A Global Fit for Thrust at NNNLL: Precision determination for $\alpha_s(M_Z)$

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Ringberg Worshop 2009

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M. Fickinger - University of Arizona
[AFHMS]

MAX-PLANCK-GESELLSCHAFT

FlaviA net

Outline

- **Motivations**
  - Fit lots of data so far unused.
  - Treat non-perturbative effects from solid grounds.
  - Perform global fits to ALL thrust data.
  - Get a precise value of $\alpha_s(M_Z)$.

- **Experimental data**
  - Definition of event shape variables: thrust.
  - Sample of data.

- **A theory for all regions**
  - SCET: fixed order results & resummation of logs & non-perturbative effects.
  - Subtraction of renormalons.
  - Non-singular terms.
  - Power Corrections.

- **Preliminary results**
  - Tail fits: a two-parameter fit.
  - Final (preliminary) value for $\alpha_s(M_Z)$.
  - Comparison with other analysis.

Determinations of $\alpha_s(M_Z)$

See M. Jamin's Talk

$\alpha_s(M_Z)$ is a key parameter for the analysis of all collider experiments.

DIS
τ decays
$\Gamma(Z \to \text{hadrons})$
$e^+e^- \to \text{jets}$
quarkonia
…

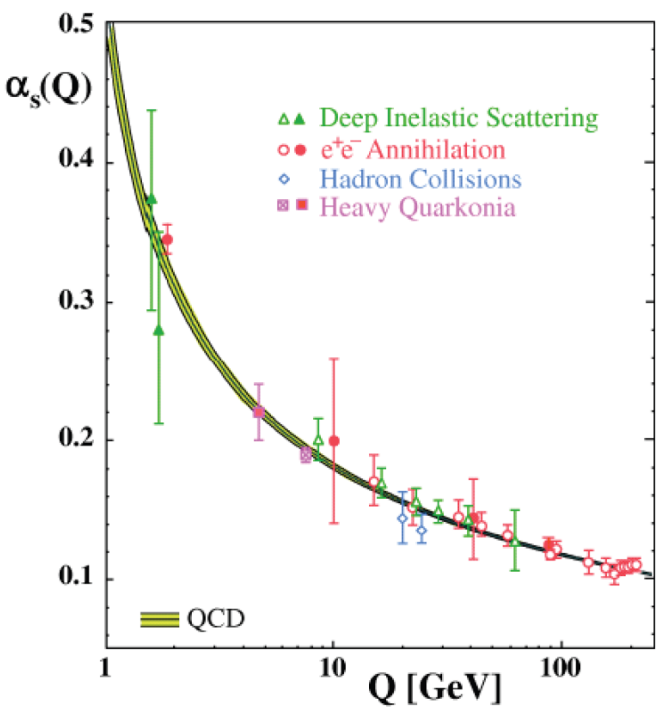
$$\alpha_s(m_Z) = 0.1176(20)$$

PDG'05 [Hinchliffe]

$$\alpha_s(m_Z) = 0.1170 \pm 0.0012$$

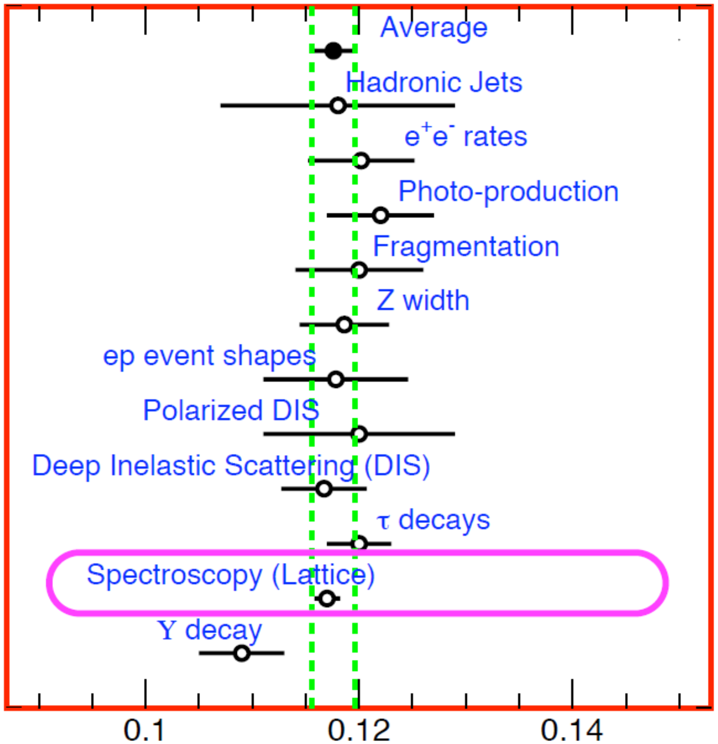
[Mason et al '05]

Staggered quarks



Determination over a wide range of energies

Must always face hadronization effects !

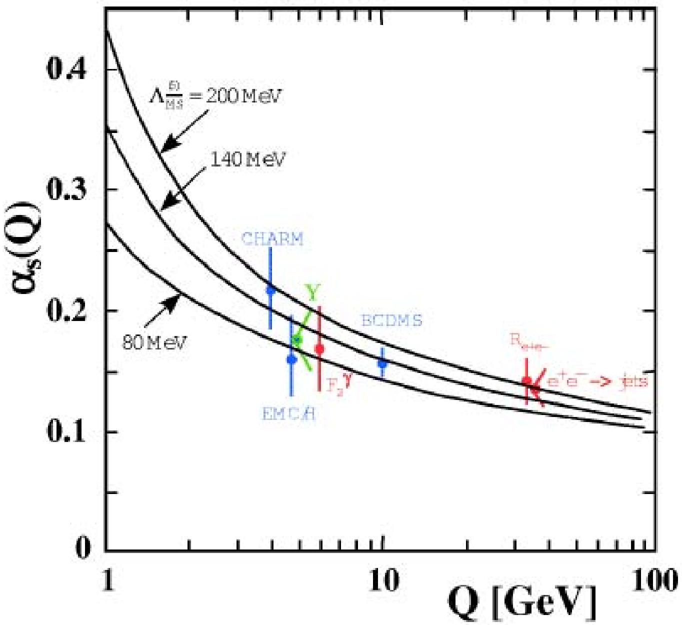


Another World Average

S. Bethke's Review

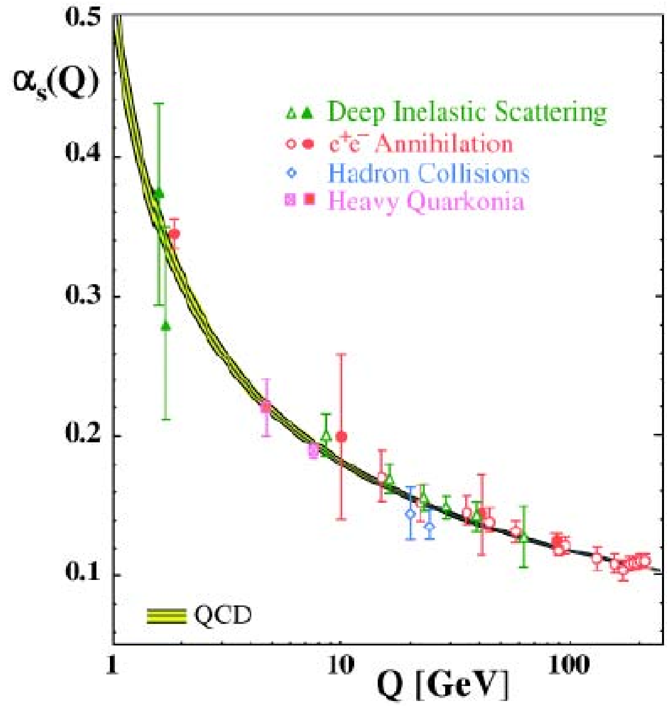
hep-ex/0606035

1989:



$$\alpha_s(m_Z) = 0.11 \pm 0.01$$

2006:



$$\alpha_s(m_Z) = 0.1189 \pm 0.0010$$

not all measurements
are used for this average

Summary of Results

theory errors dominate

measurements used in avg.

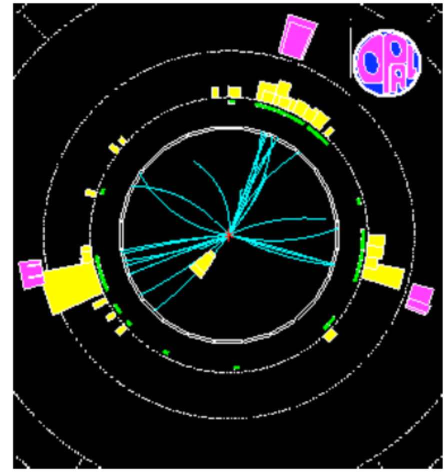
e^+e^- event shapes

Process	Q [GeV]	$\alpha_s(Q)$	$\alpha_s(M_{Z^0})$	$\Delta\alpha_s(M_{Z^0})$ exp.	$\Delta\alpha_s(M_{Z^0})$ theor.	Theory
DIS [pol. SF]	0.7 - 8		$0.113^{+0.010}_{-0.008}$	± 0.004	$^{+0.009}_{-0.006}$	NLO
DIS [Bj-SR]	1.58	$0.375^{+0.062}_{-0.081}$	$0.121^{+0.005}_{-0.009}$	–	–	NNLO
DIS [GLS-SR]	1.73	$0.280^{+0.070}_{-0.068}$	$0.112^{+0.009}_{-0.012}$	$^{+0.008}_{-0.010}$	0.005	NNLO
τ-decays	1.78	0.345 ± 0.010	0.1215 ± 0.0012	0.0004	0.0011	NNLO
DIS [ν; xF_3]	2.8 - 11		$0.119^{+0.007}_{-0.006}$	0.005	$^{+0.005}_{-0.003}$	NNLO
DIS [e/μ; F_2]	2 - 15		0.1166 ± 0.0022	0.0009	0.0020	NNLO
DIS [e-p $\rightarrow$ jets]	6 - 100		0.1186 ± 0.0051	0.0011	0.0050	NLO
Υ decays	4.75	0.217 ± 0.021	0.118 ± 0.006	–	–	NNLO
$Q\overline{Q}$ states	7.5	0.1886 ± 0.0032	0.1170 ± 0.0012	0.0000	0.0012	LGT
e^+e^- [F_2^γ]	1.4 - 28		$0.1198^{+0.0044}_{-0.0054}$	0.0028	$^{+0.0034}_{-0.0046}$	NLO
e^+e^- [σ_{had}]	10.52	0.20 ± 0.06	$0.130^{+0.021}_{-0.029}$	$^{+0.021}_{-0.029}$	0.002	NNLO
e^+e^- [jets & shps]	14.0	$0.170^{+0.021}_{-0.017}$	$0.120^{+0.010}_{-0.008}$	0.002	$^{+0.009}_{-0.008}$	resum
e^+e^- [jets & shps]	22.0	$0.151^{+0.015}_{-0.013}$	$0.118^{+0.009}_{-0.008}$	0.003	$^{+0.009}_{-0.007}$	resum
e^+e^- [jets & shps]	35.0	$0.145^{+0.012}_{-0.007}$	$0.123^{+0.008}_{-0.006}$	0.002	$^{+0.008}_{-0.005}$	resum
e^+e^- [σ_{had}]	42.4	0.144 ± 0.029	0.126 ± 0.022	0.022	0.002	NNLO
e^+e^- [jets & shps]	44.0	$0.139^{+0.011}_{-0.008}$	$0.123^{+0.008}_{-0.006}$	0.003	$^{+0.007}_{-0.005}$	resum
e^+e^- [jets & shps]	58.0	0.132 ± 0.008	0.123 ± 0.007	0.003	0.007	resum
$p\bar{p} \rightarrow b\bar{b}X$	20.0	$0.145^{+0.018}_{-0.019}$	0.113 ± 0.011	$^{+0.007}_{-0.006}$	$^{+0.008}_{-0.009}$	NLO
$p\bar{p}$, $pp \rightarrow \gamma X$	24.3	$0.135^{+0.012}_{-0.008}$	$0.110^{+0.008}_{-0.005}$	0.004	$^{+0.007}_{-0.003}$	NLO
$\sigma(p\bar{p} \rightarrow$ jets)	40 - 250		0.118 ± 0.012	$^{+0.008}_{-0.010}$	$^{+0.009}_{-0.008}$	NLO
e^+e^- $\Gamma(Z \rightarrow$ had)	91.2	$0.1226^{+0.0058}_{-0.0038}$	$0.1226^{+0.0058}_{-0.0038}$	± 0.0038	$^{+0.0043}_{-0.0005}$	NNLO
e^+e^- 4-jet rate	91.2	0.1176 ± 0.0022	0.1176 ± 0.0022	0.0010	0.0020	NLO
e^+e^- [jets & shps]	91.2	0.121 ± 0.006	0.121 ± 0.006	0.001	0.006	resum
e^+e^- [jets & shps]	133	0.113 ± 0.008	0.120 ± 0.007	0.003	0.006	resum
e^+e^- [jets & shps]	161	0.109 ± 0.007	0.118 ± 0.008	0.005	0.006	resum
e^+e^- [jets & shps]	172	0.104 ± 0.007	0.114 ± 0.008	0.005	0.006	resum
e^+e^- [jets & shps]	183	0.109 ± 0.005	0.121 ± 0.006	0.002	0.005	resum
e^+e^- [jets & shps]	189	0.109 ± 0.004	0.121 ± 0.005	0.001	0.005	resum
e^+e^- [jets & shps]	195	0.109 ± 0.005	0.122 ± 0.006	0.001	0.006	resum
e^+e^- [jets & shps]	201	0.110 ± 0.005	0.124 ± 0.006	0.002	0.006	resum
e^+e^- [jets & shps]	206	0.110 ± 0.005	0.124 ± 0.006	0.001	0.006	resum

