Disentangling Neutrino Oscillations

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Ringberg Workshop, April 27, 2009

- **Introduction**
- **The GSI measurement**
- **Neutrino oscillations**
- **Conclusions**

[A. Cohen, S. Glashow, ZL, arXiv:0810.4602]

Neutrino masses: extending the SM

- Some definition of the SM implies $m_\nu = 0$; small values are a puzzle
- Mass term similar to (up-type) quarks requires ν_R: $\mathcal{L} = Y^\nu_{ij}\,\overline{L^I_{Li}}\,\widetilde{\phi}\,\nu^I_{Rj} \;\Rightarrow\; m_\nu\,\nu\bar{\nu}$

  The ν_R fields are SM singlets, have no weak interactions (“sterile”), no evidence
- In the absence of light ν_R states, masses come from dimension-5 operators:

$$\mathcal{L}_{\text{dim-5}} = \frac{1}{\Lambda}\,(L\phi)(L\phi) \to m_\nu\,\nu\nu\,, \qquad\qquad m_\nu \propto \frac{v^2}{\Lambda} \text{ (see-saw)}$$

$\frac{Y^\nu_{ij}}{v}\,\phi\phi L_{Li}L_{Lj}$ cannot arise from loops, e, μ, τ number are accidental symm’s of SM

$B - L$ is non-anomalous, so nonperturbative terms can neither generate it

Modern view of SM: the low energy effective theory of any underlying physics

suggested scale very large $\Lambda \sim 10^{14}\,\mathrm{GeV}$

- Do neutrino mass terms violate lepton number? (To decide: $0\nu2\beta$)

Neutrinos and leptogenesis

- Two large mixing angles observed — was a real surprise
- Leptogenesis appears quite plausible:

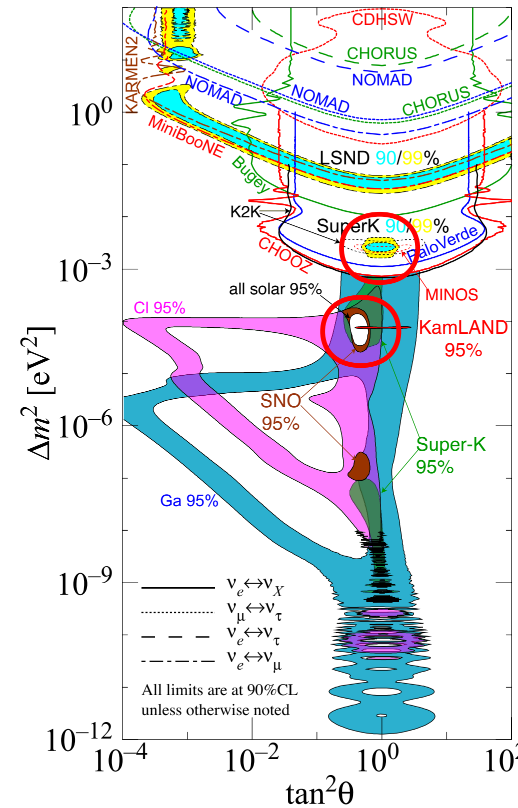
  ... generate $B-L$ by CPV decay of ν_{heavy}

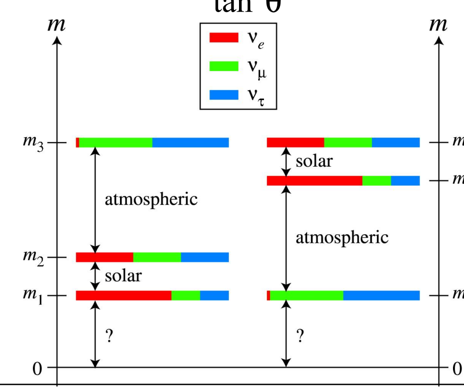
  ... ν_{heavy} lives long enough to decay when $T < m_{\nu_{\text{heavy}}}$

  Baryon asymmetry due to $B+L$ violating but $B-L$ conserving processes above electroweak phase transition

- Connection between the relevant CPV parameters and those in the light neutrino sector is model dependent

