Status of $\mathcal{B}(\bar{B} \to X_s \gamma)$

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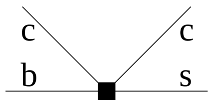
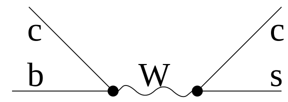
1. **Introduction**
2. **Non-perturbative effects** (main uncertainty)
3. **News on perturbative calculations at** $\mathcal{O}(\alpha_s^2)$
4. **The issue of normalization**
5. **Summary**

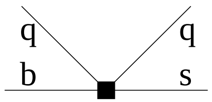
The effective Lagrangian:

$$\mathcal{L}_{\rm eff} = \mathcal{L}_{\rm QCD\times QED}(u,d,s,c,b) + \frac{4G_F}{\sqrt{2}} V_{ts}^* V_{tb} \sum_{i=1}^{8} C_i(\mu)\, Q_i + \left(\begin{array}{c}\textbf{higher-electroweak,}\\ \textbf{higher-dimensional,}\\ \textbf{on-shell vanishing,}\\ \textbf{evanescent}\end{array}\right).$$

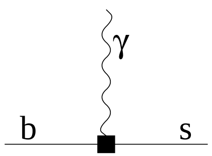
$Q_{1,2} =$ 

$= (\bar{s}\Gamma_i c)(\bar{c}\Gamma'_i b)$, **from** 

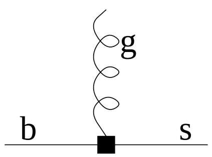
, $\quad |C_i(m_b)| \sim 1$

$Q_{3,4,5,6} =$ 

$= (\bar{s}\Gamma_i b)\Sigma_q(\bar{q}\Gamma'_i q)$, $\quad |C_i(m_b)| < 0.07$

$Q_7 =$ 

$= \frac{e m_b}{16\pi^2}\, \bar{s}_L \sigma^{\mu\nu} b_R F_{\mu\nu}$, $\quad C_7(m_b) \simeq -0.3$

$Q_8 =$ 

$= \frac{g m_b}{16\pi^2}\, \bar{s}_L \sigma^{\mu\nu} T^a b_R G^a_{\mu\nu}$, $\quad C_8(m_b) \simeq -0.15$

In the SM, the Wilson coefficients $C_1, \ldots, C_8$ are known up the the NNLO $(\mathcal{O}(\alpha_s^2))$.

Goal: Constrain new physics using the determination of C_7 from $\mathcal{B}(\bar{B} \to X_s \gamma)$ measurement.

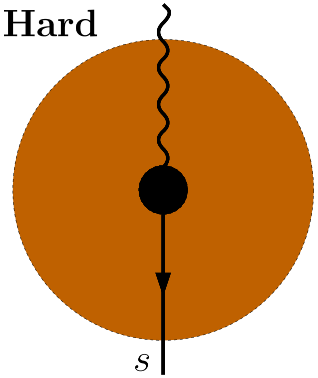
(present accuracy: $5 \div 7\%$)

Energetic photon production in charmless decays of the $\bar{B}$-meson

$(E_\gamma \gtrsim \frac{m_b}{3} \simeq 1.6\,\text{GeV})$

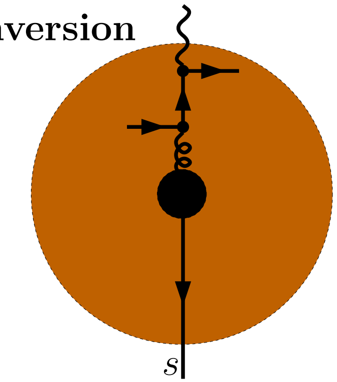
A. Without long-distance charm loops:

1. **Hard**



Dominant, well-controlled.

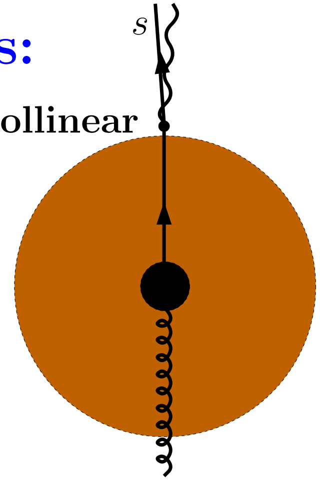
2. **Conversion**



$\mathcal{O}(\alpha_s \Lambda / m_b)$, $(-1.5 \pm 1.5)\%$.

[Lee, Neubert, Paz, 2006]

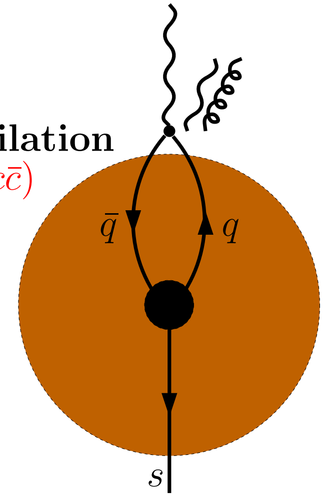
3. **Collinear**



Pert. $< 1\%$, nonp. $\sim -0.2\%$.

[Kapustin,Ligeti,Politzer, 1995]

4. **Annihilation** $(q\bar{q} \neq c\bar{c})$

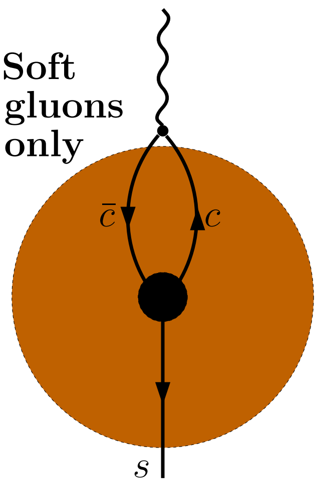


Exp. π^0, η, η', ω subtracted.

Perturbatively $\sim 0.1\%$.

B. With long-distance charm loops:

5. **Soft gluons only**

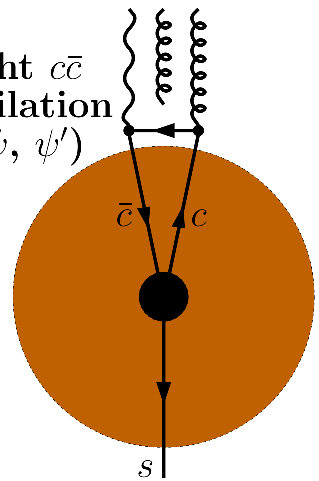


$\mathcal{O}(\Lambda^2/m_c^2)$, $\sim +3.1\%$.

[Voloshin, 1996], [...],

[Buchalla, Isidori, Rey, 1997]

6. **Boosted light $c\bar{c}$ state annihilation (e.g. η_c, J/ψ, ψ')**

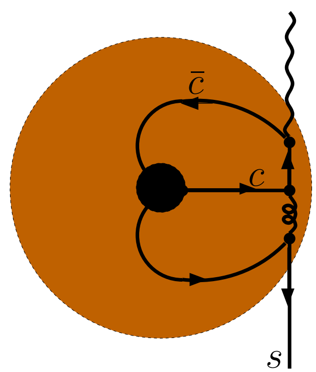


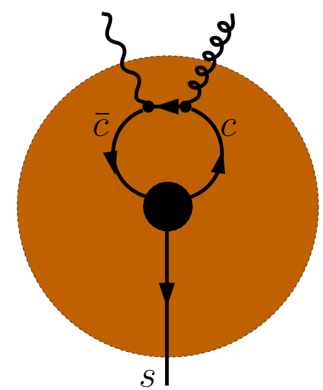
Exp. J/ψ subtracted ($< 1\%$).

Perturbatively (including hard): $\sim +3.6\%$.

$\phi_{ij}^{(1)}(\delta)$, $\phi_{ij}^{(2)\beta_0}(\delta)$, $i, j = 1, 2$

7. **Annihilation of $c\bar{c}$ in a heavy $(\bar{c}s)(\bar{q}c)$ state**





$\mathcal{O}(\alpha_s (\Lambda/M)^2)$ $\qquad$ $\mathcal{O}(\alpha_s \Lambda / M)$

$M \sim 2m_c, 2E_\gamma, m_b$.

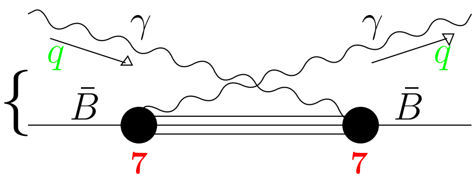
e.g. $\mathcal{B}[B^- \to D_{sJ}(2457)^- \, D^*(2007)^0 \,] \simeq 1.2\%$,
$\mathcal{B}[B^0 \to D^*(2010)^+ \, \bar{D}^*(2007)^0 K^-] \simeq 1.2\%$.