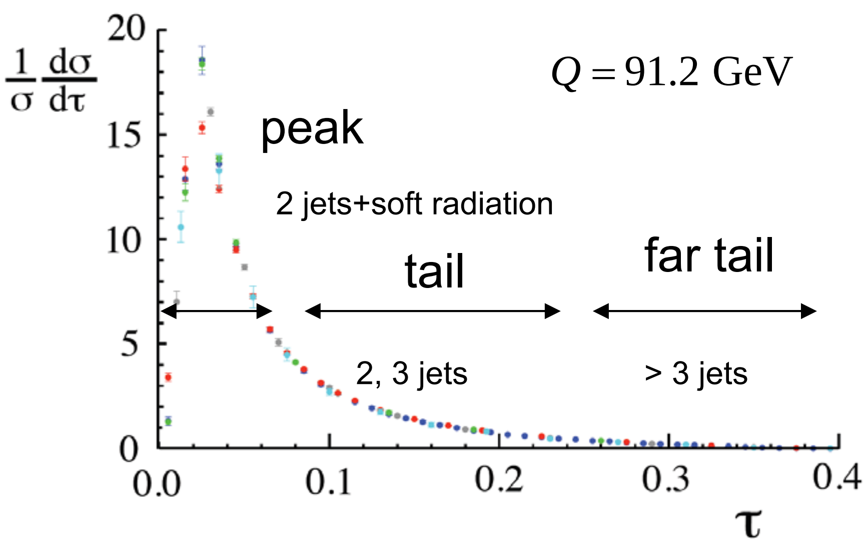
Thrust experimental data

$e^+e^- \xrightarrow{Q} \text{jets}$

LEP 2 jet event



OPAL 3 jet event

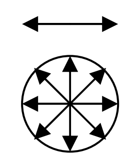


$$1-\tau = \max_{\hat{n}} \frac{\sum_i |\hat{n}\cdot\vec{p}_i|}{Q}$$

τ is an event shape variable

$\tau \to 0 \Rightarrow$ dijet

$\tau \to 0.5 \Rightarrow$ spherical

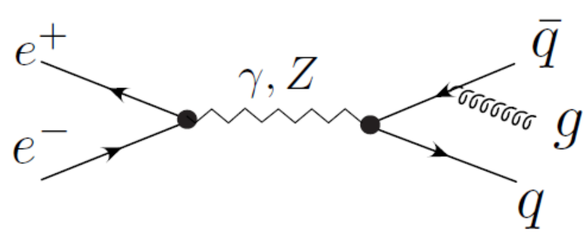


At each Q there is a distribution in τ

Experimental data

	Experiment	Values of Q
LEP	ALEPH	{91.2, 133.0, 161.0, 172.0, 183.0, 189.0, 200.0, 206.0}
	DELPHI	{45.0, 66.0, 76.0, 89.5, 91.2, 93.0, 133.0, 161.0, 172.0, 183.0, 189.0, 192.0, 196.0, 200.0, 202.0, 205.0, 207.0}
	OPAL	{91.0, 133.0, 177.0, 197.0}
	L3	{41.4, 55.3, 65.4, 75.7, 82.3, 85.1, 91.2, 130.1, 136.1, 161.3, 172.3, 182.8, 188.6, 194.4, 200.0, 206.2}
SLAC	SLD	{91.2}
DESY	TASSO	{14.0, 22.0, 35.0, 44.0}
	JADE	{35.0, 44.0}
KEK	AMY	{55.2}

Three jet events are proportional to α_s, good sensitivity



Theoretical motivation

$$\ln\left(\frac{1}{\sigma}\frac{d\sigma}{d\tau}\right) \sim (\ln\tau)\sum_{k=0}(\alpha_s \ln\tau)^{k+1} + \sum_{k=0}(\alpha_s \ln\tau)^{k+1} + \alpha_s\sum_{k=0}(\alpha_s \ln\tau)^{k} + \alpha_s^2\sum_{k=0}(\alpha_s \ln\tau)^{k} + \ldots$$

LL NLL NNLL N^3LL

For low τ, $\ln(\tau) \sim \frac{1}{\alpha_s}$ → Rearrange the perturbative series and resum logs

In fact logs dominate even for moderate τ

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LL NLL NNLL N^3LL

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In fact logs dominate even for moderate τ

singular — non-singular

$$\frac{1}{\sigma}\frac{d\sigma}{d\tau} \sim \sum_{k,l}\alpha_s^n \frac{\ln^l\tau}{\tau} + \overbrace{\sum_{k,l}\alpha_s^n \ln^l\tau + \sum_{k,l}\alpha_s f_m(\tau)}^{\text{non-singular}}$$ perturbative part

$$+ f\left(\tau, \frac{\Lambda_{QCD}}{Q}\right)$$ Non-perturbative power corrections

Improvements on earlier work

~ 1 year ago three interesting papers appear

Becher, Schwartz → NNNLL' perturbative analysis

T. Gehrmann et al

Weinzierl → NNNLO fixed order result

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In our work we have...

1. Full treatment of non-perturbative effects with field theory

   (systematic treatment of errors from power corrections).
2. Renormalon subtraction.
3. Simultaneous description of all three regions.
4. Account for factorization theorem for subleading orders.
5. b - mass effects.
6. QED effects.

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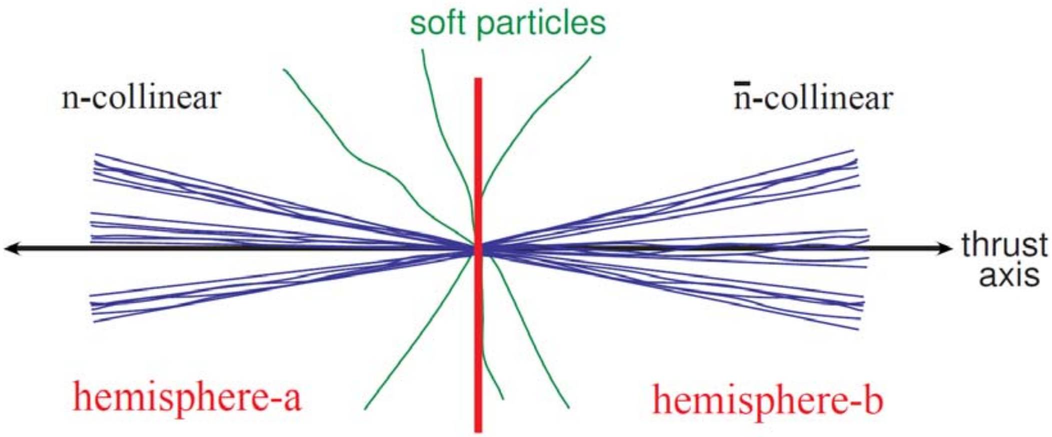
Aim

Global fit for all values of τ and Q

Fit for $\alpha_s(m_Z)$ and non-perturbative effects simultaneously

SCET basics

$$e^+e^- \xrightarrow{\gamma,Z} \text{2 jets} + X_{soft}$$



Particles in each hemisphere have

$$p_+ = E + p_n \sim O(Q)$$

$$p_- = E - p_n \sim O(\Lambda_{QCD})$$

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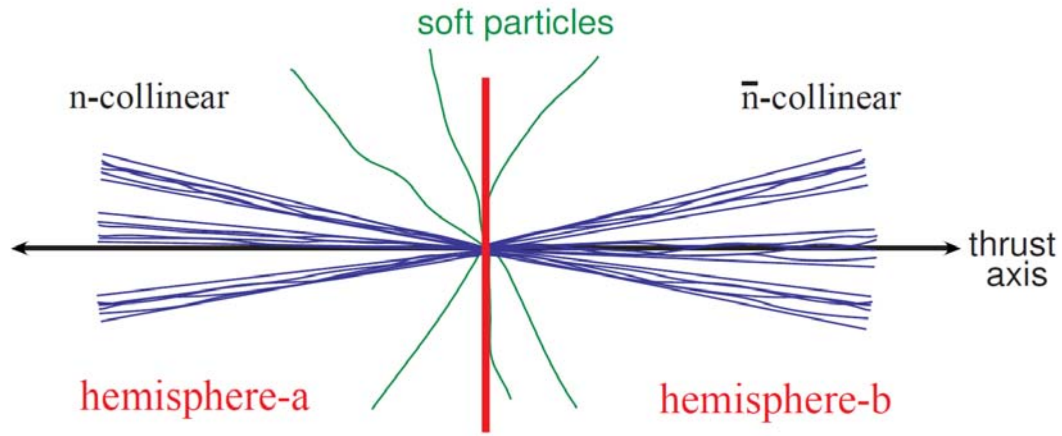

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3 energy scales ⟶ EFT

large logs !

$$Q^2 = \left(E_{e^+} + E_{e^-}\right)^2 = \left(\sum_{hem\ a+b} p_i^\mu\right)^2 \approx 2\sum_{hem\ a} E_i^2 \sim (100\ \text{GeV})^2$$

$$\mu_{Jet}^2 = \left(\sum_{hem\ a} p_i^\mu\right)^2 \approx \sum_{hem\ a} (E_i^2 - \vec{p}_i^{\,2}) \sim \sum_{hem\ a} p_i^+ p_i^- \sim Q\,\Lambda_{QCD} \sim \left(20\ \text{GeV}\right)^2$$

$$\mu_{soft}^2 = p_{hadron}^2 = \Lambda_{QCD}^2 \sim \left(2\ \text{GeV}\right)^2$$