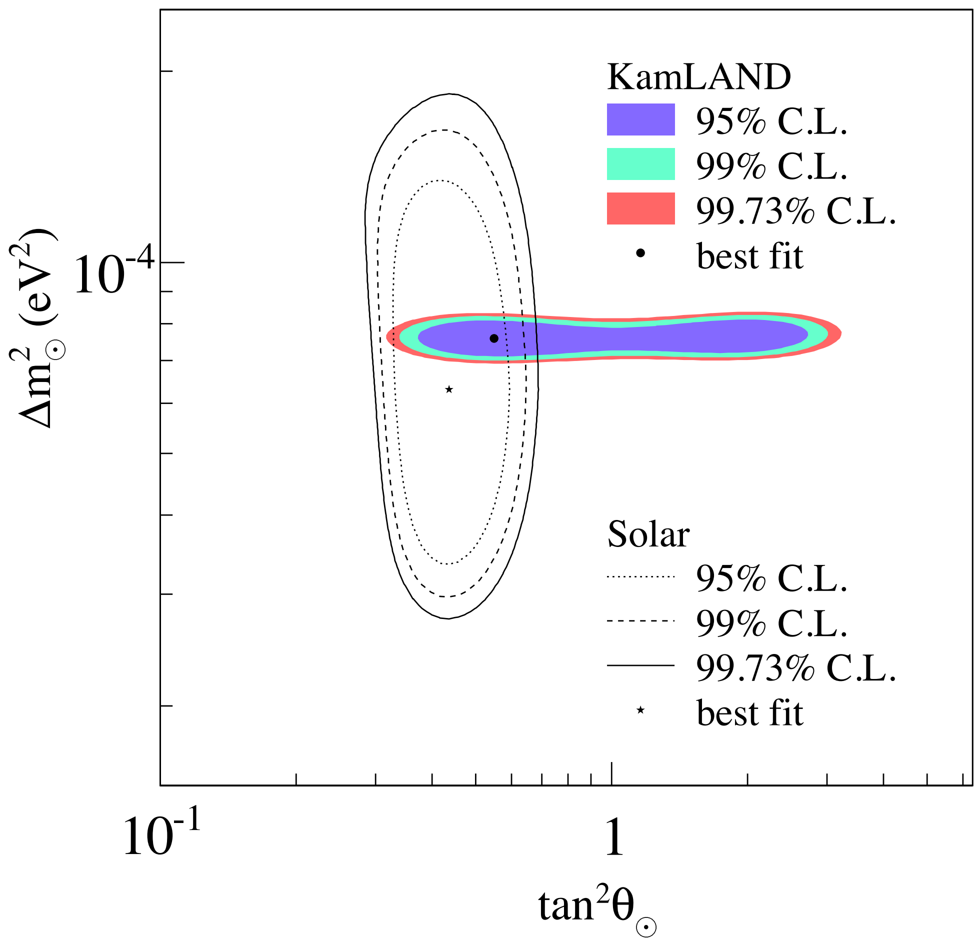
  ... Connection to TeV scale is model dependent





The solar oscillation region

- We shall be particularly interested in the solar region — almost maximal mixing



The GSI measurement

Preliminaries: Pr $=$ Praseodymium

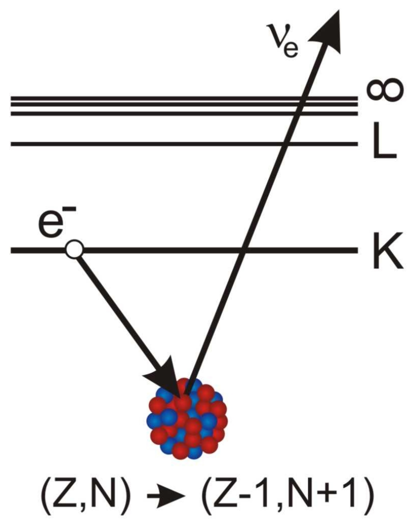
Ce $=$ Cerium

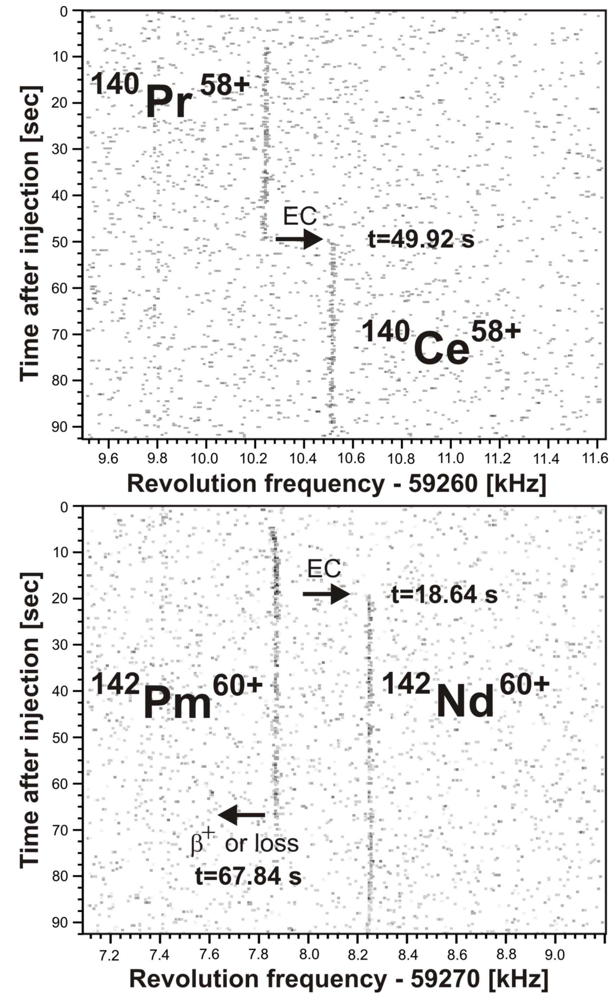
Pm $=$ Promethium

Nd $=$ Neodymium

The GSI measurement

- Electron capture decays of hydrogen-like ^{140}Pr and ^{142}Pm ions circulating in a storage ring