The "hard" contribution to $\bar{B} \to X_s \gamma$

J. Chay, H. Georgi, B. Grinstein PLB 247 (1990) 399.
A.F. Falk, M. Luke, M. Savage, PRD 49 (1994) 3367.

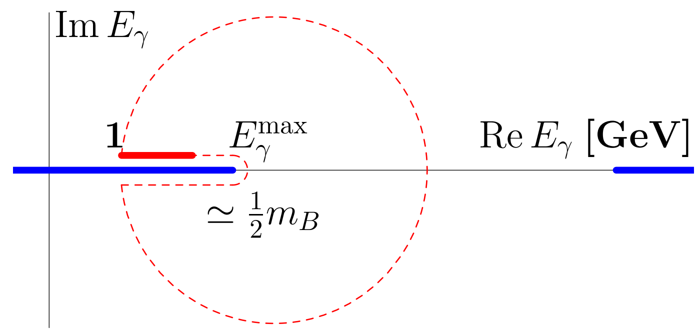
Goal: calculate the inclusive sum $\Sigma_{X_s} \left| C_7(\mu_b) \langle X_s\gamma|O_7|\bar{B}\rangle + C_2(\mu_b)\langle X_s\gamma|O_2|\bar{B}\rangle + \ldots \right|^2$

The "77" term in this sum is purely "hard". It is related via the optical theorem to the imaginary part of the elastic forward scattering amplitude $\bar{B}(\vec{p}=0)\gamma(\vec{q}) \to \bar{B}(\vec{p}=0)\gamma(\vec{q})$**:**

$\mathrm{Im}\{$  $\} \equiv \mathrm{Im}\, A$

When the photons are soft enough, $m_{X_s}^2 = |m_B(m_B - 2E_\gamma)| \gg \Lambda^2 \Rightarrow$ **Short-distance dominance** $\Rightarrow$ **OPE.**
However, the $\bar{B} \to X_s\gamma$ **photon spectrum is dominated by hard photons** $E_\gamma \sim m_b/2$**.**

Once $A(E_\gamma)$ **is considered as a function of arbitrary complex** E_γ**,** $\mathrm{Im}A$ **turns out to be proportional to the discontinuity of** A **at the physical cut. Consequently,**



$$\int_{1\ \mathrm{GeV}}^{E_\gamma^{\max}} dE_\gamma\ \mathrm{Im}\, A(E_\gamma) \sim \oint_{\mathrm{circle}} dE_\gamma\ A(E_\gamma).$$

Since the condition $|m_B(m_B - 2E_\gamma)| \gg \Lambda^2$ **is fulfilled along the circle, the OPE coefficients can be calculated perturbatively, which gives**

$$A(E_\gamma)\big|_{\mathrm{circle}} \simeq \sum_j \left[\frac{F^{(j)}_{\mathrm{polynomial}}(2E_\gamma/m_b)}{m_b^{n_j}(1-2E_\gamma/m_b)^{k_j}} + \mathcal{O}\left(\alpha_s(\mu_{\mathrm{hard}})\right) \right] \langle \bar{B}(\vec{p}=0)|Q^{(j)}_{\mathrm{local\ operator}}|\bar{B}(\vec{p}=0)\rangle.$$

Thus, contributions from higher-dimensional operators are suppressed by powers of Λ/m_b**.**

At $(\Lambda/m_b)^0$**:** $\langle \bar{B}(\vec{p})|\bar{b}\gamma^\mu b|\bar{B}(\vec{p})\rangle = 2p^\mu \quad \Rightarrow \quad \Gamma(\bar{B} \to X_s\gamma) = \Gamma(b \to X_s^{\mathrm{parton}}\gamma) + \mathcal{O}(\Lambda/m_b)$.

At $(\Lambda/m_b)^1$**:** **Nothing! All the possible operators vanish by the equations of motion.**

At $(\Lambda/m_b)^2$**:** $\langle \bar{B}(\vec{p})|\bar{h}D^\mu D_\mu h|\bar{B}(\vec{p})\rangle = -2m_B\lambda_1,$ $\quad \lambda_1 = (-0.27 \pm 0.04)\mathrm{GeV}^2$ **from** $\bar{B} \to X\ell^-\nu$ **spectrum.**
$\langle \bar{B}(\vec{p})|\bar{h}\sigma^{\mu\nu}G_{\mu\nu}h|\bar{B}(\vec{p})\rangle = 6m_B\lambda_2,$ $\quad \lambda_2 \simeq \frac{1}{4}\left(m_{B^*}^2 - m_B^2\right) \simeq 0.12\,\mathrm{GeV}^2.$

The HQET heavy-quark field $h(x)$ **is defined by** $h(x) = \frac{1}{2}(1+\not{v})b(x)\exp(im_b\, v\cdot x)$ **with** $v = p/m_B$**.**

The $\bar{B} \to X_s\gamma$ photon spectrum for $E_\gamma \sim E_\gamma^{\max} \simeq \frac{M_B}{2}$ is dominated by contributions from "hard" radiative decays of the b-quark

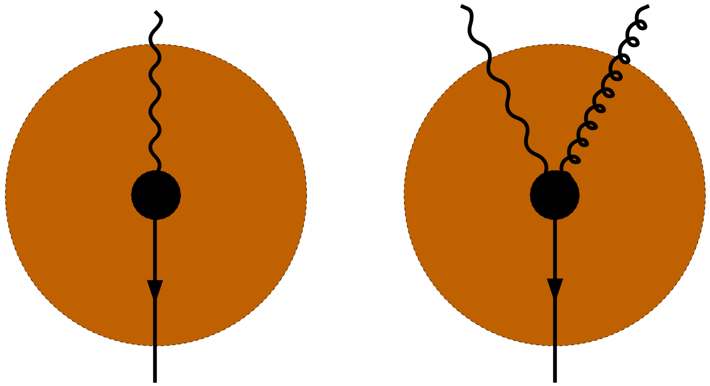
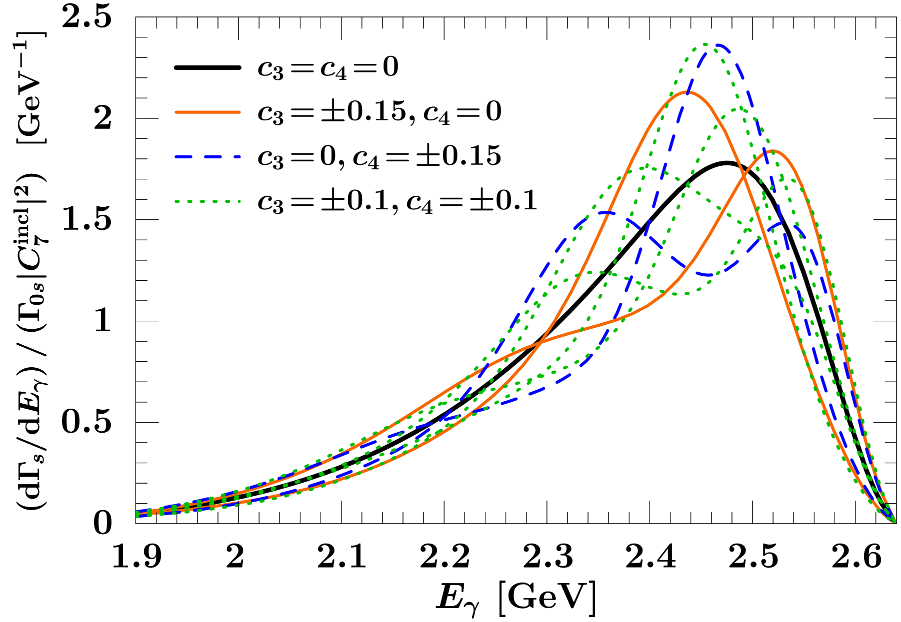


Fig. 13 from arXiv:0807.1926 by Z. Ligeti, I. Stewart and F. Tackmann.

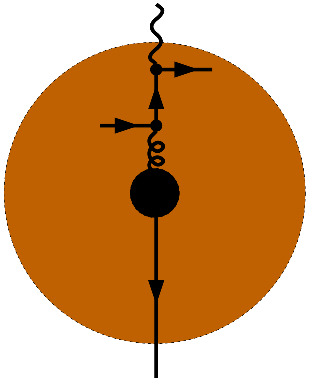


The integrated branching ratio with a lower cut E_0 on the photon energy $\mathcal{B}(\bar{B} \to X_s\gamma)_{E_\gamma > E_0}$ becomes very uncertain when E_0 is too large $(m_b - 2E_0 \sim \Lambda)$ or too small (when other than "hard" mechanisms of the photon production dominate). In a certain intermediate range of E_0:

$$\Gamma(\bar{B} \to X_s\gamma)_{E_\gamma > E_0} = \Gamma(b \to X_s^{\mathrm{parton}}\gamma)_{E_\gamma > E_0} + \mathcal{O}\left(\frac{\Lambda^2}{m_b^2}\right) + \left(\begin{array}{c}\text{small corrections due to} \\ \text{other than "hard" photons}\end{array}\right).$$

In the following, $E_0 = 1.6\,\mathrm{GeV} \simeq \frac{m_b}{3}$ is chosen as default.

Gluon-to-photon conversion in the QCD medium



This is hard gluon scattering on the valence quark or a “sea” quark that produces an energetic photon. The quark that undergoes this Compton-like scattering is assumed to remain soft in the $\bar{B}$-meson rest frame to ensure effective interference with the leading “hard” amplitude. Without interference the contribution would be negligible $(\mathcal{O}(\alpha_s^2\Lambda^2/m_b^2))$.

Suppression by Λ can be understood as originating from dilution of the target (size of the $\bar{B}$-meson $\sim \Lambda^{-1}$).

A rough estimate using vacuum insertion approximation gives

$$\Delta\Gamma/\Gamma \in [-3\%, -0.3\%] \qquad (\mathcal{O}(\alpha_s\Lambda/m_b)).$$

[Lee, Neubert, Paz, hep-ph/0609224]

However:

1. Contribution to the interference from scattering on the ”sea” quarks vanishes in the $SU(3)_{\rm flavour}$ limit because $Q_u + Q_d + Q_s = 0$.

