(Some Aspects of)

# Minimal Flavour Violation

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[based on work with Thomas Mannel, Martin Jung, Michaela Albrecht]

# Flavour in the LHC Era

**Old paradigm:**

Hints for New Physics from *B* decays *before* direct dectection of new particles.

- CKM mechanism experimentally confirmed. $\surd$
- Precision measurements of BRs and CP violation in rare decays only allow for small deviations from the SM predictions. $\surd$

- If new physics at the TeV scale and generic new flavour couplings:

  *Why NP hasn't been (clearly) observed in rare flavour decays?*

**New paradigm:**

Rare flavour decays for quarks *and* leptons to be used to constrain flavour structure of NP models *after* new particles/interactions have been established.

[CERN Workshop on Flavor in the Era of the LHC, arXiv: 0801.1800, 0801.1833, 0801.1826 ]

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# Quark flavour in the SM

- SM gauge interactions invariant under independent flavour rotations of

$$Q_L^i = \begin{pmatrix} U_L \\ D_L \end{pmatrix}^i , \qquad U_R^i , \qquad D_R^i , \qquad (i = 1..3 \text{ families})$$

- Yukawa matrices $Y_U$, $Y_D$ break flavour symmetry:

$$-\mathcal{L}_{\rm yuk} = \bar{Q}_L^i \, \widetilde{H} \, Y_U^{ij} \, U_R^j + \bar{Q}_L^i \, H \, Y_D^{ij} \, D_R^j + \text{h.c.}$$

$\rightarrow$ (global) Flavour Symmetry Group for quarks:

$$\mathcal{G}_q \quad = \quad U(3)_{Q_L} \times U(3)_{U_R} \times U(3)_{D_R} \, / \, U(1)_B$$

  - mismatch between diagonalization for $U_L$ and $D_L$ defines CKM matrix.
  - diagonal elements yield quark masses after EWSB: $m_q = y_q^{\rm diag} \, \langle H^0 \rangle$

- Parameter counting:

36 Yukawa entries − 26 symmetry generators = 10 physical parameters

# Minimal Flavour Violation

## Idea: [Buras et al., Ciuchini et al., . . . ]

- Flavour structure of NP contributions related to SM coefficients.

## Formalism: [D'Ambrosio et al., Cirigliano et al.]

- Consider Yukawa matrices as VEVs of spurion fields in an ET, which transform under flavour group:

$$Y_U \sim (3, \bar{3}, 1)\,, \quad Y_D \sim (3, 1, \bar{3}) \quad \text{of} \quad SU(3)_{Q_L} \times SU(3)_{U_R} \times SU(3)_{D_R}$$
$$Y_E \sim (3, \bar{3})\,, \quad g_\nu \sim (\bar{6}, 1) \quad \text{of} \quad SU(3)_L \times SU(3)_{E_R}$$

  - Book-keeping device for NP flavour effects with MFV $\surd$
  - Book-keeping device to classify deviations from MFV $\surd$
    [D'Ambrosio et al., Cirigliano et al., Agashe et al., TF/Mannel, Kagan et al., . . . ]
  - Dynamical effects from (local) flavour symmetries ?
  - Relation to hierarchies in fermion masses and mixings ?
    [Albrecht/TF/Jung/Mannel]

# Sequential breaking of flavour symmetry (quarks)

- generic power-counting for Yukawa entries, with $y_q \sim \lambda^{n_q}$ ($\lambda \sim 0.2$: Wolfenstein parameter)

$$(Y_U)_{ij} \sim (V_{\rm CKM}^\dagger)_{ij}\,(y_u)_j \sim \begin{pmatrix} \lambda^{n_u} & \lambda^{1+n_c} & \lambda^3 \\ \lambda^{1+n_u} & \lambda^{n_c} & \lambda^2 \\ \lambda^{3+n_u} & \lambda^{2+n_c} & 1 \end{pmatrix}$$

$$(Y_D)_{ij} \sim (V_{\rm CKM})_{ij}\,(y_d)_j \sim \begin{pmatrix} \lambda^{n_d} & \lambda^{1+n_s} & \lambda^{3+n_b} \\ \lambda^{1+n_d} & \lambda^{n_s} & \lambda^{2+n_b} \\ \lambda^{3+n_d} & \lambda^{2+n_s} & \lambda^{n_b} \end{pmatrix} .$$

- leading effect from $(Y_U)_{33} \sim y_t$
- next-to-leading effect from $(Y_D)_{33} \sim y_b$
- ...etc.

