Combining NLO Calculations with Parton Showers

Ringberg Workshop on New Physics, Flavor and Jets

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Measurements @ LHC

Goal of LHC is to determine the mechanism of EW symmetry breaking

Main question, is the SM sufficient, or do we need physics beyond the standard model (BSM)

By definition BSM is difference between true distributions in nature and SM predictions

$$\sigma_{BSM} = \sigma_{true} - \sigma_{SM}$$

Measurements @ LHC

Problem:

Measured distributions are convolutions of true distributions with detector effects

$$\sigma_{meas} = d\sigma_{true} \otimes \text{detector}$$

For a meaningful comparison between σ_{meas} and SM predictions, need to be able to calculate

$$\sigma_{pred} = d\sigma_{SM} \otimes \text{detector}$$

Why parton showers?

Detector effects depend on details of the fully hadronic events (π^+ vs π^0, details of jets)

Need $d\sigma_{SM}$ including full hadronization effects

Only known way to generate exclusive distributions is using parton shower Monte Carlos (Pythia/Herwig)

$$d\sigma_{SM} = (\text{Pert}) \otimes \text{Pythia/Herwig}$$

In order to use LHC data...

For the perturbative part, NLO calculations are the state of the art and should be viewed as mandatory

- Several processes only available at NLO
- Scale dependence only under control starting at NLO
- NLO calculations required to get to O(10%) uncertainty

Combine NLO calculations with parton showers

Outline

- Jet Observables and Monte Carlo
- The Parton Shower Algorithm
- Generics of combining with fixed order calculations
  - LO Accuracy
  - NLO Accuracy
- Some details of our calculation
- Conclusions

Jet observables and Monte Carlo

Jet cross sections

Defined with help of jet algorithm

k particles in detector $\xrightarrow[J(\Phi_k,\Phi_n)]{\text{Algorithm}}$ n jets observed

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If jet algorithm is infrared safe, can calculate perturbatively

$$\frac{d\sigma_n^{\rm jet}}{d\Phi_n} = \sum_{i\geq n} \int d\Phi_i' \, \frac{d\sigma_i^{\rm parton}}{d\Phi_i'} \, J(\Phi_i', \Phi_n)$$

Jet cross sections

Problem 1:

- Each term in sum separately divergent (cancels in sum)
- In general can only do this calculation numerically by integrating over each term in sum separately

How do we deal with the IR divergences numerically?

$$\frac{\mathrm{d}\sigma_n^{\mathrm{jet}}}{\mathrm{d}\Phi_n} = \sum_{i\geq n} \int \mathrm{d}\Phi_i' \, \frac{\mathrm{d}\sigma_i^{\mathrm{parton}}}{\mathrm{d}\Phi_i'} \, J(\Phi_i', \Phi_n)$$

Jet cross sections

Problem 2:

- Partonic calculations calculated in fixed order PT
- Presence of large ratios in phase space variables gives large logarithmic terms that destroy convergence of PT

How do we sum large logs for all i?

$$\frac{\mathrm{d}\sigma_n^{\mathrm{jet}}}{\mathrm{d}\Phi_n} = \sum_{i\geq n} \int \mathrm{d}\Phi_i' \, \frac{\mathrm{d}\sigma_i^{\mathrm{parton}}}{\mathrm{d}\Phi_i'} \, J(\Phi_i', \Phi_n)$$

Jet cross sections

Problem 3:

- Partonic calculations can only be obtained for small i
- The jet algorithm depends in general on phase space cuts and efficiencies which requires fully exclusive events

How to get expression for large i?

$$\frac{\mathrm{d}\sigma_n^{\mathrm{jet}}}{\mathrm{d}\Phi_n} = \sum_{i\geq n} \int \mathrm{d}\Phi_i' \, \frac{\mathrm{d}\sigma_i^{\mathrm{parton}}}{\mathrm{d}\Phi_i'} \, J(\Phi_i', \Phi_n)$$

Jet cross sections

Problem 4:

- Partonic calculations only give partons in final state
- Efficiencies and experimental cuts can depend on the type of hadronic final state, as well as other NP effects

How to get fully hadronized events?

$$\frac{d\sigma_n^{\text{jet}}}{d\Phi_n} = \sum_{i \geq n} \int d\Phi_i' \, \frac{d\sigma_i^{\text{parton}}}{d\Phi_i'} \, J(\Phi_i', \Phi_n)$$

Summary of the 4 problems

1. How do we implement KLN cancellation numerically?

2. How do we get expressions that resum leading logarithms?

3. How do we get expressions for large number of particles?

4. How do we get fully hadronized events?

Solution to the problems

Define "Monte Carlo cross sections"

$$\frac{d\sigma_n^{\mathrm{MC}}}{d\Phi_n} = \sum_{i \geq n} \int d\Phi_i' \, \frac{d\sigma_i^{\mathrm{parton}}}{d\Phi_i'} \, J_{\mathrm{MC}}(\Phi_i', \Phi_n)$$

And define a jet cross section calculated from these

$$\frac{d\sigma_n^{\mathrm{jet,MC}}}{d\Phi_n} = \sum_{i \geq n} \int d\Phi_i' \, \frac{d\sigma_i^{\mathrm{MC}}}{d\Phi_i'} \, J_{\overline{\mathrm{MC}}}(\Phi_i', \Phi_n)$$

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Need to define $d\sigma_i^{MC}$ such that gives measured jet cross section and solves all 4 problems

Deal with Problem 1

Divide phase space in singular and non-singular regions

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$$J(\Phi'_i,\Phi_n) = J(\Phi'_i,\Phi_n)\Theta(\Phi'_i\text{=sing}) + J(\Phi_i,\Phi_n)\Theta(\Phi'_i\text{=non-sing})$$

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$J_{MC}(\Phi'_i,\Phi_n)$

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Deal with Problem 1

Divide phase space in singular and non-singular regions

$J(\Phi'_i,\Phi_n) = \underbrace{J(\Phi'_i,\Phi_n)\Theta(\Phi'_i=\text{sing})}_{J_{MC}(\Phi'_i,\Phi_n)} + \underbrace{J(\Phi_i,\Phi_n)\Theta(\Phi'_i=\text{non-sing})}_{J_{\overline{MC}}(\Phi'_i,\Phi_n)}$

Define "Monte Carlo cross sections"

$$\frac{\mathrm{d}\sigma_n^{\mathrm{MC}}}{\mathrm{d}\Phi_n} = \sum_{i\geq n}\int \mathrm{d}\Phi'_i \, \frac{\mathrm{d}\sigma_i^{\mathrm{parton}}}{\mathrm{d}\Phi'_i} \, J_{\mathrm{MC}}(\Phi'_i,\Phi_n)$$

Since integrate over singular phase space, KLN cancellation guaranteed

Deal with Problem 2

Calculate leading logarithms to $d\sigma_n^{MC}$ to all orders in perturbation theory

Main idea is to use ideas of Sudakov factors and no-branching probabilities to construct $d\sigma_n^{MC}$

Straightforward task to obtain LL resummed result, and combination of NLO and LL can be obtained my matching

Deal with Problems 3-4

Parton shower algorithms generate phase space recursively ($\Phi_2 \rightarrow \Phi_3 \rightarrow \Phi_4 \rightarrow \ldots$)

- Each step in recursion simple $\Rightarrow$ generate arbitrarily complicated final states
- Simple known ways to implement with models of hadronization
- Gets the collinear and soft limit correct
- Does not change total cross sections

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- Simple known ways to implement with models of hadronization
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If $d\sigma_n^{MC}$ is merged with parton shower, solve all 4 Problems

Combining fixed order calculations with Parton showers

Pictoral phase space

$d\Phi_n$

$d\Phi_{n+1}$

$d\Phi_{n+2}$

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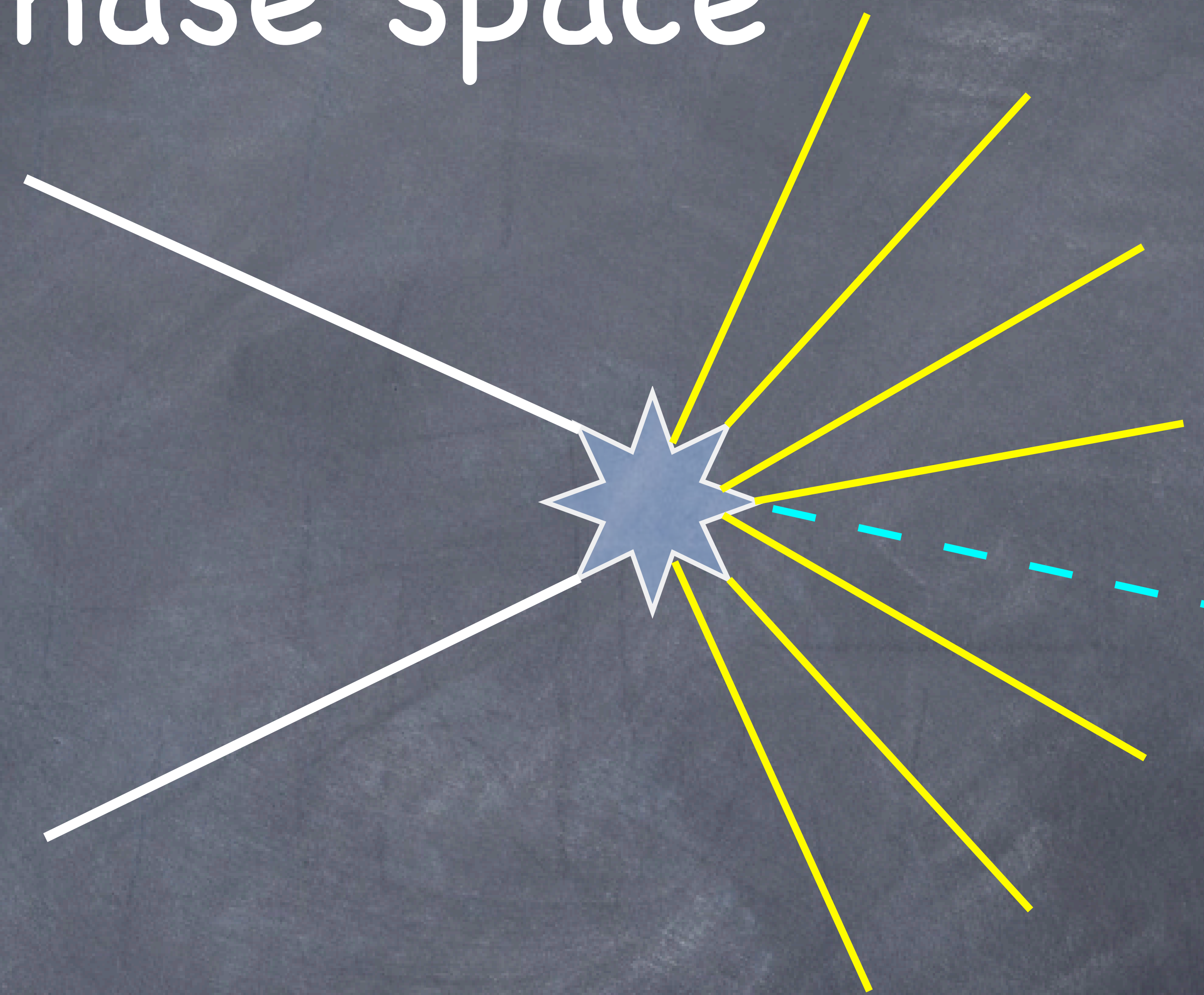
$d\Phi_n$

