Rare Kaon Decays in the SM and beyond

Ringberg Workshop: 27.4.2009

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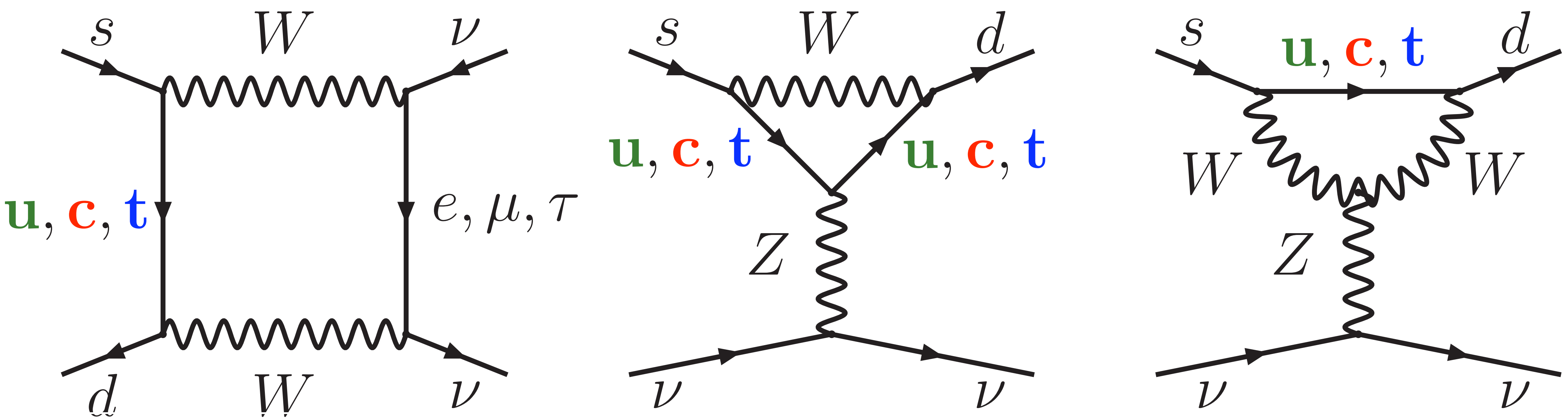
Introduction: $K \to \pi \nu \bar{\nu}$



- Dominant Operator: $Q_\nu = (\bar{s}_L \gamma_\mu d_L)(\bar{\nu}_L \gamma^\mu \nu_L)$

$$\sum_i V_{is}^* V_{id} F(x_i) = V_{ts}^* V_{td} (F(x_t) - F(x_u)) + V_{cs}^* V_{cd} (F(x_c) - F(x_u))$$

Quadratic GIM: $\lambda^5 \frac{m_t^2}{M_W^2}$ $\quad \lambda \frac{m_c^2}{M_W^2} \ln \frac{M_W}{m_c}$ $\quad \lambda \frac{\Lambda^2}{M_W^2}$

Use isospin symmetry and normalise to: $K^+ \to \pi^0 e^+ \nu$

$s \longrightarrow d$ and New Physics (NP)

$b \to s:$ $|V_{tb}^* V_{ts}| \propto \lambda^2$

$b \to d:$ $|V_{tb}^* V_{td}| \propto \lambda^3$

$s \to d:$ $|V_{ts}^* V_{td}| \propto \lambda^5$

Rare K Decays: Additional Cabbibo suppression λ^5

$$\mathcal{L}_{\text{eff}} = \frac{C(b \to s)}{\Lambda_{NP}^2}(\bar{b}\Gamma s)(\bar{\nu}\Gamma\nu) + \frac{C(b \to d)}{\Lambda_{NP}^2}(\bar{b}\Gamma d)(\bar{\nu}\Gamma\nu) + \frac{C(s \to d)}{\Lambda_{NP}^2}(\bar{s}\Gamma d)(\bar{\nu}\Gamma\nu)$$

Low NP scale $\Lambda_{NP} \simeq 1$ TeV

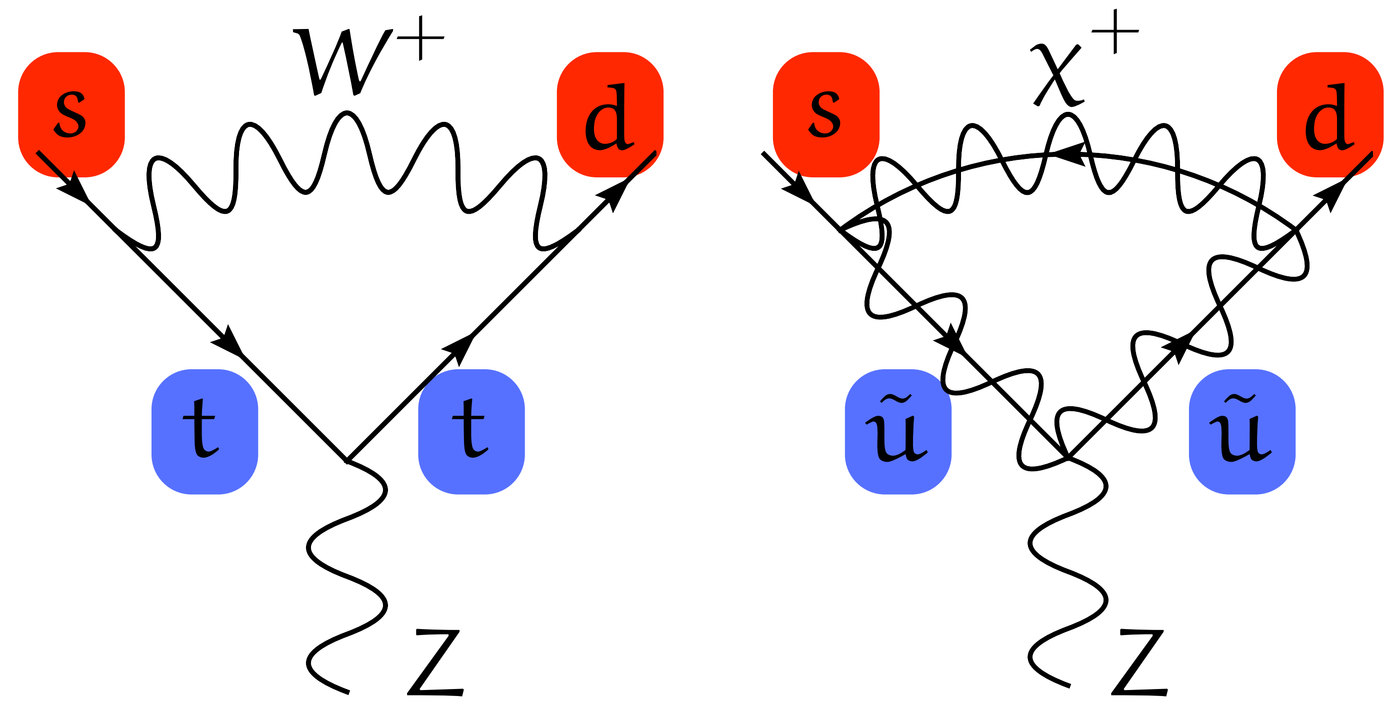
NP Flavour Sector $C(s \to d) < \lambda^5$

For Generic NP $C(s \to d) \simeq 1$

New Physics scale $\Lambda_{NP} > 75$ TeV

Rare K decays and New Physics:

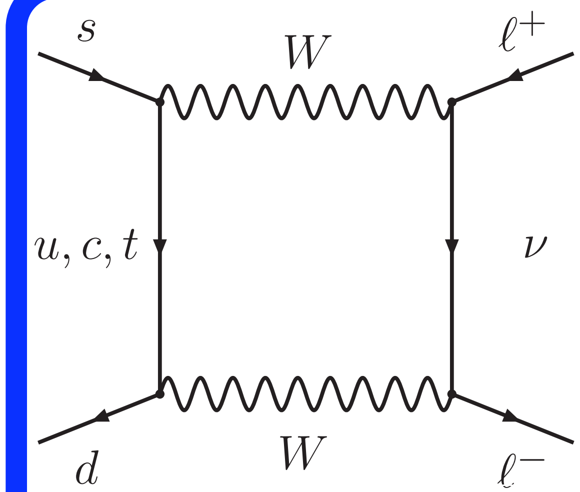
- Test deviation of flavour alignment (Minimal Flavour Violation MFV)

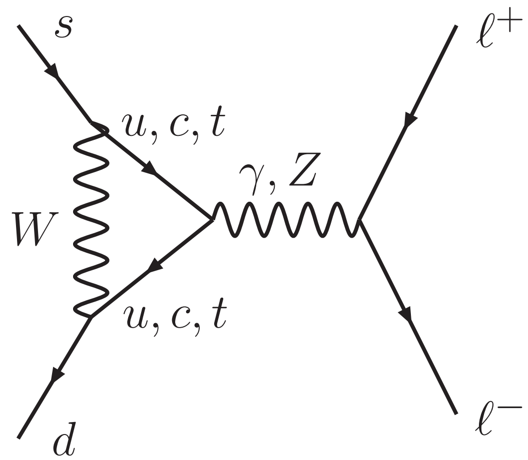


- Precise theory prediction
- Sensitive to small deviations from MFV

$$K_L \to \pi^0 \mu^+ \mu^- \qquad K_L \to \pi^0 \bar{\nu} \nu$$
$$K_L \to \pi^0 e^+ e^- \qquad K^+ \to \pi^+ \bar{\nu} \nu$$