Hierarchies of masses and mixing $\leftrightarrow$ Sequence of Flavour Symmetry Breaking

## Step 1: $SU(3)_{Q_L} \times SU(3)_{U_R} \xrightarrow{y_t} SU(2)_{Q_L} \times SU(2)_{U_R} \times U(1)$

- non-linear representation: [TF/Mannel]

$$Y_U = \mathcal{U}(\Pi_L) \begin{pmatrix} Y_U^{(2)}/\Lambda & & 0 \\ & & 0 \\ 0 & 0 & y_t \end{pmatrix} \mathcal{U}^\dagger(\Pi_{U_R}) \,, \qquad Y_D = \frac{1}{\Lambda}\, \mathcal{U}(\Pi_L) \begin{pmatrix} \tilde{Y}_D^{(2)} \\ \xi_b^\dagger \end{pmatrix}$$

  - 9 Goldstone modes $\Pi_L$ and $\Pi_{U_R}$ for broken generators.
    (with $\mathcal{U}(\pi) = \exp[i\pi^a T^a/\Lambda]$)
  - Spurions for residual flavour symmetry:

    $$Y_U^{(2)} \sim (2,2,1)\,, \qquad Y_D^{(2)} \sim (2,1,\bar{3}) \qquad \xi_b^\dagger \sim (1,1,\bar{3})$$

    under $SU(2)_{Q_L} \times SU(2)_{U_R} \times SU(3)_{D_R}$
  - sub-leading Yukawa entries supressed by $1/\Lambda$

- Next VEV $\langle \xi_b \rangle \neq 0$ at $\Lambda' \ll \Lambda$ etc.

# Example for Sequence (refers to $n_c < n_b + 2 < n_b + 3 < n_s$)

| Flavour Symmetry | | GBs | Spur. | VEVs | Symm. | Scale |
|---|---|---|---|---|---|---|
| $SU(3)_{Q_L} \times SU(3)_{U_R} \times SU(3)_{D_R} \times U(1)^2$ | | 0 | 36 | 0 | 26 | |
| $SU(2)_{Q_L} \times SU(2)_{U_R} \times SU(3)_{D_R} \times U(1)^3$ | | 9 | 26 | 1 | 17 | $\Lambda \sim y_t \, \Lambda$ |
| $SU(2)_{Q_L} \times SU(2)_{U_R} \times SU(2)_{D_R} \times U(1)^3$ | | 14 | 20 | 2 | 12 | $\Lambda' \sim y_b \, \Lambda$ |
| $SU(2)_{D_R} \times U(1)^4$ | | 19 | 14 | 3 | 7 | $\Lambda^{(2)} \sim y_c \, \Lambda$ |
| $SU(2)_{D_R} \times U(1)^3$ | | 20 | 12 | 4 | 6 | $\Lambda^{(3)} \sim y_b \lambda^2 \, \Lambda$ |
| $SU(2)_{D_R} \times U(1)^2$ | | 21 | 11 | 5 | 5 | $\Lambda^{(4)} \sim y_b \lambda^3 \, \Lambda$ |
| $U(1)^2$ | | 24 | 6 | 6 | 2 | $\Lambda^{(5)} \sim y_s \, \Lambda$ |
| $U(1)^2$ | (CP broken) | 24 | 4 | 7+1 | 2 | $\Lambda^{(6)} \sim y_s \lambda \, \Lambda$ |
| – | (CP broken) | 26 | 0 | 9+1 | 0 | $\Lambda^{(7)} \sim y_{u,d} \, \Lambda$ |

- Balance:

$$\text{\# Spurions} + \text{\# VEVs} - \text{\# Symmetries} = 10 \,,$$

$$\text{\# Goldstones} + \text{\# Spurions} + \text{\# VEVs} = 36 \,,$$

- Hierarchy of scales: e.g. for $n_b = 2$, $n_c = 3$, $n_s = 6$, $n_{u,d} = 8$

$$\Lambda^{(n)} \sim \lambda^{n+1} \, \Lambda$$

[TF/Jung/Mannel]

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- CP spontaneously broken:
  One spurion field which is uncharged w.r.t. $U(1)^2$ residual flavour symmetry

$\Rightarrow$ CP restoration may occur at relatively low scales $\sim \lambda^{6-7} \Lambda$

- Chiral $U(1)$'s allow for Peccei-Quinn solution to strong CP problem.