2. If the valence quark dominates, then the isospin-averaged $\Delta\Gamma/\Gamma$ is given by:

$$\frac{\Delta\Gamma}{\Gamma} \simeq \frac{Q_d+Q_u}{Q_d-Q_u}\Delta_{0-} = -\frac{1}{3}\Delta_{0-} = \left(+0.2 \pm 1.9_{\rm stat} \pm 0.3_{\rm sys} \pm 0.8_{\rm ident}\right)\%,$$

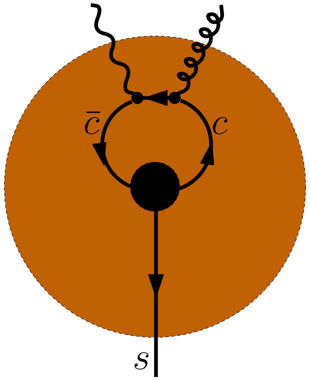
using the BABAR measurement (hep-ex/0508004) of the isospin asymmetry

$$\Delta_{0-} = [\Gamma(\bar{B}^0 \to X_s\gamma) - \Gamma(B^- \to X_s\gamma)]/[\Gamma(\bar{B}^0 \to X_s\gamma) + \Gamma(B^- \to X_s\gamma)],$$

for $E_\gamma > 1.9$ **GeV**.

Quark-to-photon conversion gives a soft s-quark and poorly interferes with the ”hard” $b \to s\gamma g$ amplitude.

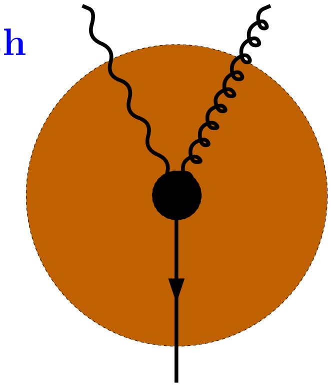
Annihilation of $c\bar{c}$ in a heavy $(\bar{c}s)(\bar{q}c)$ state



Heavy $\Leftrightarrow$ **Above the $D\bar{D}$ production threshold**

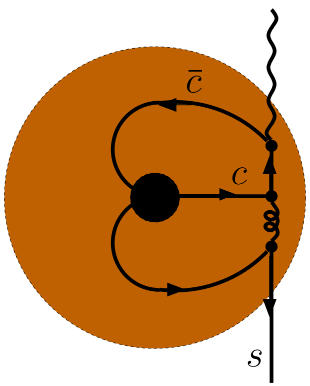
Long-distance $\Rightarrow$ **Annihilation amplitude is suppressed with respect to the open-charm decay due to the order Λ^{-1} distance between c and $\bar{c}$. By analogy to the B-meson decay constant $f_B \sim \Lambda(\Lambda/m_b)^{1/2}$, we may expect that the suppression factor scales like $(\Lambda/M)^{3/2}$, where $M \sim 2m_c, 2E_\gamma, m_b$.**

Hard gluon $\Leftrightarrow$ **Suppression by α_s of the interference with**
(non-soft)



Altogether: $\mathcal{O}\left(\alpha_s(\Lambda/M)^{3/2}\right)$.

To stay on the safe side, assume $\mathcal{O}\left(\alpha_s\Lambda/m_b\right)$ for numerical error estimates.

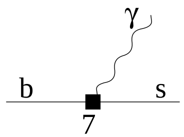


This type of amplitude interferes with the leading term but receives an additional Λ/M suppression (at least) due to participation of the s-quark in the hard annihilation.

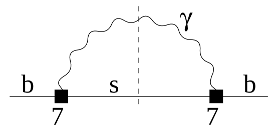
Missing ingredients in the perturbative NNLO matrix elements

$$\Gamma(b \to X_s^{\rm parton}\gamma)_{E_\gamma > E_0} = \frac{G_F^2 m_b^5 \alpha_{\rm em}}{32\pi^4} |V_{ts}^* V_{tb}|^2 \sum_{i,j=1}^{8} C_i(\mu_b) C_j(\mu_b) G_{ij}(E_0, \mu_b)$$

LO: $G_{ij} = \delta_{i7}\delta_{j7}$



$\Leftrightarrow$



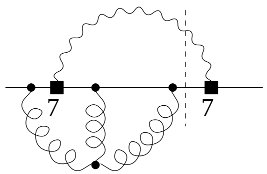
$|C_{1,2}(\mu_b)| \sim 1$, $|C_{3,4,5,6}(\mu_b)| < 0.07$,
$C_7(\mu_b) \sim -0.3$, $C_8(\mu_b) \sim -0.15$.

NLO: **The most important** G_{ij} $(i, j = 1, 2, 7, 8)$ **are known since 1996.** { [Greub, Hurth, Wyler, 1996] [Ali, Greub, 1991-1995]

The remaining G_{ij} **are known since 2002.** { [Buras, Czarnecki, MM, Urban, 2002] [Pott, 1995]

NNLO: **Only** $i, j = 1, 2, 7, 8$ **have been considered so far.**

Only G_{77} **is fully known:**



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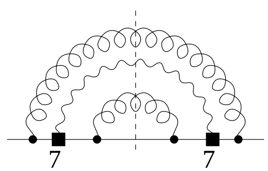
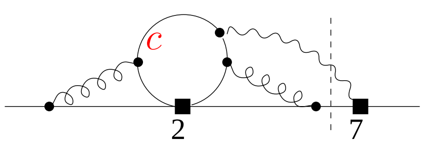


$+ \ldots$

{ [Blokland et al., 2005] [Melnikov, Mitov, 2005] [Asatrian et al., 2006-2007]

G_{27}**:**
(and analogous G_{17})



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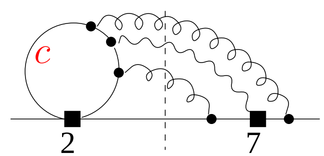


$+ \ldots$

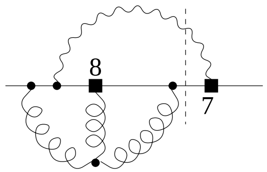
Two-particle cuts:
~ 160 **four-loop**
master integrals $(m_c = 0)$
recently completed
by T. Schutzmeier.

Three- and four-particle cuts:
R. Boughezal,
M. Czakon,
T. Schutzmeier,
in progress...

Previous status reports: arXiv:0712.1676, arXiv:0807.0915.

Diagrams with quark loops on gluon lines for $m_c \neq 0$: arXiv:0707.3090.

G_{78}:



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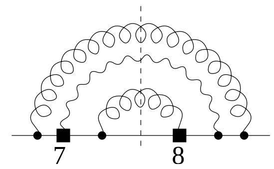


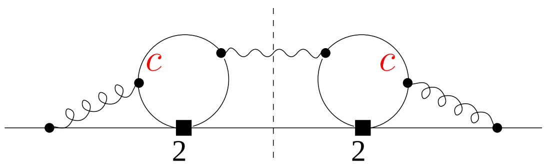
+ ...

Two-particle cuts:
finished in 2007
(unpublished)

Three- and four-particle cuts:
in progress...

H.M. Asatrian, T. Ewerth, A. Ferroglia, C. Greub, G. Ossola.

G_{22}:
(and analogous G_{11} & G_{12})



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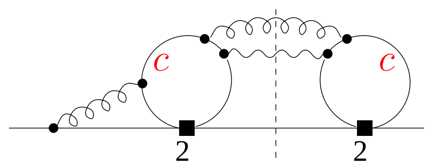

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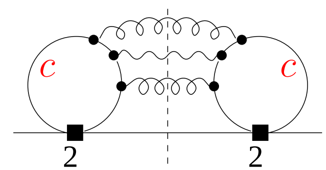


+ ...

Two-particle cuts
are known (just $|\mathrm{NLO}|^2$).

Three- and four-particle cuts
vanish at the endpoint $E_\gamma = m_b/2$.
Analogous NLO corrections are not big $(+3.6\%)$.

The current phenomenological analysis at the NNLO relies on using the BLM approximation together with the large-m_c asymptotics of the non-BLM correction. The latter correction is interpolated in m_c under the assumption that it vanishes at $m_c = 0$.

Large-m_c asymptotics of G_{ij}^{NNLO} $(m_c \gg m_b/2)$**:**

1	2	7	8	
+	+	+	+	1
	+	+	+	2
		+	−	7
			−	8

[MM, Steinhauser, 2006]

The BLM approximation for G_{ij}^{NNLO} (arbitrary m_c):

1	2	7	8	
+	+	+	−	1
	+	+	−	2
		+	+	7
			+	8

The BLM corrections to G_{78}, G_{88} are small.

G_{18} and G_{28} are small at the NLO.