$d\Phi_{n+1}$

$d\Phi_{n+2}$



Pictoral phase space

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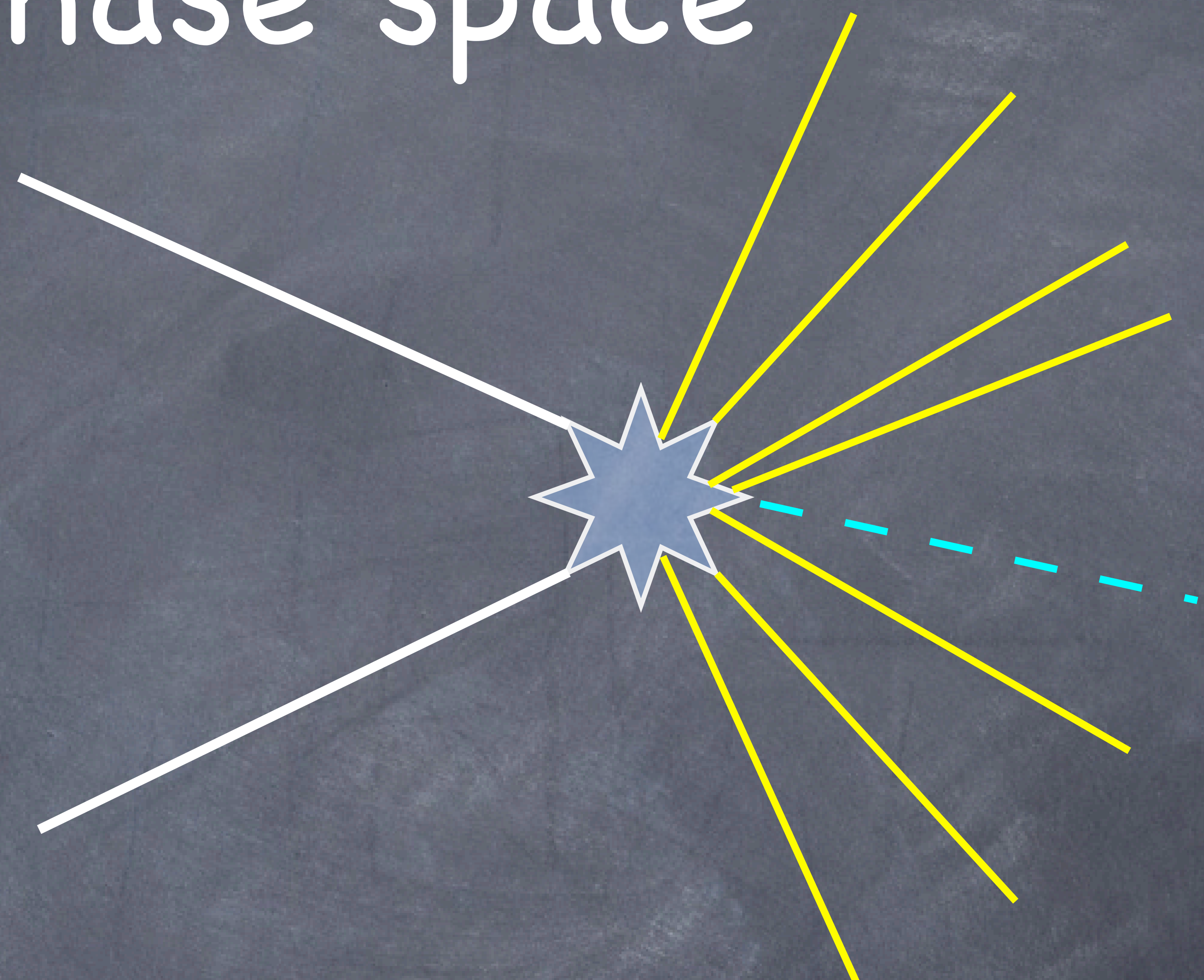

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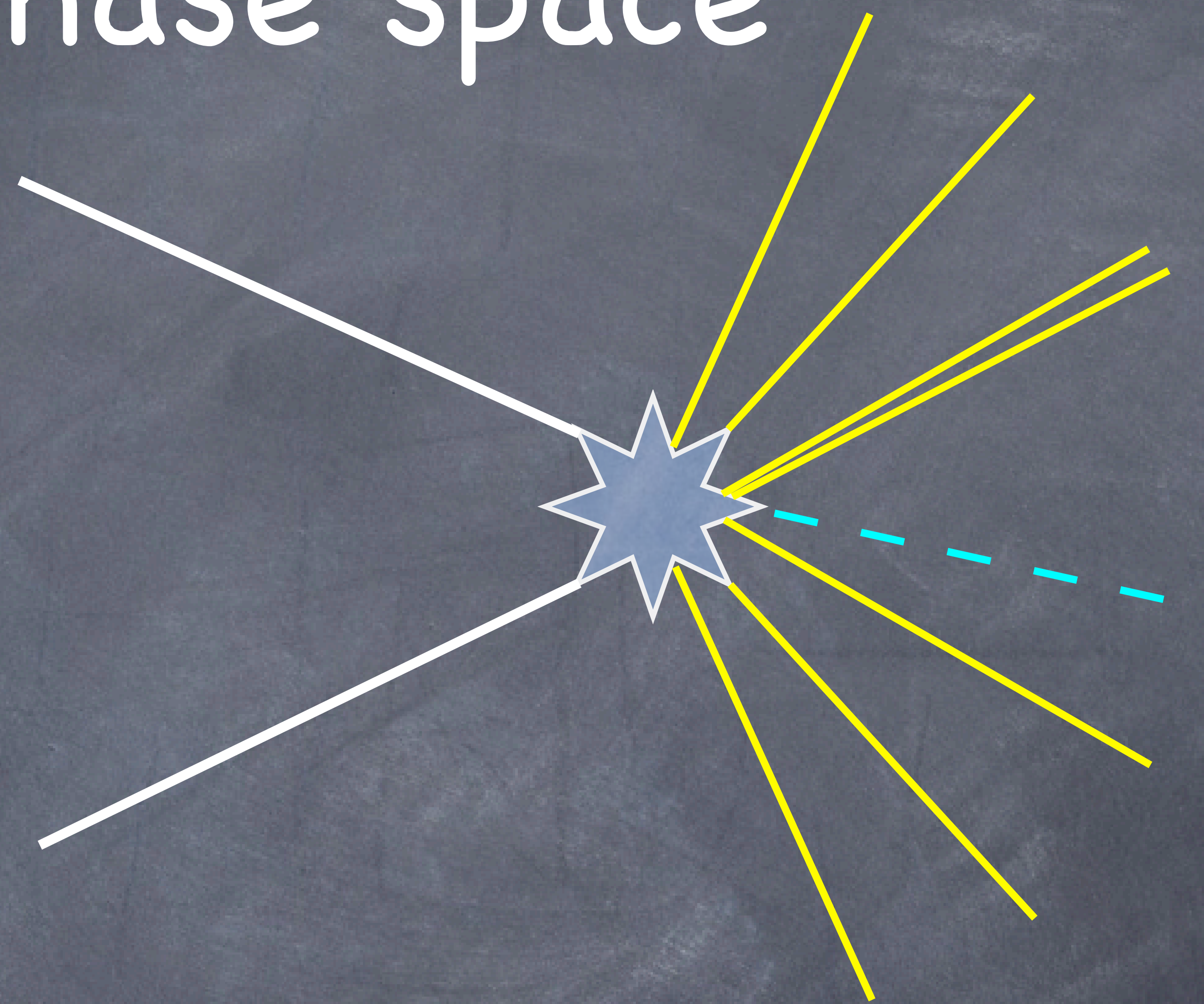

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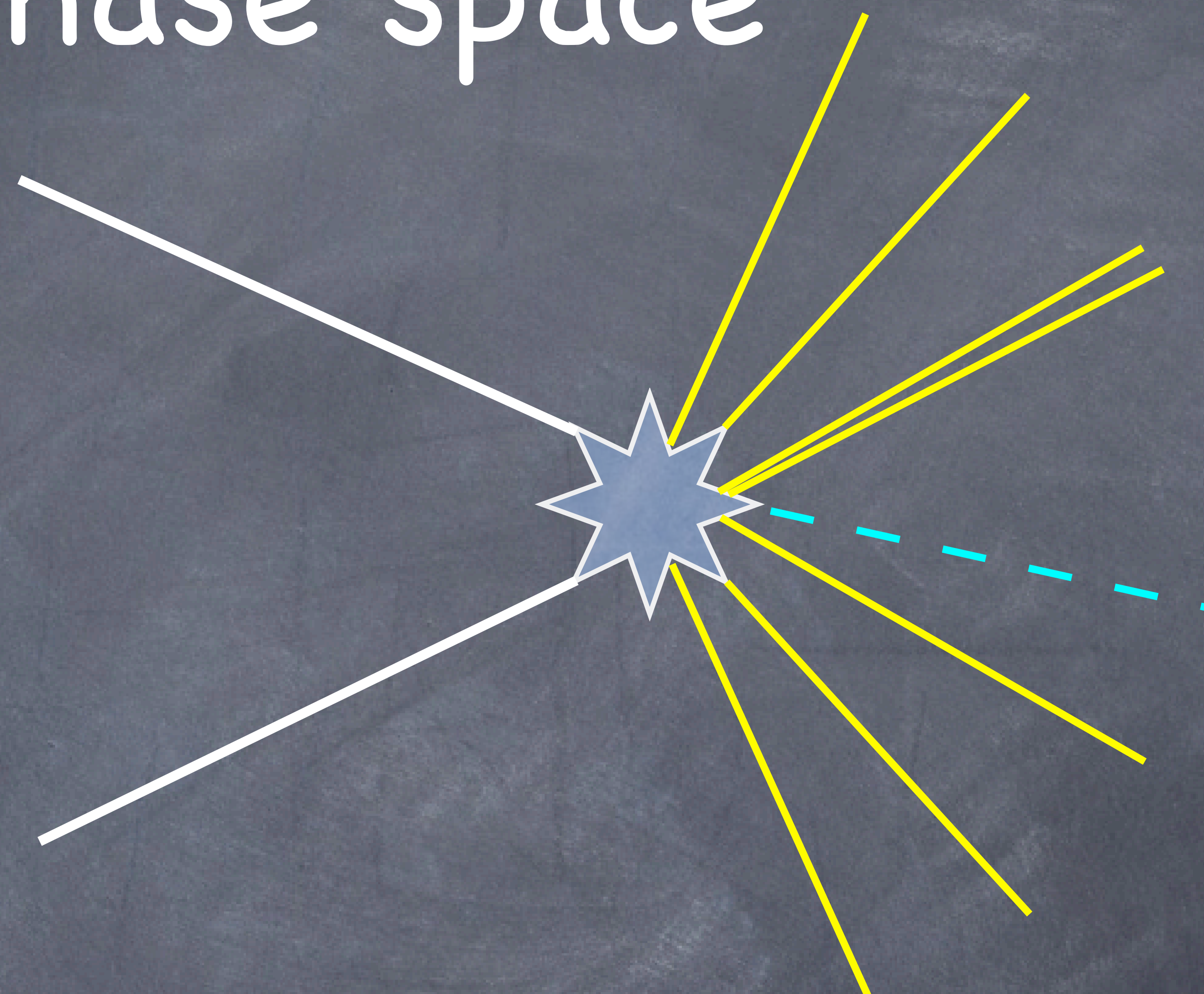