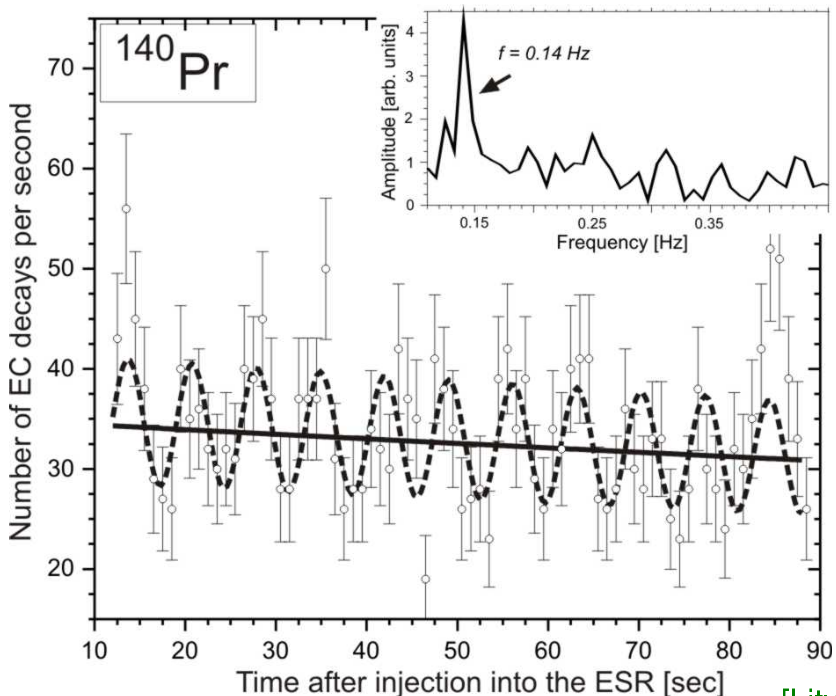
[Litvinov et al., PLB 664 (2008) 162]

- Revolution frequency ⇒ precise measurement of m/q
  - Continuous observation
  - Parent-daughter correlation
  - Detection of all EC decays
  - Modest statistics
- The created ν_e state is entangled with daughter nucleus and claimed to give a sensitive probe of ν properties

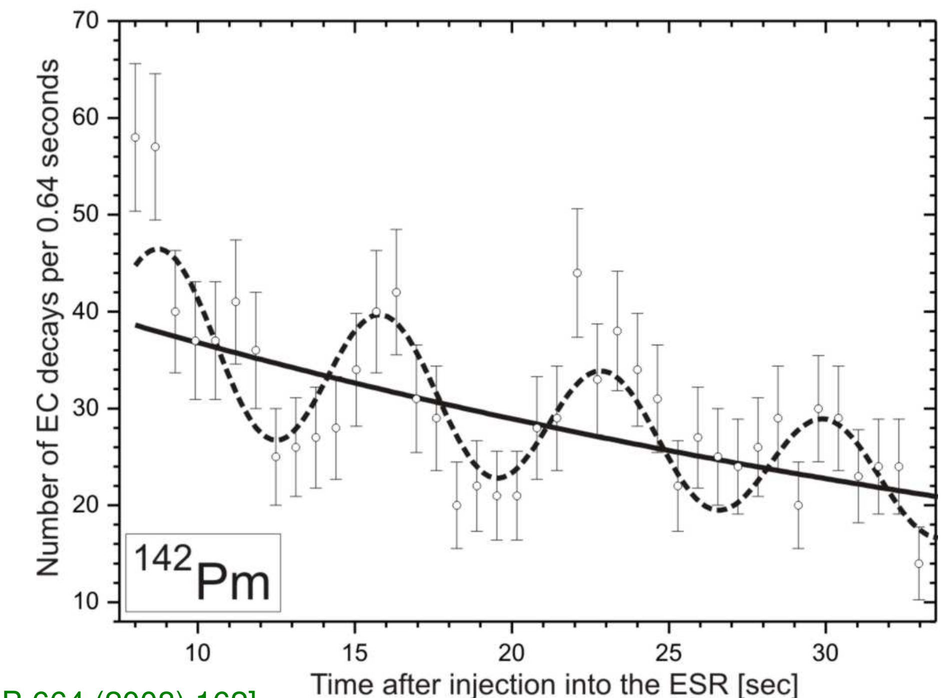
The GSI results

- Study time dependence of decay, observe deviations from simple exponential

$^{140}_{59}\mathrm{Pr}^{58+} \longrightarrow {}^{140}_{58}\mathrm{Ce}^{58+}$



$^{142}_{61}\mathrm{Pm}^{60+} \longrightarrow {}^{142}_{60}\mathrm{Nd}^{60+}$



[Litvinov et al., PLB 664 (2008) 162]

- Can these signals be due to neutrino oscillations?