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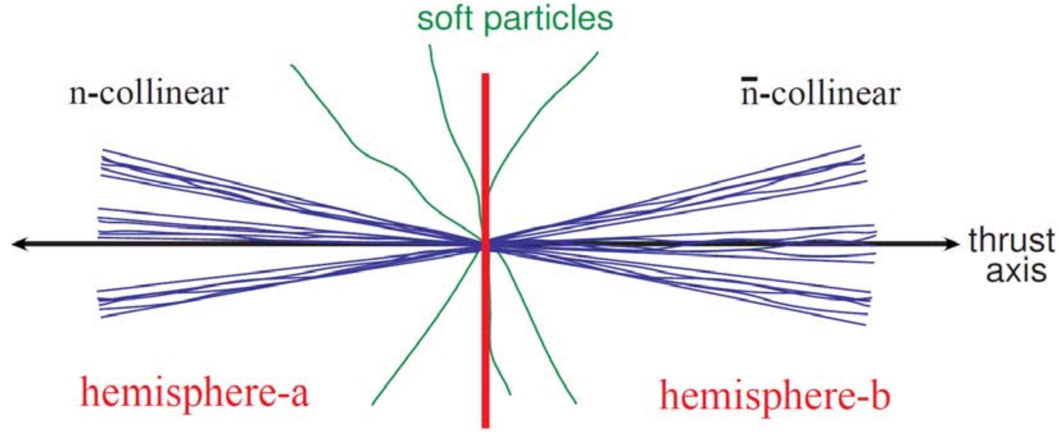

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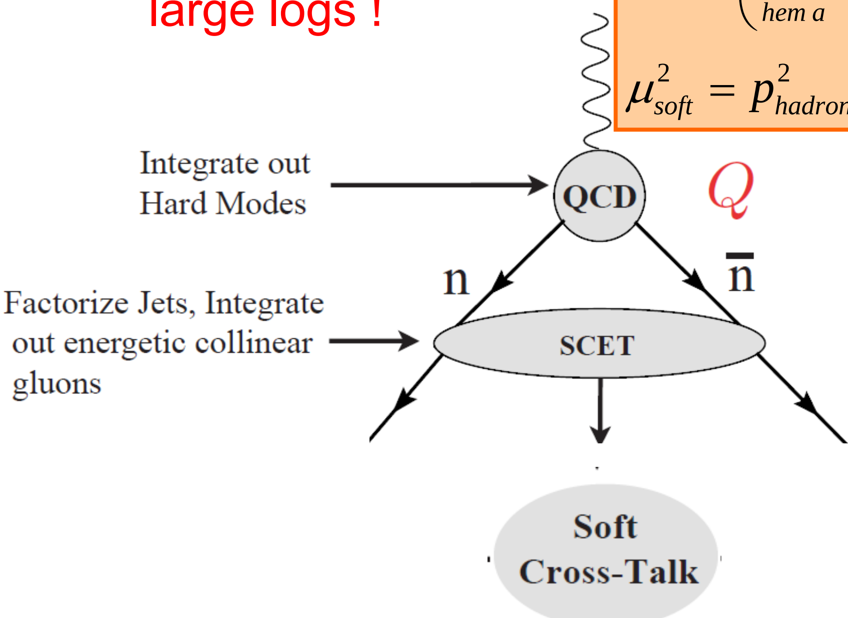

Matching

QCD SCET representation

$$\bar{q}\,\Gamma^\mu q \;\rightarrow\; \bar{\xi}_n W_n Y_n^\dagger \Gamma^\mu Y_{\bar{n}} W_{\bar{n}} \xi_{\bar{n}}$$

Collinear Wilson lines

Soft Wilson lines

SCET basics

SCET $\lambda \sim \sqrt{\tau}$			
	n-collinear	$(\xi_n,\, A_n^\mu)$	$p_n^\mu \sim Q(\lambda^2, 1, \lambda)$
	$\bar{n}$-collinear	$(\xi_{\bar{n}},\, A_{\bar{n}}^\mu)$	$p_{\bar{n}}^\mu \sim Q(1, \lambda^2, \lambda)$
Crosstalk:	soft	$(q_s,\, A_s^\mu)$	$p_s^\mu \sim Q(\lambda^2, \lambda^2, \lambda^2)$

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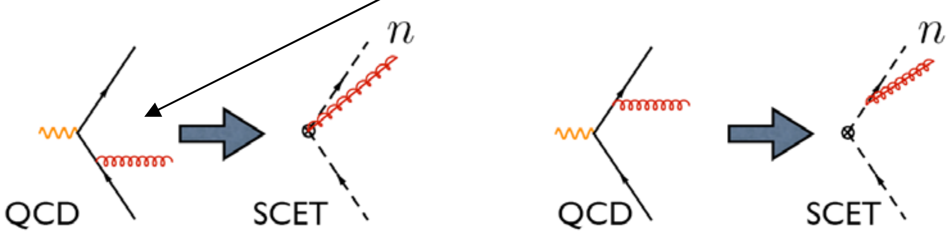
$$Y_n^\dagger(x) = \mathrm{P}\ \exp\left(ig\int_0^\infty ds\, n\cdot A_s(ns+x)\right)$$

$$W_n = P\exp\left(ig\int_0^\infty ds\, \bar{n}\cdot A_n(s\bar{n})\right)$$

Soft & collinear Wilson lines

$$\bar{q}\,\Gamma^\mu q \quad \rightarrow \quad C(Q,\mu_h)\, \bar{\xi}_n W_n Y_n^\dagger\, \Gamma^\mu\, Y_{\bar{n}} W_{\bar{n}}\, \xi_{\bar{n}}$$



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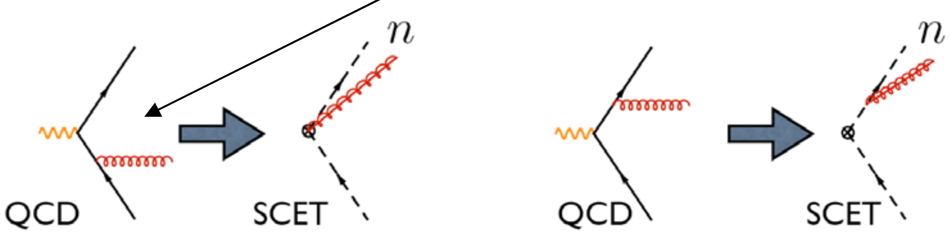

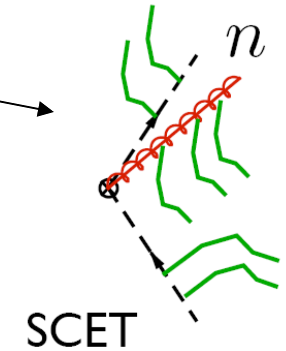

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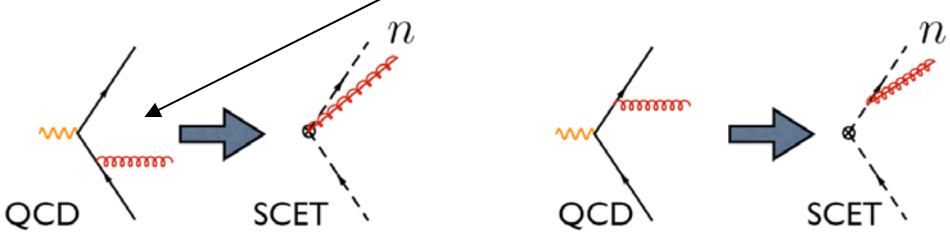

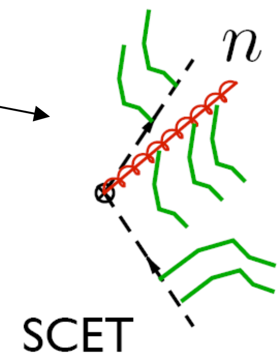

$$S(\ell^+,\ell^-,\mu) \equiv \frac{1}{N_c}\sum_{X_s}\delta(\ell^+ - k_s^{+a})\delta(\ell^- - k_s^{-b})\langle 0|(\overline{Y}_{\bar{n}})^{cd}\,(Y_n)^{ce}(0)|X_s\rangle\langle X_s|(Y_n^\dagger)^{ef}\,(\overline{Y}_{\bar{n}}^\dagger)^{df}(0)|0\rangle$$

$$J_n(Qr_n^+,\mu) = \frac{-1}{8\pi N_c Q}\,\mathrm{Disc}\int d^4x\, e^{ir_n\cdot x}\,\langle 0|\mathrm{T}\,\bar{\chi}_{n,Q}(0)\bar{\not{n}}\chi_n(x)|0\rangle$$

$$H(Q,\mu_h) = \left|C(Q,\mu_h)\right|^2$$

Factorization theorem

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\tau} = \sigma_0 \, H(Q,\mu) \int \mathrm{d}\ell \, \mathrm{d}\ell' \, J_T(Q\tau - \ell, \mu) \, S_p(\ell - \ell' - 2\Delta, \mu) \, S_{\mathrm{model}}(\ell', \lambda)$$

Still has large logs

Electroweak factor (family dependent)

Hard matching coefficient (function)

Jet function (distribution)

Partonic Soft function (distribution + renormalon)

Model Soft function (non-perturbative)

Valid at leading order in power counting. Efficient for the dijet region ! (SCET)

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There is a u=1/2 renormalon ambiguity in S_{partonic}

$$\Delta S_{\mathrm{partonic}}(\ell) \sim \Lambda_{\mathrm{QCD}} \frac{\mathrm{d}}{\mathrm{d}\ell} S_{\mathrm{partonic}}(\ell)$$

Hoang & Stewart

The renormalon arises from separating perturbative and non-perturbative effects in dim-reg. Subtracting the renormalon gives stability to perturbation theory, and here achieves a positive cross section at very small tau.

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Model Soft function (non-perturbative)

Valid at leading order in power counting. Efficient for the dijet region ! (SCET)

Complete basis!

$$S_{\text{model}}(\ell) = \frac{1}{\lambda}\left[\sum_{n=0}^{\infty} c_n\, f_n\!\left(\frac{\ell}{\lambda}\right)\right]^2$$

$$\Delta = \delta(R,\mu) + \bar{\Delta}(R,\mu)$$

← Renormalon-free gap shift

$$\delta(R,\mu) = \sum_n \left(\frac{\alpha}{4\pi}\right)^n \delta_n(R,\mu)$$

Infrared scheme parameter

[See A. Jain's Talk]

Resummation of Logs

Resummation of large logs!