Region of Φ_n looks like Φ_{n-1}

Pictoral phase space

$d\Phi_n$

$d\Phi_{n+1}$

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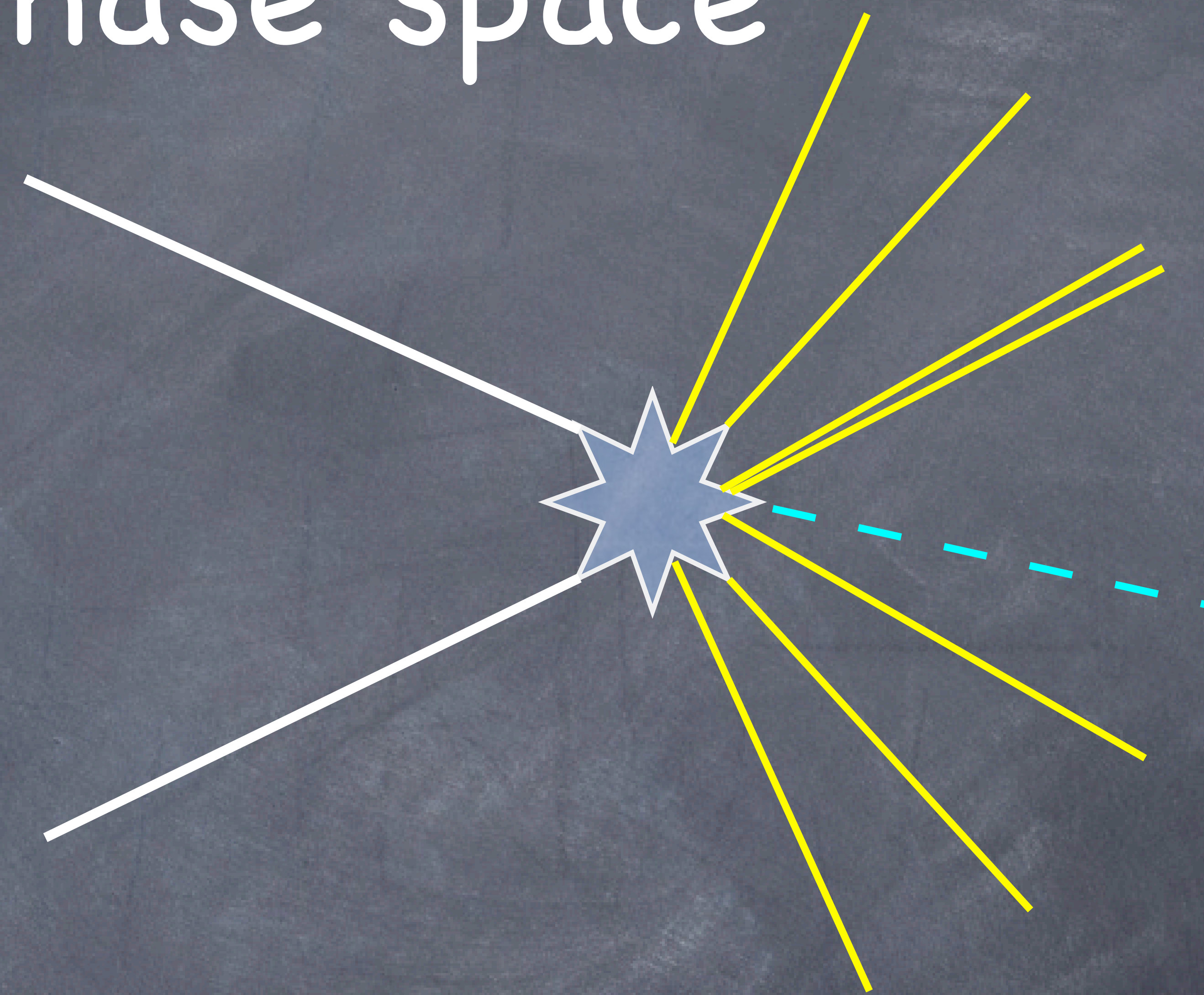
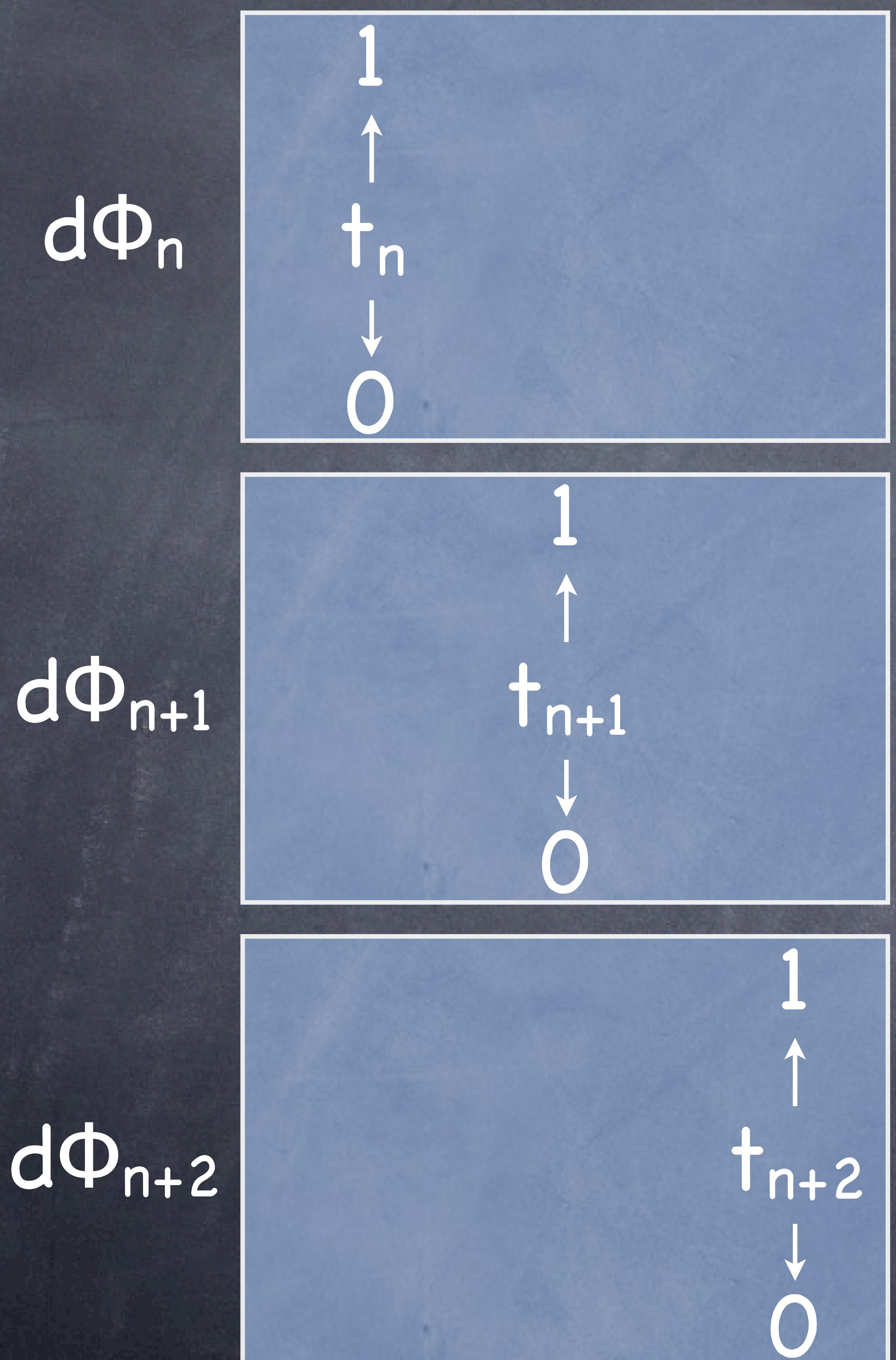


