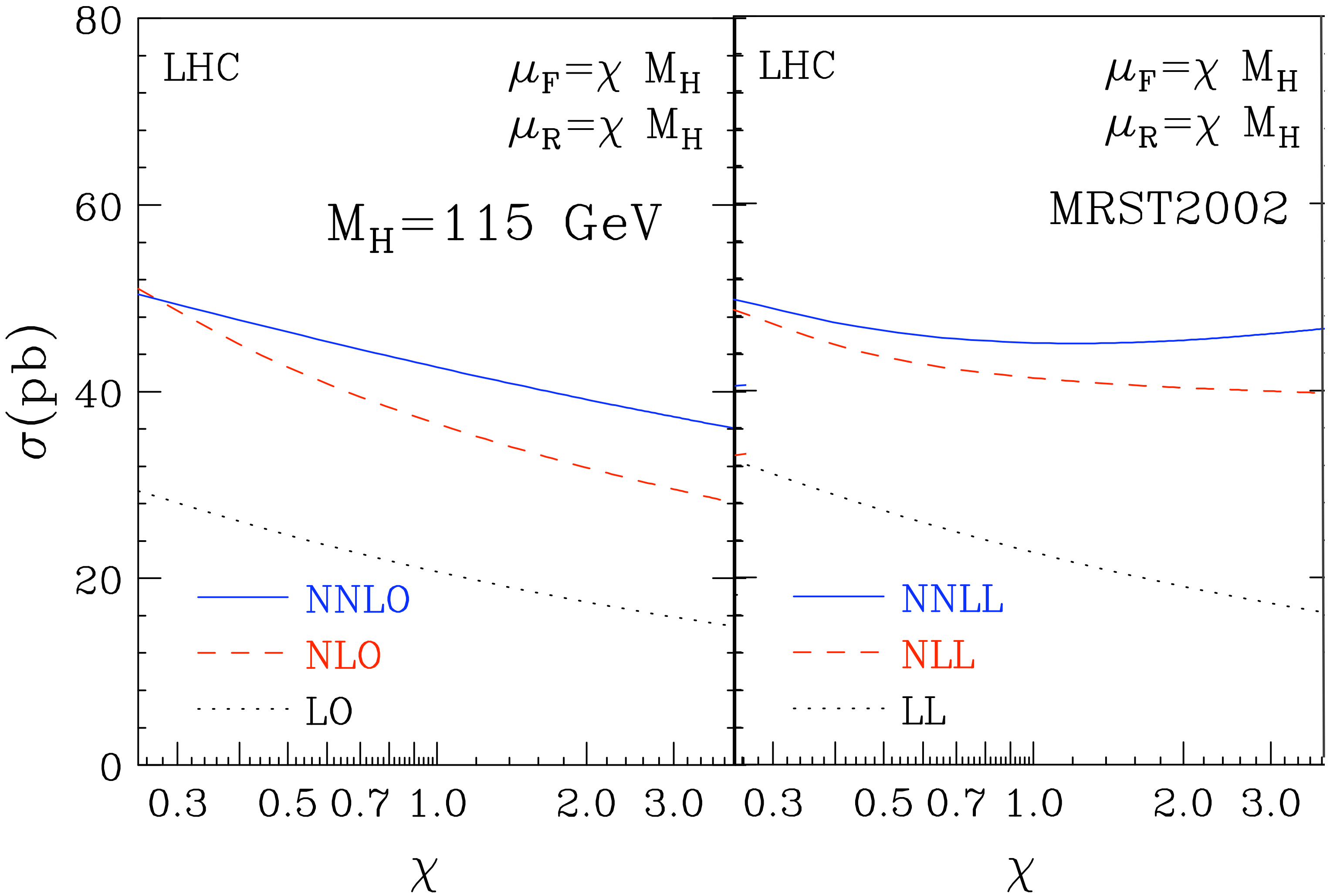
$$4\pi\,\mathrm{Im}\Pi_{q\bar{q}}(-s+i\epsilon) = 1 + d_1\frac{\alpha_s(s)}{4\pi}\frac{\arctan(a_s)}{a_s} + \frac{d_2}{1+a_s^2}\left(\frac{\alpha_s(s)}{4\pi}\right)^2\left[1 + \frac{d_1\beta_1}{d_2\beta_0}\left(\frac{\arctan(a_s)}{a_s} - \frac{\ln(1+a_s^2)}{2} - 1\right)\right] + \dots,$$

$$= 1 + 0.872\,d_1\frac{\alpha_s(s)}{4\pi} + d_2\left(\frac{\alpha_s(s)}{4\pi}\right)^2\left(0.671 - 0.219\frac{d_1\beta_1}{d_2\beta_0}\right) + \dots,$$

$a_s = 0.7$ for $\mu = 1.5\,\mathrm{GeV}$

  - same as contour-improved PT
  - not much of an effect at higher energies

Traditional soft-gluon resummation



Catani, de Florian, Grazzini, Nason '03
numerical update: de Florian and Grazzini '09

Moment space resummation

- Our soft-gluon resummation effects are numerically smaller than what Catani et al. find.
- After matching (for m_H=120GeV at LHC) 50.4pb [trad. method] vs. 48.5pb [RGI] vs. 47.6pb [FO].
- Differences
  - they work in Mellin moment space, expand in 1/*N* instead of (1-z): different power corr's
  - scale choice $\mu_s \sim m_H/N$ is built into traditional framework (this leads to Landau-pole ambiguities)
  - NNLL vs. NNNLL

EFT analysis in moment space

- Can easily do EFT analysis in moment space.

$$C_N(m_t, m_H, \mu_f) = \left[C_t(m_t^2, \mu_t^2)\right]^2 \left|C_S(-m_H^2 - i\epsilon, \mu_h^2)\right|^2$$
$$\times\, U(m_H, \mu_t, \mu_h, \mu_s, \mu_f)\, \widetilde{s}_{\text{Higgs}}\left(\ln\frac{m_H^2}{\mu_s^2} + \partial_\eta, \mu_s^2\right) \bar{N}^{-2\eta} + \mathcal{O}\left(\frac{1}{N}\right)$$

- Numerical results close to traditional approach.
- Mellin inversion

$$\frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty} dN\, z^{-N}\, \bar{N}^{-2\eta} = (-\ln z)^{-1+2\eta}\, \frac{e^{-2\gamma_E \eta}}{\Gamma(2\eta)}$$
$$= \sqrt{z}\, \frac{z^{-\eta}}{(1-z)^{1-2\eta}}\, \frac{e^{-2\gamma_E \eta}}{\Gamma(2\eta)} \left[1 + O\Big((1-z)^2\Big)\right]$$

  - difference is factor of $\sqrt{z}$

Moment space

- Multiplying resummed kernel in momentum space by $\sqrt{z}$ gives numerical result close to moment space approach, [50.1pb].
- Factor $\sqrt{z}$ is power correction near threshold $z \to 1$.
- Appears artificial since the factor is not there in the fixed order result, however, including it makes singular terms larger (96% of full result instead of 80%).