Lepton Flavour Violation in Tau Decays

Sascha Turczyk

in collaboration with

Benjamin Dassinger Thorsten Feldmann Thomas Mannel

based on [JHEP **0710** (2007) 039]

Theoretische Physik 1
Universität Siegen

Workshop on New Physics, Flavors and Jets, Ringberg Castle
April 27, 2009

Content

Motivation

Why New Physics?

- First evidence of new physics (NP): Neutrino oscillation
- Incorporating neutrino oscillation into SM

$\Rightarrow$ Lepton flavour violating (LFV) decays, but practically unobservable in the SM

- New physics at the TeV scale

$\Rightarrow$ Potentially sizeable LFV decays

How to Identify Beyond SM Effects

- Reliable experimental bounds on LFV decay modes

$\Rightarrow$ Requires knowledge on phase space dependence

- Compare particular NP models (SUSY, Little Higgs, ...)
- Or: Model-independent approach (this work)

Motivation

Why New Physics?

- First evidence of new physics (NP): Neutrino oscillation
- Incorporating neutrino oscillation into SM

$\Rightarrow$ Lepton flavour violating (LFV) decays, but practically unobservable in the SM

- New physics at the TeV scale

$\Rightarrow$ Potentially sizeable LFV decays

How to Identify Beyond SM Effects

- Reliable experimental bounds on LFV decay modes

$\Rightarrow$ Requires knowledge on phase space dependence

- Compare particular NP models (SUSY, Little Higgs, ...)
- Or: Model-independent approach (this work)

Idea of the Analysis

Bottom-Up Approach

- Regard SM as an EFT: Expand NP effects in terms of $1/\Lambda$
- All NP models can be matched onto EFT

Goals

- Effective operators differ in chirality, Dirac structure, ...
- $\Rightarrow$ Depending on NP model: Dominance of particular operators
- $\Rightarrow$ Consistent exp. constraints on NP for a given scenario
- Experimental bounds have to take into account different phase space distributions

Choose Decay Modes

- Muons have excellent experimental signature, at e.g. LHC
- $\Rightarrow$ Here: Concentrate on $\tau^\pm \to \mu^\pm \mu^+ \mu^-$

Idea of the Analysis

Bottom-Up Approach

- Regard SM as an EFT: Expand NP effects in terms of $1/\Lambda$
- All NP models can be matched onto EFT

Goals

- Effective operators differ in chirality, Dirac structure, ...

$\Rightarrow$ Depending on NP model: Dominance of particular operators

$\Rightarrow$ Consistent exp. constraints on NP for a given scenario

- Experimental bounds have to take into account different phase space distributions

Choose Decay Modes

- Muons have excellent experimental signature, at e.g. LHC

$\Rightarrow$ Here: Concentrate on $\tau^{\pm} \to \mu^{\pm}\mu^{+}\mu^{-}$

Idea of the Analysis

Bottom-Up Approach

- Regard SM as an EFT: Expand NP effects in terms of $1/\Lambda$
- All NP models can be matched onto EFT

Goals

- Effective operators differ in chirality, Dirac structure, ...

$\Rightarrow$ Depending on NP model: Dominance of particular operators

$\Rightarrow$ Consistent exp. constraints on NP for a given scenario

- Experimental bounds have to take into account different phase space distributions

Choose Decay Modes

- Muons have excellent experimental signature, at e.g. LHC

$\Rightarrow$ Here: Concentrate on $\tau^{\pm} \to \mu^{\pm}\mu^{+}\mu^{-}$

Leading Operators

Structure

- Respect gauge symmetry of SM
- Group leptons into (incomplete) doublets v: Higgs VEV

$$L = \begin{pmatrix} \nu_L \\ \ell_L \end{pmatrix}, \quad R = \begin{pmatrix} 0 \\ \ell_R \end{pmatrix}, \quad H = \frac{1}{\sqrt{2}} \begin{pmatrix} v + h_0 + i\chi_0 & \sqrt{2}\phi_+ \\ -\sqrt{2}\phi_- & v + h_0 - i\chi_0 \end{pmatrix}$$

dim=6 leptonic:

$$O_1 = (\bar{L}\gamma_\mu L)(\bar{L}\gamma^\mu L)$$
$$O_2 = (\bar{L}\tau^a\gamma_\mu L)(\bar{L}\tau^a\gamma^\mu L)$$
$$O_3 = (\bar{R}\gamma_\mu R)(\bar{R}\gamma^\mu R)$$
$$O_4 = (\bar{R}\gamma_\mu R)(\bar{L}\gamma^\mu L)$$

dim=6 radiative:

$$R_1 = g'(\bar{L}H\sigma_{\mu\nu}R)B^{\mu\nu}$$
$$R_2 = g(\bar{L}\tau^a H\sigma_{\mu\nu}R)W^{\mu\nu,a}$$

Ingredients

- $B^{\mu\nu}$: $U(1)_Y$ and $W^{\mu\nu,a}$: $SU(2)_L$ gauge field
- Only tree-level contribution for simplicity

General Case

Leading Operators

Structure

- Respect gauge symmetry of SM
- Group leptons into (incomplete) doublets

v: Higgs VEV

$$L = \begin{pmatrix} \nu_L \\ \ell_L \end{pmatrix}, \quad R = \begin{pmatrix} 0 \\ \ell_R \end{pmatrix}, \quad H = \frac{1}{\sqrt{2}} \begin{pmatrix} v + h_0 + i\chi_0 & \sqrt{2}\phi_+ \\ -\sqrt{2}\phi_- & v + h_0 - i\chi_0 \end{pmatrix}$$

dim=6 leptonic:

$$O_1 = (\bar{L}\gamma_\mu L)(\bar{L}\gamma^\mu L)$$
$$O_2 = (\bar{L}\tau^a\gamma_\mu L)(\bar{L}\tau^a\gamma^\mu L)$$
$$O_3 = (\bar{R}\gamma_\mu R)(\bar{R}\gamma^\mu R)$$
$$O_4 = (\bar{R}\gamma_\mu R)(\bar{L}\gamma^\mu L)$$

dim=6 radiative:

$$R_1 = g'(\bar{L}H\sigma_{\mu\nu}R)B^{\mu\nu}$$
$$R_2 = g(\bar{L}\tau^a H\sigma_{\mu\nu}R)W^{\mu\nu,a}$$

Ingredients

- $B^{\mu\nu}$: $U(1)_Y$ and $W^{\mu\nu,a}$: $SU(2)_L$ gauge field
- Only tree-level contribution for simplicity

▸ General Case

Subleading Operators: Dimension 8

dim=8 leptonic:

$$P_1 = (\bar{L}HR)(\bar{L}HR)$$

$$P_2 = (\bar{L}\tau^a HR)(\bar{L}\tau^a HR)$$

$$Q_1 = (\bar{L}HR)(\bar{R}H^\dagger L)$$

$$Q_2 = (\bar{L}\tau^a HR)(\bar{R}H^\dagger \tau^a L)$$

$$P_1^{(T)} = (\bar{L}H\sigma_{\mu\nu}R)(\bar{L}H\sigma^{\mu\nu}R)$$

$$P_2^{(T)} = (\bar{L}\tau^a H\sigma_{\mu\nu}R)(\bar{L}\tau^a H\sigma^{\mu\nu}R)$$

Ingredients

- Neglect covariant derivates: small lepton Yukawa coupling
- $\tan\beta$ enhancement in 2HDM models possible
- Neglect Dim 8 operators due to v^2/Λ^2 suppression

Relevant Hamilton-Operators

Most General Effective Hamilton-Operator

- Sum over all operators with flavour dependent couplings
- Project out relevant charged LFV operators

Leading Operators O_{1-4} with Coupling Constants

$$H_{\text{eff}}^{(LL)(LL)} = g_V^{(LL)(LL)} \frac{(\bar{\ell}_L \gamma_\mu \tau_L)(\bar{\ell}'_L \gamma^\mu \ell''_L)}{\Lambda^2}, \quad H_{\text{eff}}^{(RR)(RR)} = g_V^{(RR)(RR)} \frac{(\bar{\ell}_R \gamma_\mu \tau_R)(\bar{\ell}'_R \gamma^\mu \ell''_R)}{\Lambda^2}$$

$$H_{\text{eff}}^{(LL)(RR)} = g_V^{(LL)(RR)} \frac{(\bar{\ell}_L \gamma_\mu \tau_L)(\bar{\ell}'_R \gamma^\mu \ell''_R)}{\Lambda^2} + g_V^{(RR)(LL)} \frac{(\bar{\ell}_R \gamma_\mu \tau_R)(\bar{\ell}'_L \gamma^\mu \ell''_L)}{\Lambda^2}$$