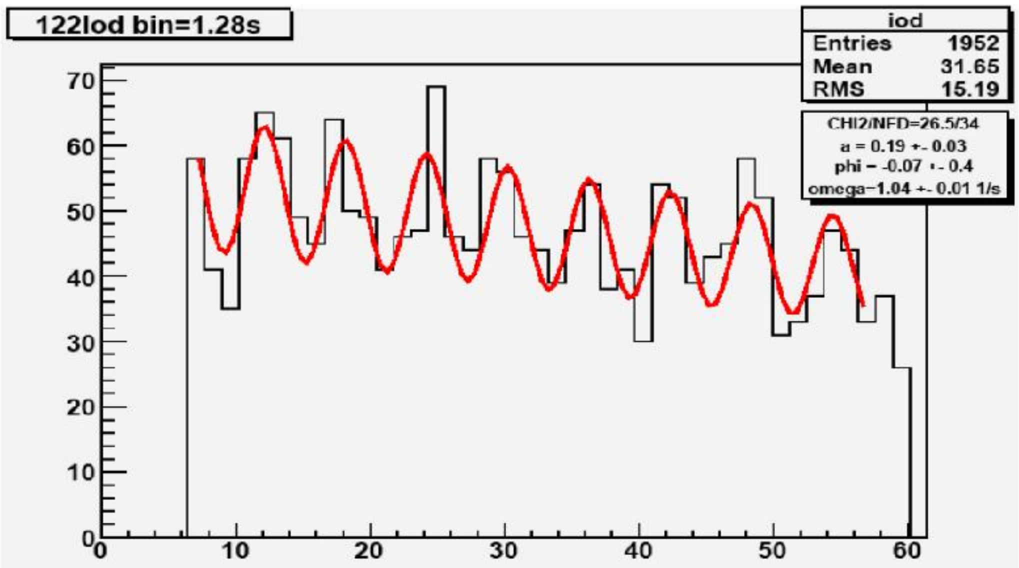
(Similar preliminary results for ^{122}I electron capture)

Claim: $\omega_{EC} = \frac{2\pi}{T_{EC}} = \frac{\delta m^2}{2\gamma M}$

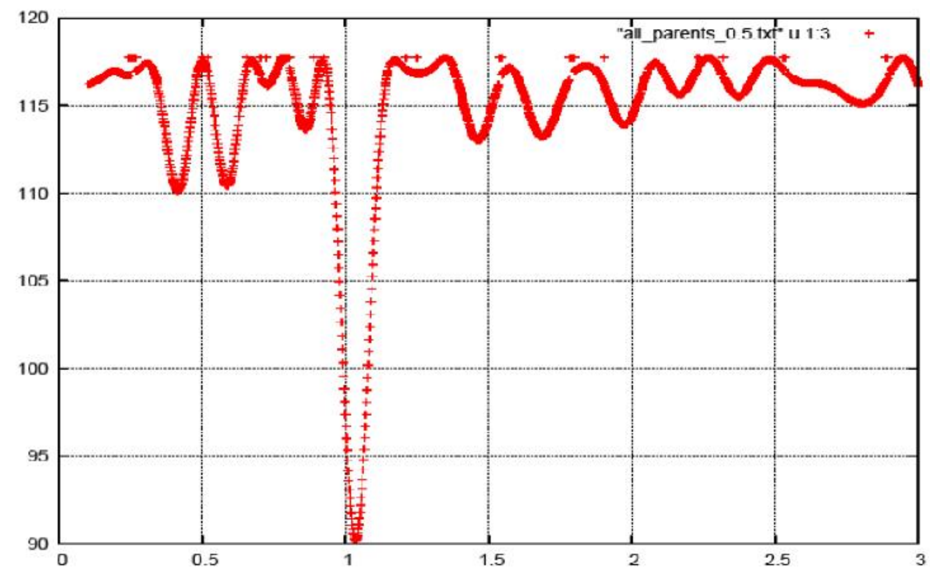
Preliminary ^{122}I results

- Test the scaling of the modulation frequency with the parent mass

"Very preliminary" results using 1/5 of the data [P. Kienle, PANIC 2008, Eilat]



Time dependence



χ^2 as function of ω

- Measured period: $T = 6.04(6)$ s vs. $T = 6.14$ s expected from M_m scaling

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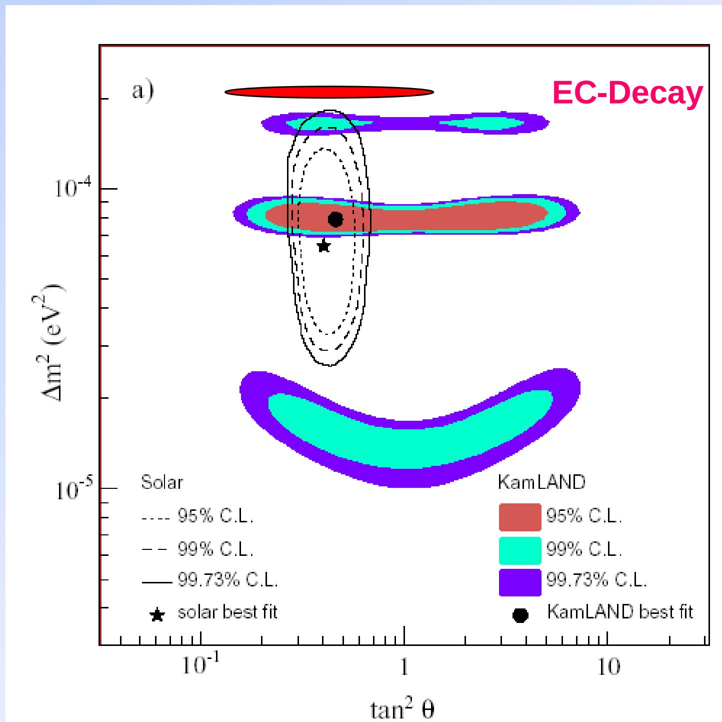
The GSI interpretation