No large logs any more

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\tau} = \sigma_0 H(Q,\mu_Q) U_H(Q,\mu_Q,\mu_s) \int \mathrm{d}\ell \mathrm{d}\ell' \, U_J(Q\tau - \ell - \ell', \mu_Q, \mu_s) J_T(Q\ell', \mu_j) S_T(\ell, \mu_s)$$

$$\mu_h \sim Q \qquad \mu_j \sim Q\sqrt{\tau} \qquad \mu_s \sim Q\tau$$

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$$\mu_h \sim Q \qquad \mu_j \sim Q\sqrt{\tau} \qquad \mu_s \sim Q\tau$$

In Fourier space all convolutions turn into product of functions

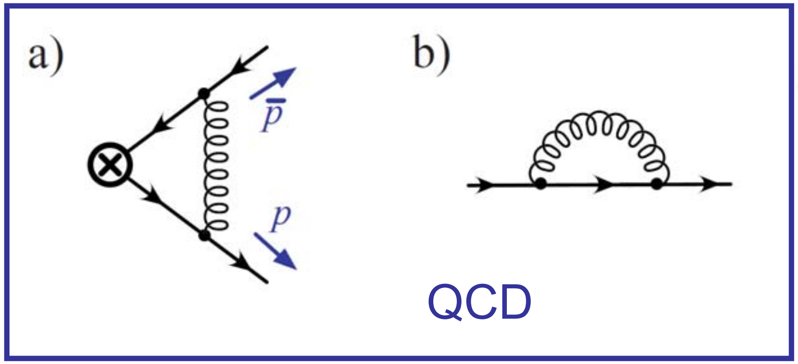
Removes renormalon

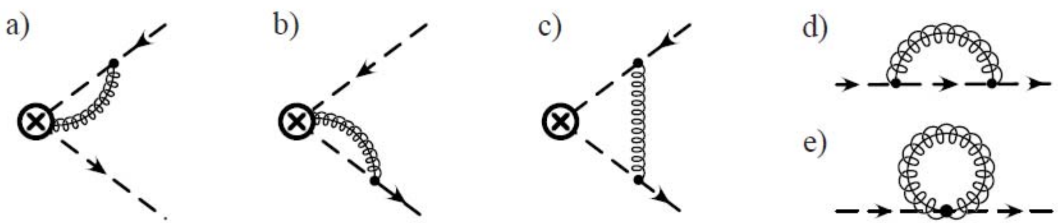
$$\frac{d\sigma}{d\tau} = \sigma_0 \, H(Q,\mu_Q) U_H(Q,\mu_Q,\mu_s) \int dx \, e^{ix(Q\tau - \bar{\Delta})} \, \tilde{U}_J(x,\mu_Q,\mu_s) \, \tilde{J}_T\left(\frac{x}{Q},\mu_j\right) e^{-2ix\delta} \tilde{S}_p(x,\mu_s) \tilde{S}_{\rm model}(x,\lambda)$$

Shifts τ

Expand out in α_s

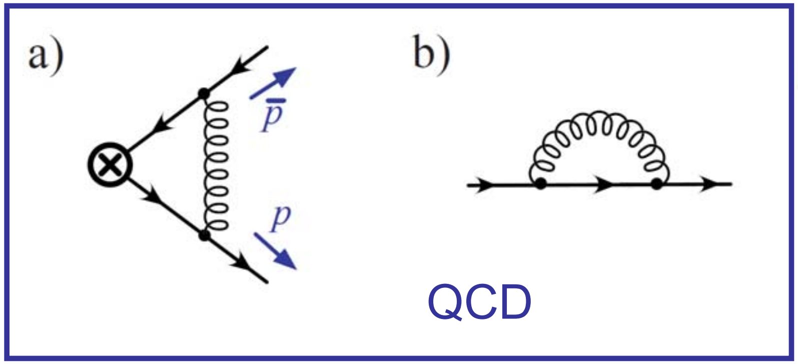
Hard, jet and soft pieces

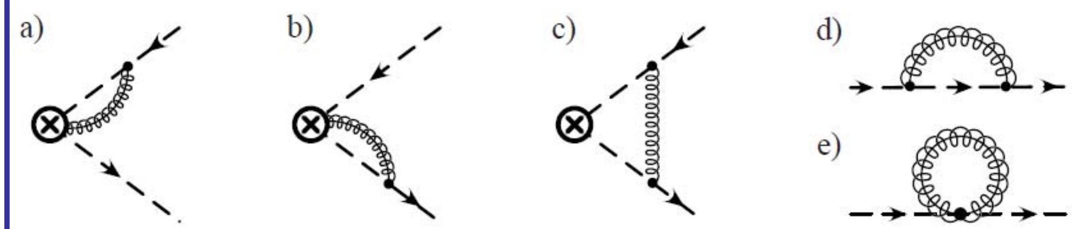


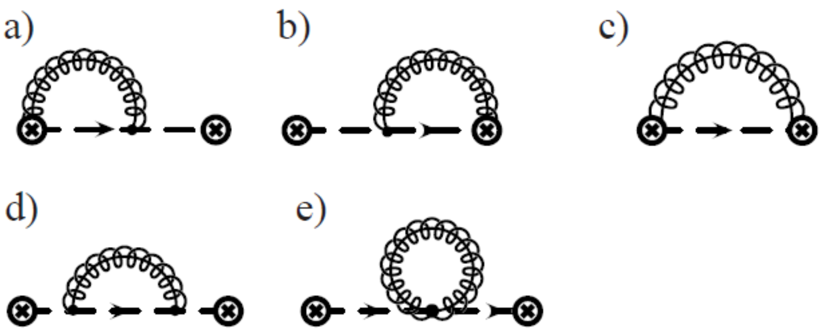


Hard coefficient: from matching

Hard, jet and soft pieces







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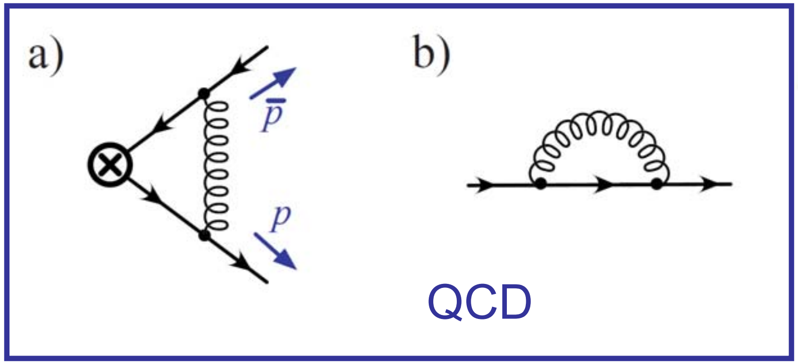

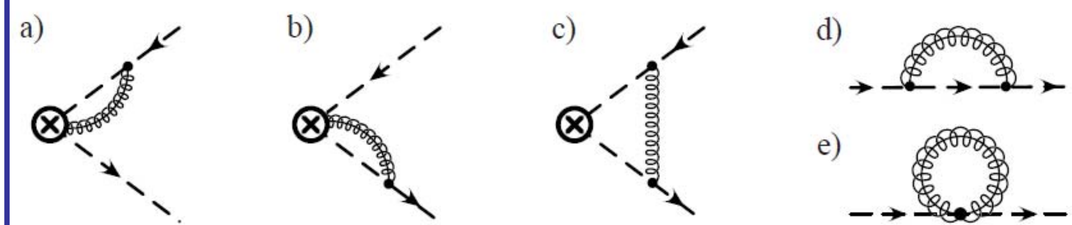

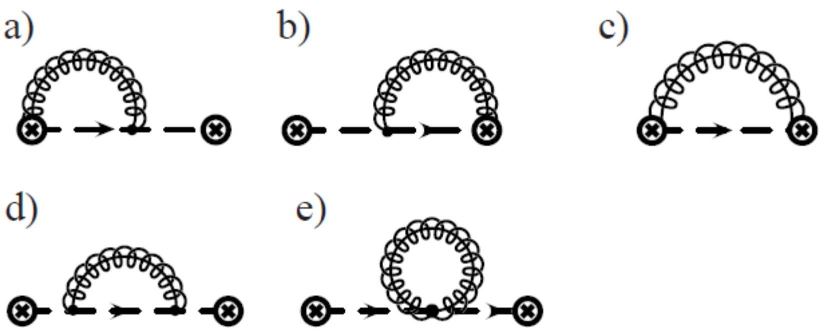

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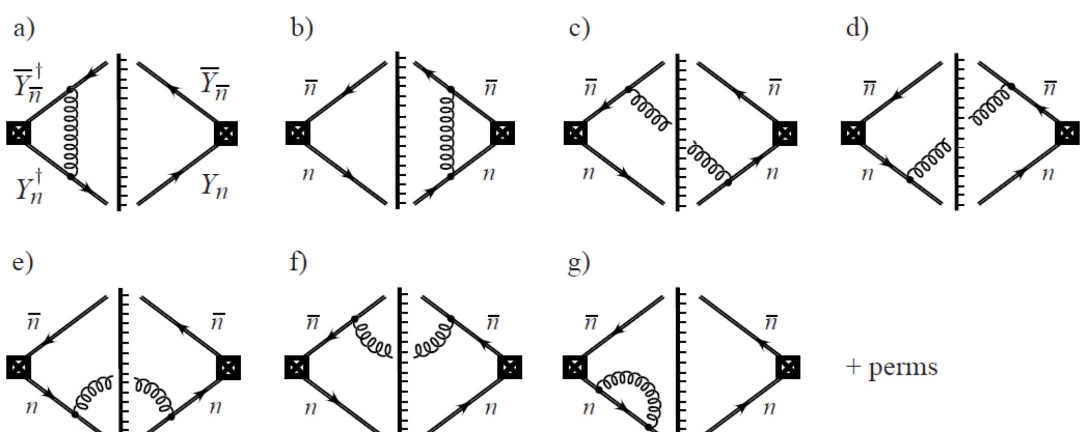

Soft function:

It is non-perturbative

Perturbative for large tau

Ingredients for the calculation

Order or the analysis

Matrix elements

IR and UV running of gap parameter

Renormalon subtraction terms

	cusp	non-cusp	matching	alphas	γ_Δ^μ	γ_Δ^R	$\delta(\Delta)_i$	non-singular
LL	1	–	tree	1	tree	tree	–	–
NLL	2	1	tree	2	1	1	–	–
NNLL	3	2	1	3	2	2	1	1
N^3LL	4^P	3	2	4	3	3	2	2
N^3LL’	4^P	3	3^*	4	3	3	3	3

From a Padé approximant

log information known, and sum of non-log terms known.

From fixed order full QCD calculations

Ingredients for the calculation

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From a Padé approximant

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From fixed order full QCD calculations

The renormalon subtractions $\delta_i(R,\mu)$, and the UV and IR running of the gap $\Delta(R,\mu)$ are determined from the soft function

The ‘primed’ analysis enhances the matching order by one. Relevant !

Ingredients for the calculation

	Known up to $O(\alpha_s^2)$ NNLO	Moch, Vermaresen & Vogt
Hard coefficient	Anomalous dimension known up to $O(\alpha_s^3)$	
NNLL	Cusp anomalous dimension known up to $O(\alpha_s^3)$	

Ingredients for the calculation

Hard coefficient

NNLL

- Known up to $O(\alpha_s^2)$ NNLO
- Anomalous dimension known up to $O(\alpha_s^3)$
- Cusp anomalous dimension known up to $O(\alpha_s^3)$

} Moch, Vermaresen & Vogt

Jet function

NNNLL

- Known up to $O(\alpha_s^2)$ NNLO — Becher & Neubert
- Anomalous dimension known up to $O(\alpha_s^3)$ — Moch, Vermaresen & Vogt

Ingredients for the calculation

Hard coefficient — NNLL

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Moch, Vermaresen & Vogt

Jet function — NNNLL

- Known up to $O(\alpha_s^2)$ NNLO — Becher & Neubert
- Anomalous dimension known up to $O(\alpha_s^3)$ — Moch, Vermaresen & Vogt

Soft function

- Analytically known up to $O(\alpha_s)$ — Schwartz; Fleming et al
- Numerically known up to $O(\alpha_s^2)$ NNLO — Becher & Schwartz; Hoang & Kluth

Adding the $O(\alpha_s^4)$ cusp with a Padé approximation $\longrightarrow$ NNNLL' analysis

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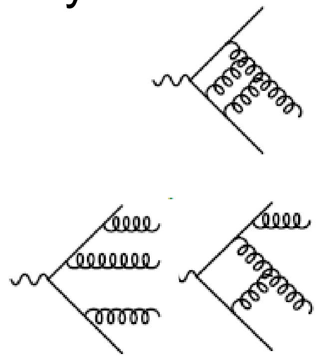
Adding the $O(\alpha_s^4)$ cusp with a Padé approximation → NNNLL' analysis

Fixed order		
1 - loop	Known analytically	Ellis et al.
2 -, 3 - loops	Known only numerically	Glover et al, Gehrmann et al.
		Weinzierl



Power corrections

$$\frac{\Lambda_{QCD}}{\mu_s} \approx \frac{\Lambda_{QCD}}{Q\tau}$$

$$H \; J_T \otimes S_T^{pert} \otimes S_T^{\text{model}}$$

$$\frac{\mu_s^2}{\mu_J^2} \approx \tau$$

$$\frac{d\sigma_{\text{nonsingular}}}{d\tau} \otimes S_T^{\text{model}}$$

$$\frac{\Delta_{QCD}}{\mu_h} \approx \frac{\Lambda_{QCD}}{Q}$$

Requires SCET subleading calculation

Same effect for all tau

Numerically irrelevant

The relative importance is region-dependent.