$K_L \to \pi^0 l^+ l^-$: Three Contributions





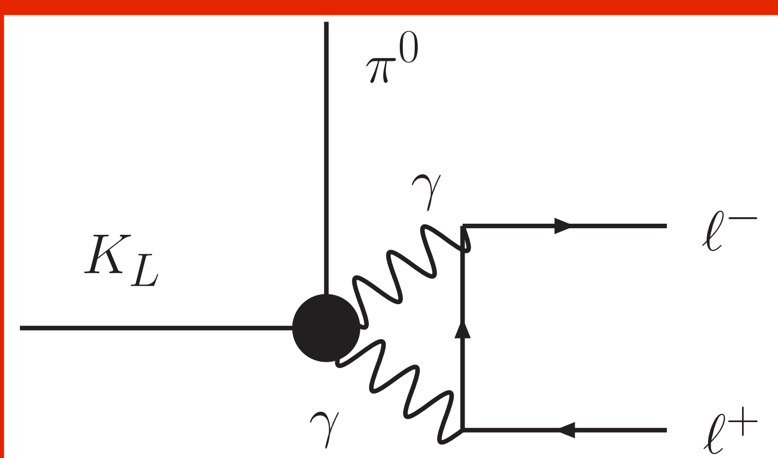
$$Q_{7V} = (\bar{s}_L \gamma_\mu d_L)(\bar{l}\gamma^\mu l) \to 1^{--}$$

$$Q_{7A} = (\bar{s}_L \gamma_\mu d_L)(\bar{l}\gamma^\mu \gamma_5 l)$$

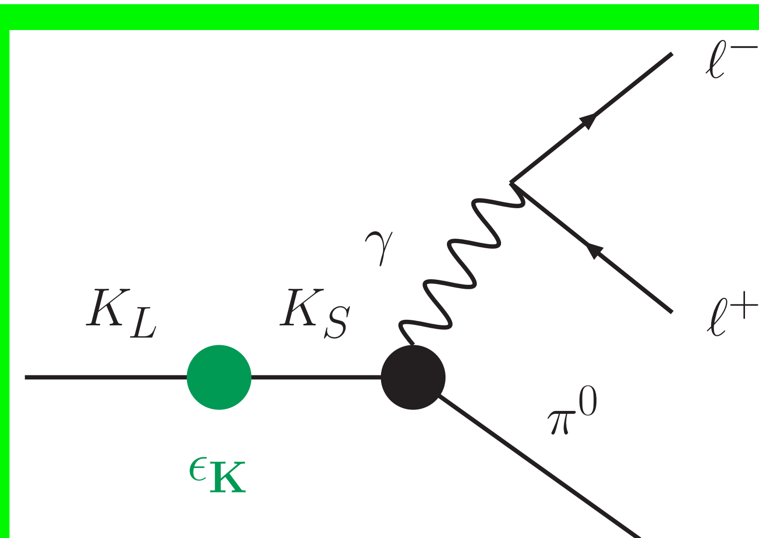
$$\to 1^{++},\ 0^{-+}$$

Direct CP Violating

Wilson Coefficients: $y_{7V},\ y_{7A}$ at NLO [Buchalla et al. ´96]

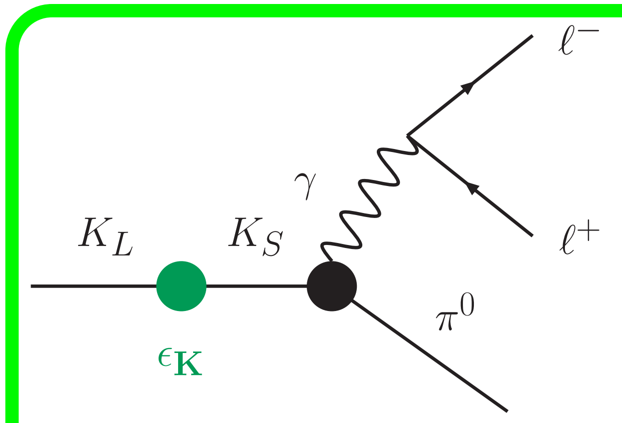


CP Conserving



Indirect CP Violating

$K_L \to \pi^0 l^+ l^-$: Three Contributions

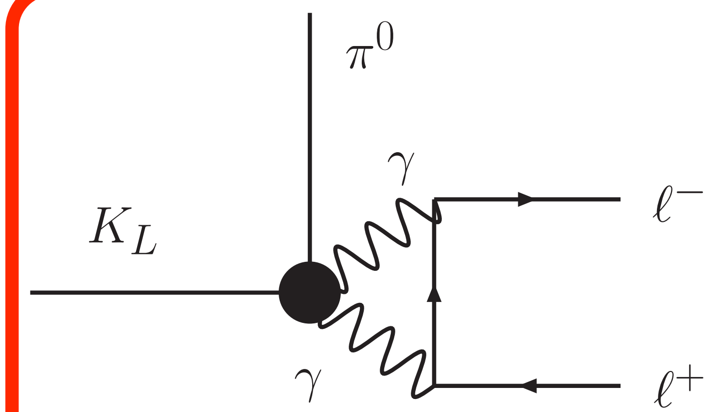


Counterterm $|a_S| = 1.2 \pm 0.2$ from $K_S \to \pi^0 l^+ l^-$

[D´Ambrosio et. al. ´98, Mescia et. al. ´06]

For 1^{--} interference with Q_{7V}

[Buchalla et. al. ´03, Friot et al. ´04]



Estimate from $K_L \to \pi^0 \gamma\gamma$

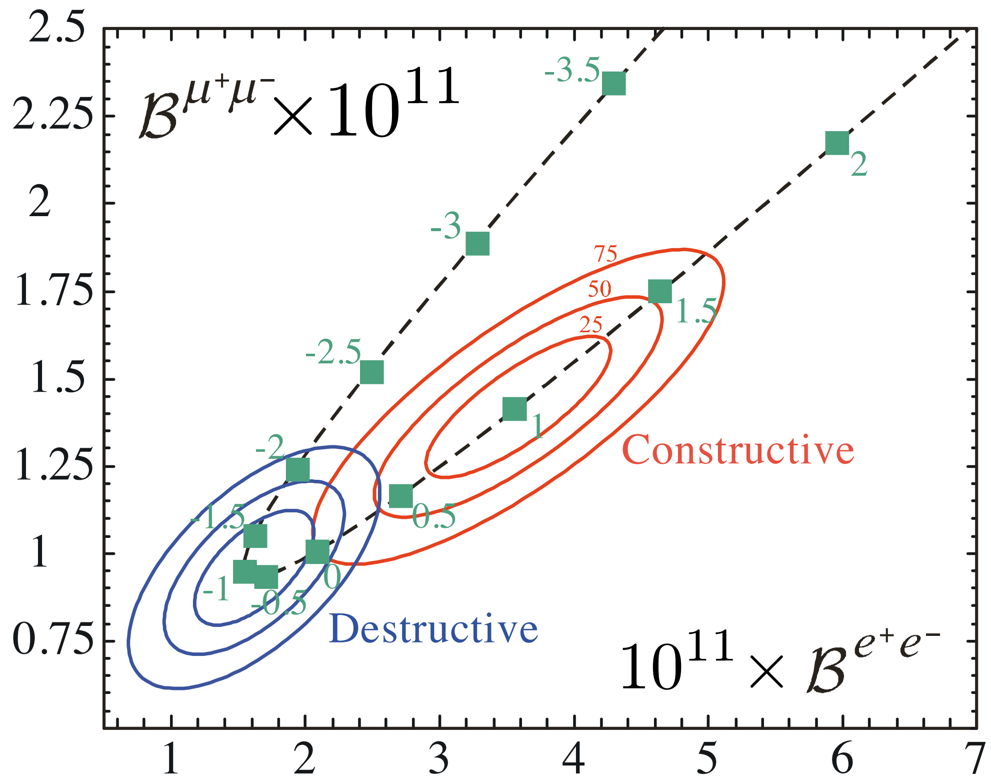
[Isidori et. al. ´04]

$$\mathcal{B}r(K_L \to \pi^0 \ell^+ \ell^-) = \left(C_{dir}^{\ell} \pm C_{int}^{\ell} |a_S| + C_{mix}^{\ell} |a_S|^2 + C_{\gamma\gamma}^{\ell}\right) \times 10^{-12}$$

ℓ	C_{dir}^{ℓ}	C_{int}^{ℓ}	C_{mix}^{ℓ}	$C_{\gamma\gamma}^{\ell}$
e	$(4.62 \pm 0.24)(y_V^2 + y_A^2)$	$(11.3 \pm 0.3) y_V$	14.5 ± 0.5	≈ 0
μ	$(1.09 \pm 0.05)(y_V^2 + 2.32 y_A^2)$	$(2.63 \pm 0.06) y_V$	3.36 ± 0.20	5.2 ± 1.6

$K_L \to \pi^0 l^+ l^-$: Improvements

- Measure both $\mathcal{B}r_{e^+e^-}$ and $\mathcal{B}r_{\mu^+\mu^-}$: [Mescia et. al. ´06]
  Disentangle short distance contribution $(y_{7V},\ y_{7A})$
- Dominant theory error in a_s:
  Forward backward asymmetry. [Mescia, Smith, Trine ´06]
  Better measurement of $K_S \to \pi^0 l^+ l^-$. [Smith ´07]