Solar, KamLAND, EC
Results on Δm^2-$\tan^2\theta$



PANIC 2008 Eilat

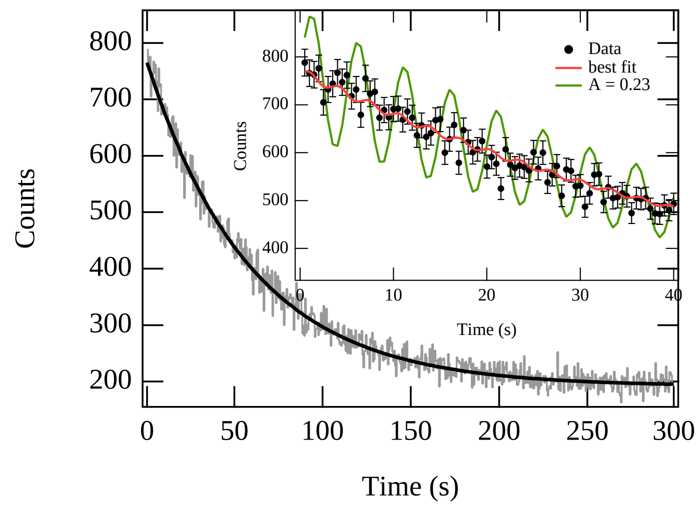
P.Kienle

The Berkeley measurement

- Produce ^{142}Pm by bombarding ^{124}Sn with ^{23}Na^{5+} for a short time compared to claimed 7 sec modulation

  Much smaller production of other $Z = 61$ isotopes

  The EC branching ratio of neutral ^{142}Pm is 22%, $^{142}_{61}\text{Pm}^{60+} \to {}^{142}_{60}\text{Nd}^{60+}$, then measure the K_α x-rays for a long time (from K-shell vacancy in Nd)



[Vetter et al., PLB 670 (2008) 196]

- Resulting bound much stronger than the claimed GSI signal

- Difference due to interactions with phonons? Seemed pretty strange...

  It is argued that orbital electron-capture decays of neutral ^{142}Pm atoms implanted into the lattice of a solid (LBNL experiment) do not fulfil the constraints of *true two-body beta decays*, since momentum as well as energy of the final state are distributed among three objects, namely the electron neutrino, the recoiling daughter atom and *the lattice phonons*. To our understanding, this could be a reason for the non-observation of a periodic time modulation in the number of electron-capture decays of implanted neutral ^{142}Pm atoms. [arXiv:0807.2308]

Neutrino oscillations

Conventional analysis

- Consider only two neutrino flavors: $|\nu_e\rangle = \cos\theta\,|\nu_1\rangle + \sin\theta\,|\nu_2\rangle$

  Time evolution: $|\nu_e(t)\rangle = \cos\theta\,|\nu_1\rangle\, e^{-iE_1 t} + \sin\theta\,|\nu_2\rangle\, e^{-iE_2 t}$

- Assume same momentum, $p_1 = p_2$

$$E_2 - E_1 = \sqrt{p^2 + m_2^2} - \sqrt{p^2 + m_1^2} = \frac{m_2^2 - m_1^2}{2p} \equiv \frac{\delta m^2}{2p}$$

Arguements ~ 10 yrs ago, same energy

$$p_2 - p_1 = \sqrt{E^2 - m_2^2} - \sqrt{E^2 - m_1^2} = \frac{m_1^2 - m_2^2}{2E} = -\frac{\delta m^2}{2E}$$

- Oscillation phase: $\delta(q\cdot x) = \delta(E\,t - p^i x^i) = \dfrac{\delta m^2\, t}{2p} \approx \dfrac{\delta m^2\, t}{2E} \left(\approx \dfrac{\delta m^2 L}{2E}\right)$

– No difference for ultrarelativistic neutrinos — which is correct in principle?
(both violate energy-momentum conservation; oscillations in space or time?)

– Is there dependence on details of the source and detector?

– Can experiments decide / discriminate between different approaches?