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It is the same model function!

- In the peak region one cannot expand
- Tail and far tail: expansion causes shift

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Numerically irrelevant

Same effect for all tau

The relative importance is region-dependent.

Theoretical uncertainty $\quad \frac{\delta\alpha_s}{\alpha_s} \sim \frac{\Lambda_{QCD}}{Q} \sim 0.5\%$

Concerning renormalons and gap subtraction, we treat the non-singular in the same way as the singular, to ensure complete cancellation in the far tail and it is consistent with the subleading factorization theorem. We convolute it with the same Soft function model.

Subleading perturbative contributions

$$\frac{d\sigma}{d\tau} = \sigma_0 \, H(Q,\mu) \int d\ell \; J_T(Q\tau - \ell, \mu) \, S_T(\ell,\mu) + \left.\frac{d\sigma^{partonic}}{d\tau}\right|_{\text{non-singular}} \otimes \; S_{\text{model}}$$

Peak and tail

Tail & far tail

In the far tail region, fixed order perturbation theory is the optimal description.

We no longer have three scales, resummation messes up cancellation.

Subleading perturbative contributions

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\tau} = \sigma_0\, H(Q,\mu)\int \mathrm{d}\ell\, J_T(Q\,\tau-\ell,\mu)\, S_T(\ell,\mu) + \frac{\mathrm{d}\sigma^{partonic}}{\mathrm{d}\tau}\bigg|_{\text{non-singular}} \otimes \; S_{\text{model}}$$

Peak and tail — Tail & far tail

In the far tail region, fixed order perturbation theory is the optimal description.

We no longer have three scales, resummation messes up cancellation.

Use of factorization theorems at subleading order derived with SCET.

We define the non-singular terms as the full QCD fix order result with the expanded SCET subtracted.

How do we determine non-singular?

$$\left.\frac{d\sigma}{d\tau}\right|_{\text{fixed order}} = \left.\frac{d\sigma}{d\tau}\right|_{\text{SCET(no resummation)}} + O(\tau^{0,1\ldots})$$

Full QCD SCET expanded out

← Subleading terms (do not have a resummation of logs but do know factorization theorem)

$$\left.\frac{d\sigma}{d\tau}\right|_{\text{non-singular}} \equiv \left.\frac{d\sigma}{d\tau}\right|_{\text{fixed order}} - \left.\frac{d\sigma}{d\tau}\right|_{\text{SCET(no resummation)}}$$

← known analytically to three-loop accuracy*

known numerically at 2 -, 3 - loops

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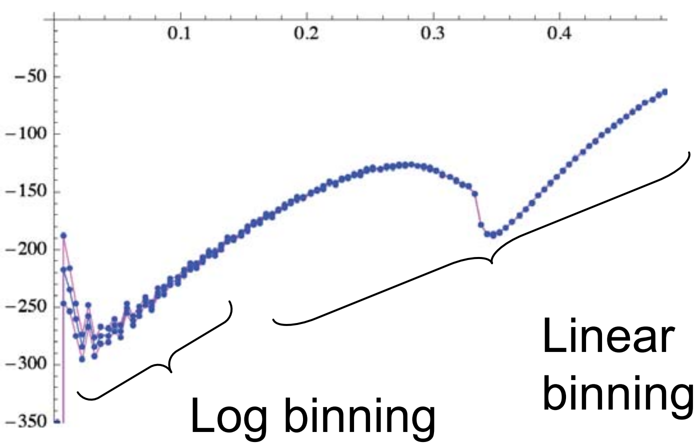
← known analytically to three-loop accuracy*

known numerically at 2 -, 3 - loops

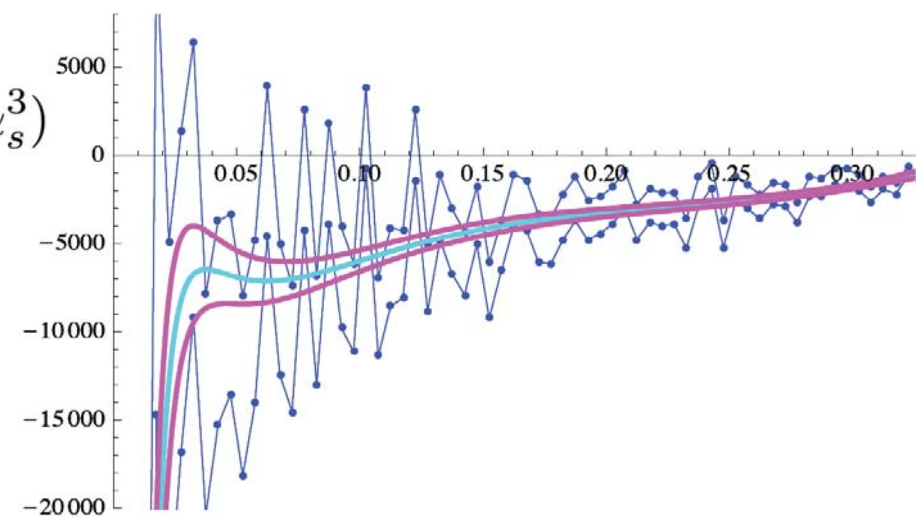
Gets very noisy at small τ !

→ We use a fit function at small τ and an interpolating function for the rest

$\mathcal{O}(\alpha_s^2)$



$\mathcal{O}(\alpha_s^3)$



Non-perturbative contributions *Hoang & Stewart*

$$S(\ell,\mu) = \int d\ell' \, \underbrace{S_{\text{part}}(\ell - \ell' - 2\delta, \mu)}_{\text{Gives the right anomalous dimension}} \, \underbrace{S_{\text{model}}(\ell' - 2\bar{\Delta})}_{\text{Parametrizes the non-perturbative effects}}$$

Non-perturbative contributions

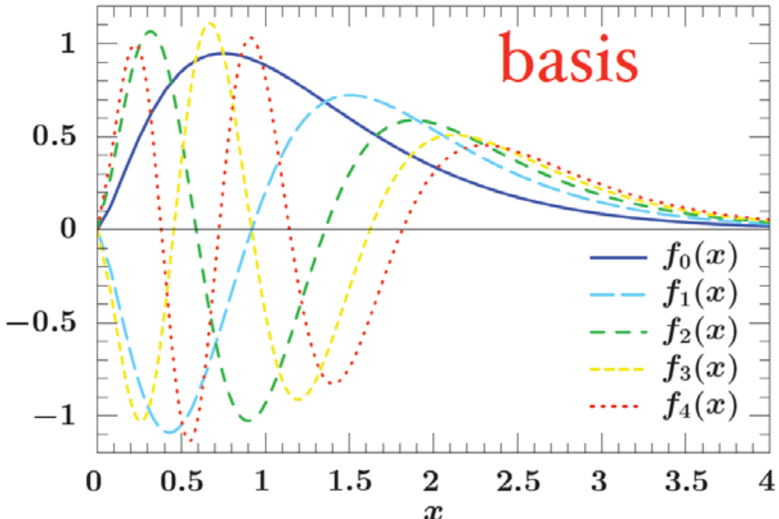
Hoang & Stewart

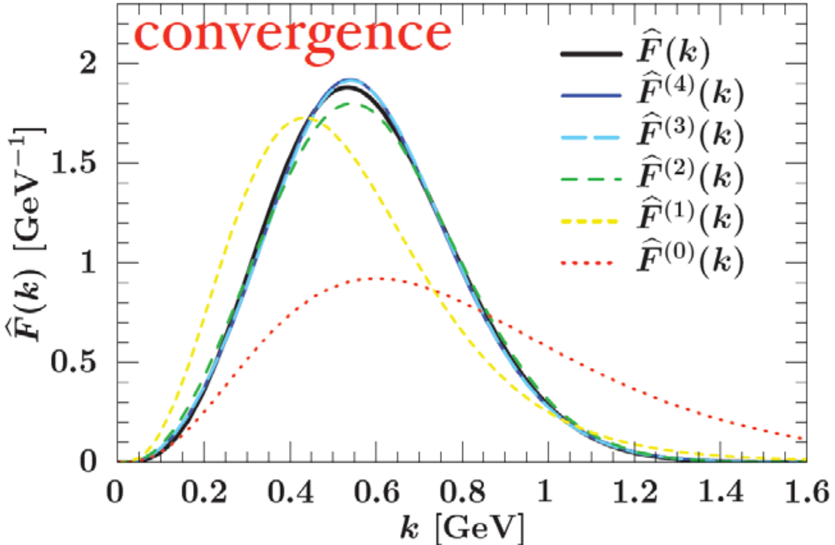
$$S(\ell,\mu)=\int \mathrm{d}\ell'\, S_{\text{part}}(\ell-\ell'-2\delta,\mu)\, S_{\text{model}}(\ell'-2\bar{\Delta})$$

Gives the right anomalous dimension

Parametrizes the non-perturbative effects

Ligeti , Stewart & Tackmann $S_{\text{model}}(\ell)=\frac{1}{\lambda}\left[\sum_{n=0}^{\infty} c_n\, f_n\!\left(\frac{\ell}{\lambda}\right)\right]^2$ [See talk by F. Tackmann]





Non-perturbative contributions

Hoang & Stewart

Contains an infrared renormalon

Removes renormalon

"Infrared subtraction scale" dependence 'R'

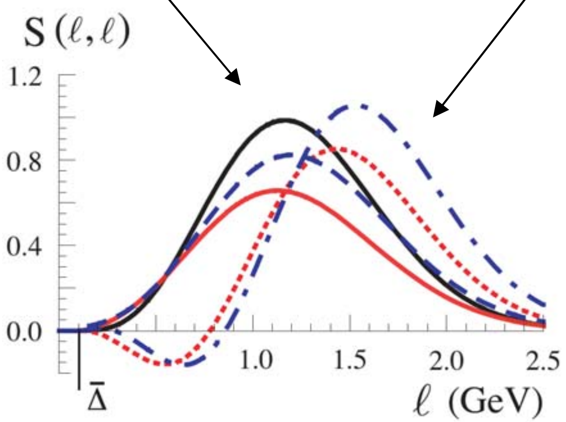
$$S(\ell,\mu) = \int \mathrm{d}\ell' \, S_{\text{part}}(\ell - \ell' - 2\delta, \mu) \, S_{\text{model}}(\ell' - 2\bar{\Delta})$$

Gives the right anomalous dimension

Parametrizes the non-perturbative effects

Renormalon subtracted

No subtraction



$$\delta_1(\mu_S, R) = R e^{\gamma_E} \frac{\alpha_s(\mu_S)}{\pi} \left[-2C_F \ln \frac{\mu_S}{R} \right]$$

$$R(\tau) \lesssim \mu_s(\tau)$$

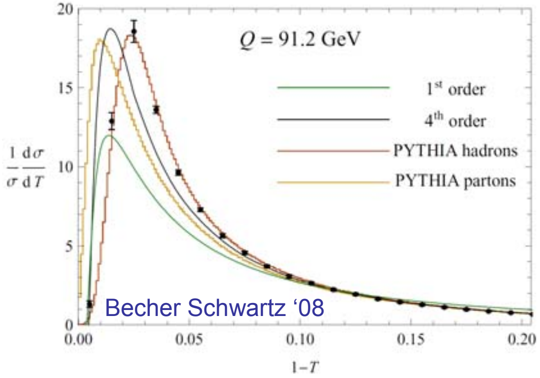
$\bar{\Delta}(R,\mu)$ must be evolved to avoid large logs.
We evolve from $\bar{\Delta}(R_0,\mu_0) \equiv \Delta_0$ to $\bar{\Delta}(R,\mu)$ to sum them up.