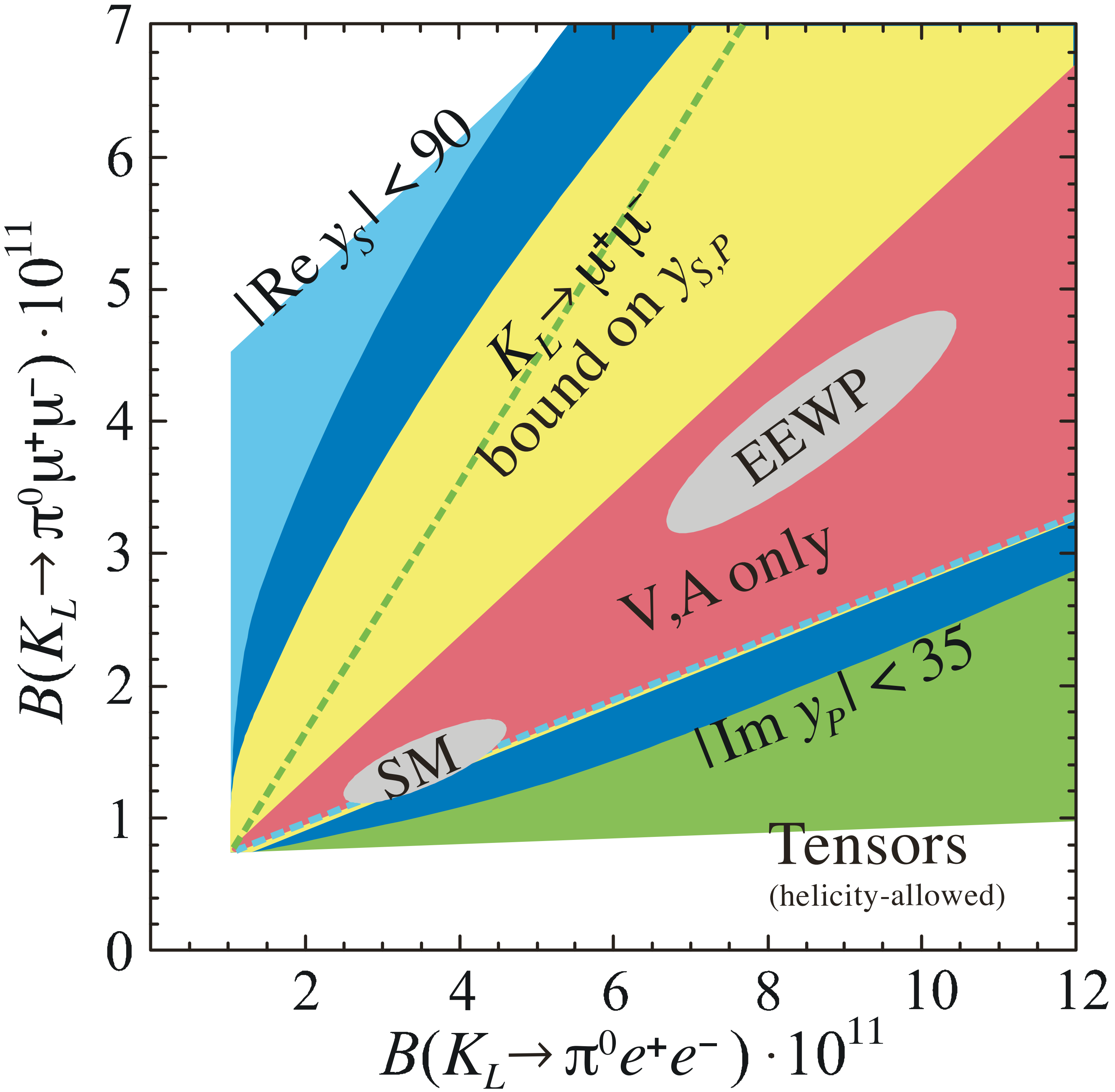


[Mescia et. al. ´06]

[KTEV ´04]	[KTEV ´00]
$Br_{e^+e^-}$	$Br_{\mu^+\mu^-}$
$< 28 \times 10^{-11}$	$< 38 \times 10^{-11}$

$K_L \to \pi^0 l^+ l^-$: New Physics

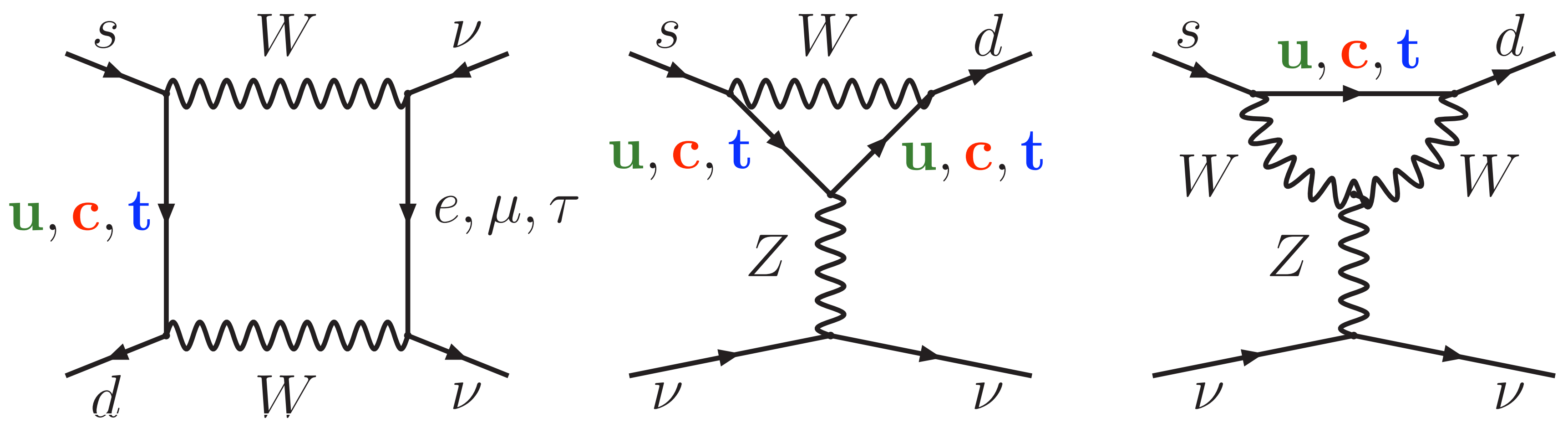
- EW penguin operators correlated with $K \to \pi \bar{\nu} \nu$, while QCD ones fixed by data



- $Q_s = (\bar{s}d)(\bar{l}l)$ and $Q_p = (\bar{s}d)(\bar{l}\gamma_5 l)$ generated in the MSSM with large tan beta.
- Effect only $K_L \to \pi^0 \mu^+ \mu^-$ correlated with $K_L \to \mu^+ \mu^-$
- Sensitive to tensor operators

[Mescia, Smith, Trine ´06]

$K_L \rightarrow \pi^0 \bar{\nu} \nu$: Effective Hamiltonian



CP violating: DCPV : ICPV : CPC = 1 : 10^{-2} : < 10^{-4}

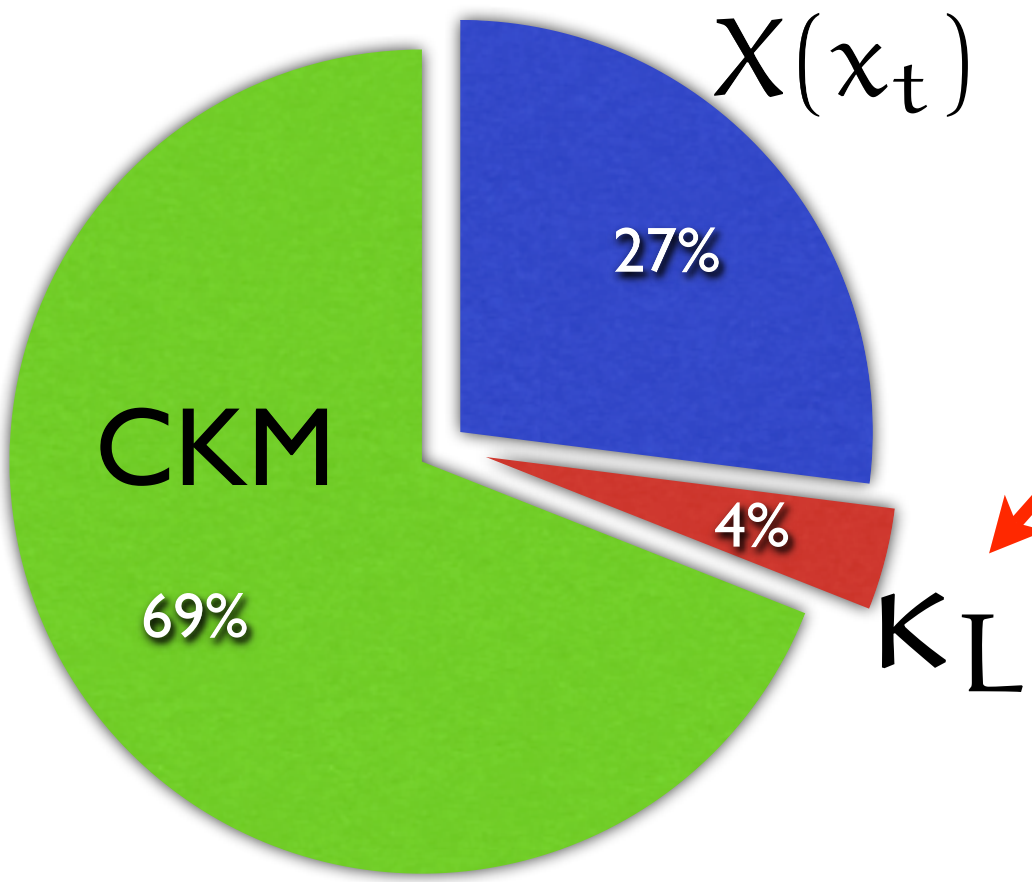
[Buchalla, Isidori ´96]

Only top quark contributes: $H_{eff} = \frac{4G_F}{\sqrt{2}} \frac{\alpha V_{ts}^* V_{td}}{2\pi \sin^2 \Theta_W} X(x_t) Q_\nu$

Use isospin symmetry and normalise to: $K^+ \rightarrow \pi^0 e^+ \nu$

$$\mathcal{B}r(K_L \rightarrow \pi^0 \bar{\nu} \nu) = \kappa_L \left(\frac{\mathrm{Im}(V_{ts}^* V_{td})}{\lambda^5} X(x_t) \right)^2$$

$K_L \to \pi^0 \bar{\nu} \nu$: Theoretical Status



Matrix element extracted from K_{l3} decays. $N^{\frac{3}{2}}LO$ χPT
[Mescia, Smith ´07; Bijnens, Ghorbani ´07]

No further long distance uncertainty

$X(x_t)$: NLO QCD calculation: ±1% error
[Misiak, Urban ´99; Buchalla, Buras ´99]

$X(x_t)$: Electroweak (EW) corrections: ±2% error
[Buchalla, Buras ´99]

Reduce error with 2 loop electroweak calculation

$X(x_t)$: Electroweak Corrections

- $X(x_t)$: Dominant theoretical uncertainty for $K_L \to \pi^0 \bar{\nu} \nu$

- For example a change $\sin^{OS}\theta_W \leftrightarrow \sin^{\overline{MS}}\theta_W$ results in 5% uncertainty