[Bieri, Greub, Steinhauser, 2003]
[Ligeti, Luke, Manohar, Wise, 1999]
[Ferroglia, Haisch, 2007]

The issue of global normalization.

$$\mathcal{B}(\bar{B} \to X_s\gamma)_{E_\gamma > E_0} = \mathcal{B}(\bar{B} \to X_c e\bar{\nu})_{\rm exp} \left|\frac{V_{ts}^* V_{tb}}{V_{cb}}\right|^2 \frac{6\alpha_{\rm em}}{\pi\, C} \left[\underbrace{P(E_0)}_{\text{pert.}} + \underbrace{N(E_0)}_{\text{non-pert}}\right]$$

$$\frac{\Gamma[b \to X_s\gamma]_{E_\gamma > E_0}}{|V_{cb}/V_{ub}|^2\, \Gamma[b \to X_u e\bar{\nu}]} = \left|\frac{V_{ts}^* V_{tb}}{V_{cb}}\right|^2 \frac{6\alpha_{\rm em}}{\pi}\, P(E_0),$$

The semileptonic phase-space factor:

$$C = \left|\frac{V_{ub}}{V_{cb}}\right|^2 \frac{\Gamma[\bar{B} \to X_c e\bar{\nu}]}{\Gamma[\bar{B} \to X_u e\bar{\nu}]}$$

$$C = \begin{cases} 0.582 \pm 0.016, & \text{C. W. Bauer et al., hep-ph/0408002, 1S scheme,} \\ 0.546^{+0.023}_{-0.033}, & \text{P. Gambino and P. Giordano, arXiv:0805.0271, kinetic scheme.} \end{cases}$$

$$\overline{m}_c(\overline{m}_c) = \begin{cases} 1.224 \pm 0.057, & \text{1S scheme,} \\ 1.267 \pm 0.056, & \text{kinetic scheme.} \end{cases}$$

$\frac{\partial}{\partial m_c} P(E_0) < 0 \quad \Rightarrow$ **The differences tend to cancel in the radiative branching ratio.**

Numerical results for the SM branching ratio:

$$\mathcal{B}(\bar{B} \to X_s\gamma)^{\rm NNLO}_{E_\gamma>1.6\ {\rm GeV}} = \begin{cases} (3.15 \pm 0.23) \times 10^{-4}, & \text{hep-ph/0609232, using the 1S scheme,} \\ (3.26 \pm 0.24) \times 10^{-4}, & \text{following the kin scheme analysis of arXiv:0805.0271, but } \overline{m}_c(\overline{m}_c)^{\rm 2loop} \text{ rather than } \overline{m}_c(\overline{m}_c)^{\rm 1loop} \text{ in } P(E_0). \end{cases}$$

Contributions to the total uncertainty:

5% non-perturbative, mainly $\mathcal{O}\left(\alpha_s \frac{\Lambda}{m_b}\right)$ → Improved measurements of Δ_{0-} should help.

3% parametric $\left(\alpha_s(M_Z),\ \mathcal{B}^{\rm exp}_{\rm semileptonic},\ m_c\ \&\ C,\ \ldots\right)$

- $\alpha_s(M_Z)$: 2.0%
- $\mathcal{B}^{\rm exp}_{\rm semileptonic}$: 1.6%
- m_c & C: 1.1% (1S), 2.5% (kin)

3% m_c-interpolation ambiguity → The calculation of G_{17} and G_{27} for $m_c = 0$ should help a lot.

3% higher order $\mathcal{O}(\alpha_s^3)$ → This uncertainty will stay with us.

Currently known contributions to $\mathcal{B}(\bar{B} \to X_s\gamma)$ that have not been included in the estimate $(3.15 \pm 0.23) \times 10^{-4}$ in hep-ph/0609232:

$(\pm 7.3\%)$

- **New/old large-β_0 bremsstrahlung effects**
  [Ligeti, Luke, Manohar, Wise, 1999]
  [Ferroglia, Haisch, 2007, to be published] $\Rightarrow$ $+2.0\%$ **in the BR**

- **Four-loop mixing into the $b \to sg$ operator Q_8**
  [Czakon, Haisch, MM, hep-ph/0612329] $\Rightarrow$ -0.3% **in the BR**

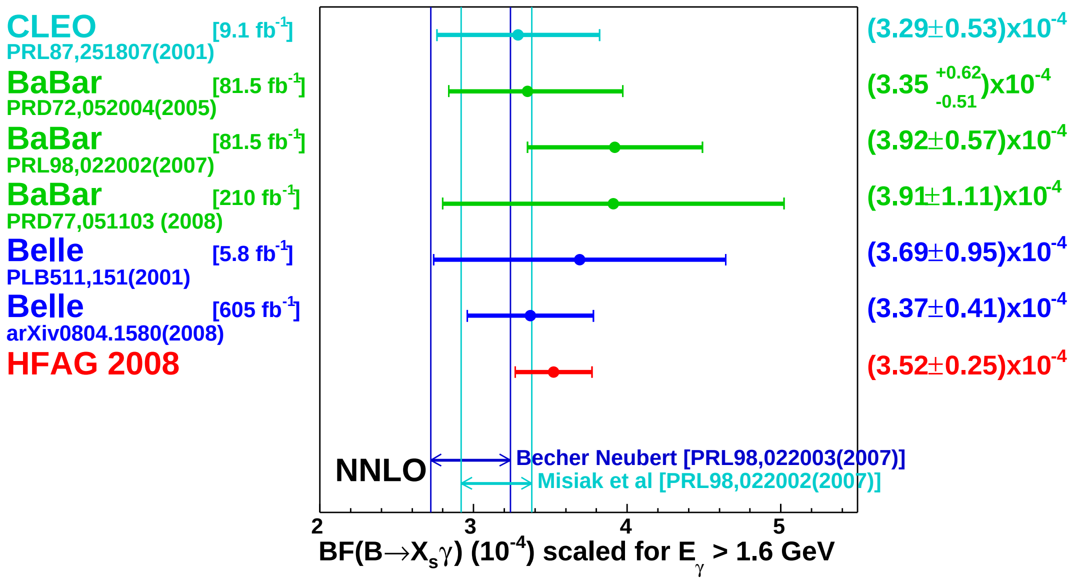
- **Effects of m_c and m_b in loops on gluon lines**
  [Asatrian, Ewerth, Gabrielyan, Greub, hep-ph/0611123]
  [Boughezal, Czakon, Schutzmeier, arXiv:0707.3090] $\Rightarrow$ $+1.6\%$ **in the BR**
  [Pak, Czarnecki, arXiv:0803.0960]
  [Ewerth, arXiv:0805.3911]

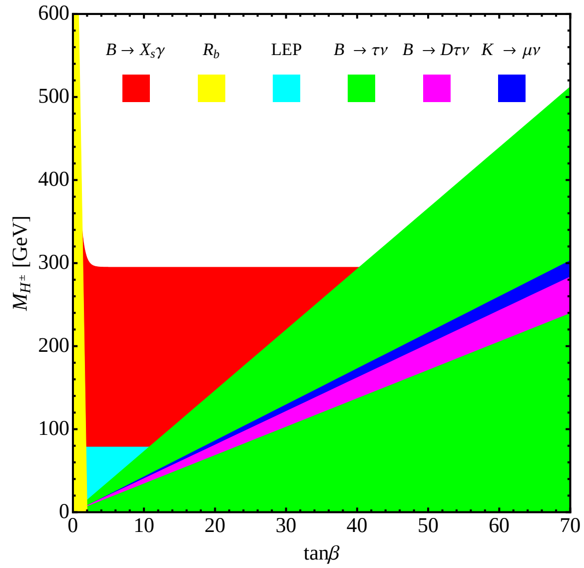
- **Non-perturbative $\mathcal{O}\left(\alpha_s \frac{\Lambda}{m_b}\right)$ effects in the term $\sim C_7 C_8$**
  [Lee, Neubert, Paz, hep-ph/0609224] $\Rightarrow$ -1.5% **in the BR**

- **Non-perturbative collinear effects**
  [Kapustin, Ligeti, Politzer, hep-ph/9507248] $\Rightarrow$ -0.2% **in the BR**

Total: $+1.6\%$ **in the BR**

Comparison with the measurements
(Slide from the talk of M. Nakao (KEK) at Moriond 2009)





HFAG average: $\mathcal{B}(B \to X_s\gamma)_{E_\gamma>1.6\ \mathrm{GeV}} = (3.52 \pm 0.25) \times 10^{-4}$

(scaling down to 1.6 GeV may be controvertial — motivation to lower E_γ)

- Agreement with latest NNLO calculation
- Strong constraints on generic 2HDM charged Higgs
  (MSSM charged Higgs case is more complicated due to possible destructive interference)
- Also strong constraints on various new physics scenarios
  (but bigger room than before as data $\mathcal{B}$ is now higher than SM)

Estimate of $\mathcal{B}(\bar{B} \to X_s\gamma)$ at $O(\alpha_s^2)$

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(Received 24 September 2006; published 12 January 2007)

Combining our results for various $O(\alpha_s^2)$ corrections to the weak radiative B-meson decay, we are able to present the first estimate of the branching ratio at the next-to-next-to-leading order in QCD. We find $\mathcal{B}(\bar{B} \to X_s\gamma) = (3.15 \pm 0.23) \times 10^{-4}$ for $E_\gamma > 1.6$ GeV in the $\bar{B}$-meson rest frame. The four types of uncertainties: nonperturbative (5%), parametric (3%), higher-order (3%), and m_c-interpolation ambiguity (3%) have been added in quadrature to obtain the total error.