[See A. Jain's Talk]

R is a scheme parameter

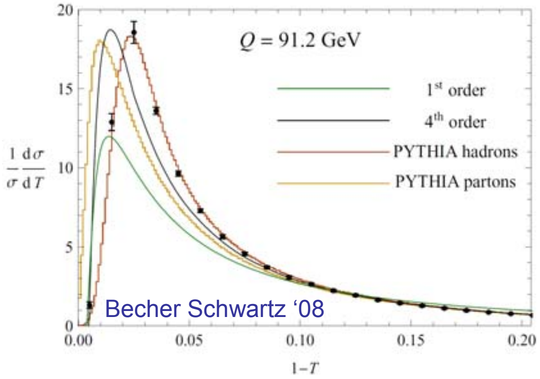
Δ_0 is a model parameter

Treatment of non-perturbative effects



Most LEP analyses used montecarlo generators to estimate NP corrections

Treatment of non-perturbative effects



Most LEP analyses used montecarlo generators to estimate NP corrections

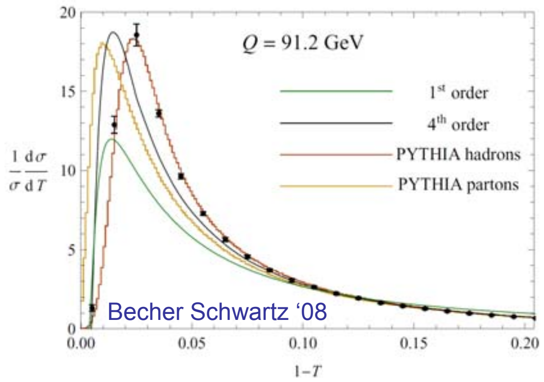
Davison & Webber's Model: freezes α below some scale μ_I

$$\left.\frac{d\sigma}{d\tau}\right|_{\tau} = \left.\left(\frac{d\sigma}{d\tau}\right)^{\text{perturbation theory}}\right|_{\tau+\Delta\tau}$$

$$\Delta\tau \sim \alpha_{\text{eff}}\frac{\mu_I}{Q} \sim \frac{\Lambda_{QCD}}{Q}$$

Definite pattern in Q

Treatment of non-perturbative effects



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$$\Delta\tau \sim \alpha_{\text{eff}} \frac{\mu_I}{Q} \sim \frac{\Lambda_{QCD}}{Q}$$

Definite pattern in Q

Predictible from QCD/SCET factorization !

In the tail region $\ell_{\text{soft}} \sim Q\,\tau \gg \Lambda_{QCD}$

and we can expand the soft function

$$S(\tau) = S_{\text{pert}}(\tau) - S'_{\text{pert}}(\tau)\frac{2\Omega_1}{Q} \approx S_{\text{pert}}\left(\tau - \frac{2\Omega_1}{Q}\right)$$

$\Omega_1 \sim \Lambda_{QCD}$ Is a non-perturbative parameter

Ω_1 is defined in field theory

Lee & Sterman

Shifts distributions to the right !

Treating all regions together

The renormalon subtraction introduces a scheme parameter R(τ).

The gap function $\bar{\Delta}$ has both ultraviolet (μ) and infrared (R) running.

We must turn off the resummation in the multijet region

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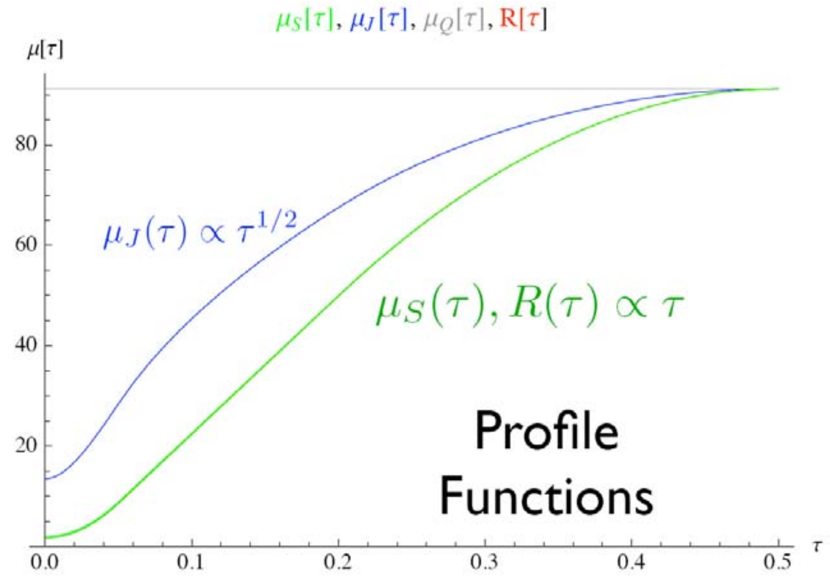
Let us μ_J, μ_s and R depend on τ



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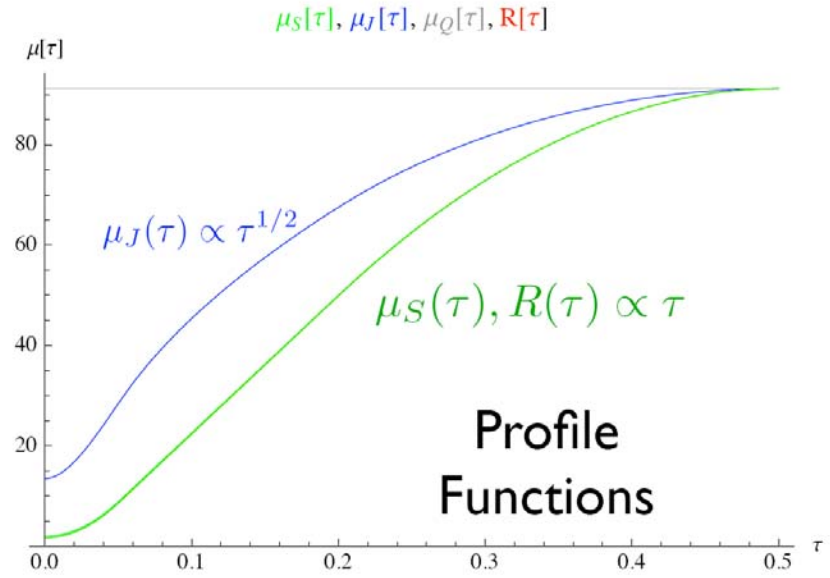

So all scales merge to the hard scale for the multijet region, and the transition occurs smoothly.

Still there is room for estimating the perturbative uncertainties by varying these profile functions within the constraints.

Mass corrections

Fleming , Hoang, Mantry & Stewart

Now the electroweak effects tell apart the up- and down-type quarks

Our theoretical uncertainty for a massless production is at the 1% level

Hard, soft and running kernels unchanged.

Use $\overline{\text{MS}}$ running mass at the jet scale.

Shifts thresold to $\frac{2m^2}{Q^2} \longrightarrow 1-\sqrt{1-\frac{4m^2}{Q^2}}$

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$$J_T(p,\mu) \to J_T(p,m,\mu) = \begin{cases} J_{\text{distribution}}(p,m,\mu) \to \text{ Fourier transform} \\ J_{\text{function}}(p,m,\mu) \to \text{ Treat in momentum space} \end{cases}$$

Non-singular piece analytically known [AFHMS]

Important for bottom

Small for charm

The one-loop mass corrections are about a 2% effect

One-loop level suffices

$$\text{Theoretical distribution} = \frac{\sum_{q=u,d,s,c,b} \left.\frac{\mathrm{d}\sigma}{\mathrm{d}\tau}\right|_q}{\sum_{q=u,d,s,c,b} (\sigma_0)_q}$$

QED corrections

[AFHMS]

Affect all matrix elements: Hard, Jet and Soft

$\alpha_s(\mu)$ and $\alpha(\mu)$ have coupled evolution equations.
One can solve them perturbatively.

The non-singular term is trivially obtained

All running kernels are affected. There are interesting QED – QCD mixing effects !

Different for up and down quarks! Need to take into account electroweak factors.

One can consider $\alpha \leq \mathrm{O}(\alpha_s^2)$

No renormalon subtractions for QED

QED corrections

[AFHMS]

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All running kernels are affected. There are interesting QED – QCD mixing effects !

Stick to NLO in Matrix elements and NNLL for the running.

They have a small effect on $\alpha_s(m_Z)$

They shift Ω_1

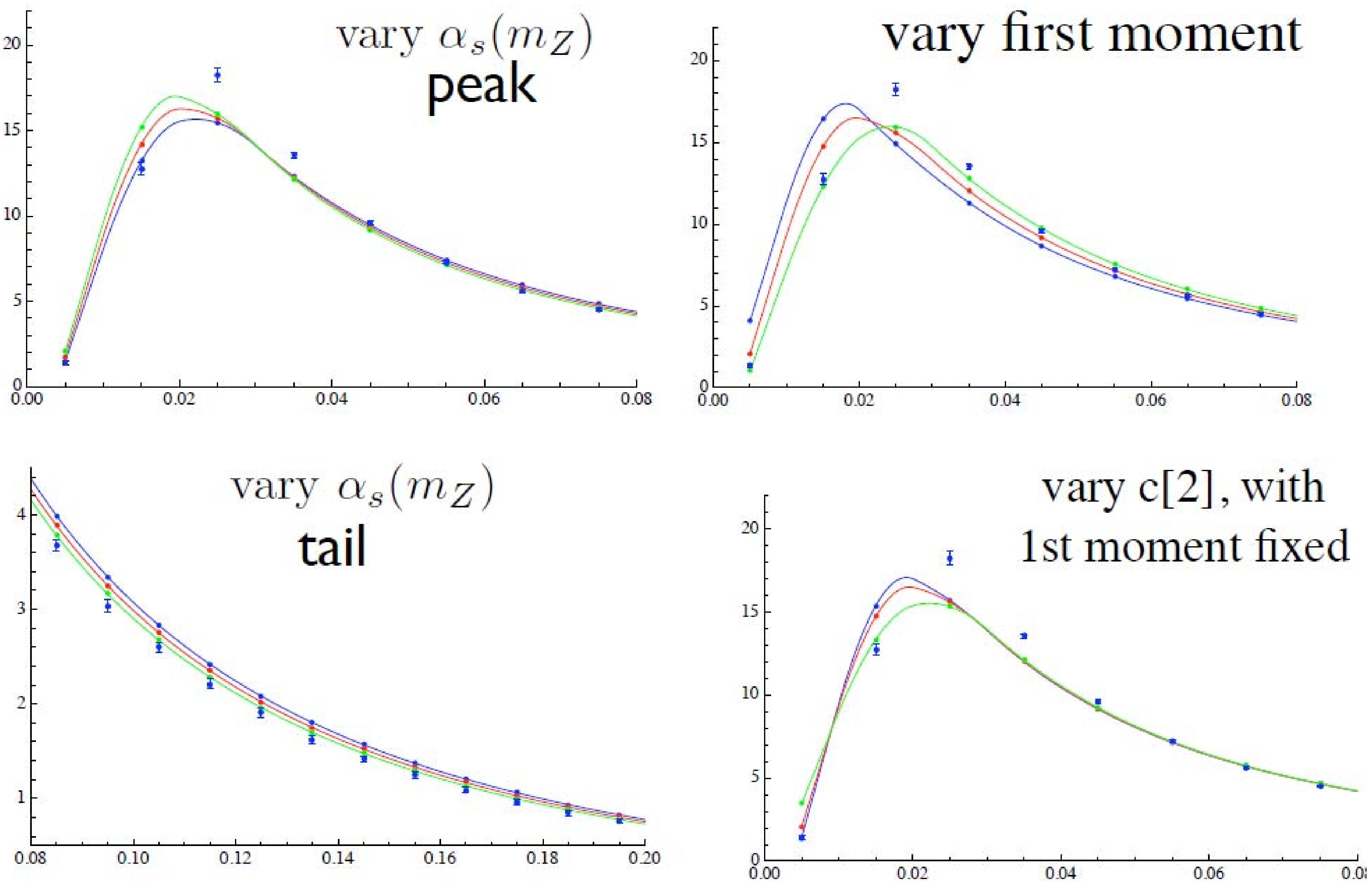
Different for up and down quarks! Need to take into account electroweak factors.