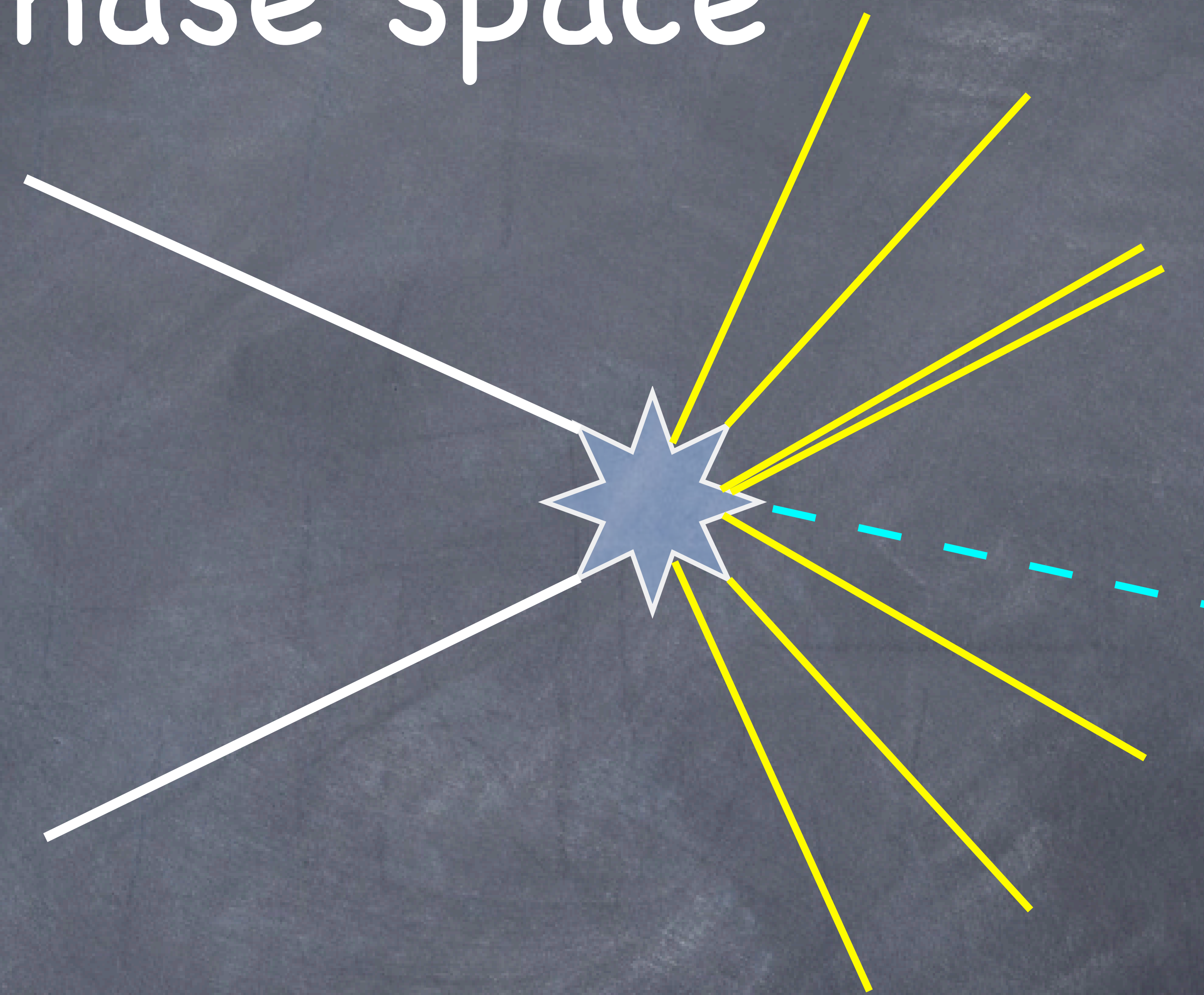
Region of Φ_n looks like Φ_{n-1}

Define resolution variable t_n

($t_n \to 0$ in collinear region)

Pictoral phase space





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The parton shower

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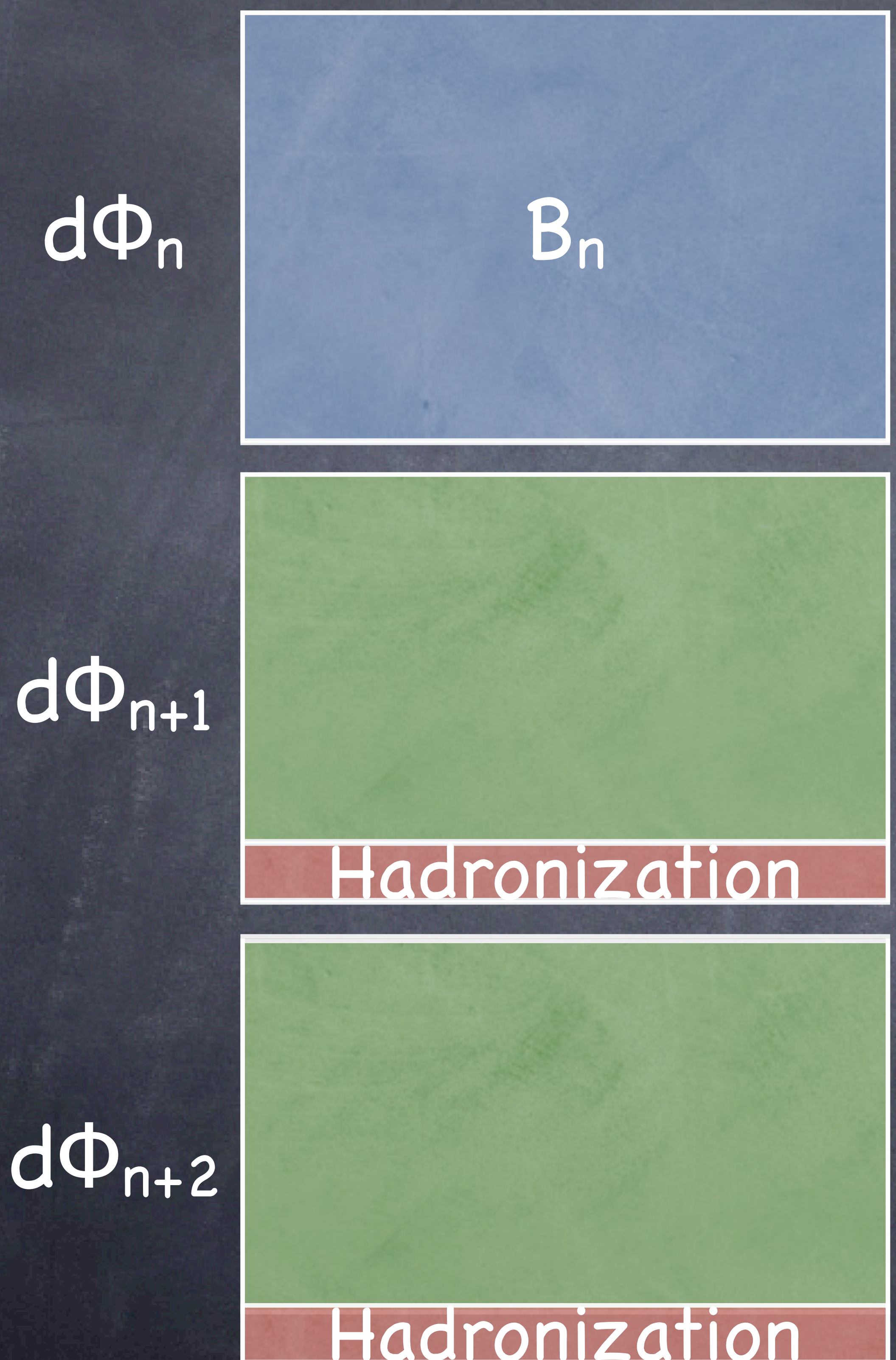
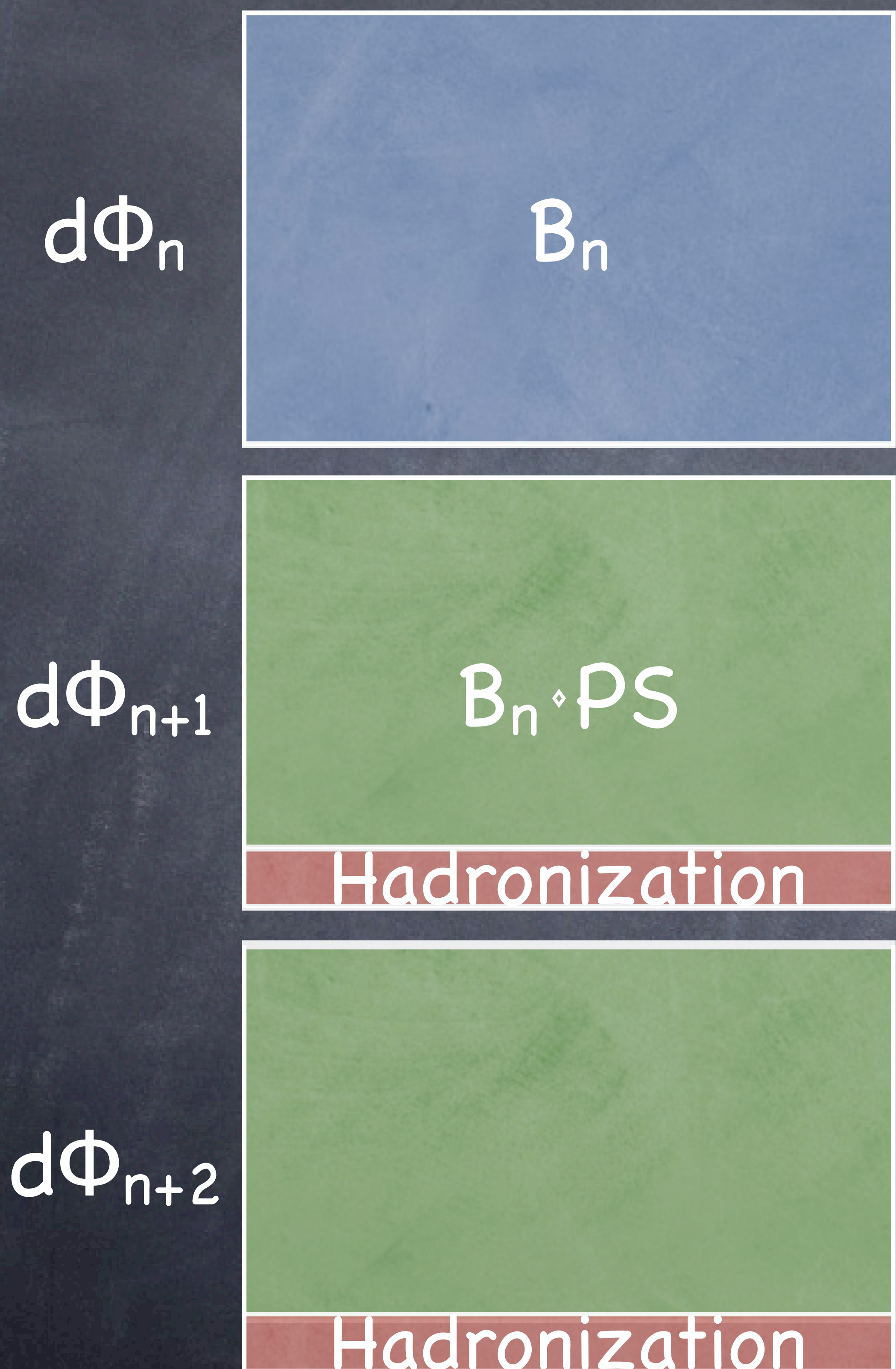