Leading Operators R_{1-2} with Coupling Constants

$$\langle \ell' \bar{\ell} \ell | H_{\text{eff}}^{\text{rad}} | \tau \rangle = \alpha_{em} \frac{v}{\Lambda^2} \frac{q^\nu}{q^2} \sum_{h,s} g_{\text{rad}}^{(s,h)} \langle \ell' \bar{\ell} \ell | \left(\bar{\ell}'_h (-i\sigma_{\mu\nu}) \tau_s \right) \left(\bar{\ell} \gamma^\mu \ell \right) | \tau \rangle, \quad (s,h) = \begin{matrix} (L,R) \\ (R,L) \end{matrix}$$

Radiative Operators

Relevant Hamilton-Operators

Most General Effective Hamilton-Operator

- Sum over all operators with flavour dependent couplings
- Project out relevant charged LFV operators

Leading Operators O_{1-4} with Coupling Constants

$$H_{\rm eff}^{(LL)(LL)} = g_V^{(LL)(LL)} \frac{(\bar\ell_L \gamma_\mu \tau_L)(\bar\ell'_L \gamma^\mu \ell''_L)}{\Lambda^2}\,, \quad H_{\rm eff}^{(RR)(RR)} = g_V^{(RR)(RR)} \frac{(\bar\ell_R \gamma_\mu \tau_R)(\bar\ell'_R \gamma^\mu \ell''_R)}{\Lambda^2}$$

$$H_{\rm eff}^{(LL)(RR)} = g_V^{(LL)(RR)} \frac{(\bar\ell_L \gamma_\mu \tau_L)(\bar\ell'_R \gamma^\mu \ell''_R)}{\Lambda^2} + g_V^{(RR)(LL)} \frac{(\bar\ell_R \gamma_\mu \tau_R)(\bar\ell'_L \gamma^\mu \ell''_L)}{\Lambda^2}$$

Leading Operators R_{1-2} with Coupling Constants

$$\langle \ell' \bar\ell \ell | H_{\rm eff}^{\rm rad} | \tau \rangle = \alpha_{em} \frac{v}{\Lambda^2} \frac{q^\nu}{q^2} \sum_{h,s} g_{\rm rad}^{(s,h)} \langle \ell' \bar\ell \ell | \left(\bar\ell'_h (-i\sigma_{\mu\nu}) \tau_s \right) \left(\bar\ell \gamma^\mu \ell \right) | \tau \rangle\,, \quad (s,h) = \binom{L,R}{R,L}$$

Radiative Operators

Relevant Hamilton-Operators

Most General Effective Hamilton-Operator

- Sum over all operators with flavour dependent couplings
- Project out relevant charged LFV operators

Leading Operators O_{1-4} with Coupling Constants

$$H_{\text{eff}}^{(LL)(LL)} = g_V^{(LL)(LL)} \frac{(\bar{\ell}_L \gamma_\mu \tau_L)(\bar{\ell}'_L \gamma^\mu \ell''_L)}{\Lambda^2}, \quad H_{\text{eff}}^{(RR)(RR)} = g_V^{(RR)(RR)} \frac{(\bar{\ell}_R \gamma_\mu \tau_R)(\bar{\ell}'_R \gamma^\mu \ell''_R)}{\Lambda^2}$$

$$H_{\text{eff}}^{(LL)(RR)} = g_V^{(LL)(RR)} \frac{(\bar{\ell}_L \gamma_\mu \tau_L)(\bar{\ell}'_R \gamma^\mu \ell''_R)}{\Lambda^2} + g_V^{(RR)(LL)} \frac{(\bar{\ell}_R \gamma_\mu \tau_R)(\bar{\ell}'_L \gamma^\mu \ell''_L)}{\Lambda^2}$$