[TF/Jung/Mannel]

# Construction of a Spurion Potential [TF/Jung/Mannel]

- Build 10 invariants in terms of $U = Y_U Y_U^\dagger \Lambda^2$ and $D = Y_D Y_D^\dagger \Lambda^2$ :

$$I_1^{(2)} = \mathrm{tr}[U]\,, \qquad I_1^{(4)} = \mathrm{tr}[U^2] - (I_1^{(2)})^2\,, \qquad I_1^{(6)} = \mathrm{tr}[U^3] - \frac{3}{2}\, I_1^{(4)} I_1^{(2)} - (I_1^{(2)})^3$$

$$I_2^{(2)} = \mathrm{tr}[D]\,, \qquad I_2^{(4)} = \mathrm{tr}[D^2] - (I_1^{(2)})^2\,, \qquad I_2^{(6)} = \mathrm{tr}[D^3] - \frac{3}{2}\, I_2^{(4)} I_2^{(2)} - (I_2^{(2)})^3$$

$$\begin{aligned} I_3^{(4)} &= \mathrm{tr}[UD] - I_1^{(2)} I_2^{(2)} \\ I_3^{(6)} &= \mathrm{tr}[U^2 D] - \frac{1}{2}\, I_1^{(4)} I_2^{(2)} - I_3^{(4)} I_1^{(2)} - I_2^{(2)}\,(I_1^{(2)})^2 \\ I_4^{(6)} &= \mathrm{tr}[UD^2] - \frac{1}{2}\, I_2^{(4)} I_1^{(2)} - I_3^{(4)} I_2^{(2)} - I_1^{(2)}\,(I_2^{(2)})^2 \\ I_1^{(8)} &= \mathrm{tr}\,[U(UD - DU)D] \end{aligned}$$

- Expand potential around assumed minimum:

$$V = \sum_{k,m} \sum_{i,j} \frac{1}{\Lambda^{m+k-4}} \left( I_i^{(m)} - v_i^{(m)} \right) M_{ij}^{(m,k)} \left( I_j^{(k)} - v_j^{(k)} \right) ,$$

# Construction of a Spurion Potential [TF/Jung/Mannel]

Comments:

- Restriction to dim-2 and dim-4 operators

$$V^{(4)} = \sum_i m_i^2 \, I_i^{(2)} + \sum_{i,j} 2\rho_{ij} \, I_i^{(2)} I_j^{(2)} + \sum_i \lambda_i \, I_i^{(4)} \,,$$

  not sufficient to break flavour symmetry completely.

- First step:

$$v_1^{(2)} \; (\simeq y_t^2 \, \Lambda^2) \; \neq 0 \,, \qquad I_i^{(m)} = 0 \quad \text{otherwise}$$

  $\longrightarrow$ Potential re-expressed in terms of 9 invariants for residual flavour group.

- Construction can be repeated at each step of the sequence of FSB
  - $\longrightarrow$ matching conditions between potential parameters.
  - $\longrightarrow$ radiative corrections between $\Lambda^{(i)} \to \Lambda^{(i+1)}$.

# Dynamical Interpretation of Goldstone Bosons

[Albrecht/TF/Mannel]

- Promote (part) of flavour symmetry to local gauge invariance:
  - massive gauge bosons, $M^{(n)} \sim g\Lambda^{(n)}$
  - new FCNCs at low scales with MFV incorporated
  - anomalies to be treated within EFT context [Preskill 1991]
- Keep chiral $U(1)$ symmetries global:
  - Axion solution to strong CP problem: [Peccei/Quinn, Weinberg, Wilczek,. . . ]

standard: Higgs-doublets charged under PQ symmetry
here: Yukawa spurions charged under PQ symmetry

# Example: $[SU(2)_{D_R} \times U(1)_X]_{\rm local} \times U(1)_{u_R} \times U(1)_{D_R^{(2)}}$

- corresponds to intermediate step, with Yukawa VEVs

$$\langle Y_U \rangle = \begin{pmatrix} 0 & 0 & 0 \\ 0 & \bullet & 0 \\ 0 & 0 & \bullet \end{pmatrix} , \qquad \langle Y_D \rangle = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & \bullet \\ 0 & 0 & \bullet \end{pmatrix}$$