DOI: 10.1103/PhysRevLett.98.022002 PACS numbers: 12.38.Bx, 13.20.He

The inclusive radiative B-meson decay provides important constraints on the minimal supersymmetric standard model and many other theories of new physics at the electroweak scale. The power of such constraints depends on the accuracy of both the experiments and the standard model (SM) calculations. The latest measurements by Belle and *BABAR* are reported in Refs. [1,2]. The world average performed by the Heavy Flavor Averaging Group [3] for $E_\gamma > 1.6$ GeV reads

$$\mathcal{B}(\bar{B} \to X_s\gamma) = (3.55 \pm 0.24^{+0.09}_{-0.10} \pm 0.03) \times 10^{-4}. \quad (1)$$

The combined error in the above result is of the same size as the expected $O(\alpha_s^2)$ next-to-next-to-leading order (NNLO) QCD corrections to the perturbative decay width $\Gamma(b \to X_s^{\rm parton}\gamma)$, and larger than the known nonperturbative corrections to the relation $\Gamma(\bar{B} \to X_s\gamma) \simeq \Gamma(b \to X_s^{\rm parton}\gamma)$ [4–6]. Thus, calculating the SM prediction for the b-quark decay rate at the NNLO is necessary for taking full advantage of the measurements.

Evaluating the $O(\alpha_s^2)$ corrections to $\mathcal{B}(b \to X_s^{\rm parton}\gamma)$ is a very involved task because hundreds of three-loop on-shell and thousands of four-loop tadpole Feynman diagrams need to be computed. In a series of papers [7–14], we have presented partial contributions to this enterprise. The purpose of the present Letter is to combine all the existing results and obtain the first estimate of the branching ratio at the NNLO. We call it an estimate rather than a prediction because some of the numerically important contributions have been found using an interpolation in the charm quark mass, which introduces uncertainties that are difficult to quantify.

Let us begin with recalling that the leading-order (LO) contribution to the considered decay originates from one-loop diagrams in the SM. An example of such a diagram is shown in Fig. 1. Dressing this diagram with one or two virtual gluons gives examples of diagrams that one encounters at the next-to-leading order (NLO) and the NNLO. In addition, one should include diagrams describing the bremsstrahlung of gluons and light quarks.

An additional difficulty in the analysis of the considered decay is the presence of large logarithms $(\alpha_s \ln M_W^2/m_b^2)^n$ that should be resummed at each order of the perturbation series in α_s. To do so, one employs a low-energy effective theory that arises after decoupling the top quark and the heavy electroweak bosons. Weak interaction vertices (operators) in this theory are either of dipole type ($\bar{s}\sigma^{\mu\nu}bF_{\mu\nu}$, $\bar{s}\sigma^{\mu\nu}T^a bG^a_{\mu\nu}$) or contain four quarks ($[\bar{s}\Gamma b][\bar{q}\Gamma' q]$).

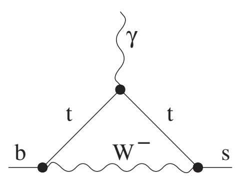

FIG. 1. Sample LO diagram for the $b \to s\gamma$ transition.

Analysis of $\mathcal{B}(\bar{B} \to X_s\gamma)$ at Next-to-Next-to-Leading Order with a Cut on Photon Energy

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(Received 9 October 2006; published 12 January 2007)

By combining a recent estimate of the total $\bar{B} \to X_s\gamma$ branching fraction at $O(\alpha_s^2)$ with a detailed analysis of the effects of a cut $E_\gamma \geq 1.6$ GeV on photon energy, a prediction for the partial $\bar{B} \to X_s\gamma$ branching fraction at next-to-next-to-leading order in renormalization-group improved perturbation theory is obtained, in which contributions from all relevant scales are factorized. The result $\mathcal{B}(\bar{B} \to X_s\gamma) = (2.98 \pm 0.26) \times 10^{-4}$ is about 1.4σ lower than the experimental world average. This opens a window for significant new physics contributions in rare radiative B decays.

DOI: 10.1103/PhysRevLett.98.022003 PACS numbers: 12.38.Bx, 12.39.St, 13.20.−v, 13.20.He

Introduction.—The inclusive decay $\bar{B} \to X_s\gamma$ is an important example of a flavor-changing neutral current process, which has been used to test the flavor sector of the standard model. Many groups have worked on improving the theoretical analysis of this process so as to keep pace with refinements in the measurements of its branching fraction. The effective weak Hamiltonian at next-to-next-to-leading order (NNLO) has been obtained by calculating multiloop matching coefficients and anomalous dimensions [1–4]. While the fermionic NNLO corrections to the $b \to s\gamma$ matrix elements have been known for some time [5], complete NNLO corrections are presently available only for the electromagnetic dipole operator [6,7]. However, an approximate result for the NNLO charm-penguin contributions has just been published [8]. Combining these ingredients, a first estimate of the $\bar{B} \to X_s\gamma$ branching ratio at NNLO has been presented in [8,9].

What about an erratum to this paper?

A complication in the analysis arises from the fact that measurements of the $\bar{B} \to X_s\gamma$ branching fraction impose stringent cuts on photon energy (defined in the B-meson rest frame), $E_\gamma > E_0$, with E_0 in the range between 1.8 to 2.0 GeV. The standard treatment is to extrapolate different measurements to a common reference point $E_0 = 1.6$ GeV using phenomenological models [10]. In that way, the experimental world average $\mathcal{B}(\bar{B} \to X_s\gamma) = (3.55 \pm 0.24^{+0.09}_{-0.10} \pm 0.03) \times 10^{-4}$ has been derived [11]. The first error combines statistical and systematic uncertainties, the second one is due to the extrapolation from high E_0 to the reference value, and the last error accounts for the subtraction of $\bar{B} \to X_d\gamma$ background. A theoretical result for the branching ratio with a cut at $E_0 = 1.6$ GeV has been derived in [8,9] using two-loop calculations of the photon-energy spectrum in fixed-order perturbation theory [12,13]. It has been argued that the extrapolation from the total to the partial branching fraction does not introduce additional theoretical uncertainties. This assertion is questionable because of the dynamical relevance of a soft scale $\Delta = m_b - 2E_0 \approx 1.4$ GeV, whose value is significantly lower than the b-quark mass.

Accounting for the photon-energy cut properly requires one to disentangle contributions associated with the hard scale $\mu_h \sim m_b$, the soft scale $\mu_0 \sim \Delta$, and an intermediate scale $\mu_i \sim \sqrt{m_b\Delta}$ set by the typical final-state hadronic invariant mass. When the cut value E_0 is chosen sufficiently low, renormalization-group (RG) improved perturbation theory can be employed to calculate the effects of the photon-energy cut using a multiscale operator product expansion [14]. In the process, logarithms of the ratio Δ/m_b are resummed to all orders. More importantly, this approach allows us to isolate the contributions associated with the lowest scale Δ, which become nonperturbative if the cut E_0 is chosen too high. We have recently performed a systematic analysis of the cut effects at NNLO. Two-loop corrections at the soft scale were calculated in [15], while those at the intermediate scale were computed in [16]. Here, the analysis is completed by extracting the two-loop hard matching corrections from a comparison with fixed-order calculations of the photon spectrum [12,13].

Using this method, we compute the fraction of all $\bar{B} \to X_s\gamma$ events with $E_\gamma \geq 1.6$ GeV with a perturbative precision of 5%. At this level of accuracy several other, nonperturbative effects need to be evaluated carefully. The event fraction receives hadronic power corrections $\sim(\Lambda_{\rm QCD}/\Delta)^n$ governed by B-meson matrix elements of local operators. The leading correction ($n = 2$) is known and turns out to be small, but terms with $n \geq 3$ are presently unknown. Recently, a new class of enhanced, nonlocal $\Lambda_{\rm QCD}/m_b$ corrections to the $\bar{B} \to X_s\gamma$ decay rate has been identified [17]. A model analysis indicates that they can affect the total decay rate at the level of a few percent.

Combining our result for the event fraction with the prediction for the total branching fraction from [8,9], we obtain

$$\mathcal{B}(\bar{B} \to X_s\gamma) = (2.98 \pm 0.26) \times 10^{-4} \quad (1)$$

for $E_0 = 1.6$ GeV, where we have added in quadrature the uncertainties from higher-order perturbative effects

Arguments on unreliability: MM, arXiv:0808.3134.