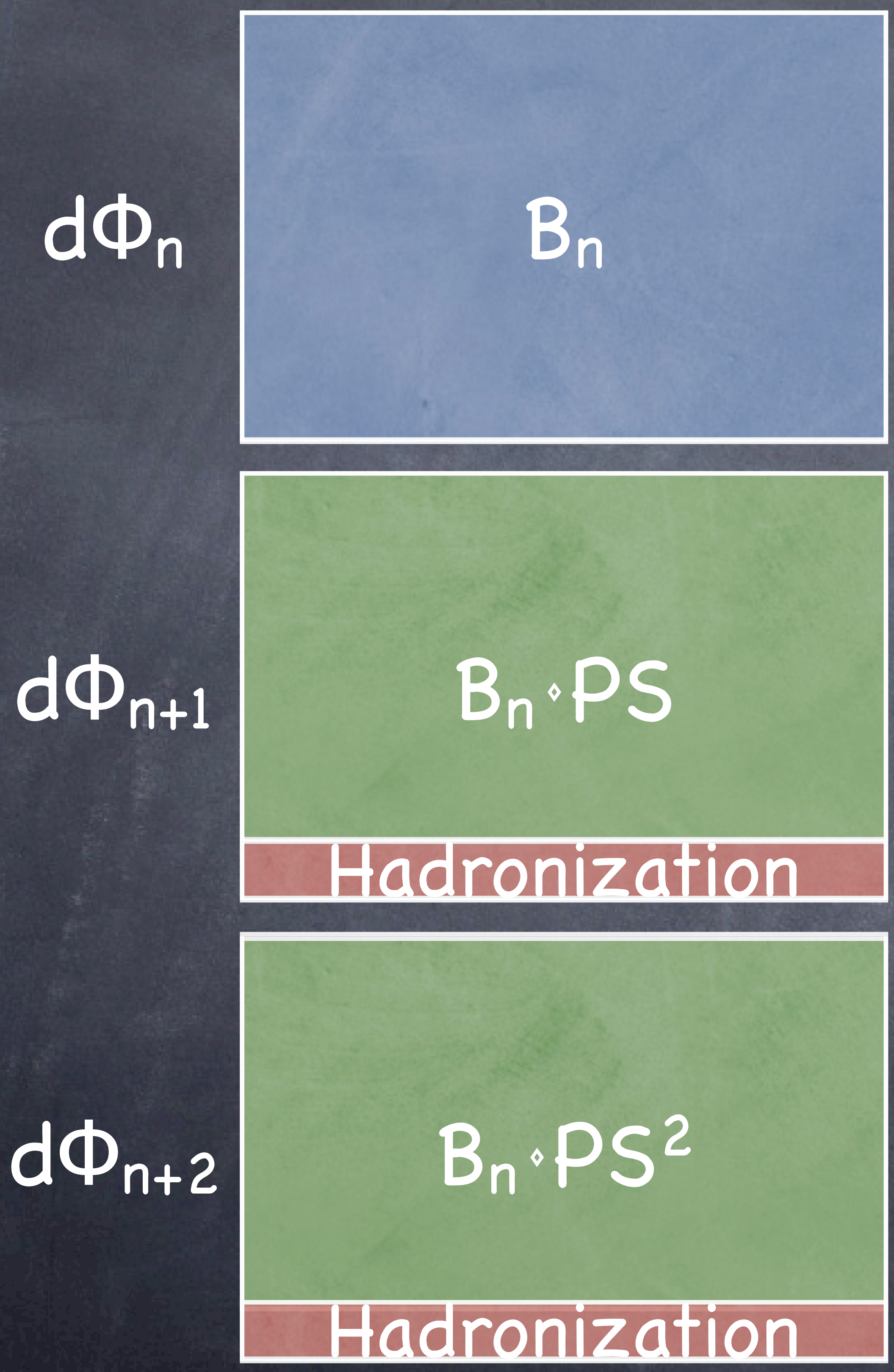
The parton shower



The parton shower

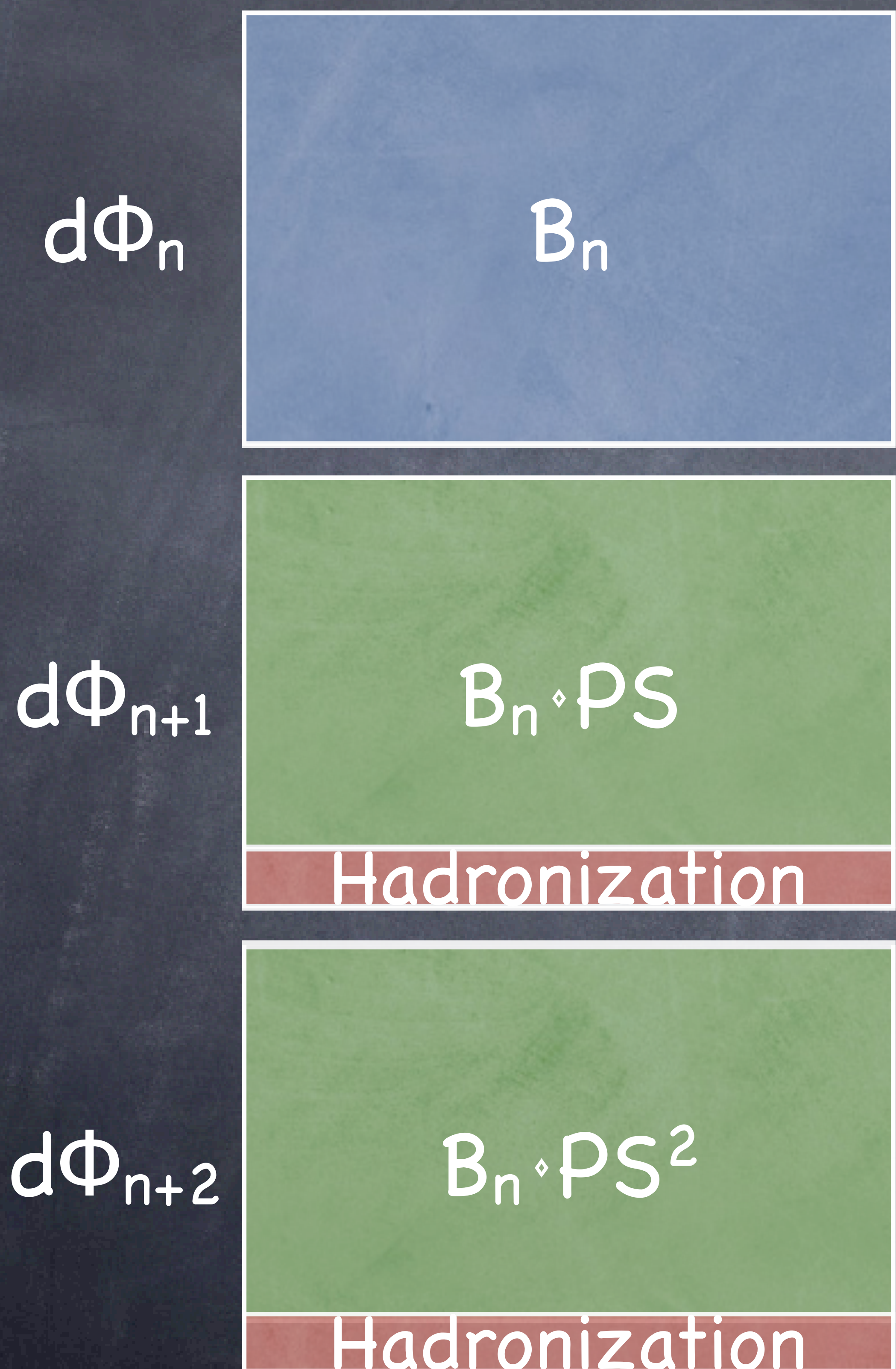
$d\Phi_n$	B_n
$d\Phi_{n+1}$	$B_n \cdot PS$ Hadronization
$d\Phi_{n+2}$	$B_n \cdot PS^2$ Hadronization

The parton shower



- Starts from known B_n
- Adds extra emissions via simple algorithm
- Is probabilistic (always sums to the answer started from)
- Simple way to attach hadronization at t_{had}

The parton shower



- Starts from known B_n
- Adds extra emissions via simple algorithm
- Is probabilistic (always sums to the answer started from)
- Simple way to attach hadronization at t_{had}

Solves Problems 3-4 as advertised

Combining with LO

$d\Phi_n$	B_n
$d\Phi_{n+1}$	$B_n \diamond PS$
$d\Phi_{n+2}$	$B_n \diamond PS^2$

How do we correct higher dim phase space to LO results?

Double up phase space

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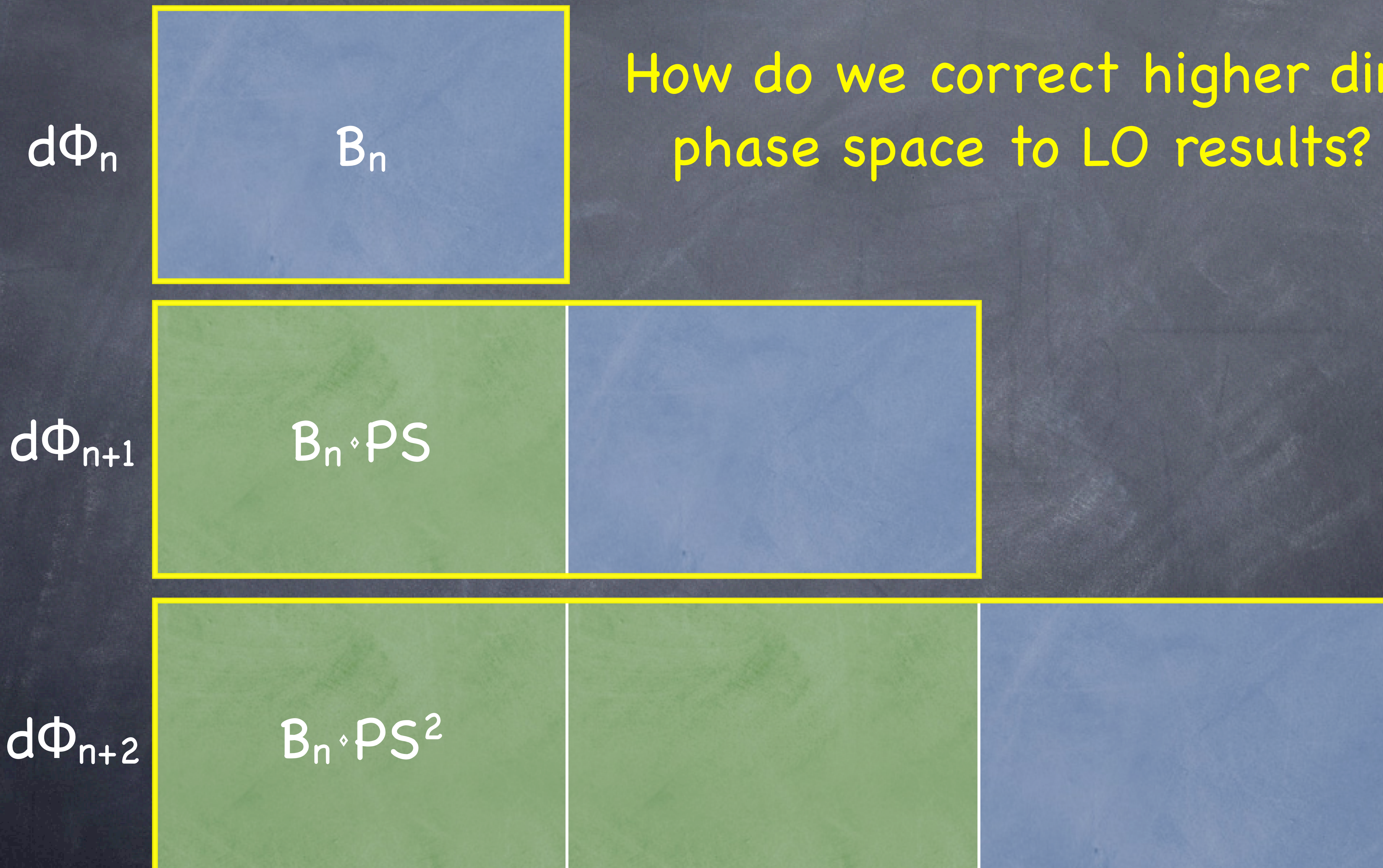

Double up phase space

How do we correct higher dim phase space to LO results?