Leading Operators R_{1-2} with Coupling Constants

$$\langle \ell' \bar{\ell} \ell | H_{\text{eff}}^{\text{rad}} | \tau \rangle = \alpha_{em} \frac{v}{\Lambda^2} \frac{q^\nu}{q^2} \sum_{h,s} g_{\text{rad}}^{(s,h)} \langle \ell' \bar{\ell} \ell | \left(\bar{\ell}'_h (-i\sigma_{\mu\nu}) \tau_s \right) \left(\bar{\ell} \gamma^\mu \ell \right) | \tau \rangle, \quad (s,h) = \begin{smallmatrix} (L,R) \\ (R,L) \end{smallmatrix}$$

Radiative Operators

The Decay Modes and Couplings

Leading Coupling Matrices

- Matrices in flavour space
- Vector current: $g_V^{(L_i L^j)(L_k L^l)}$, $g_V^{(R_i R^j)(R_k R^l)}$, $g_V^{(L_i L^j)(R_k R^l)}$, $g_V^{(R_i R^j)(L_k L^l)}$
- Radiative current $g_{\mathrm{rad}}^{(L_i R^j)}$, $g_{\mathrm{rad}}^{(R_i L^j)}$

The Different Decay Modes

$$\tau^- \to e^- e^- e^+ \qquad \tau^- \to \mu^- \mu^- e^+$$
$$\tau^- \to \mu^- \mu^- \mu^+ \qquad \tau^- \to \mu^- e^- e^+$$
$$\tau^- \to e^- e^- \mu^+ \qquad \tau^- \to e^- \mu^- \mu^+$$

- Take into account Pauli principle for two identical particles
- Red marked: Contribution from $H_{\mathrm{eff}}^{\mathrm{rad}}$

$$\tau^- \to \ell'^- \gamma^* \to \ell'^- (\ell^+ \ell^-)$$

The Decay Modes and Couplings

Leading Coupling Matrices

- Matrices in flavour space
- Vector current: $g_V^{(L_i L^j)(L_k L^l)}$, $g_V^{(R_i R^j)(R_k R^l)}$, $g_V^{(L_i L^j)(R_k R^l)}$, $g_V^{(R_i R^j)(L_k L^l)}$
- Radiative current $g_{\mathrm{rad}}^{(L_i R^j)}$, $g_{\mathrm{rad}}^{(R_i L^j)}$

The Different Decay Modes

$\tau^- \to e^- e^- e^+$	$\tau^- \to \mu^- \mu^- e^+$
$\tau^- \to \mu^- \mu^- \mu^+$	$\tau^- \to \mu^- e^- e^+$
$\tau^- \to e^- e^- \mu^+$	$\tau^- \to e^- \mu^- \mu^+$

- Take into account Pauli principle for two identical particles
- Red marked: Contribution from $H_{\mathrm{eff}}^{\mathrm{rad}}$

$$\tau^- \to \ell'^- \gamma^* \to \ell'^- (\ell^+ \ell^-)$$

The Decay Modes and Couplings

Leading Coupling Matrices

- Matrices in flavour space
- Vector current: $g_V^{(L_i L^j)(L_k L^l)}$, $g_V^{(R_i R^j)(R_k R^l)}$, $g_V^{(L_i L^j)(R_k R^l)}$, $g_V^{(R_i R^j)(L_k L^l)}$
- Radiative current $g_{\mathrm{rad}}^{(L_i R^j)}$, $g_{\mathrm{rad}}^{(R_i L^j)}$

The Different Decay Modes

$$\tau^- \to e^- e^- e^+ \qquad \tau^- \to \mu^- \mu^- e^+$$
$$\tau^- \to \mu^- \mu^- \mu^+ \qquad \tau^- \to \mu^- e^- e^+$$
$$\tau^- \to e^- e^- \mu^+ \qquad \tau^- \to e^- \mu^- \mu^+$$