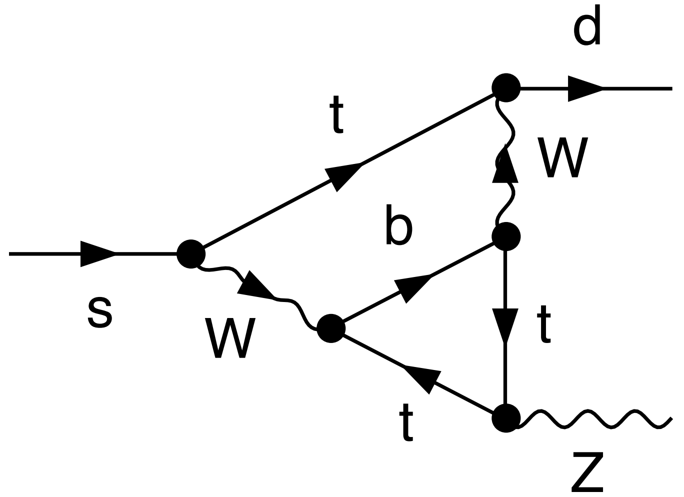
$$H_{eff} = \frac{4G_F}{\sqrt{2}} \frac{\alpha V_{ts}^* V_{td}}{2\pi \sin^2 \Theta_W} X(x_t) Q_\nu$$

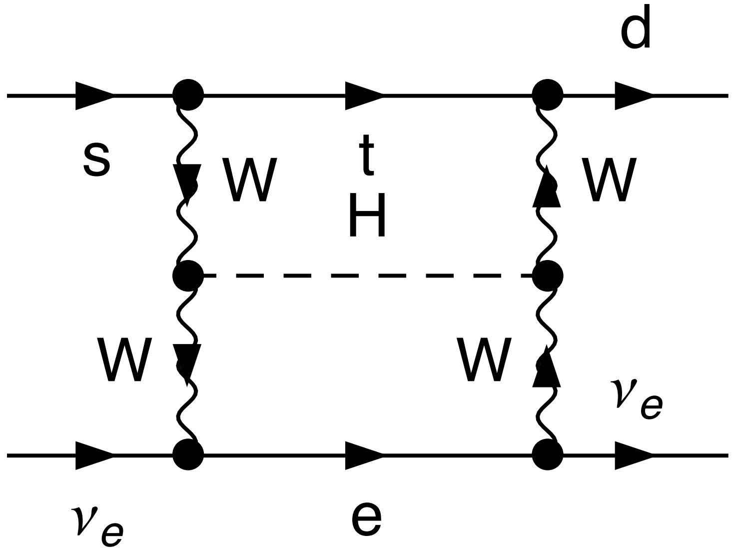
- Uncertainty estimated in the large m_t limit $\sim 2\%$
  [Buchalla, Buras '99]

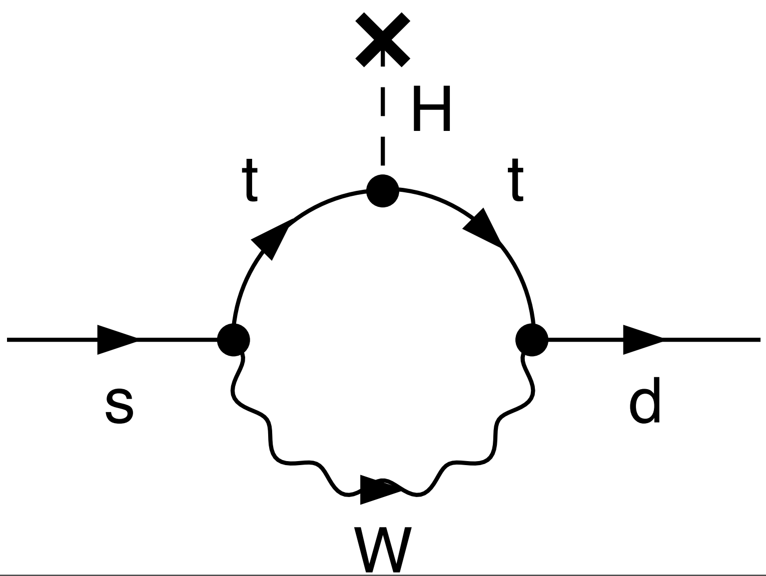
- Dominant uncertainty: Do the calculation!
  [Brod, MG, Stamou]

$X(x_t)$: Electroweak Corrections

- Use the $\overline{\text{MS}}$ scheme
- VEV minimises renormalised potential: include tadpoles
- Traces with γ_5: use HV scheme
- Renormalisation:
  locality, UV OK
  To do: check gauge independence
- Numerics: Rather long expressions - not yet stable.







$K^+ \to \pi^+ \bar{\nu}\nu$ and $K_L \to \pi^0 \bar{\nu}\nu$

Different from $K_L \to \pi^0 \bar{\nu}\nu$

- CP conserving: Top & charm contribute

$$\mathcal{B}r\left(K^+ \to \pi^+\nu\bar{\nu}(\gamma)\right) = \kappa_+(1+\Delta_{EM})$$

$$\times \left| \frac{V_{ts}^* V_{td} X_t(m_t^2) + \lambda^4 \mathrm{Re} V_{cs}^* V_{cd} \left(P_c(m_c^2) + \delta P_{c,u}\right)}{\lambda^5} \right|^2 .$$

$\frac{m_c^2}{M_W^2}$ suppression lifted by $\log(\frac{m_c}{M_W})\frac{1}{\lambda^4}$

Like in $K_L \to \pi^0 \bar{\nu}\nu$

- Only Q_ν: Quadratic GIM & Isospin symmetry
- Top quark contribution like in $K_L \to \pi^0 \bar{\nu}\nu$

$K^+ \to \pi^+ \bar{\nu}\nu$ Long distance

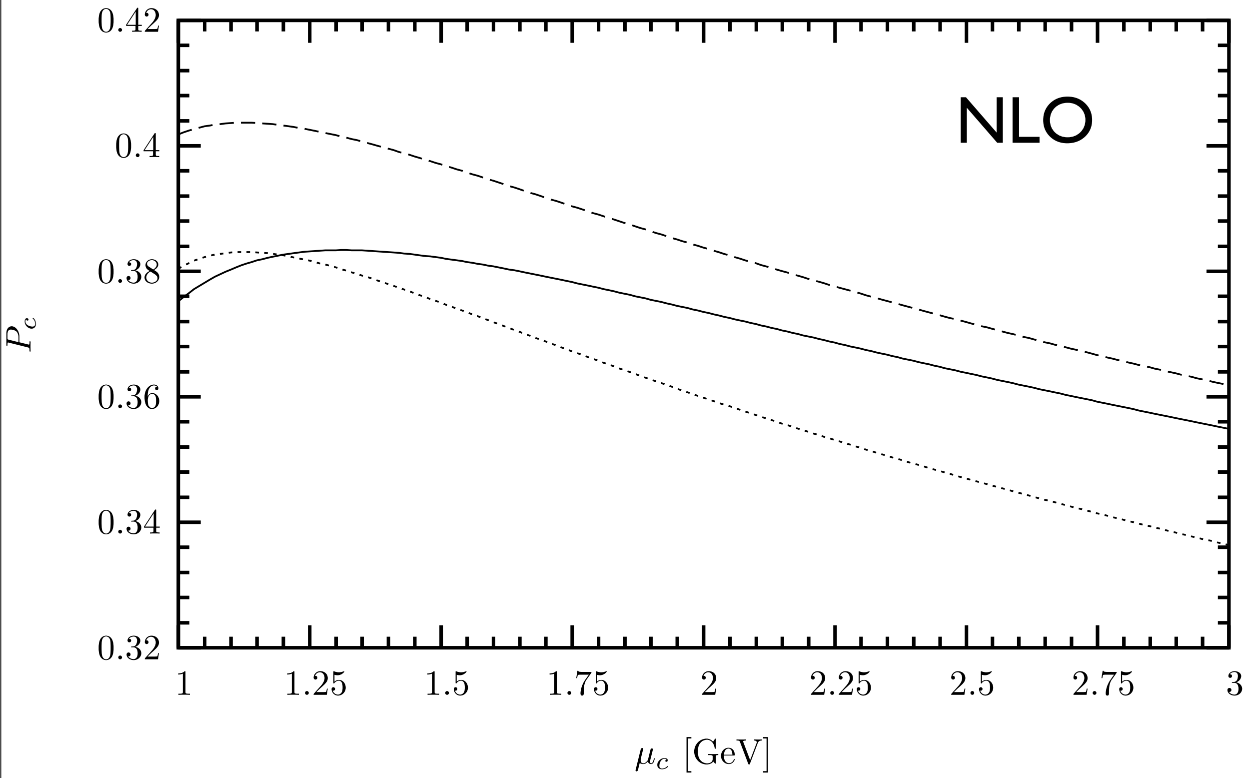
- Matrix element extracted from K_{l3} decays
  [Mescia, Smith ´07]
- $K^+ \to \pi^+ \nu\bar{\nu}$ is $K^+ \to \pi^+ \nu\bar{\nu}(\gamma)$
  QED radiative corrections included:

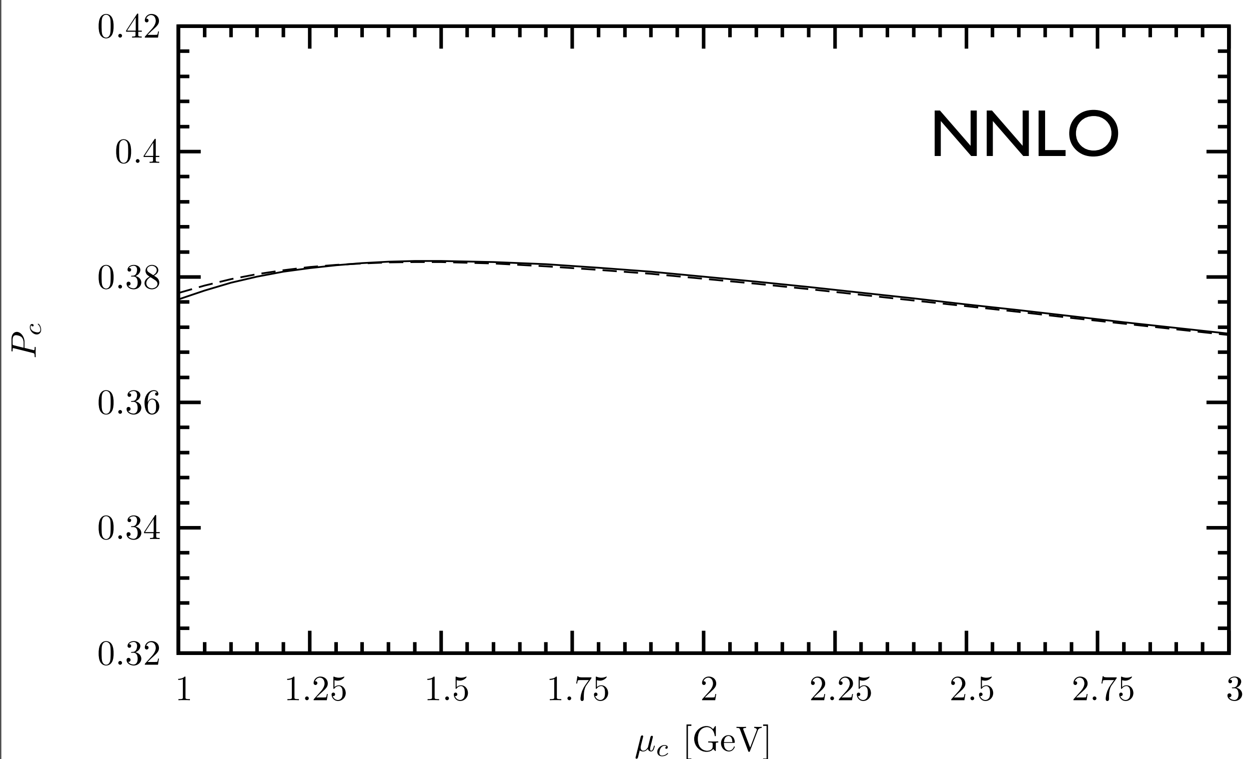
$$\Delta_{EM}(E_\gamma < 20\text{MeV}) = -0.003$$

- Uncertainty in $\kappa_+(1-\Delta_{EM})$ reduced by $\frac{1}{7}$

- Below charm scale: Dimension 8 operators
  [Falk et. al. ´01]
- Together with light quarks: $\delta P_{c,u} = 0.04 \pm 0.02$
  [Isidori, Mescia, Smith ´05]
- Could be Improved by Lattice [Isidori et. al. ´05]

$K^+ \to \pi^+ \bar{\nu}\nu$ charm contr. (QCD)





- Resum $\log \frac{m_c}{M_W}$ in P_c

P_c at NLO: ±10% (theory)
[Buras, MG, Haisch, Nierste ´05]

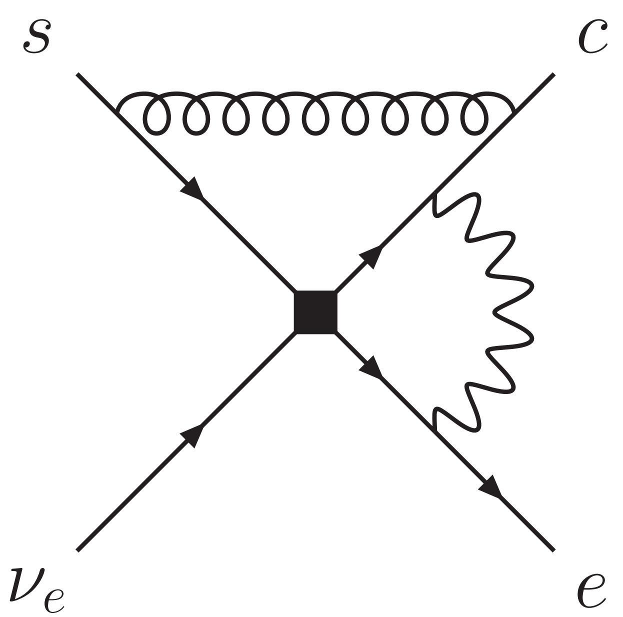
P_c at NNLO: ±2.5% (theory)
[Buras, MG, Haisch, Nierste ´05]

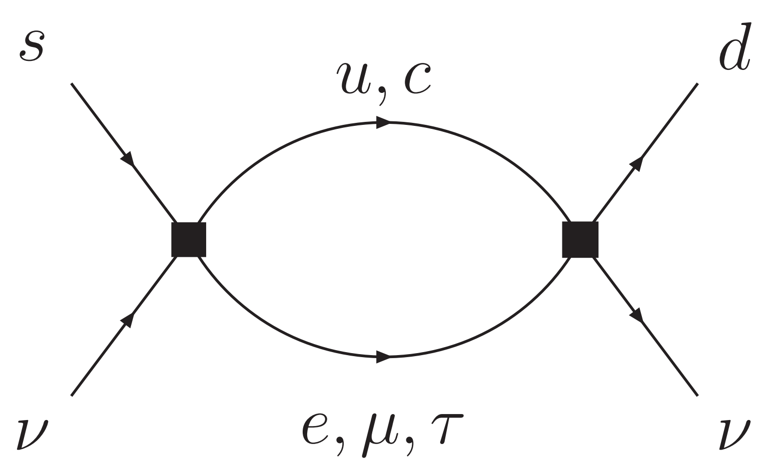
- Dominant parametric uncertainty in P_c:

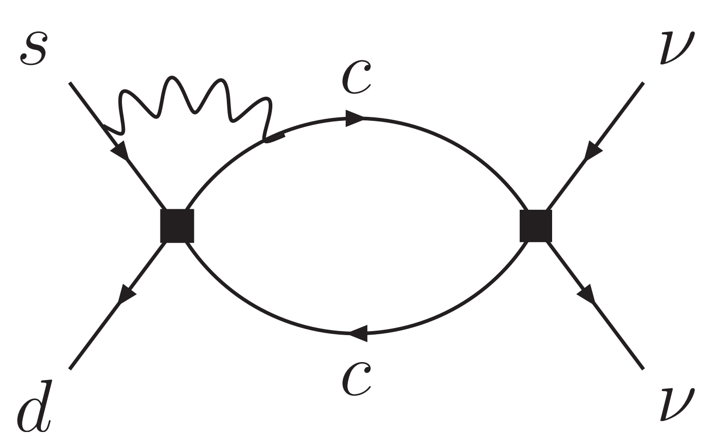
$m_c = (1.3 \pm 0.05)\text{GeV}$ **75%**

$\alpha_s = 0.1187 \pm 0.002$ **25%**

$K^+ \to \pi^+ \bar{\nu}\nu$ charm contr. (EW)

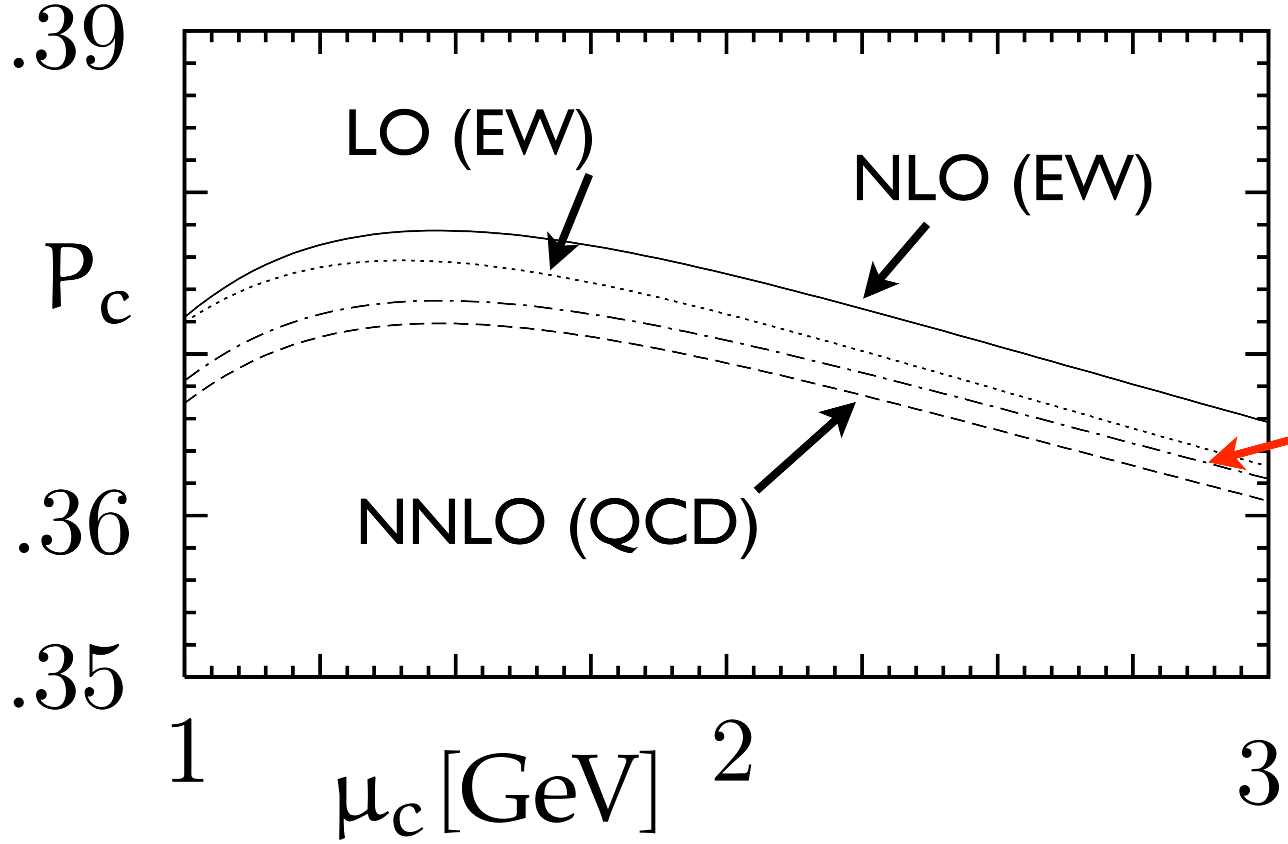






- Large QED logs? Does Q_ν run?
- Semileptonic operator has QED running and mixes into Q_ν.
- No $\mathcal{O}(\alpha/\alpha_s)$ but $\mathcal{O}(\alpha)$ corrections: NLO QEDxQCD calculation
- Bilocal mixing is $\mathcal{O}(G_F^2)$
- What is the parameter $x_c = \frac{m_c^2}{M_W^2}$
- EW corrections define M_W