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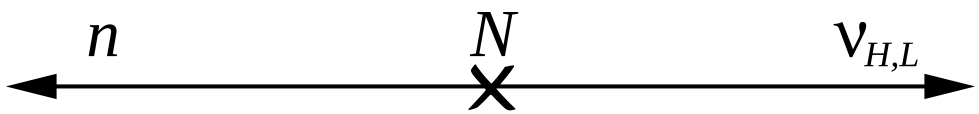
Start as simple as possible

- Simplest scenario: $N \to n\,\nu_{H,L}$, label ν mass eigenstates heavy / light



- As an idealized starting point, assume (semiclassical):
  – Parent is arbitrarily long lived, daughter is stable, treat decay perturbatively
  – Momenta of parent and decay products arbitrarily well known
  – Ignore spins and possible internal excitations

- What is the neutrino wave function after decay? Cannot be $\cos\theta\,|\nu_L\rangle + \sin\theta\,|\nu_H\rangle$

  Decays to distinct mass eigenstates to be viewed as separate channels

  Usual names of quarks refer to mass eigenstates, write Q^I_L, d^I_R for interaction eigenstates in $\mathcal{L}$; the mass eigenstate neutrinos should also have simple names...

The correct picture

- State after decay $(N \to n\,\nu_{H,L}$, e.g., $\pi \to \mu\nu_{H,L}$ or multi-body)

$$|\psi\rangle = \frac{1}{\sqrt{\mathcal{N}}}\left[\int D_2(k_l, q_l)\cos\theta\,|n(k_l)\nu_L(q_l)\rangle + \int D_2(k_h, q_h)\sin\theta\,|n(k_h)\nu_H(q_h)\rangle\right]$$

$$D_2(k, q) = \frac{\mathrm{d}^3 k}{(2\pi)^3 2E_k}\,\frac{\mathrm{d}^3 q}{(2\pi)^3 2E_q}\,(2\pi)^4\,\delta^4(P - k - q)$$

The mass eigenstates' energies & momenta differ (just like that of e, μ in π decay)

- All particles are on-shell, and $|\psi\rangle$ is eigenstate of energy & momentum

Only possible because $\nu_{H,L}$ are entangled with the daughter n

Impossible to conserve energy-momentum if ν state is $|\nu\rangle = \cos\theta\,|\nu_L\rangle + \sin\theta\,|\nu_H\rangle$

- Phase space in amplitude, not in decay rate

Same result as dPS$\times|A|^2$, because particles in different directions do not interfere

The density matrix

- Construct density matrix for ν by tracing over undetected n (most experiments)

$$\rho_\nu = \frac{1}{\sqrt{\mathcal{N}}}\Big[\int D_2(k_l,q_l)D_2(\tilde{k}_l,\tilde{q}_l)\cos^2\theta\,\langle n(k_l)|n(\tilde{k}_l)\rangle\,|\nu_L(q_l)\rangle\langle\nu_L(\tilde{q}_l)|$$
$$+\int D_2(k_l,q_l)D_2(\tilde{k}_h,\tilde{q}_h)\cos\theta\,\sin\theta\,\langle n(k_l)|n(\tilde{k}_h)\rangle\,|\nu_L(q_l)\rangle\langle\nu_H(\tilde{q}_h)| + \text{h.c.}$$
$$+\int D_2(k_h,q_h)D_2(\tilde{k}_h,\tilde{q}_h)\sin^2\theta\,\langle n(k_h)|n(\tilde{k}_h)\rangle\,|\nu_H(q_h)\rangle\langle\nu_H(\tilde{q}_h)|\Big]$$

Interference term in the middle line vanishes, because:

$$\langle n(k_l)|n(\tilde{k}_h)\rangle = (2\pi)^3\,2E'_{k_l}\,\delta^3(\mathbf{k}_l - \tilde{\mathbf{k}}_h) \qquad E'_k \equiv \sqrt{\mathbf{k}^2 + M'^2}$$

- Had to find no oscillation, because if we could measure the daughter n arbitrarily precisely, we could reconstruct $\nu_{H,L}$ $\Rightarrow$ If decay products of an initial state of well-defined momentum evolve without further interaction, oscillation cannot appear

- In realistic situations, these assumptions have to be (and are) violated

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How oscillations arise

- Something must allow $\langle n(k_l)|n(\tilde{k}_h)\rangle \neq 0$

  In $\pi \to \mu\nu$ the effect of $\Gamma_\pi \neq 0$ is by far sufficient; Or localization: $2\,\mathrm{km} = 10^{-10}\,\mathrm{eV}$
  (Or parent in a state which is superposition of spatial momenta in a narrow band)

  Momentum difference $\sim \dfrac{\delta m^2}{m_\pi} \sim 10^{-12}\,\mathrm{eV} \;\ll\; \Gamma_\pi \sim 3\times 10^{-8}\,\mathrm{eV}$

- Choose emission as origin, detection at $z = (t, d) \simeq t\left(1, \dfrac{|\vec{q}_h| + |\vec{q}_l|}{E_h + E_l}\right) = t\,\dfrac{q_h + q_l}{E_h + E_l}$

  Can be justified in stationary phase approximation

- Oscillation phase: $\varphi = (q_h - q_l)\cdot z = \dfrac{t}{E_h + E_l}(q_h - q_l)\cdot(q_h + q_l) = t\,\dfrac{\delta m^2}{E_h + E_l}$