- Take into account Pauli principle for two identical particles
- Red marked: Contribution from $H_{\mathrm{eff}}^{\mathrm{rad}}$

$$\tau^- \to \ell'^- \gamma^* \to \ell'^- (\ell^+ \ell^-)$$

Conventions

Kinematical Variables

- Neglect electron mass
- Decay distribution in

$$m_{--}^2 \equiv m_{12}^2 = (p'_{e^-,\mu^-} + p_{\mu^-})^2$$
$$m_{+-}^2 \equiv m_{23}^2 = (p_{\mu^-} + p_{\mu^+})^2$$

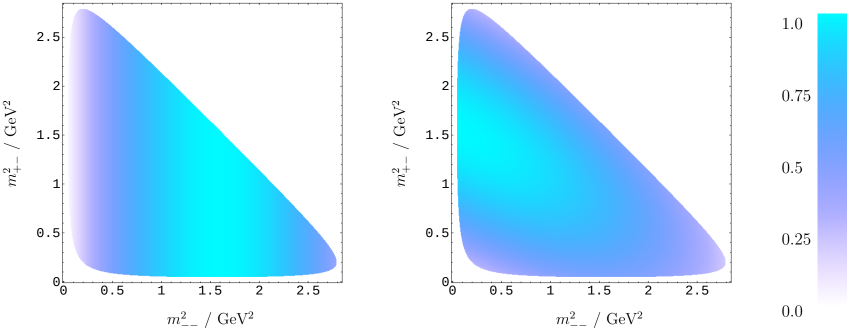
- Third combination is given by

$$m_{13}^2 = m_\tau^2 + 2m_\mu^2 + m_{e^-,\mu^-}^2 - m_{--}^2 - m_{+-}^2$$

Decay Distributions

- Consider Dalitz distributions in m_{--}^2 and m_{+-}^2
- Neglect interference terms with different helicities (suppression due to small lepton masses)

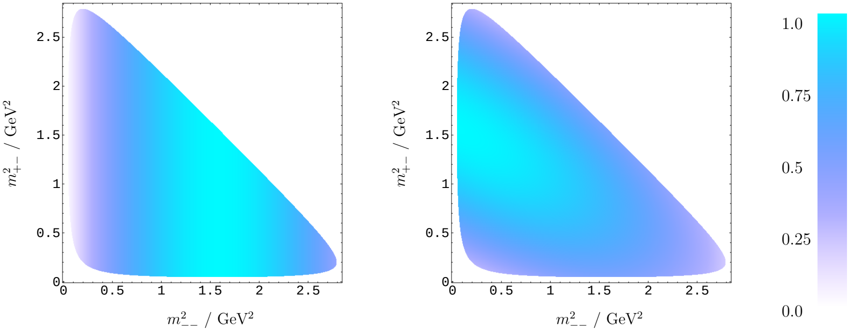
Vector Current



Decay Distribution for Left Figure: $\propto |g_V^{(L_\mu L^\tau)(L_\mu L^\mu)}|^2$

$$\frac{d^2\Gamma_V^{(LL)(LL)}}{dm_{23}^2\, dm_{12}^2} = \frac{|g_V^{(L_\mu L^\tau)(L_\mu L^\mu)}|^2}{\Lambda^4} \frac{(m_\tau^2 - m_\mu^2)^2 - (2m_{12}^2 - m_\tau^2 - 3m_\mu^2)^2}{256\,\pi^3\, m_\tau^3}$$

Vector Current

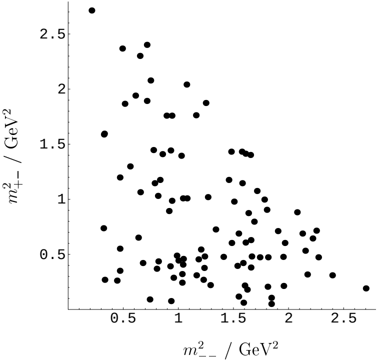


Decay Distribution for Right Figure: $\propto |g_V^{(L_\mu L^\tau)(R_\mu R^\mu)}|^2$