$d\Phi_n$	B_n		
$d\Phi_{n+1}$	$B_n \diamond PS$	$B_{n+1} - B_n \diamond PS$	
$d\Phi_{n+2}$	$B_n \diamond PS^2$		

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$d\Phi_{n+1}$	$B_n \diamond PS$	$B_{n+1} - B_n \diamond PS$	
$d\Phi_{n+2}$	$B_n \diamond PS^2$	$[B_{n+1}-B_n \diamond PS] \diamond PS$	$B_{n+2} - [B_{n+1}-B_n \diamond PS] \diamond PS - B_n \diamond PS^2$

Double up phase space

Need Shower analytically

$d\Phi_n$	B_n		
$d\Phi_{n+1}$	$B_n \diamond PS$	$B_{n+1} - B_n \diamond PS$	
$d\Phi_{n+2}$	$B_n \diamond PS^2$	$[B_{n+1} - B_n \diamond PS] \diamond PS$	$B_{n+2} - [B_{n+1} - B_n \diamond PS] \diamond PS - B_n \diamond PS^2$

Double up phase space

Need Shower analytically

Gives negative weights

$d\Phi_n$	B_n		
$d\Phi_{n+1}$	$B_n \diamond PS$	$B_{n+1} - B_n \diamond PS$	
$d\Phi_{n+2}$	$B_n \diamond PS^2$	$[B_{n+1} - B_n \diamond PS] \diamond PS$	$B_{n+2} - [B_{n+1} - B_n \diamond PS] \diamond PS - B_n \diamond PS^2$

Double up phase space

- Need Shower analytically
- Gives negative weights
- Almost impossible to extend

$d\Phi_n$	B_n		
$d\Phi_{n+1}$	$B_n \diamond PS$	$B_{n+1} - B_n \diamond PS$	
$d\Phi_{n+2}$	$B_n \diamond PS^2$	$[B_{n+1} - B_n \diamond PS] \diamond PS$	$B_{n+2} - [B_{n+1} - B_n \diamond PS] \diamond PS - B_n \diamond PS^2$

Double up phase space

Separate phase space

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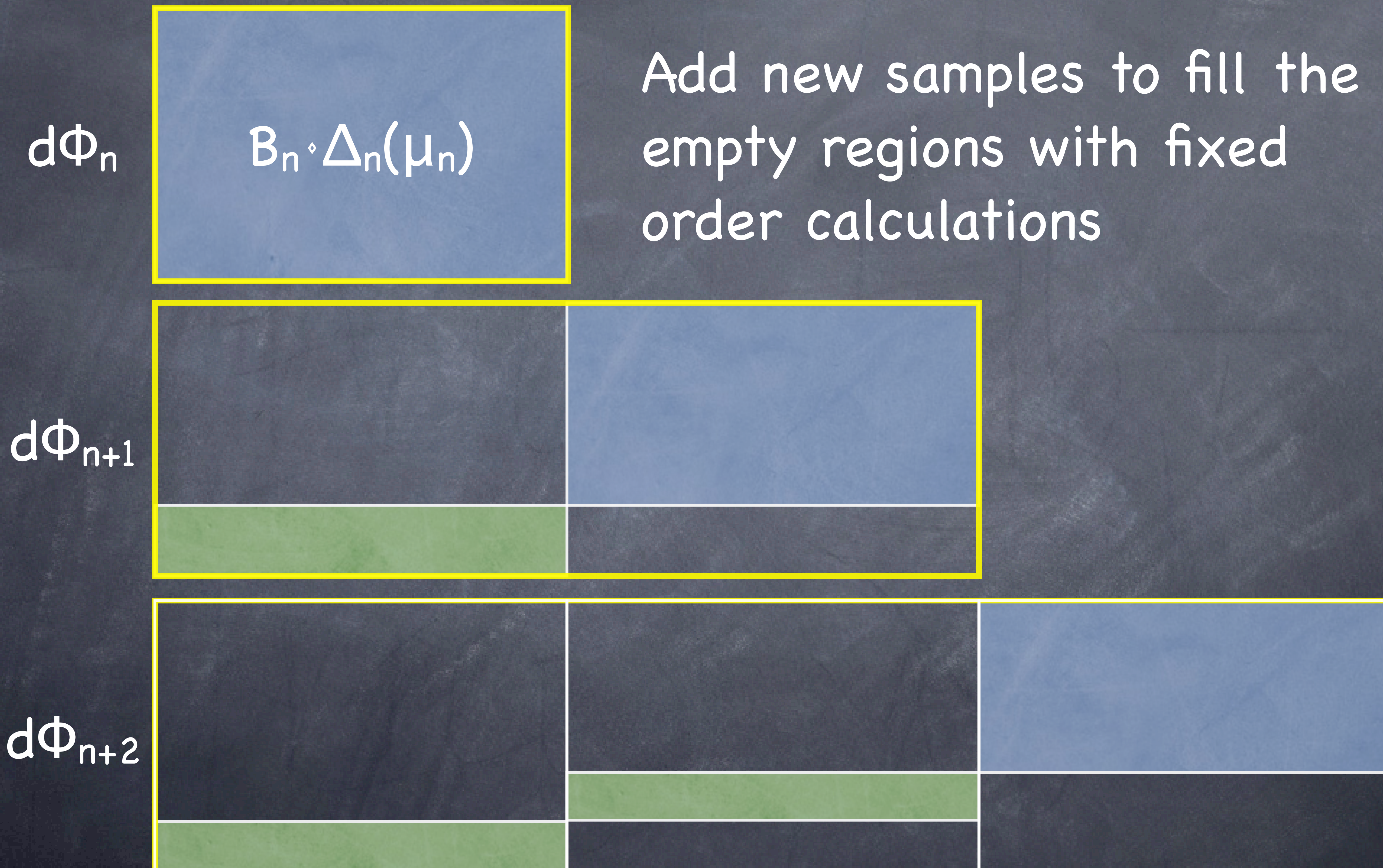
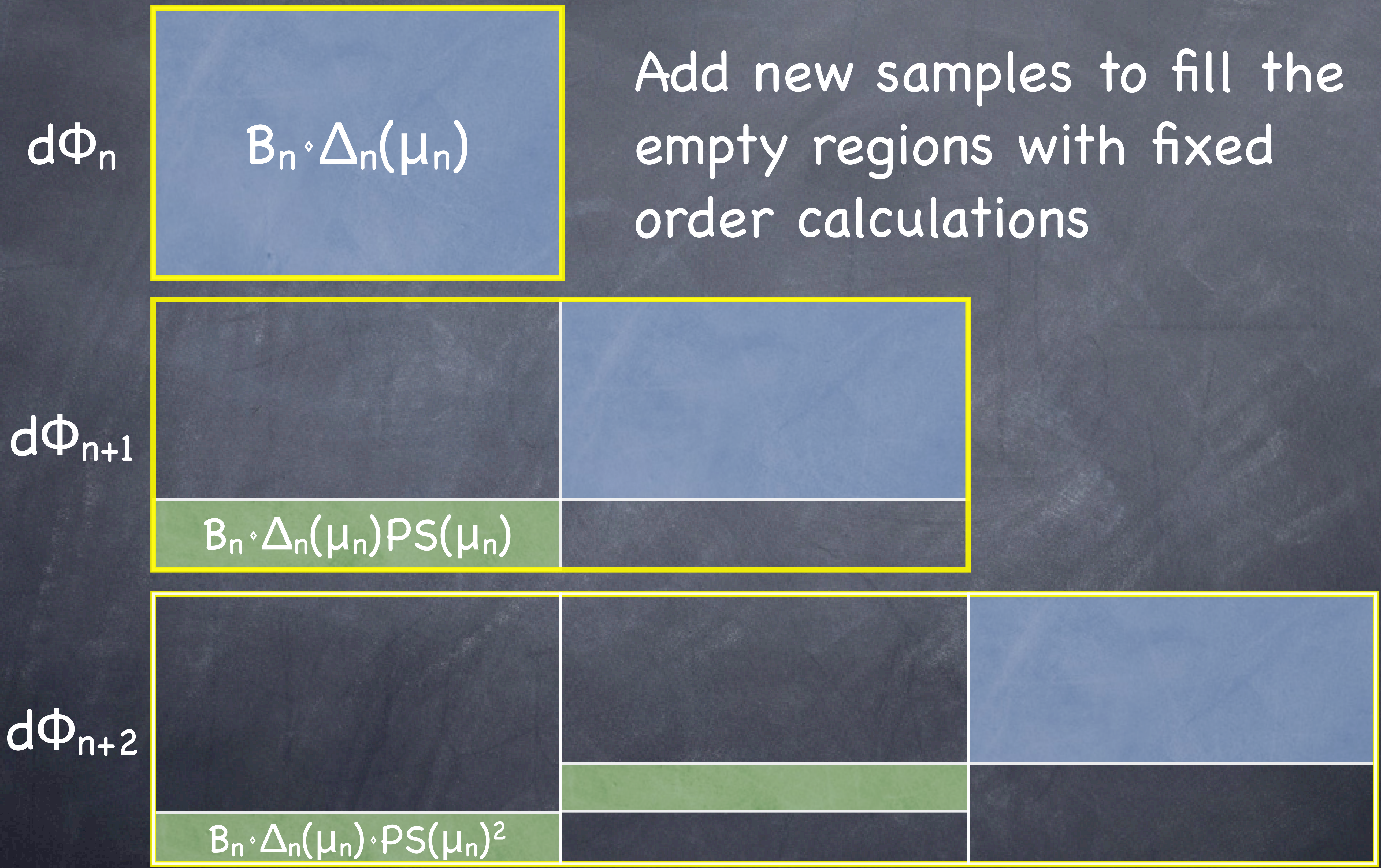