- $U(1)_X$ charge: $X = \frac{1}{\sqrt{3}} \left( T^8_{Q_L} + T^8_{U_R} + T^8_{D_R} \right) + \left( T^3_{Q_L} + T^3_{U_R} \right)$

$\rightarrow$ Anomaly coefficients:

$$\mathrm{tr}[X^3] = 3/4 \,, \qquad \mathrm{tr}[X^2\, Y] = -1$$

Consider $U(1)_X$ Goldstone boson:

$$\pi_X(x) \to \pi_X(x) + \omega_X(x) \,, \qquad \text{for} \quad X_\mu \to X_\mu + \frac{1}{g_X}\, \partial_\mu \omega_X(x)$$

→ Effective Lagrangian contains:

$$\begin{aligned}\mathcal{L}_\pi &= -\frac{F_X^2}{2}\,(\partial_\mu \pi_X(x) - g_X X_\mu(x))^2 \\ &\quad - \pi_X(x)\,\mathrm{tr}[X^3]\,\frac{g_X^2}{16\pi^2}\,X_{\mu\nu}(x)\,\widetilde{X}^{\mu\nu}(x)\,, \\ &\quad - \pi_X(x)\,\mathrm{tr}[X^2\,Y]\,\frac{g_Y g_X}{8\pi^2}\,X_{\mu\nu}(x)\,\widetilde{Y}^{\mu\nu}(x)\,,\end{aligned}$$

compensates for anomalous change of fermionic path integral measure.

- With gauge-fixing term

$$\mathcal{L}_{\mathrm{g.f.}} = -\frac{1}{2\alpha_X}\left(\partial_\mu X^\mu(x) - \alpha_X\, g_X\, F_X^2\, \pi_X(x)\right)^2 \,,$$

  - removes mixing between $\pi_X$ and $X_\mu$
  - yields massive gauge bosons $M_X = g_X F_X$.

# Examples for induced 4-quark operators (tree level, unit. gauge)

$$\Lambda \gg m_\chi \sim y_b |V_{ub}| \Lambda > M_X = g_X F_X > m_\xi \sim y_s \Lambda > M_D = g_D F_D \gg M_W$$

- Integrating out the spurion field $\chi_{13}$ (related to $(Y_D)_{13}$)

$$\mathcal{L}_{4\mathrm{q}}^{\chi} = \frac{1}{2 m_\chi^2 \Lambda^2} \left( \bar{Q}_L^{(1)} H \, b_R + \mathrm{h.c.} \right)^2$$

coefficient scales like

$$\frac{v^2}{\Lambda^2 \, m_\chi^2} \sim \frac{|V_{ub}|^2 \, m_b^2}{m_\chi^4} \ll \frac{v^2}{m_\chi^4}$$

  - $|V_{ub}|$ suppresses mixing between 1. and 3. generation (MFV √)
  - $m_b$ suppresses currents with right-handed *b*-quarks (MFV √)
  - (mixing with $Q_L^{(2)}$ further suppressed after SM CKM-rotation) (MFV √)

$$\Lambda \gg m_\chi \sim y_b |V_{ub}| \Lambda > M_X = g_X F_X > m_\xi \sim y_s \Lambda > M_D = g_D F_D \gg M_W$$

- Integrating out the massive $U(1)_X$ gauge boson:

$$\mathcal{L}_{4q}^X = -\frac{1}{2F_X^2} \left[\bar{\psi}_L \, \gamma_\mu \, X \, \psi_L\right] \left[\bar{\psi}_L \, \gamma^\mu \, X \, \psi_L\right]$$

where $\psi_L = \{Q_L^{(1,2,3)}, u_R^*, c_R^*, t_R^*, (d_R^*, s_R^*), b_R^*\}$ and $X$ is the associated charge.

  - violate quark universality (→ like lepto-phobic $Z'$ models)
  - $\Delta F = 1$ and $\Delta F = 2$ operators after SM CKM rotation (MFV $\surd$)

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- Integrating out the spurion field $\xi_s$ (related to $(Y_D)_{22}$)

$$\mathcal{L}_{4q}^{\xi} = \frac{1}{2 m_\xi^2 \Lambda^2} \left( \bar{Q}_L^{(2)} H s_R + \mathrm{h.c.} \right)^2$$

  coefficient scales like

$$\frac{v^2}{\Lambda^2 m_\xi^2} \sim \frac{m_s^2}{m_\xi^4} \ll \frac{v^2}{m_\xi^4}$$