  Result is manifestly Lorentz invariant

  For neutral K, B, D mixing, useful to think in rest frame $(\delta m \ll \delta E)$, then $\varphi = \delta m\, t$

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Stationary phase approximation

- $e^{i(q_h - q_l)\cdot z} = e^{i[(E_h - E_l)t - (q_h^i - q_l^i)x_i]}$ $\qquad q = (E, \vec{q}\,) = (\sqrt{m^2 + \vec{q}^{\,2}}, \vec{q}\,),\ z = (t, \vec{x})$

$$\frac{\partial E}{\partial q_i} = \frac{q_i}{E} \quad \Rightarrow \quad \frac{q_i}{E} t - x_i = 0 \quad \Rightarrow \quad \frac{\vec{q}}{E} = \frac{\vec{x}}{t}$$ for both heavy and light states

$$z = (t, \vec{x}) = t\left(1, \frac{\vec{q}_{h,l}}{E_{h,l}}\right) = \frac{t}{E_h + E_l}(E_h + E_l,\ \vec{q}_h + \vec{q}_l) = \frac{t}{E_h + E_l}(q_h + q_l)$$

So: $(q_h - q_l)\cdot z = \dfrac{\delta m^2\, t}{E_h + E_l}$

- This derivation holds equally for neutrinos and neutral mesons

Wave packets

- In the idealized case: infinitely long lived parent $+$ plane-wave states of decay products, at $z=(t,d)$ the neutrino detection amplitude is (Heisenberg picture)

$$\mathcal{A} \sim \cos^2\theta\, e^{iq_l\cdot z} + \sin^2\theta\, e^{iq_h\cdot z}$$

Realistic experiments involve amplitudes with space-time support localized around the trajectory $d = v\,t$; the $\nu_{H,L}$ particles have slightly different velocities

- The two amplitudes can only interfere if they have common support at detector; i.e., wave packets do not separate prior to detection (size of wave packet $= v\,\Delta T$)

$$\left|\frac{v_h - v_l}{v_h + v_l}\right| = \left|\frac{\sigma E\,\delta q - \delta E\,\sigma q}{\sigma E\,\sigma q - \delta E\,\delta q}\right| \ll \frac{\Delta T}{t}$$

If this condition is satisfied, oscillations are possible (supernovae...)

The GSI experiment

- Contrary to most experiments, the neutrino is not detected, oscillation is observed in the decay rate of parent nucleus (so in this case, trace over neutrinos)

  Consider squared amplitude; cut propagator of physical neutrino $\delta(q^2 - m_{h,l}^2)\,\theta(q^0)$

  $|\nu_L\rangle$ and $|\nu_H\rangle$ have different masses, and are orthogonal states with any momenta

- Rate of disappearance of parent particle: $\Gamma(N \to n\,\nu_H) + \Gamma(N \to n\,\nu_L)$

- Cannot observe mixing of mass eigenstates without detecting the neutrino

  Argue that signal is due to Thomas precession (coupling of rotation to spin of electron and nuclei)
  Two hyperfine levels of parent, only one decays via EC ($1_{\rm nucl} + \frac{1}{2}{}_e \to \frac{1}{2}$); magnetic field causes precession, mixes two initial states... not easy to get 7 sec oscillation period... [arXiv:0811.2302]

Mössbauer neutrino oscillations

- Resonant capture of $\bar{\nu}_e$ from bound-state tritium decay [Raghavan, hep-ph/0511191, etc.]

  Bound state beta decay: ^{3}H $\rightarrow$ ^{3}He $+ \bar{\nu}_e + e^-$ (in bound state)

  Capture by reverse reaction: ^{3}He $+ \bar{\nu}_e + e^-$ (in bound state) $\rightarrow$ ^{3}H

  If neutrino emission and absorption are recoilless $\Rightarrow$ resonance enhancement

  Tremendous challenges: gravity ($\Delta E/E \sim 10^{-18}/\mathrm{cm}$), chemistry, ... etc.

- If it can be realized, it will be a powerful tool to study neutrino oscillations

  No ambiguity about the approach to neutrino oscillations to be resolved

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Bilenky et al.: such oscillations may or may not occur, and the Raghavan experiment "provides the unique possibility to discriminate basically different approaches to oscillations of flavor neutrinos."

Akhmedov et al.: "a proper interpretation of the time-energy uncertainty relation is fully consistent with oscillations of Mössbauer neutrinos."

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Conclusions

Conclusions

- Neutrino oscillations can be understood using usual field theoretical tools
- Many of the confusions in the literature can be avoided
  Energy and momentum are conserved...
- Details of the source, detector, and decoherence are not always essential
- The GSI results cannot be attributed to mixing of neutrino mass eigenstates
- The Mössbauer neutrino experiment, if feasible, could probe neutrino oscillations

Backup slides

Aside: the factor-of-two mistake

From: ....