One can consider $\alpha \leq \mathrm{O}(\alpha_s^2)$

No renormalon subtractions for QED

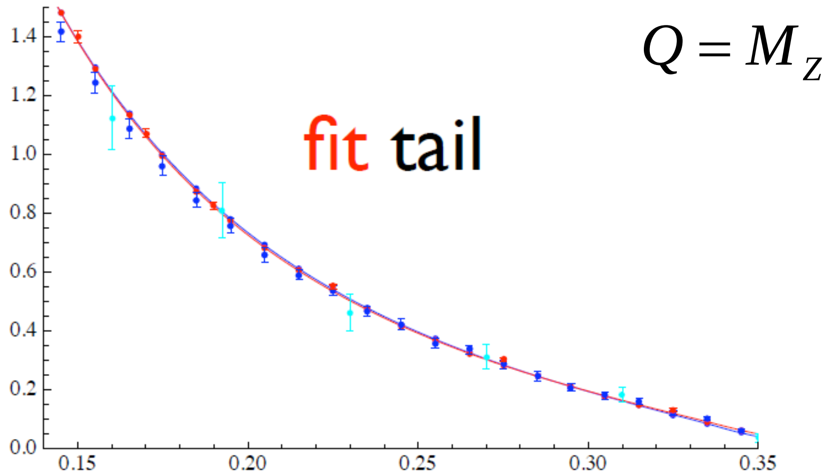
Do not need to worry about initial state radiation, since these effects are minimized in the normalization procedure.

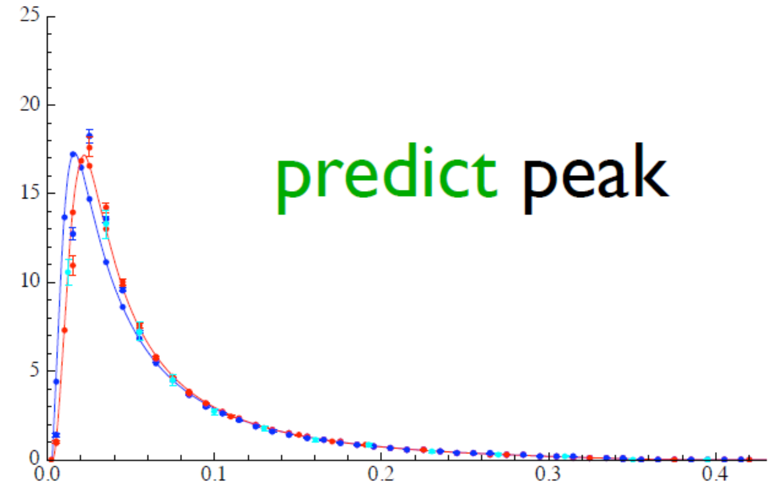
Effects of various pieces

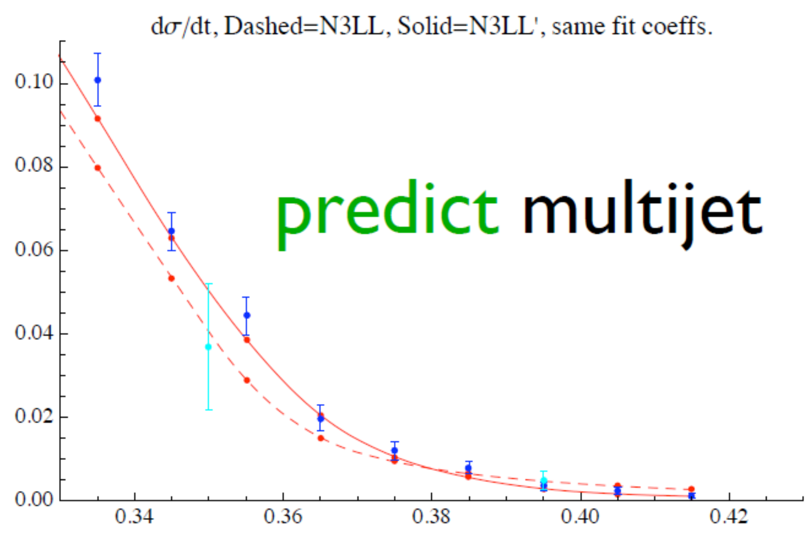


Two parameter fit in tail region

In the tail $[\tau, 0.14, 0.33]$ only the shift matters. So we fit for Ω_1 & $\alpha(M_Z)$ simultaneously







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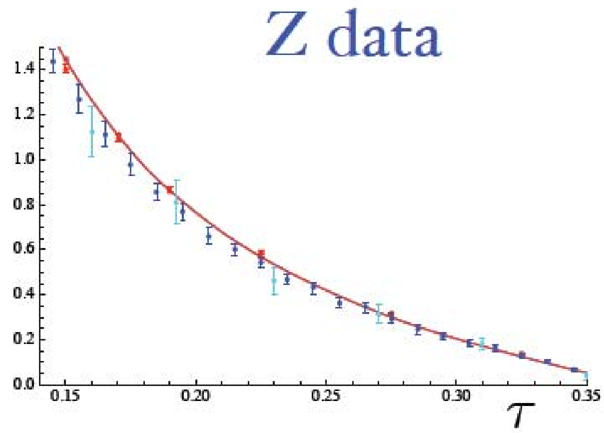
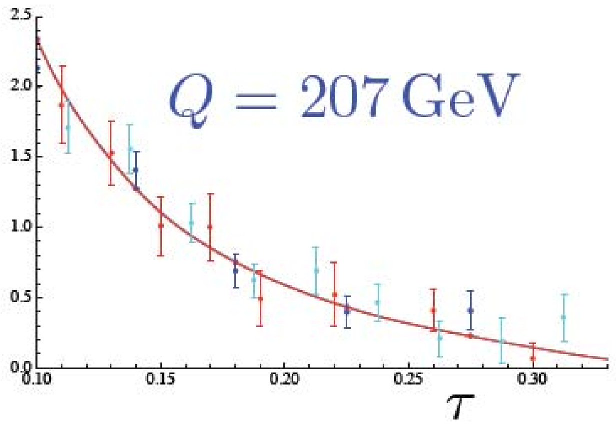
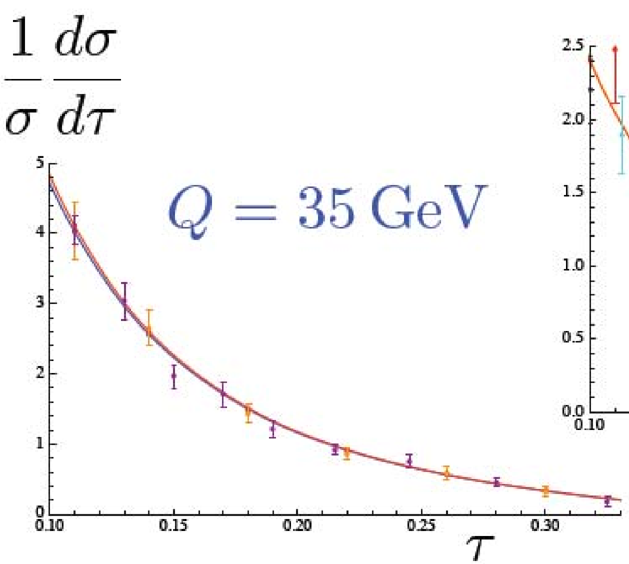
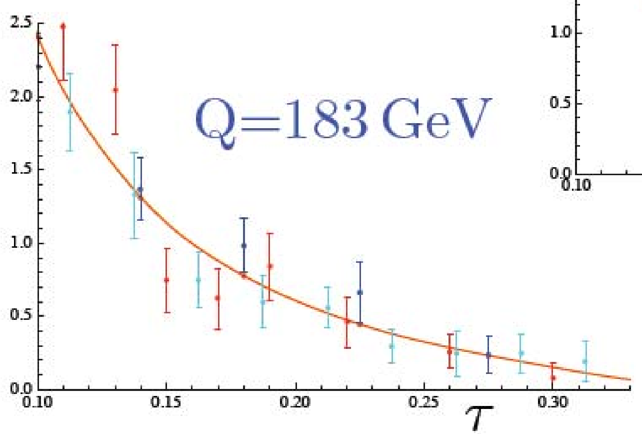


The two-parameter fit gives an excellent description for many Q's







Estimate of theory uncertainties

- Four-loop anomalous cusp coefficient (small)
- Three-loop unknown non-logarithmic terms (subdominamt)
- Non-singular fit functions errors (small)
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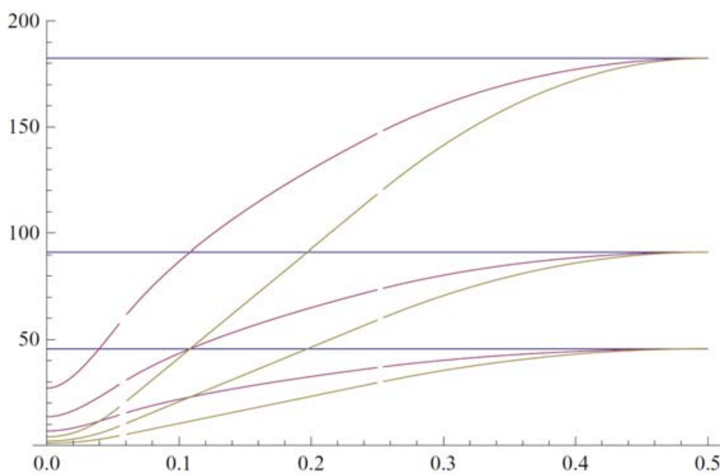
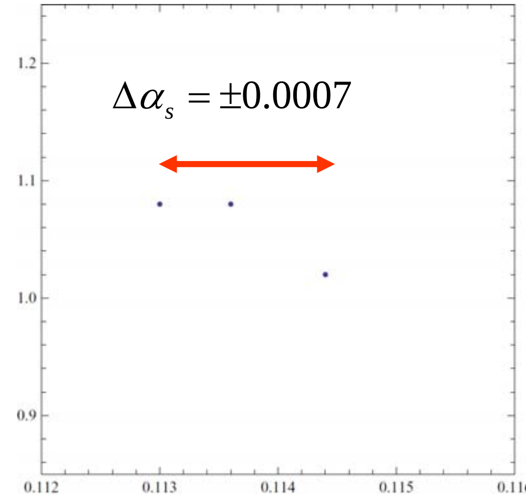
Final error here is work in progress