$K^+ \to \pi^+ \bar{\nu}\nu$ charm contr. (EW)



[Brod, MG ´08]

- Use $\overline{MS}$ scheme
- Normalise to G_F
- use

$$x_c = \sqrt{2}\frac{\sin^2\theta_W}{\pi\alpha} G_F m_c^2(\mu_c)$$

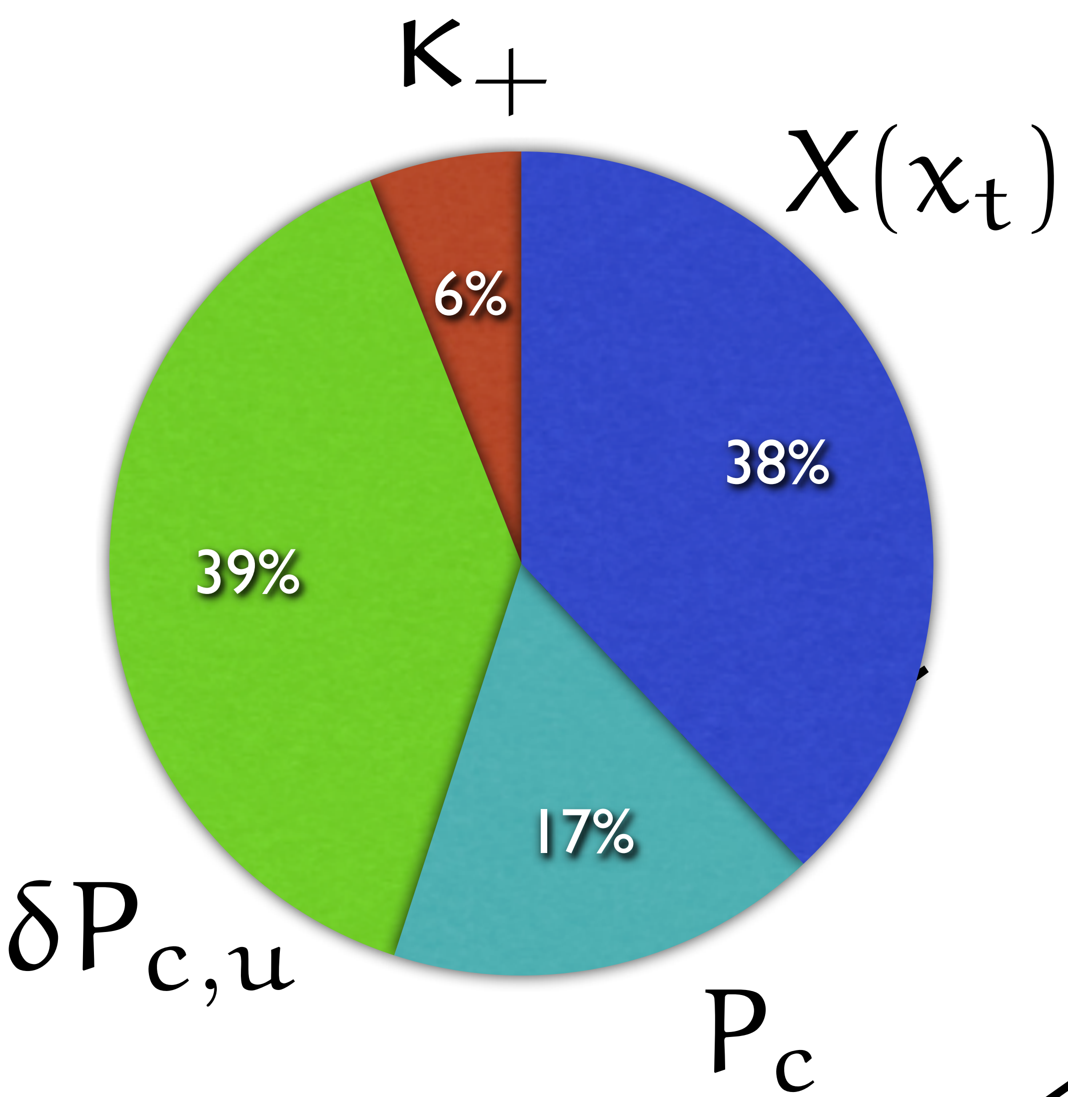
- instead of

$$x_c = \frac{m_c(\mu)^2}{M_W^2}$$

- P_c enhanced by up to 2% for all EW

$K^+ \to \pi^+ \bar{\nu}\nu$ Error budget

Theory error budget



NA62 10% error

for $m_c(m_c) = (1286 \pm 13)\mathrm{MeV}$
[Kühn et. al. ´07]

$$\mathcal{B}r_{K^+} = (0.85 \pm 0.07) \times 10^{-10}$$

Theory error: 10%x30% = 3%

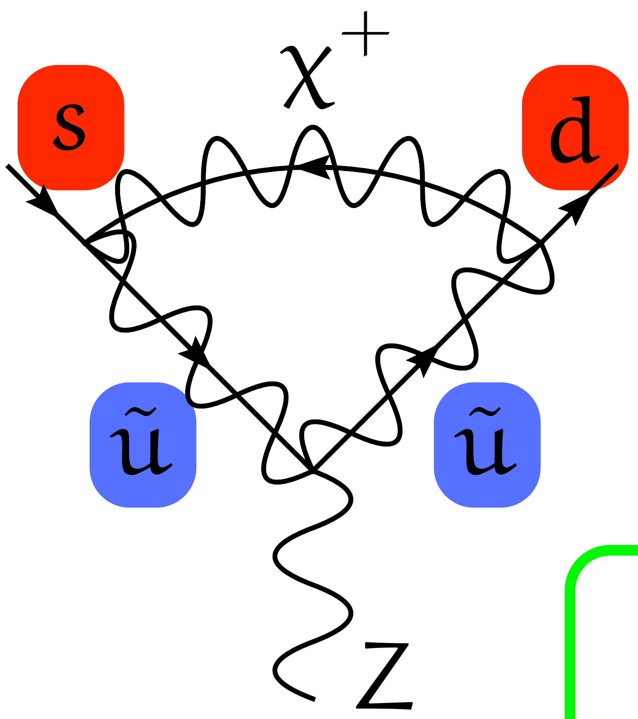
for $m_c(m_c) = (1224 \pm 57)\mathrm{MeV}$
[Hoang et. al. ´05]

$$\mathcal{B}r_{K^+} = (0.80 \pm 0.08) \times 10^{-10}$$

Experiment [E787, E949 ´08]

$$\mathcal{B}r_{K^+} = (1.73^{+1.15}_{-1.05}) \times 10^{-10}$$

$K \to \pi \bar{\nu} \nu$ in the MSSM



New physics in: $X(x_t) \to X(x_t, \tilde{m}, \tilde{M})$

MSSM is a 2HDM of Type II:

$$\mathcal{L} = -Y^d_{ij} H_d \bar{d}^i_R q^j - Y^u_{ij} H_u \bar{u}^i_R q^j + \text{h.c.}$$

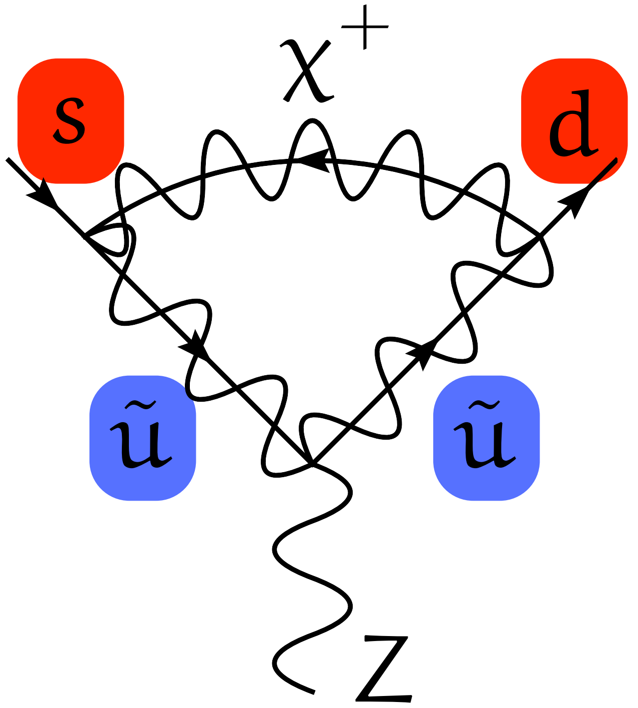
for small $\tan\beta = v_u / v_d = \mathcal{O}(1)$

Yukawa and mass-matrix aligned

Flavour Violation in squark mass matrix

$$\hat{\mathcal{M}}^2_{\tilde{u}} = \begin{pmatrix} \hat{M}^2_{\tilde{u}_L} & v_u \hat{A}^\dagger_u - v_d \mu \hat{Y}^\dagger_u \\ v_u \hat{A}_u - v_d \mu^* \hat{Y}_u & \hat{M}^2_{\tilde{u}_R} \end{pmatrix}$$

$K \to \pi \bar{\nu} \nu$ in the MSSM: MFV



Minimal Flavour Violation:
Aligned squarks and quarks

No strong enhancement possible.
Interesting correlations with other observable

[Buras, Gambino, MG, Jäger, Silvestrini `00; Isidori, Mescia, Paradisi, Smith, Trine `06]