To: slg@bu.edu

CC: cohen@bu.edu, ligeti@lbl.gov

Date: 2009-01-13 06:33

Dear Professor Glashow,

I noted with interest your recent arXiv article on the neutrino oscillation problem. There are some recent related articles, which you do not cite, and which you may find of interest:

The formulae you give for the oscillation phase are not correct because, as explained in (I), the production event in each interfering amplitude occurs at the same time, which is physically impossible if the distance between the source and production events is the same for both mass eigenstates and their velocites are different. ... the neglect of this time difference leads to the factor two error in the interference phase.

Aside: the factor-of-two mistake

- Claims about the standard oscillation formula being wrong by a factor of 2

  Claim: $|\nu_{H,L}\rangle$ mass eigenstates have different velocities $\Rightarrow$ different travel times from emission to detection locations $\quad (E^2 - |\vec{q}\,|^2 = m^2 \;\Rightarrow\; E/v - |\vec{q}\,| = m^2/|\vec{q}\,|)$

  Wrong phase: $q_2 \cdot z_2 - q_1 \cdot z_1 = (E_2 t_2 - E_1 t_1) - (q_2 - q_1) d$

  $$= \left(\frac{E_2}{v_2} - q_2 - \frac{E_1}{v_1} + q_1\right) d = \left(\frac{m_2^2}{q_2} - \frac{m_1^2}{q_1}\right) d = t\,\frac{\delta m^2}{E} + \ldots$$

- Oscillation occurs even in single decay events, as in double slit experiment with only one particle at a time in the apparatus $\Rightarrow$ there cannot be a $\Delta\, t$ contribution to the phase difference

- t is a parameter of local operators that describe the decay and detection events, which are the same for all terms in amplitude — there is no time operator in QM

Aside: states and fields

- Usual names of quarks refer to mass eigenstates, write Q_L^I, d_R^I for interaction eigenstates in $\mathcal{L}$; the mass eigenstate neutrinos should also have simple names...
- Flavor-charge operators, such as the $e\,(\mu)$ number, are well-defined in the SM augmented with neutrino mixing, but do not commute with the Hamiltonian
- The lepton flavor conserving weak interactions are simplest to write in terms of the $\nu_e\,(\nu_\mu)$ *field* with definite flavor, which acts on a *state* altering its electron (muon) number by one unit; however, since time evolution alters the flavor, it is not too fruitful to consider *states* of definite flavor
- Although the *fields* that create and annihilate mass eigenstates are linear combinations of the *fields* of definite flavor, the corresponding construction for *states* is not helpful — similar to *chirality* (useful property of fields) and *helicity* (measurable property of states)

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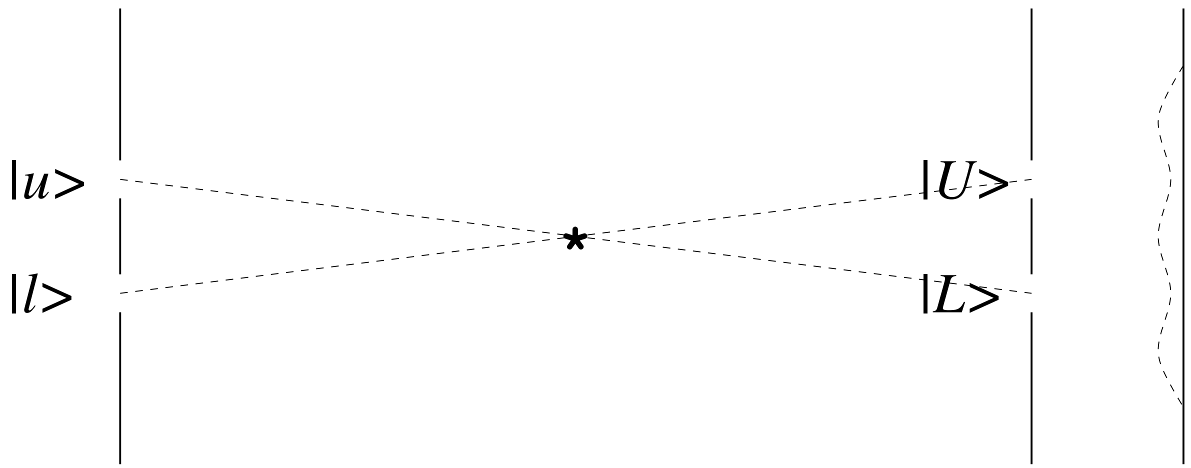
Apparent paradoxes about causality violation?

- Consider $P \to dd$ decay at rest, with double-slit experiments in opposite directions



1) Only r.h.s. (slits on l.h.s. removed): $|\psi\rangle = \frac{1}{\sqrt{2}}\big(|U\rangle + |L\rangle\big)$

... interference from $\langle U|L\rangle + \langle L|U\rangle$ terms in density matrix

2) Both sides observed: $|\psi\rangle = \frac{1}{\sqrt{2}}\big(|U\rangle\,|l\rangle + |L\rangle\,|u\rangle\big)$

... no interference, because $\langle u|l\rangle = \langle l|u\rangle = 0$

- Can one transfer information from l.h.s. to r.h.s by having or not having the slits?