Summary

- **More work is necessary to estimate non-perturbative corrections to the total decay rate that originate from diagrams where the photon is emitted far away from the decaying b-quark.**

- **Perturbative NNLO calculations for $m_c = 0$ are extremely difficult but noticeably moving forward.**

- **An intriguing tension occurs between the 1S- and kinetic-scheme determinations of the normalization factor C.**

- **The discussion on "MSOPE" in $\mathcal{B}(\bar{B} \to X_s \gamma)$ for** $E_0 \in [1,\ 1.6]\,\mathrm{GeV}$ **has timed-out.**

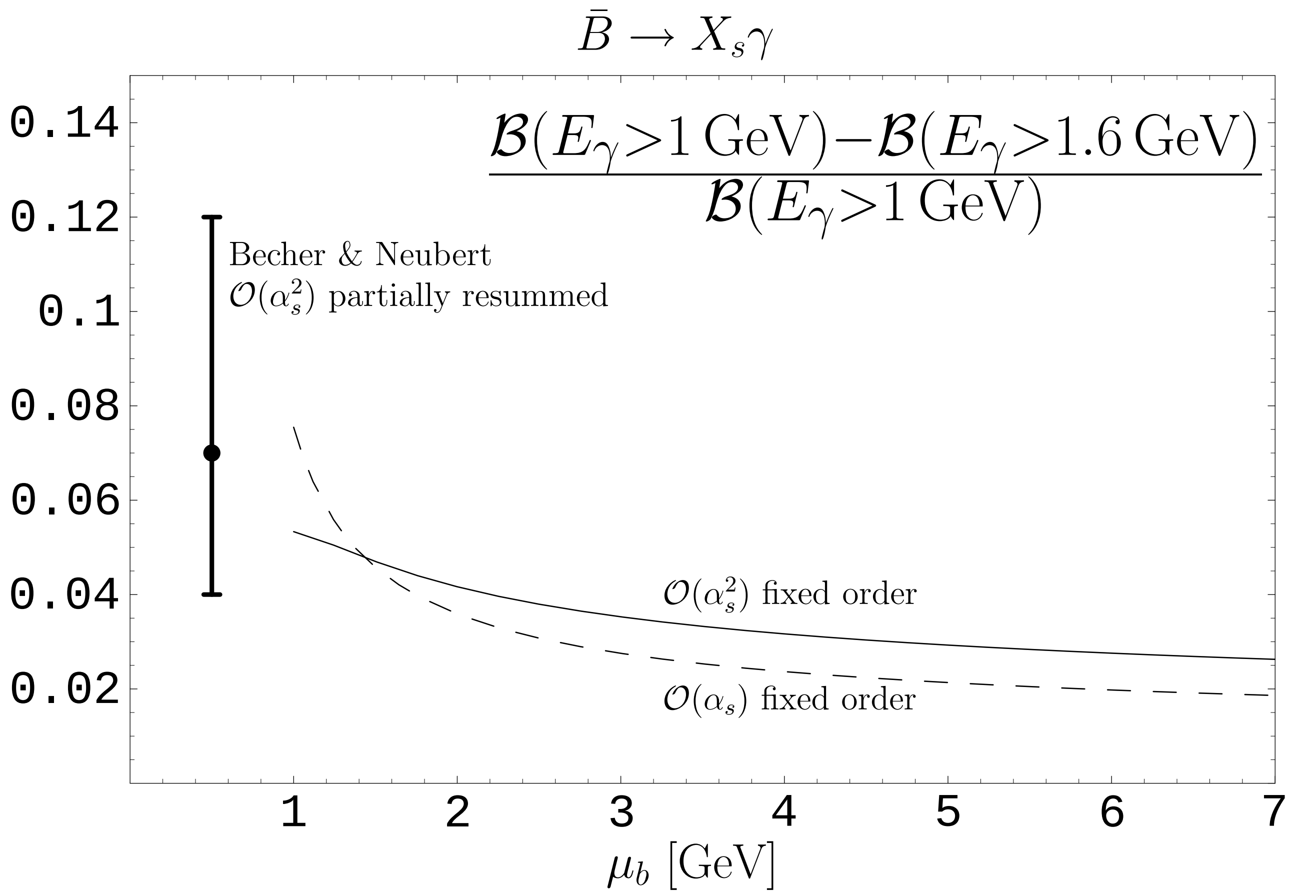
BACKUP SLIDES

Comments on the Multi-Scale OPE (MSOPE) calculation by T. Becher and M. Neubert, PRL 98 (2007) 022003 [hep-ph/0610067].

	$\mathcal{B}(E_\gamma > 1 GeV)$	$\mathcal{B}(E_\gamma > 1.6 GeV)$
hep-ph/0609232 ("fixed order")	3.27×10^{-4}	3.15×10^{-4}
hep-ph/0610067 ("MSOPE")	3.27×10^{-4} **(adopted from above)**	3.05×10^{-4}

before adding the -1.5% of $\mathcal{O}(\alpha_s \Lambda / m_b)$.

There is almost a factor-of-two difference in:



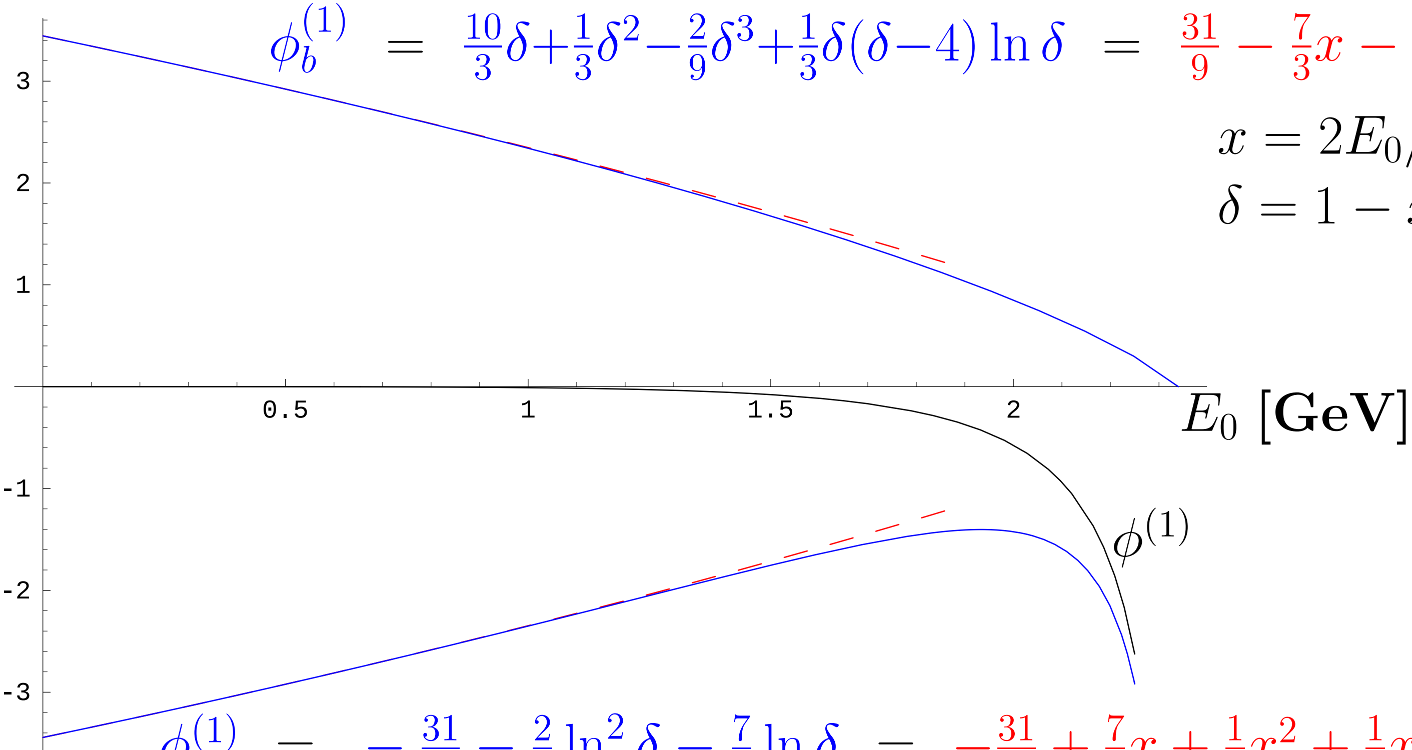
For simplicity, let us set $C_i(\mu_b) \to 0$ for $i \neq 7$. Then, in the “fixed order”:

$$\mathcal{B}(E_\gamma > E_0)/\mathcal{B}_{\rm total} = 1 + \frac{\alpha_s(\mu_b)}{\pi}\phi^{(1)}(E_0) + \left(\frac{\alpha_s(\mu_b)}{\pi}\right)^2 \phi^{(2)}(E_0) + \ldots$$

$$\phi^{(1)}(E_0) = \phi_a^{(1)}(E_0) + \phi_b^{(1)}(E_0)$$

$$\phi_b^{(1)} = \frac{10}{3}\delta + \frac{1}{3}\delta^2 - \frac{2}{9}\delta^3 + \frac{1}{3}\delta(\delta-4)\ln\delta = \frac{31}{9} - \frac{7}{3}x - \frac{1}{2}x^2 - \frac{1}{9}x^3 - \frac{5}{36}x^4 + \mathcal{O}(x^5)$$

$$x = 2E_0/m_b$$
$$\delta = 1 - x$$

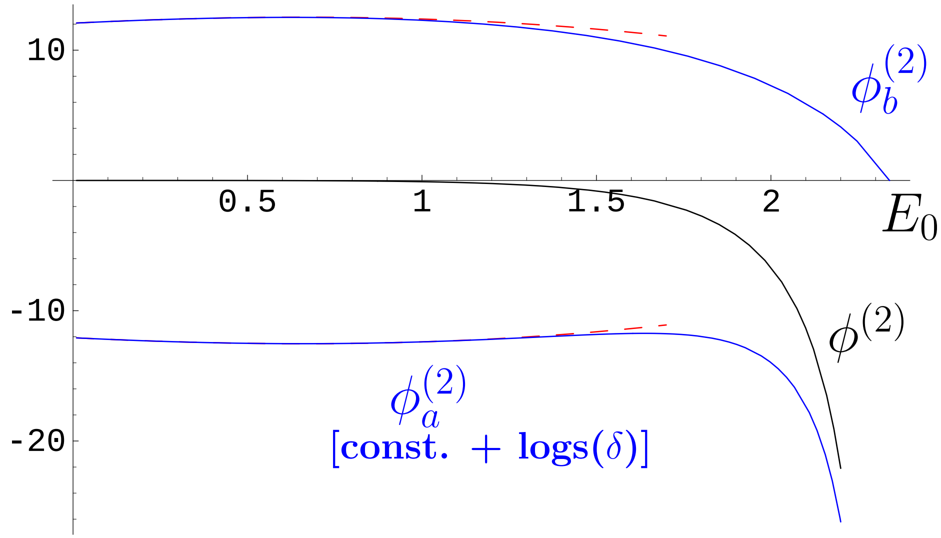


$$\phi_a^{(1)} = -\frac{31}{9} - \frac{2}{3}\ln^2\delta - \frac{7}{3}\ln\delta = -\frac{31}{9} + \frac{7}{3}x + \frac{1}{2}x^2 + \frac{1}{9}x^3 - \frac{1}{36}x^4 + \mathcal{O}(x^5)$$

Terms up to $\mathcal{O}(x^3)$ must cancel out in $\phi_a^{(1)} + \phi_b^{(1)}$. In the current MSOPE results, the higher-order corrections to $\phi_a^{(1)}$ are resummed, but $\phi_b^{(1)}$ is retained in the “fixed order”.