Diagonal

Flavour Violation in squark mass matrix

$$\hat{\mathcal{M}}^2_{\tilde{u}} = \begin{pmatrix} \hat{M}^2_{\tilde{u}_L} & v_u \hat{A}_u^\dagger - v_d \mu \hat{Y}_u^\dagger \\ v_u \hat{A}_u - v_d \mu^* \hat{Y}_u & \hat{M}^2_{\tilde{u}_R} \end{pmatrix}$$

$K \to \pi \bar{\nu} \nu$ and non MFV

Offdiagonal squark mass-matrix: Extra Flavour Violation

Diagonalisation:
Mass insertions

$$\delta^{u}_{LR_{ij}} = \frac{\hat{\mathcal{M}}^2_{\tilde{u}_{i_R j_L}}}{\hat{\mathcal{M}}^2_{\tilde{u}}}$$

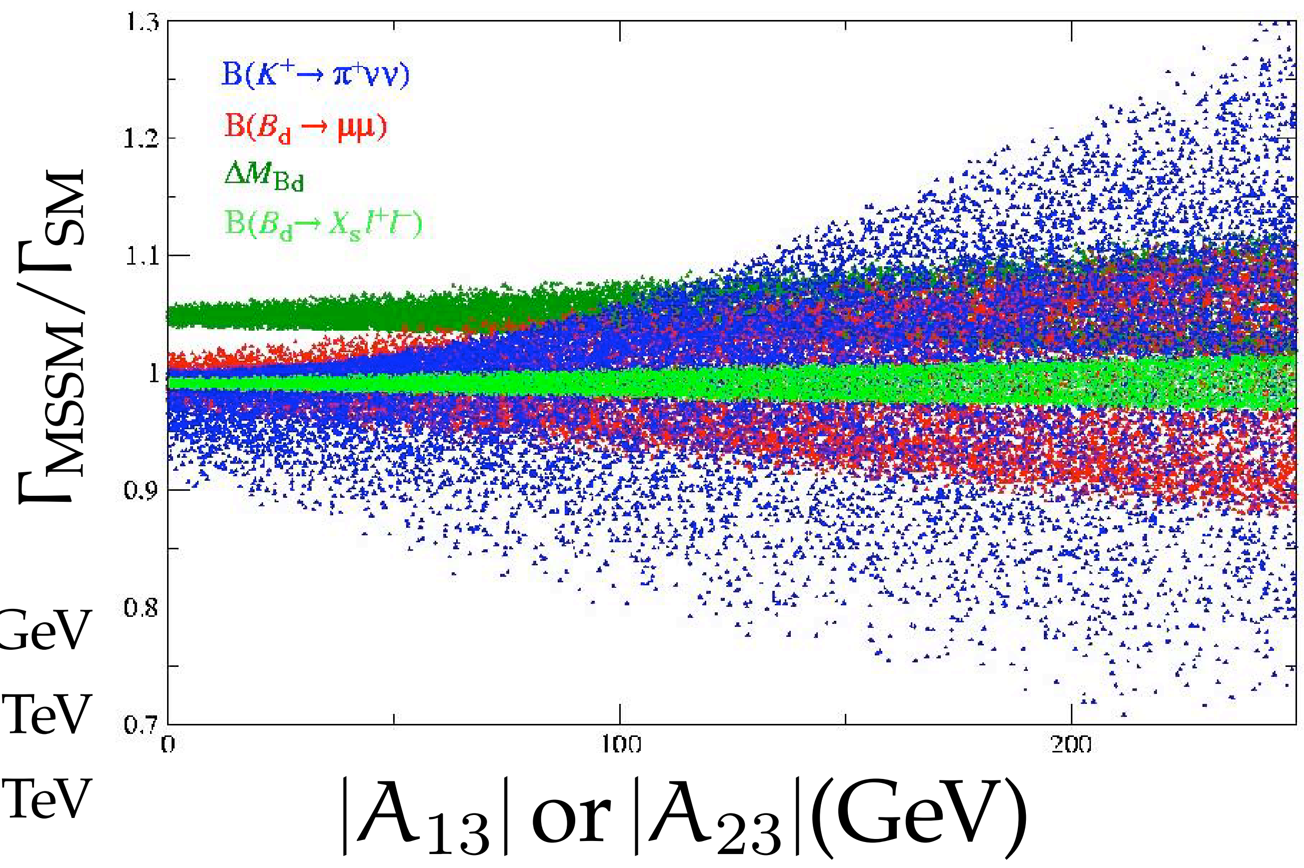
sensitive to A_u [Isidori, Mescia, Paradisi, Smith, Trine `06]

2 LR insertions dominant

[Colangelo, Isidori ´98]

$$X^2_{LL} \propto \delta^{u*}_{LR_{ts}} \delta^{u}_{LR_{td}}$$

No CKM suppression



$\tan\beta = 2-4 \quad \mu = 500 \pm 10\text{GeV} \quad M_2 = 300 \pm 10\text{GeV}$

$M_{\tilde{u}_R} = 600 \pm 20\text{GeV} \quad M_{\tilde{q}_L} = 800 \pm 20\text{GeV} \quad A_0 = 1\text{TeV}$

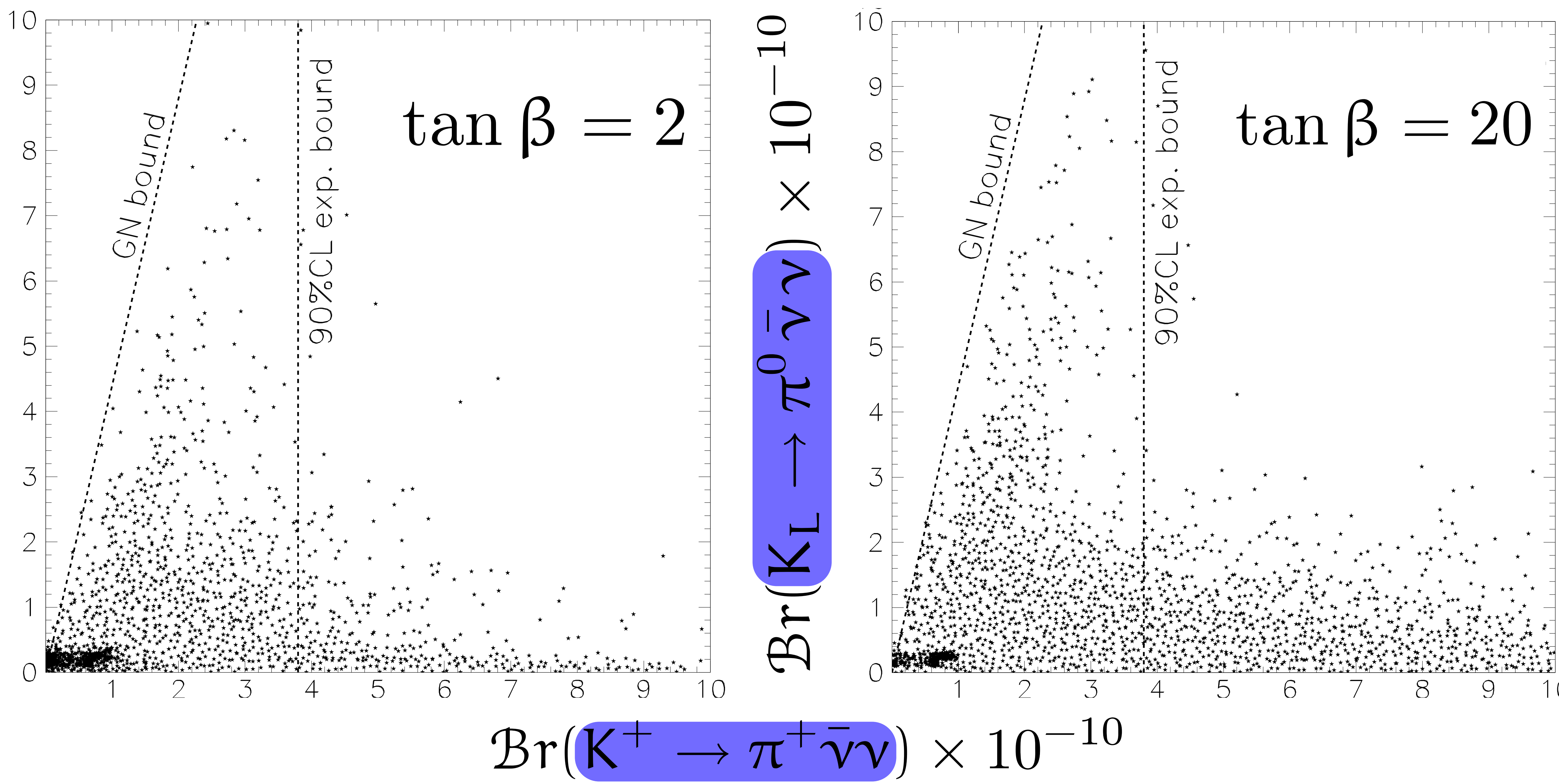
$M_1 = 500\text{GeV} \quad M_{\tilde{d}_R} = M_{\tilde{l}} = M_3 = M_{\tilde{H}^+} = 2\text{TeV}$

$K \to \pi \bar{\nu} \nu$ can reach the Bounds

MSSM 66 parameter scan: GN bound saturated

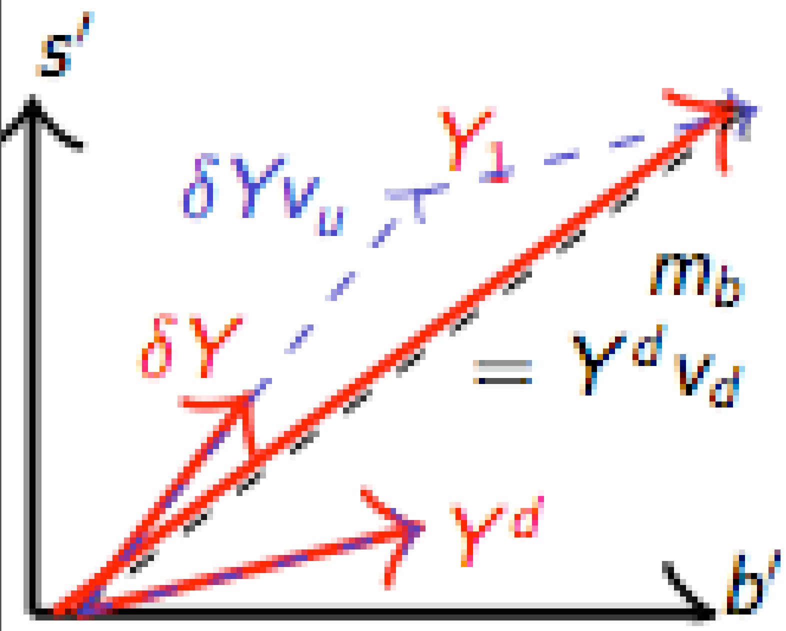
[Buras, Ewerth, Jäger, Rosiek´05]