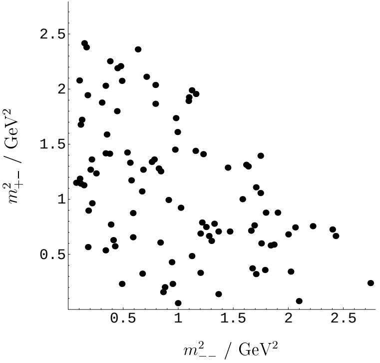
$$\frac{d^2 \Gamma_V^{(LL)(RR)}}{dm_{23}^2 dm_{12}^2} = \frac{|g_V^{(L_\mu L^\tau)(R_\mu R^\mu)}|^2}{\Lambda^4} \left[\frac{(m_\tau^2 - m_\mu^2)^2 - 4 m_\mu^2 (m_\tau^2 + m_\mu^2 - m_{12}^2)}{512 \pi^3 m_\tau^3} - \frac{(2m_{13}^2 - m_\tau^2 - 3m_\mu^2)^2 + (2m_{23}^2 - m_\tau^2 - 3m_\mu^2)^2}{1024 \pi^3 m_\tau^3} \right]$$

Simulation for 100 Events: Vector Current

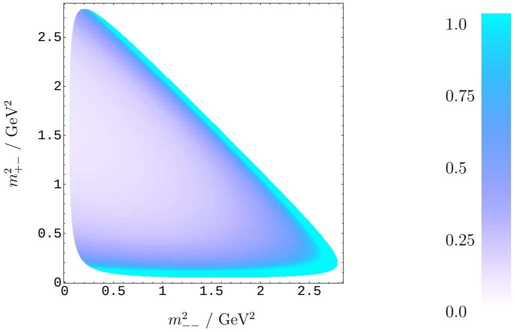
$$\frac{d^2\Gamma_V^{(LL)(LL)}}{dm_{23}^2\, dm_{12}^2}$$



$$\frac{d^2\Gamma_V^{(LL)(RR)}}{dm_{23}^2\, dm_{12}^2}$$



Radiative Transition



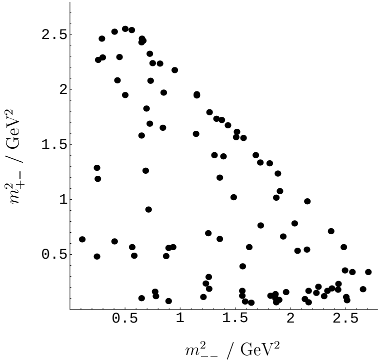
Decay Distribution for Radiative Mediated Decay

$$\frac{d^2\Gamma^{(LR)}_{\text{rad}}}{dm^2_{23}\,dm^2_{12}} = \alpha^2_{\text{em}}\,\frac{|g^{(L_\mu R^\tau)}_{\text{rad}}|^2\,v^2}{\Lambda^4}\left[\frac{m^2_\mu\,(m^2_\tau - m^2_\mu)^2}{128\,\pi^3\,m^3_\tau}\left(\frac{1}{m^4_{13}} + \frac{1}{m^4_{23}}\right) + \frac{2m^2_{12} - 3m^2_\mu}{128\,\pi^3\,m^3_\tau}\right.$$

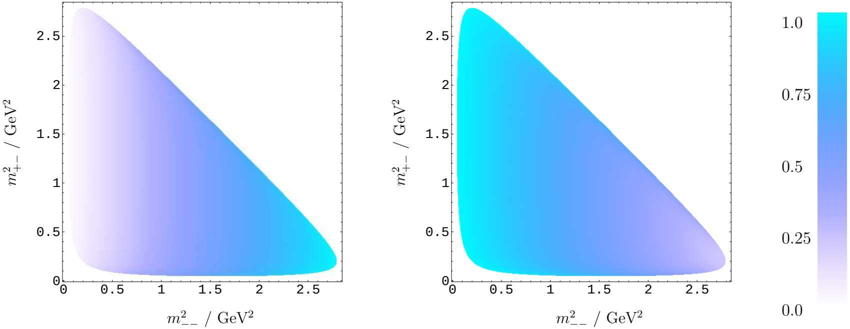
$$\left. + \frac{m^2_\mu(m^4_\tau - 3m^2_\tau m^2_\mu + 2m^4_\mu)}{128\,\pi^3\,m^2_{13}\,m^2_{23}\,m^3_\tau} + \frac{(m^2_{13} + m^2_{23})(m^4_{12} + m^4_{13} + m^4_{23} - 6m^2_\mu(m^2_\mu + m^2_\tau))}{256\,\pi^3\,m^2_{13}\,m^2_{23}\,m^3_\tau}\right]$$