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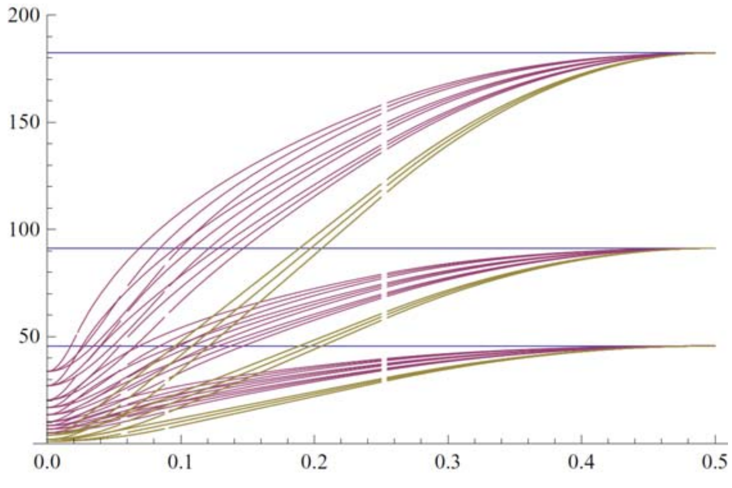

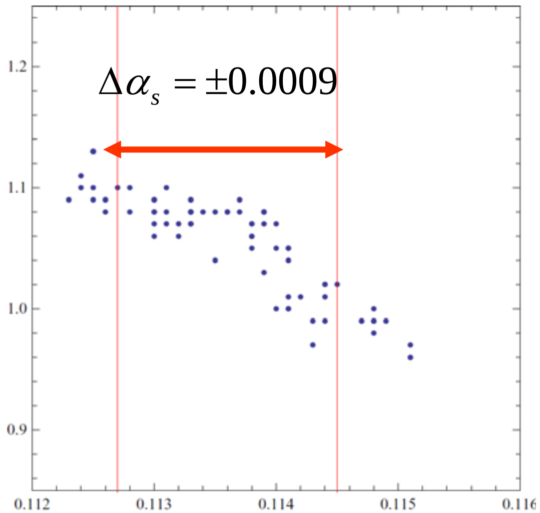

Two parameter fit in tail region

In the tail $[\tau, 0.14, 0.33]$ only the shift matters. So we fit for Ω_1 & $\alpha(M_Z)$ simultaneously

We use LEP working groups correlation model for systematic errors

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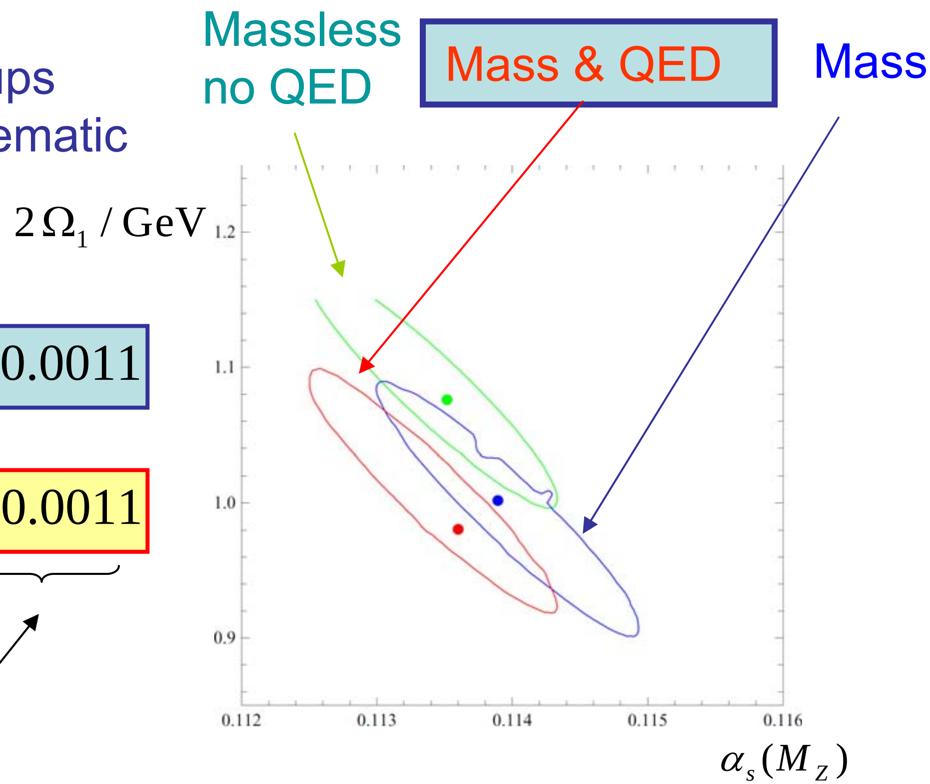

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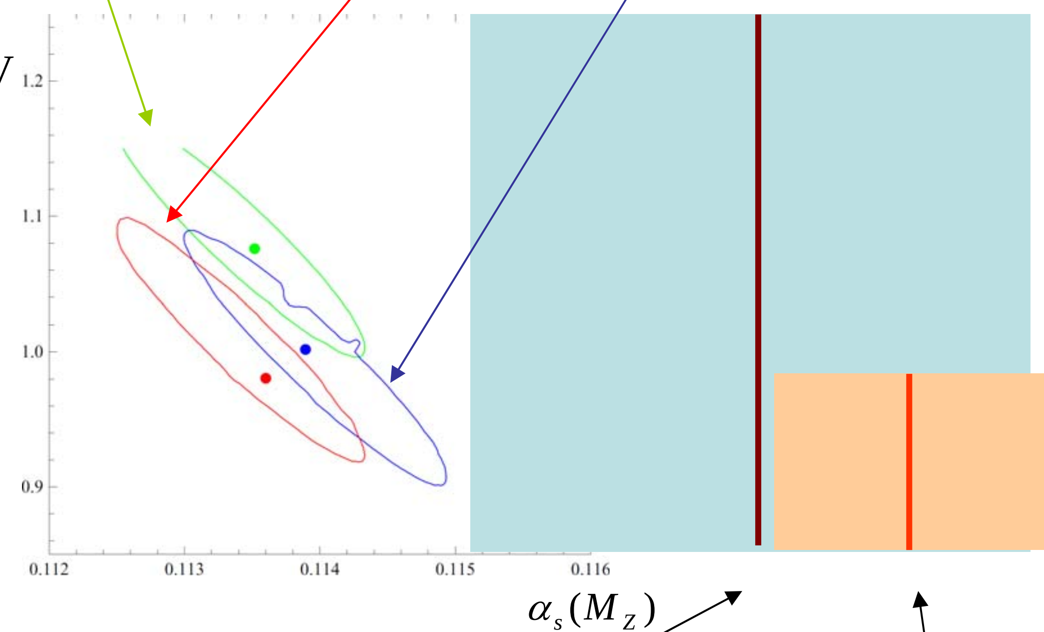
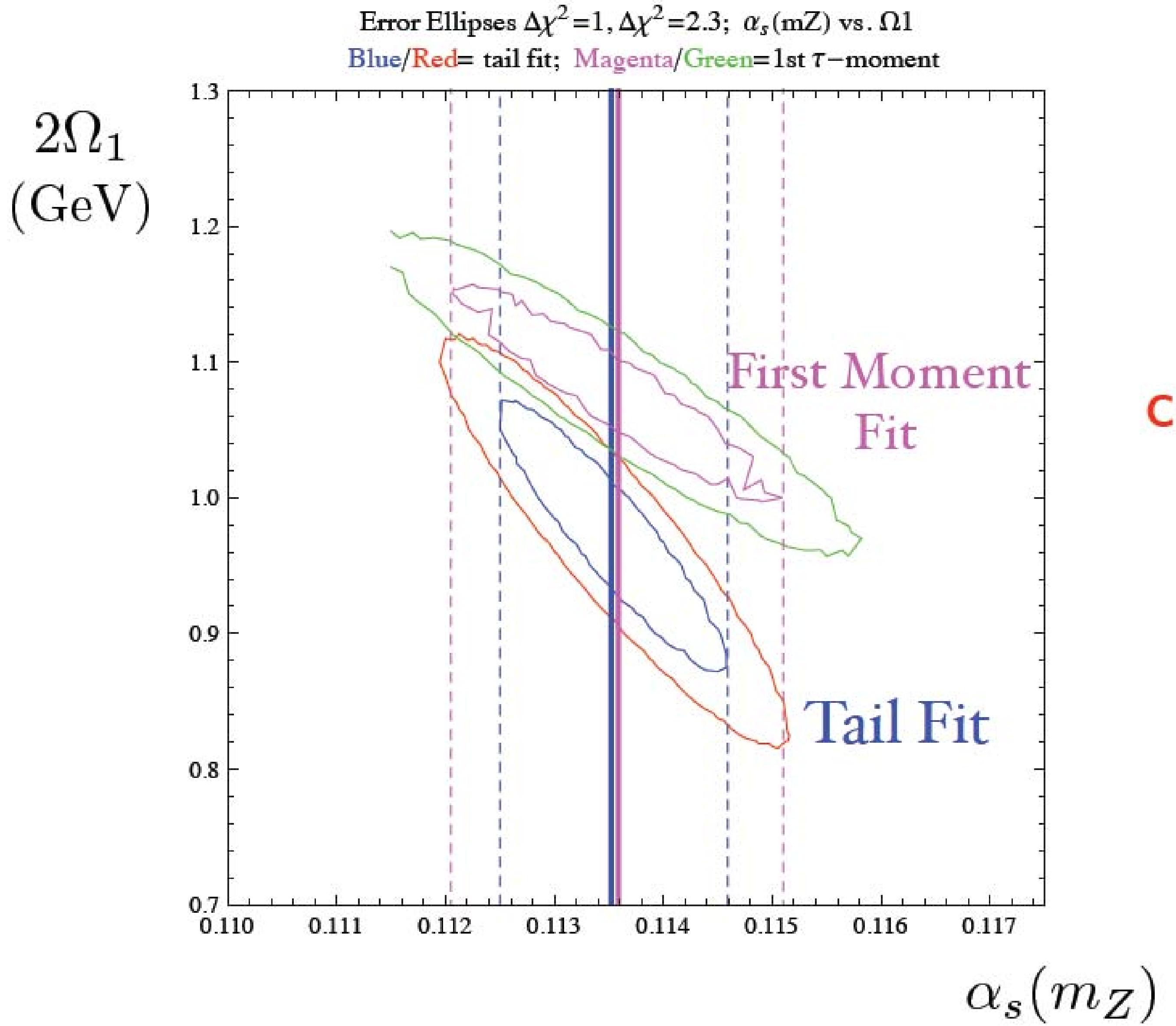


PDG world average

Bethke average

Our value is lower than the World average → montecarlo hadronization gives larger values

Tail Fit & first Moment Fit Results



consistent

Comparison with other α^3 analyses

Becher & Schwartz

- Data: Aleph & Opal
- Resummation: In the amplitude (EFT RG Equations)
- No non-perturbative effects in central value
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Dissertori, Gehrmann-De Ridder Gehrmann, Glover & Heinrich

- Data: Aleph
- Resummation: fixed order
- Non-perturbative effects treatment: Montecarlo generator
- Result $\alpha(M_Z) = 0.1240 \pm 0.0029$

Conclusions

- The Soft-Collinear Effective Theory provides a powerful formalism for deriving factorization theorems and analyzing processes with Jets.

- SCET has finally provided theorists with a mean to catch up to the experimental precision of LEP.

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The future for high precision determinations of the strong coupling constant looks good!

Thanks for your attention !