Add new samples to fill the empty regions with fixed order calculations

Separate phase space

Add new samples to fill the empty regions with fixed order calculations



Separate phase space

Add new samples to fill the empty regions with fixed order calculations

$d\Phi_n$

$B_n \cdot \Delta_n(\mu_n)$

$d\Phi_{n+1}$

$B_{n+1} \cdot \Delta_{n+1}(\mu_{n+1})$

$B_n \cdot \Delta_n(\mu_n) PS(\mu_n)$

$d\Phi_{n+2}$

$B_n \cdot \Delta_n(\mu_n) \cdot PS(\mu_n)^2$

Separate phase space

$d\Phi_n$

$B_n \cdot \Delta_n(\mu_n)$

Add new samples to fill the empty regions with fixed order calculations

$d\Phi_{n+1}$

$B_{n+1} \cdot \Delta_{n+1}(\mu_{n+1})$

$B_n \cdot \Delta_n(\mu_n) PS(\mu_n)$

$d\Phi_{n+2}$

$B_{n+1} \cdot \Delta_{n+1}(\mu_{n+1}) \cdot PS\,(\mu_{n+1})$

$B_n \cdot \Delta_n(\mu_n) \cdot PS(\mu_n)^2$

Separate phase space

Add new samples to fill the empty regions with fixed order calculations

$d\Phi_n$

$B_n \cdot \Delta_n(\mu_n)$

$d\Phi_{n+1}$

$B_{n+1} \cdot \Delta_{n+1}(\mu_{n+1})$

$B_n \cdot \Delta_n(\mu_n) PS(\mu_n)$

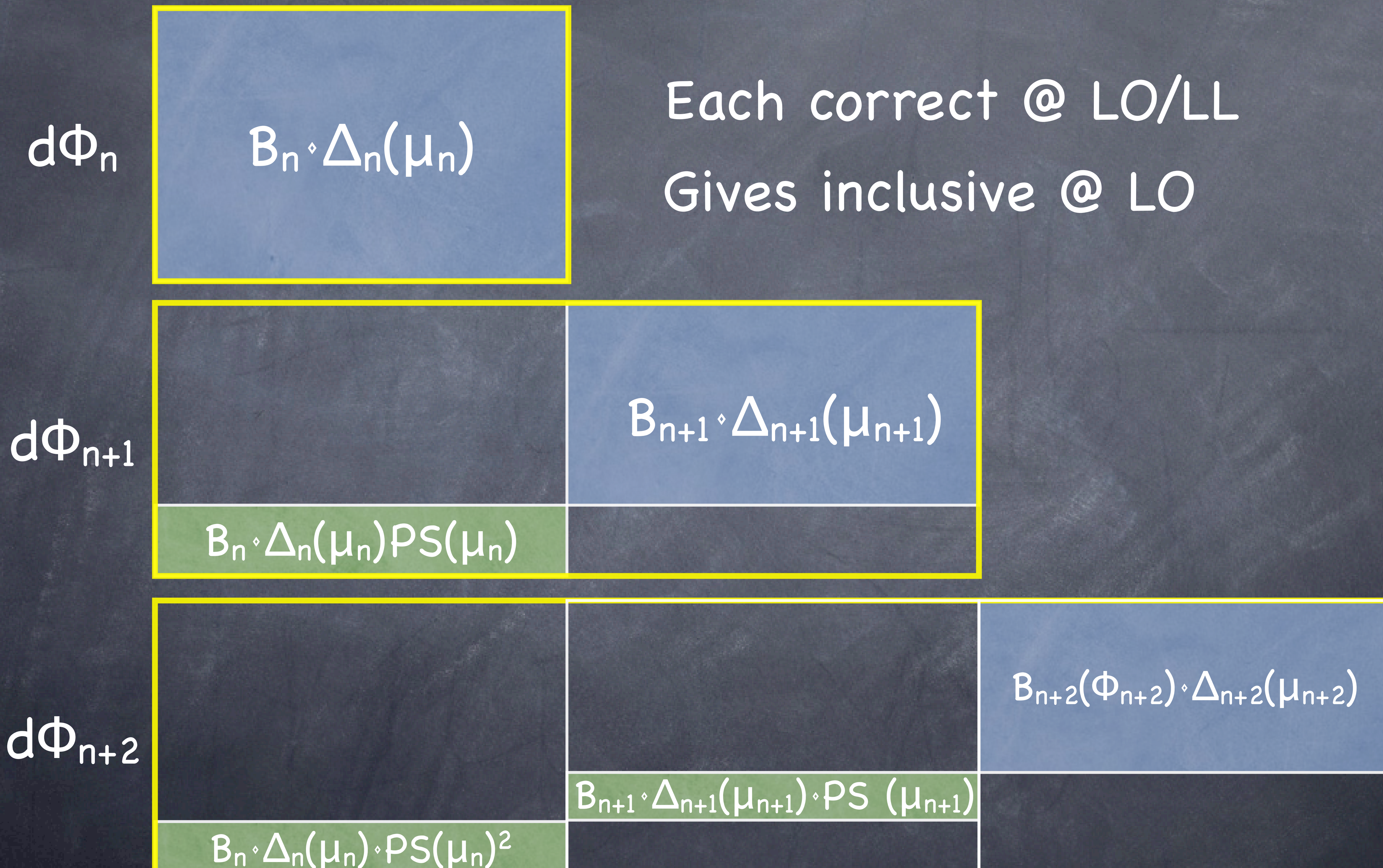
$d\Phi_{n+2}$

$B_{n+2}(\Phi_{n+2}) \cdot \Delta_{n+2}(\mu_{n+2})$

$B_{n+1} \cdot \Delta_{n+1}(\mu_{n+1}) \cdot PS\ (\mu_{n+1})$

$B_n \cdot \Delta_n(\mu_n) \cdot PS(\mu_n)^2$

Separate phase space



Each correct @ LO/LL

Gives inclusive @ LO

Separate phase space

$d\Phi_n$	$B_n \cdot \Delta_n(\mu_n)$		
$d\Phi_{n+1}$	$B_n \cdot \Delta_n(\mu_n) PS(\mu_n)$	$B_{n+1} \cdot \Delta_{n+1}(\mu_{n+1})$	
$d\Phi_{n+2}$	$B_n \cdot \Delta_n(\mu_n) \cdot PS(\mu_n)^2$	$B_{n+1} \cdot \Delta_{n+1}(\mu_{n+1}) \cdot PS\ (\mu_{n+1})$	$B_{n+2}(\Phi_{n+2}) \cdot \Delta_{n+2}(\mu_{n+2})$

Each correct @ LO/LL

Gives inclusive @ LO

CKKW

MLM

The same at NLO

Double up phase space

Double up phase space

$d\Phi_n$	$B_n+V_n+\int B_n \diamond PS$		
$d\Phi_{n+1}$	$[B_n+V_n+\int B_n \diamond PS] \diamond PS$	$B_{n+1} - B_n \diamond PS$	
$d\Phi_{n+2}$			

Double up phase space

$d\Phi_n$	$B_n+V_n+\int B_n \diamond PS$		
$d\Phi_{n+1}$	$[B_n+V_n+\int B_n \diamond PS] \diamond PS$	$B_{n+1} - B_n \diamond PS$	
$d\Phi_{n+2}$			

MC@NLO

Double up phase space

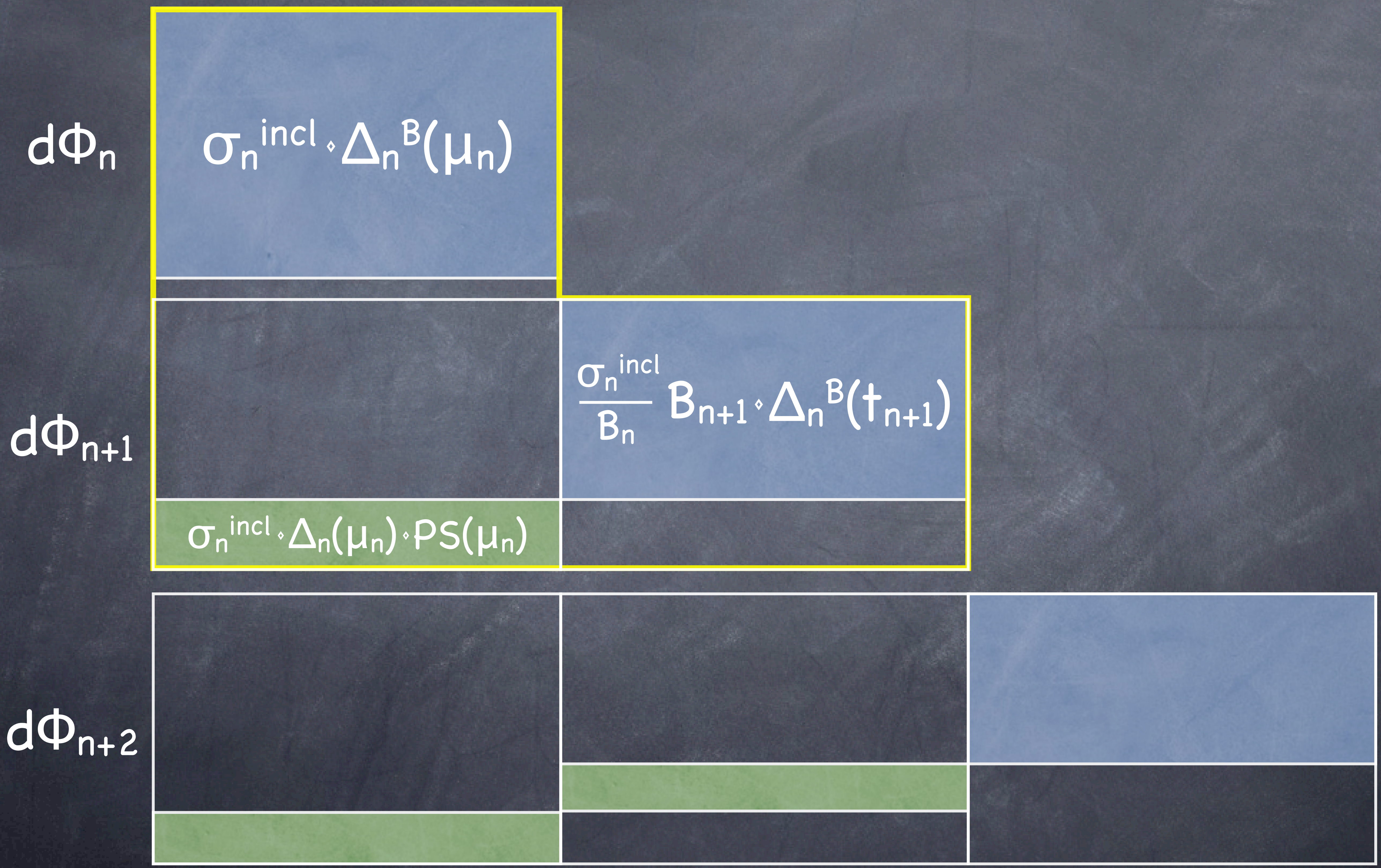
- Need Shower analytically
- Gives negative weights
- Almost impossible to extend

$d\Phi_n$	$B_n + V_n + \int B_n \diamond PS$		
$d\Phi_{n+1}$	$[B_n + V_n + \int B_n \diamond PS] \diamond PS$	$B_{n+1} - B_n \diamond PS$	
$d\Phi_{n+2}$			

MC@NLO

Separate phase space

Separate phase space



Separate phase space

$d\Phi_n$

$\sigma_n^{incl} \cdot \Delta_n^B(\mu_n)$

$d\Phi_{n+1}$

$\frac{\sigma_n^{incl}}{B_n} B_{n+1} \cdot \Delta_n^B(t_{n+1})$

$\sigma_n^{incl} \cdot \Delta_n(\mu_n) \cdot PS(\mu_n)$

POWHEG

$d\Phi_{n+2}$

Separate phase space

Need to calculate integral

Need to calculate Sudakov

Not obvious how to extend

$d\Phi_n$ $\quad \sigma_n^{\text{incl}} \cdot \Delta_n^B(\mu_n)$

$d\Phi_{n+1}$ $\quad \dfrac{\sigma_n^{\text{incl}}}{B_n} B_{n+1} \cdot \Delta_n^B(t_{n+1})$