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  - (mixing with $Q_L^{(1,3)}$ further suppressed after SM CKM-rotation) (MFV $\surd$)

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- Integrating out the massive $SU(2)_{D_R}$ gauge bosons:

$$\mathcal{L}_{4q}^D = -\frac{1}{2F_D^2} \left[ \bar{D}_R^{(2)} \gamma_\mu \frac{\sigma^a}{2} D_R^{(2)} \right]^2 ,$$

  - violates quark universality between $d_R$ and $s_R$
  - $\Delta F = 1$ and $\Delta F = 2$ operators after SM CKM rotation (MFV $\surd$)

# Summary

- MFV provides book-keeping device for NP Flavour Sector.
- Here: Dynamical effects within Effective Theory Framework for MFV:
  - Sequential FSB $\leftrightarrow$ Hierarchies in masses and mixings.
  - Spurion Potential.
  - Goldstone modes and spontaneously broken local flavour symmetries.
  - Induced 4-quark operators.
  - not discussed: PQ mechanism and axion phenomenology
- Possible generalizations:
  - 2-Higgs-Models with large tan $\beta$ [Kagan et al., . . . ]
  - Lepton sector [TF/Mannel, . . . ]
  - Models with different gauge groups.
  - . . .

# Lepton flavour in the SM: (a) massless neutrinos

- SM gauge interactions invariant under independent flavour rotations of

$$L^i = \begin{pmatrix} \nu_L \\ \ell_L \end{pmatrix}^i , \qquad E_R^i$$

$$\longrightarrow \qquad \mathcal{G}_\ell^{(a)} = U(3)_L \times U(3)_{E_R} \, / \, (U(1)_e \times U(1)_\mu \times U(1)_\tau)$$

- Yukawa matrix $Y_E$ breaks flavour symmetry:

$$-\mathcal{L}^\ell_{\text{yuk}} = \bar{L}^i \, H \, Y_E^{ij} \, E_R^j + \text{h.c.}$$

  - $m_E = y_E^{\text{diag}} \, \langle H^0 \rangle$
  - individual lepton flavour conserved

- Parameter counting:

18 Yukawa entries − 15 symmetry generators = 3 physical parameters

# Lepton flavour in the SM: (b) minimally extended SM

- Yukawa matrix $Y_E$ and dim-5 couplings $g_\nu$ break flavour symmetry:

$$-\mathcal{L}^{\ell}_{\rm yuk} = \bar{L}^i\, H\, Y_E^{ij}\, E_R^j + \frac{1}{\Lambda_{\rm LN}}\, (H^T L^i)^T\, g_\nu^{ij}\, (H^T L^j) + \text{h.c.}$$

  - $m_E = y_E^{\rm diag}\, \langle H^0 \rangle$
  - mismatch between diagnolization of $Y_E$ and $g_\nu$ determines PMNS matrix
  - $m_\nu \sim \langle H_0 \rangle^2 / \Lambda_{\rm LN}$ (see-saw), lepton-number violated!

- Now: $\mathcal{G}_\ell^{(b)} = U(3)_L \times U(3)_{E_R}$

- Parameter counting:
  - $18 + 12 = 30$ real parameters in Yukawa matrix $Y_E$ and $g_\nu$ (symmetric)
  - $9 + 9 = 18$ flavour symmetry generators in $\mathcal{G}_\ell^{(b)}$

  $\Rightarrow$ 12 physical parameters = 6 masses + 3 PMNS angles + 3 CP phases.

- Effective potential gets contributions from non-trivial QCD vacuum:

$$V = V_0 - K\, v^6\, \mathrm{Re}\left[\det Y_U \, \det Y_D \, e^{i\theta_0}\right] + \ldots \qquad (K > 0)$$

- After local flavour symmetry spontaneously broken, global $U(1)_{u_R} \times U(1)_{d_R}$ remains, such that

$$\det Y_U = y_u(x)\, y_c\, y_t\, e^{-i\pi_u(x)}\,, \qquad \det Y_D = y_{dd}(x)\, y_{ss}\, y_{bb}\, e^{-i\pi_d(x)}$$

- Potential minimized for

$$\langle \theta_{\rm eff} \rangle = \theta_0 - \langle \pi_u(x) + \pi_d(x) \rangle = 0$$

i.e. no CP-violation from strong interactions.

$\Rightarrow$ Axion Phenomenology (to be worked out)