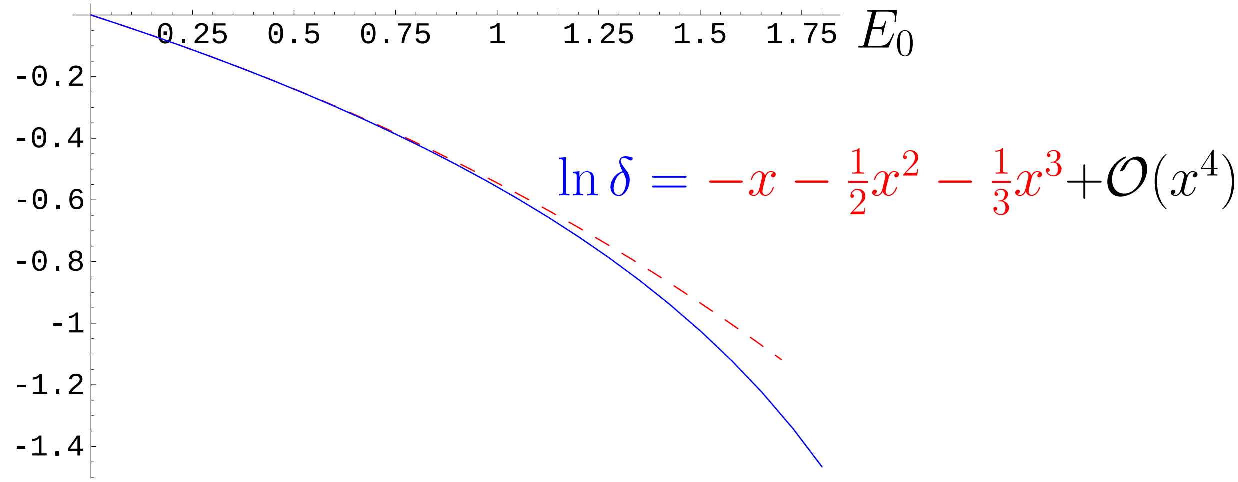
$\Rightarrow$ These results are unreliable for 1 GeV $< E_0 <$ 1.6 GeV.

The same pattern arises at $\mathcal{O}(\alpha_s^2)$:



$x = 2E_0/m_b$
$\delta = 1 - x$

It must be the case also at higher orders because:



However, only "const + logs(δ)" have been included at orders $\mathcal{O}(\alpha_s^3)$ and higher in hep-ph/0610067.

Interpolation in m_c

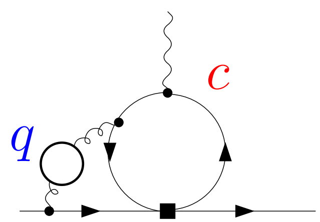
$$\mathcal{B}(\bar{B} \to X_s \gamma)_{E_\gamma > E_0} = \underbrace{X}_{\text{normalization}} \left[\underbrace{P(E_0)}_{\text{perturbative}} + \underbrace{N(E_0)}_{\text{non-perturbative}} \right]$$

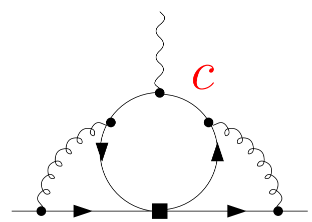
Expansion of $P(E_0)$:

$$P = \underbrace{P^{(0)} + \frac{\alpha_s(\mu_b)}{4\pi}\left(P_1^{(1)} + P_2^{(1)}(r)\right) + \left(\frac{\alpha_s(\mu_b)}{4\pi}\right)^2 \left(P_1^{(2)}}_{\textbf{known}} + P_2^{(2)}(r) + \underbrace{P_3^{(2)}(r)}_{\textbf{known}}\right)$$

$$P_1^{(1)}, P_3^{(2)} \sim C_i^{(0)} C_j^{(1)}, \qquad P_2^{(1)}, P_2^{(2)} \sim C_i^{(0)} C_j^{(0)}, \qquad P_1^{(2)} \sim \left(C_i^{(0)} C_j^{(2)},\ C_i^{(1)} C_j^{(1)}\right)$$

Moreover: $P_2^{(2)} = A\, n_f + B = -\frac{3}{2}(11 - 2/3 n_f) A + \frac{33}{2} A + B = P_2^{(2)\beta_0} + P_2^{(2)\text{rem}}$



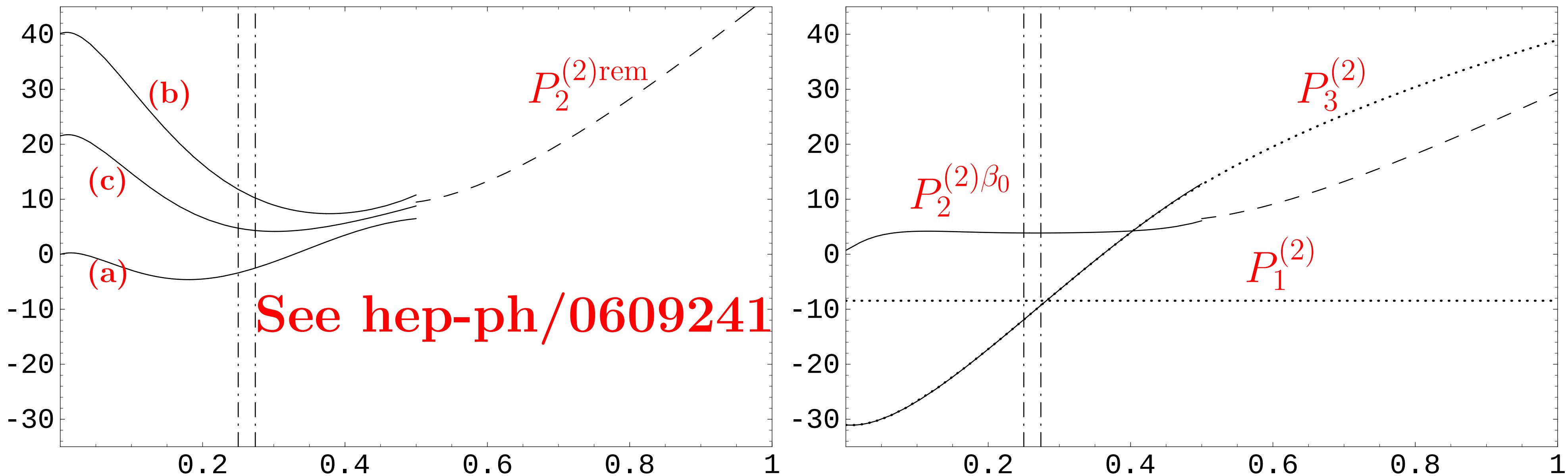


$$r = \frac{m_c(m_c)}{m_b^{1S}}$$

$P_2^{(2)\beta_0}$ **known for all r**

The complete $P_2^{(2)}$ has been calculated only for $r \gg \frac{1}{2}$.

The NNLO corrections $P_k^{(2)}$ as functions of $r = m_c(m_c)/m_b^{1S}$



Dotted: exact, Solid: small-r expansions, Dashed: leading large-r asymptotics.