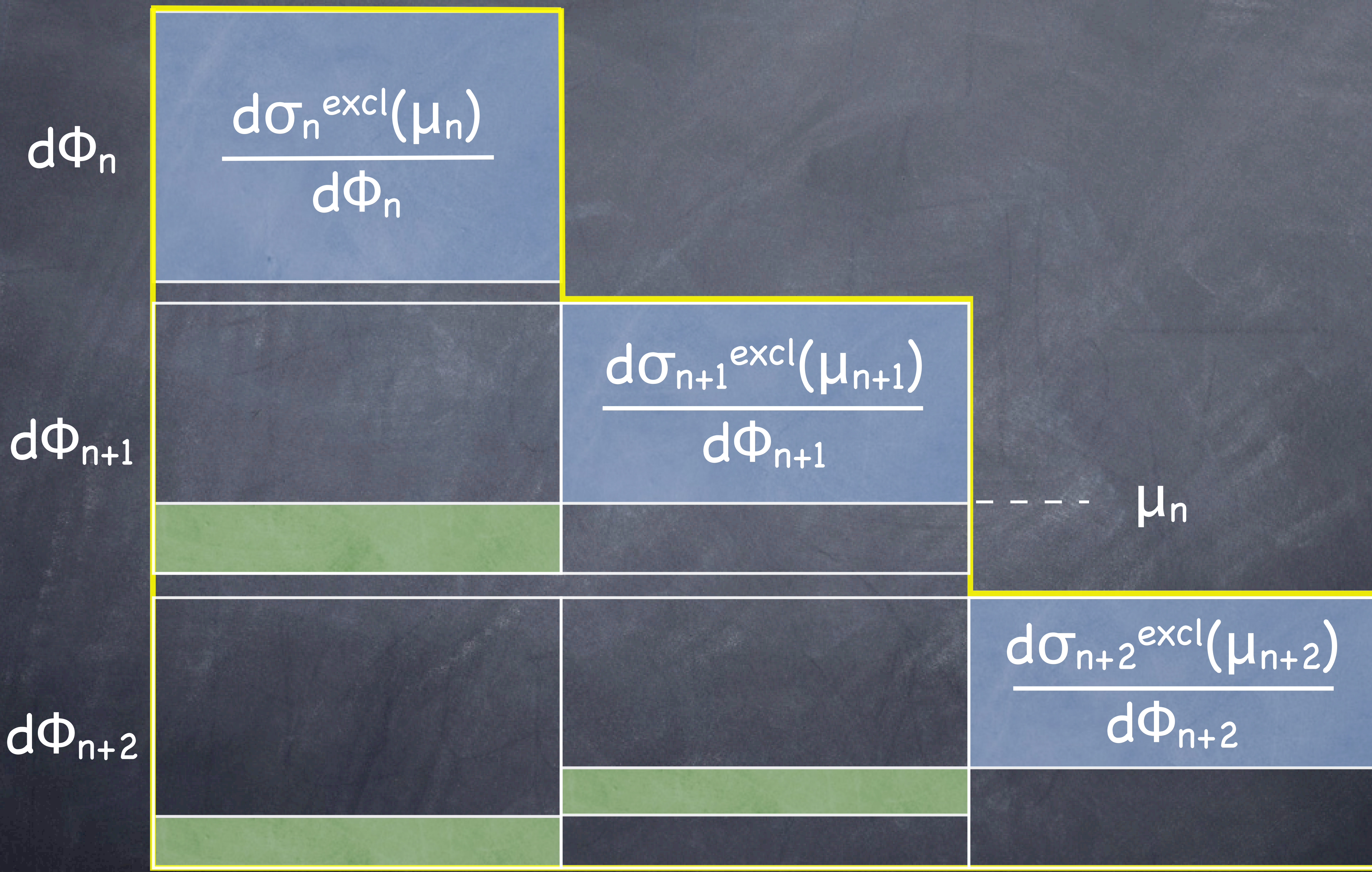
$\sigma_n^{\text{incl}} \cdot \Delta_n(\mu_n) \cdot PS(\mu_n)$

$d\Phi_{n+2}$

POWHEG

Our Method

Our Method



$d\Phi_n$
$\frac{d\sigma_n^{excl}(\mu_n)}{d\Phi_n}$
$d\Phi_{n+1}$
$\frac{d\sigma_{n+1}^{excl}(\mu_{n+1})}{d\Phi_{n+1}}$
μ_n
$d\Phi_{n+2}$
$\frac{d\sigma_{n+2}^{excl}(\mu_{n+2})}{d\Phi_{n+2}}$

Our Method

Each correct @ NLO/LL

Gives inclusive @ NLO

No gratuitous num integrals

$d\Phi_n$ $$\frac{d\sigma_n^{excl}(\mu_n)}{d\Phi_n}$$

$d\Phi_{n+1}$ $$\frac{d\sigma_{n+1}^{excl}(\mu_{n+1})}{d\Phi_{n+1}}$$

μ_n

$d\Phi_{n+2}$ $$\frac{d\sigma_{n+2}^{excl}(\mu_{n+2})}{d\Phi_{n+2}}$$

Determining the σ^{excl}

- Obtain the correct expression at fixed order
  - Need careful definition of J_{MC} to have analytical results
- Write expression that has correct logarithmic structure
  - Use parton shower ideas as a guidline
- Combine the two results by a simple matching

Fixed order results

Deriving a generic expression

$$\frac{\mathrm{d}\sigma_n^{\mathrm{excl}}(\mu_n)}{d\Phi_n} = \frac{\mathrm{d}\sigma_n^{\mathrm{parton}}}{d\Phi_n} + \int \mathrm{d}\Phi'_{n+1} \frac{\mathrm{d}\sigma_{n+1}^{\mathrm{parton}}}{d\Phi_n} J_{\mathrm{MC}}(\Phi'_{n+1}, \Phi_n, \mu_n)$$

Fixed order results

Deriving a generic expression

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$$= B_n(\Phi_n) + V_n(\Phi_n) + \int \mathrm{d}\Phi'_{n+1}\, B_{n+1}(\Phi'_{n+1}) J_{\mathrm{MC}}(\Phi'_{n+1}, \Phi_n, \mu_n)$$

Fixed order results

Deriving a generic expression

$$
\begin{aligned}
\frac{\mathrm{d}\sigma_n^{\mathrm{excl}}(\mu_n)}{d\Phi_n} &= \frac{\mathrm{d}\sigma_n^{\mathrm{parton}}}{d\Phi_n} + \int \mathrm{d}\Phi'_{n+1} \, \frac{\mathrm{d}\sigma_{n+1}^{\mathrm{parton}}}{d\Phi_n} J_{\mathrm{MC}}(\Phi'_{n+1}, \Phi_n, \mu_n) \\
&= B_n(\Phi_n) + V_n(\Phi_n) + \int \mathrm{d}\Phi'_{n+1} \, B_{n+1}(\Phi'_{n+1}) J_{\mathrm{MC}}(\Phi'_{n+1}, \Phi_n, \mu_n) \\
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Fixed order results

Deriving a generic expression

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Final result for small μ_n

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Write S as sum over different terms

$$S_{n+1}(\Phi'_{n+1}) = \sum_i S^{(i)}_{n+1}(\Phi'_{n+1})$$

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For each i can find $J_{MC}^{(i)}$ that allows to integrate

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Therefore, can choose

$$J_{\mathrm{MC}}(\Phi'_{n+1}, \Phi_n, \mu_n) = \sum_i \frac{S^{(i)}_{n+1}(\Phi'_{n+1})}{S_{n+1}(\Phi'_{n+1})} J^{(i)}_{\mathrm{MC}}(\Phi'_{n+1}, \Phi_n, \mu_n)$$

Example: Catani-Seymour

Different term for each of three partons [(i)→ij,k]

singularity $p_i \cdot p_j \to 0$ with k recoil

$$\sum_i \equiv \sum_{ij,k}$$

Factorization for each {ij,k}

$$S^{(i)}_{n+1}(\Phi_{n+1}) \equiv \mathcal{D}^{ij,k}_{n+1}(\Phi_{n+1})$$

$$\mathrm{d}\Phi_{n+1} \equiv \mathrm{d}\Phi^{ij,k}_n \mathrm{d}\Phi^{ij,k}_{\mathrm{rad}}$$

$$dy^{ij,k} dz^{ij,k} d\phi^{ij,k}$$

This allows to define

$$J^{ij,k}_{\mathrm{MC}}(\Phi_{n+1}, \Phi_n, \mu_n) = \delta(\Phi_n - \Phi^{ij,k}_n)\Theta(y^{ij,k} < \mu_n)$$

Gives rise to analytically calculable integrals

Nagy, Trocsanyi ('98)

Correct logarithmic structure

Use the fact that parton shower resums leading logarithmic terms

Write cross section in recursive form

$$\left[\frac{\mathrm{d}\sigma_n^{\mathrm{PS}}}{\mathrm{d}\Phi_n}\right] = \left[\frac{\mathrm{d}\sigma_{n-1}^{\mathrm{PS}}}{\mathrm{d}\Phi_{n-1}}\right] \times PS$$

Several subtleties, but can be done

Combine results

Use slightly generalized LL result

$$\left[\frac{\mathrm{d}\sigma_n^{\mathrm{MC}}(\mu_n)}{\mathrm{d}\Phi_n}\right] = \sum_i \left(\left[\frac{\mathrm{d}\sigma_{n-1}^{\mathrm{MC}}}{\mathrm{d}\Phi_{n-1}}\right] Q^{(i)}(\Phi_{n-1\to n}) + M_n^{(i)}(\Phi_n) \right) \Delta_n(\mu_n)$$

Choose splitting functions as

$$Q^{(i)}(\Phi_{n-1\to n}) = \frac{S_n^{(i)}(\Phi_n)}{B_{n-1}(\Phi_{n-1})}$$

Determine matching coefficient by explicit comparison with previous NLO result

Determining the σ^{excl}

By expanding to NLO order and comparing with known results, can obtain M_n

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The tree level diagrams

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Known subtraction functions

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The virtual (1-loop) diagrams

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The tree level diagrams	Known subtraction functions	The virtual (1-loop) diagrams	Expansion of the Sudakov function

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The tree level diagrams	Known subtraction functions	The virtual (1-loop) diagrams	Expansion of the Sudakov function

Everything known analytically!

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Hope to have first numerical results by the summer

Conclusions

- Both NLO calculations and parton shower algorithms crucial to have detailed understanding of signals and backgrounds
- Need to merge the two approaches to get reliable and trustworthy results
- Four main problems that need to be addressed
- Believe we have a fast and efficient algorithm that should give us first results by the summer