Potential large effects for: $K_L \to \pi^0 \bar{\nu} \nu$



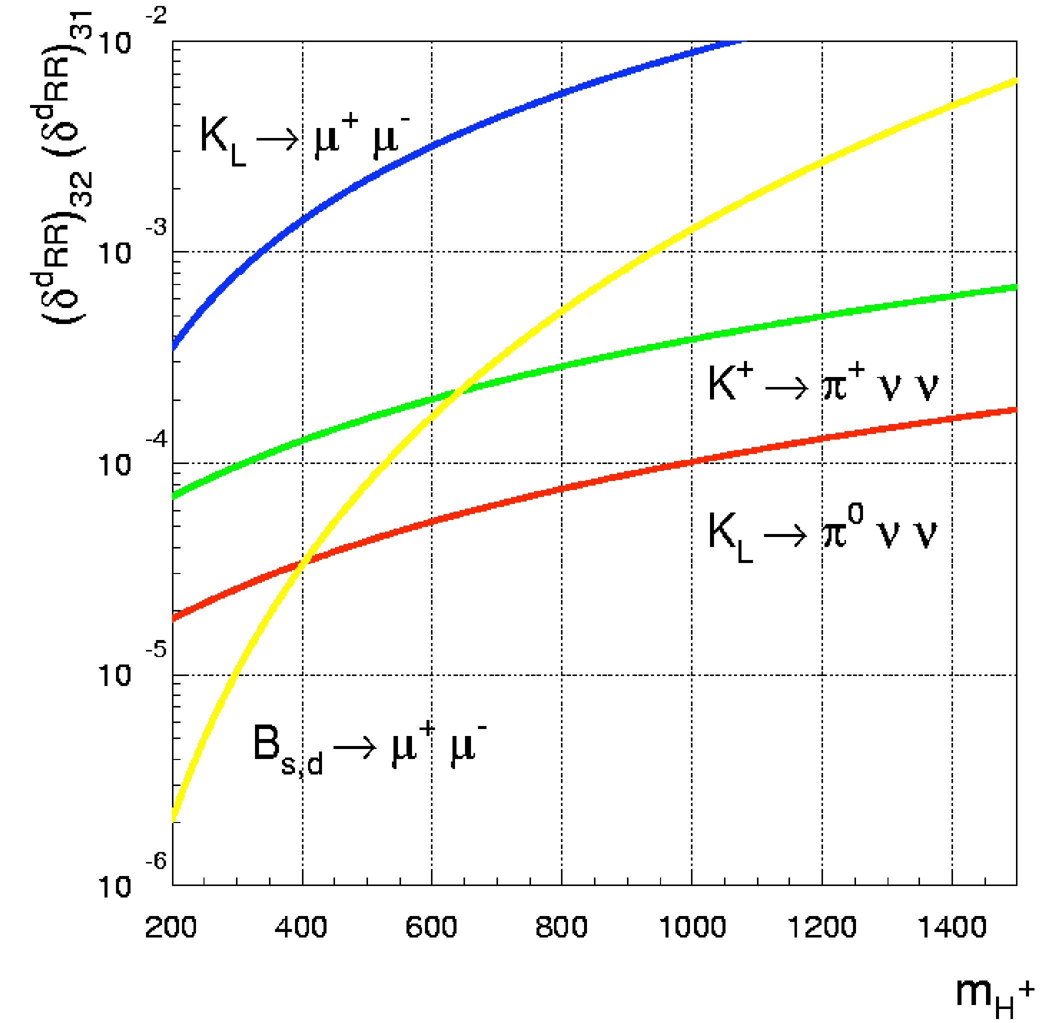
$K \to \pi \bar{\nu} \nu$ and large $\tan \beta$

1-loop: H_u couples to down quarks



$$\mathcal{L}^{Y}_{\text{eff}} = \delta Y_{ij}\, \bar{d}^{i}_{R} H^{*}_{u} \cdot Q^{j}_{L}$$

$$\to \cancel{Y^{d}_{ij} \propto M^{d}_{ij}}$$



$\to \tan \beta$ Enhanced effects

$B_s \to \mu^+\mu^-$ in MFV: $(\tan \beta)^6$
[Babu, Kolda `00]

$B \to \mu^+\mu^-$:improved bound

$K \to \pi \bar{\nu} \nu$ Beyond MFV:

$$(\tan \beta)^4 (\delta^{d}_{RR_{ts}} \delta^{d}_{RR_{td}})^2$$

[Isidori, Paradisi `06]

$K \to \pi \bar{\nu} \nu$:decouples slower

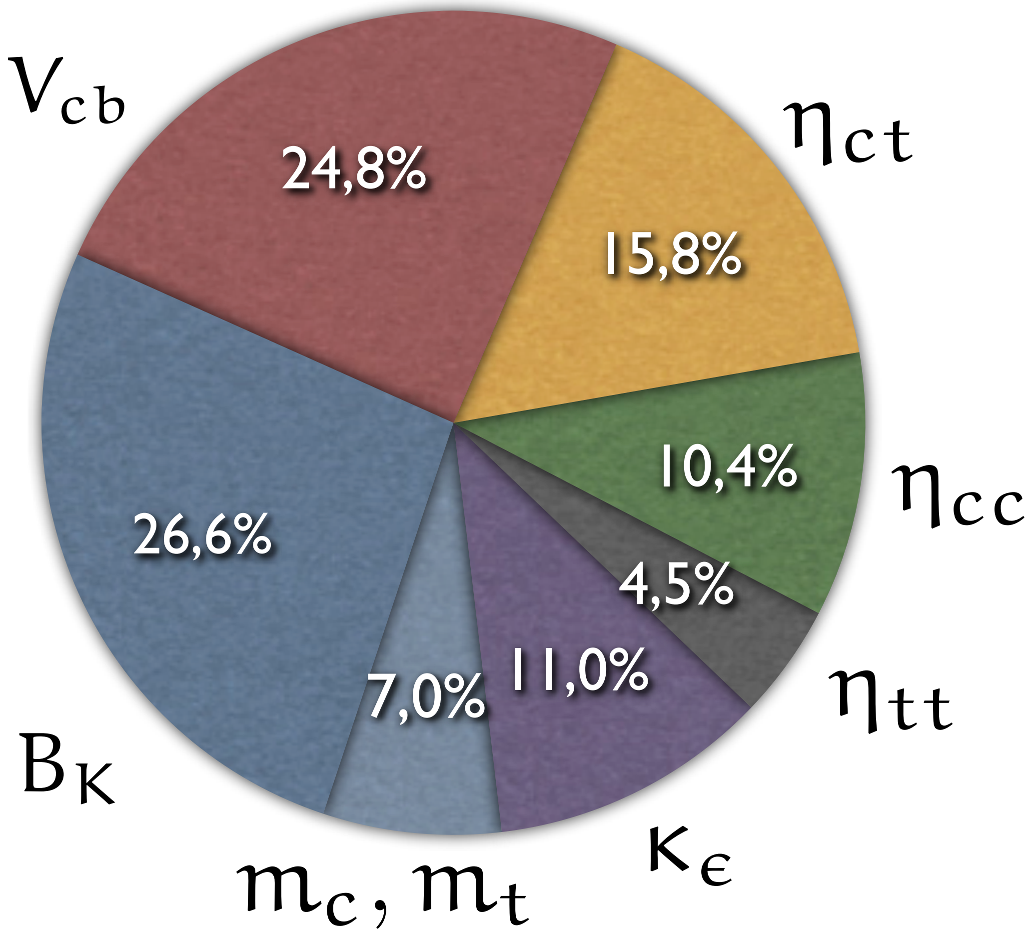
Complementary Information

Some Comments on ϵ_K

- ϵ_K is in particular sensitive to deviations from MFV
- Talk by Diego Guadagnoli: will discuss ϵ_K

ϵ_K error budget:

$$\mathcal{H} \propto [\lambda_c^2 \eta_{cc} S_0(x_c) + \lambda_t^2 \eta_{tt} S_0(x_t) + \lambda_c \lambda_t \eta_{tc} S_0(x_c, x_t)] Q_K b_K(\mu)$$



error for B_K and η_{ij} of similar size

Perform 3 loop RGE analysis of η_{ct}

3 loop matching for η_{cc}

[Brod, MG]

Conclusions

$K \to \pi \bar{\nu} \nu$ and $K_L \to \pi^0 l^+ l^-$ provide a unique test of the SM and its extensions

$K \to \pi \bar{\nu} \nu$ the cleanest + future improvements

$K_L \to \pi^0 l^+ l^-$ different sensitivity to New Physics
Theory prediction could be improved by exp.

	Theory	Experiment
$K_L \to \pi^0 e^+ e^-$	$(3.54\,^{+0.98}_{-0.85}) \times 10^{-11}$	$< 28 \times 10^{-11}$ KTEV
$K_L \to \pi^0 \mu^+ \mu^-$	$(1.41\,^{+0.28}_{-0.26}) \times 10^{-11}$	$< 38 \times 10^{-11}$ KTEV
$K_L \to \pi^0 \nu \bar{\nu}$	$(2.76 \pm 0.40) \times 10^{-11}$	$< 6.7 \times 10^{-8}$ E391a
$K^+ \to \pi^+ \nu \bar{\nu}$	$(8.51 \pm 0.70) \times 10^{-11}$	$(1.73\,^{+1.15}_{-1.05}) \times 10^{-10}$ E787 E949