Interpolation:

$$P_2^{(2)\text{rem}}(r) = x_1 + x_2\, P_2^{(1)}(r) + x_3\, r\, \frac{d}{dr} P_2^{(1)}(r) + x_4\, P_2^{(2)\beta_0}(r) + x_5 |A_{\text{NLO}}(r)|^2$$

The coefficients x_k are determined from the asymptotic behaviour at large r

and from the requirement that either **(a)** $P_2^{(2)\text{rem}}(0) = 0$,

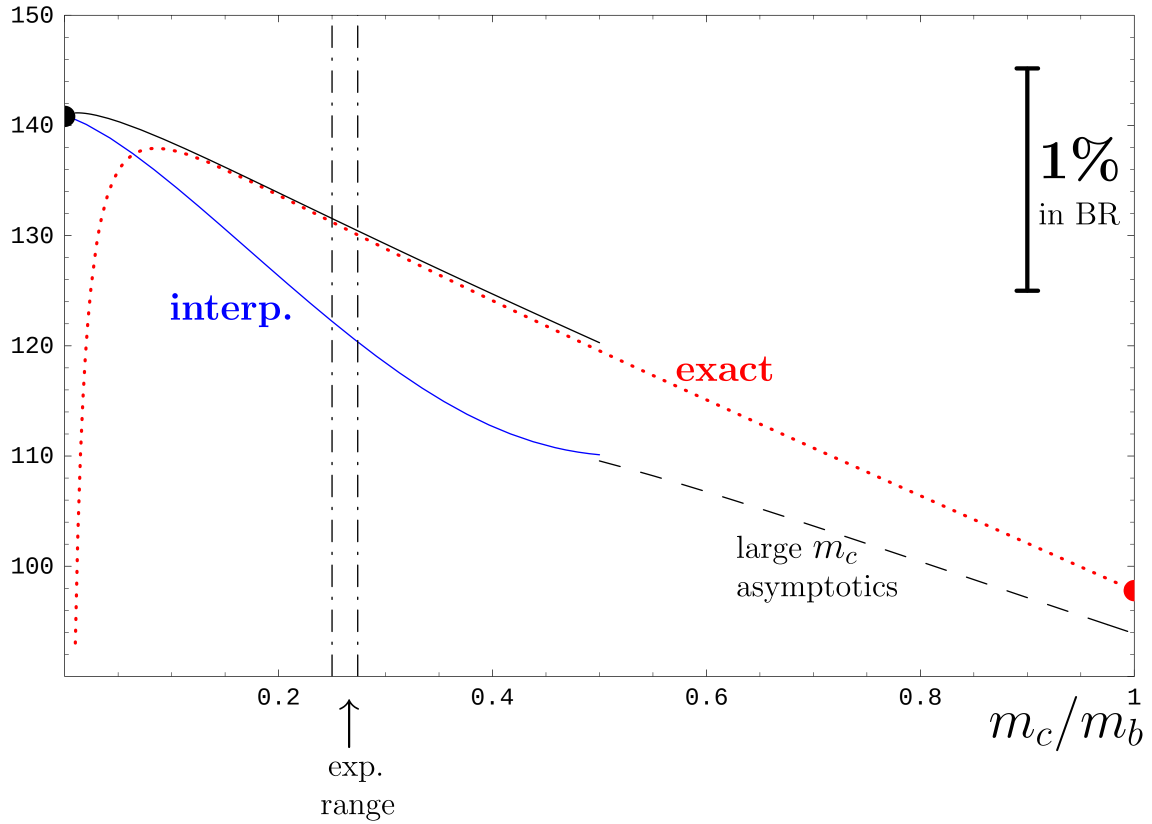
or **(b)** $P_1^{(2)} + P_2^{(2)\text{rem}}(0) + P_3^{(2)}(0) = 0$,

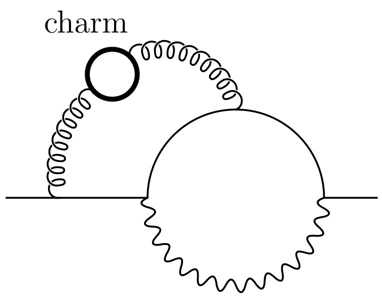
or **(c)** $P_2^{(2)\text{rem}}(0) = \left[P_2^{(2)\text{rem}}(0)\right]_{77}$.

The average of (a) and (b) is chosen to determine the central value of the NNLO branching ratio. The difference between these two cases is used to estimate the interpolation ambiguity.

The m_c-dependence of $P_2^{(2)\mathrm{rem}} = C_i^{(0)}(\mu_b) C_j^{(0)}(\mu_b) K_{ij}^{(2)\mathrm{rem}}(\mu_b, E_0)$.

Example: $K_{77}^{(2)\mathrm{rem}}(2.5\ \mathrm{GeV}, 1.6\ \mathrm{GeV})$ **as a function of** m_c/m_b:





Value at $m_c = 0$**:** **Blokland et al., hep-ph/0506055** ($c\bar{c}$ production included).

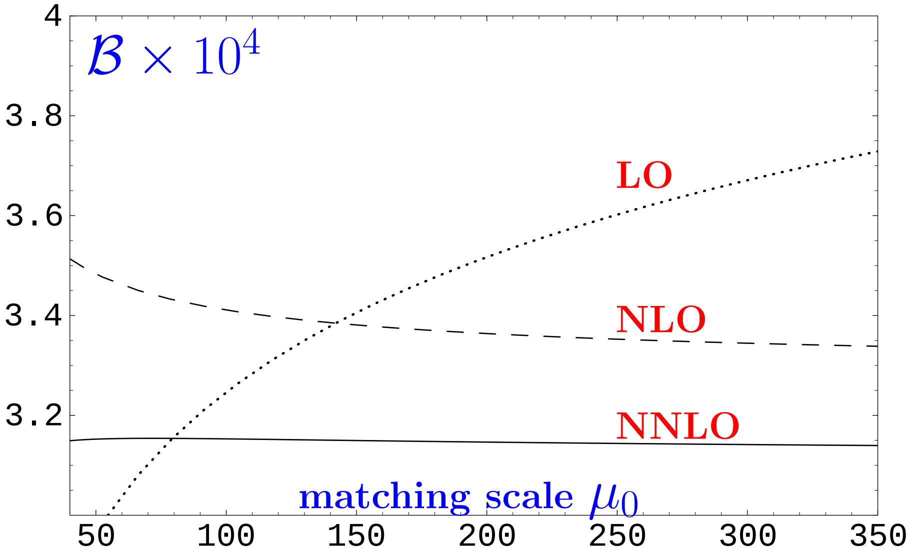
Large-m_c **asymptotics:** **Steinhauser, MM, hep-ph/0609241.**

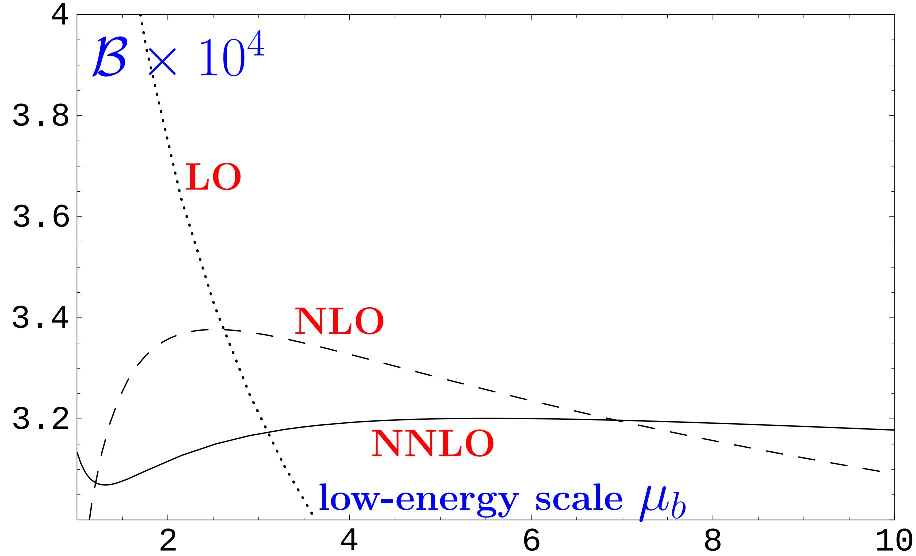
Interpolation: " " " ($c\bar{c}$ production included).

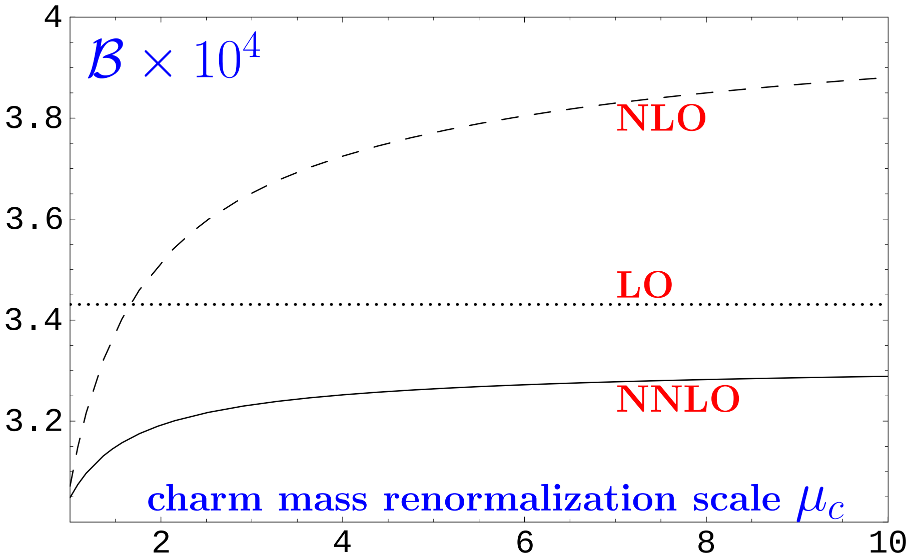
Exact $b \to X_s\gamma$**:** **Asatrian et al, hep-ph/0611123** ($c\bar{c}$ production excluded).

Exact $b \to X_u e\bar{\nu}$**:** **Pak, Czarnecki, arXiv:0803.0960** ($c\bar{c}$ production included).

Renormalization scale dependence of $\mathcal{B}(\bar{B} \to X_s\gamma)_{E_\gamma > 1.6\ \mathrm{GeV}}$







"Central" values:

$\mu_0 = 160$ **GeV**

$\mu_b = 2.5$ **GeV**

$\mu_c = 1.5$ **GeV**