Simulation for 100 Events: Radiative Transition

$$\frac{d^2\Gamma_{\text{rad}}^{(LR)}}{dm_{23}^2\, dm_{12}^2}$$



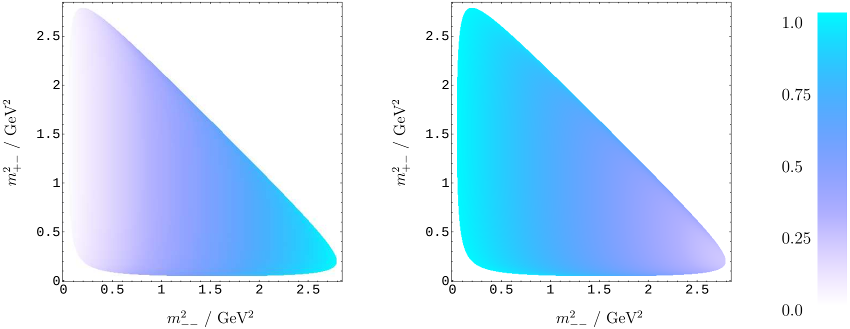
Interference Terms



Decay Distribution for Left Figure: $\propto \mathrm{Re}[g_V^{(L_\mu L^\tau)(L_\mu L^\mu)}\, g_{\mathrm{rad}}^{*(L_\mu R^\tau)}]$

$$\frac{\mathrm{d}^2\Gamma^{(LL)(LL)}_{\mathrm{mix}}}{\mathrm{d}m^2_{23}\,\mathrm{d}m^2_{12}} = \alpha_{\mathrm{em}}\,\frac{2\,v\,\mathrm{Re}[g_V^{(L_\mu L^\tau)(L_\mu L^\mu)}\, g_{\mathrm{rad}}^{*(L_\mu R^\tau)}]}{\Lambda^4} \times \left[\frac{m^2_{12} - 3m^2_\mu}{64\,\pi^3\,m^2_\tau} + \frac{m^2_\mu(m^2_\tau - m^2_\mu)(m^2_{13} + m^2_{23})}{128\,\pi^3\,m^2_\tau\,m^2_{13}\,m^2_{23}}\right]$$

Interference Terms

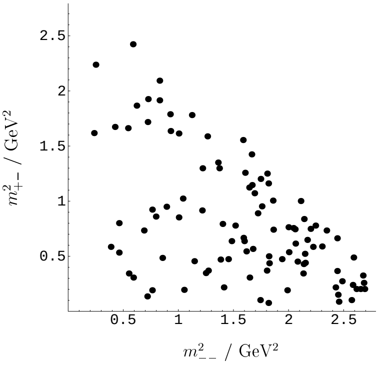


Decay Distribution for Right Figure: $\propto \mathrm{Re}[g_V^{(L_\mu L^\tau)(R_\mu R^\mu)}\, g_{\mathrm{rad}}^{*(L_\mu R^\tau)}]$

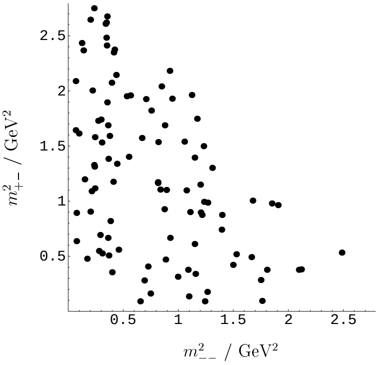
$$\frac{d^2\Gamma_{\mathrm{mix}}^{(LL)(RR)}}{dm_{23}^2\, dm_{12}^2} = \alpha_{\mathrm{em}} \frac{2\, v\, \mathrm{Re}[g_V^{(L_\mu L^\tau)(R_\mu R^\mu)}\, g_{\mathrm{rad}}^{*(L_\mu R^\tau)}]}{\Lambda^4} \times \left[\frac{m_\tau^2 - m_{12}^2 - 3m_\mu^2}{256\,\pi^3\, m_\tau^2} + \frac{m_\mu^2 (m_\tau^2 - m_\mu^2)(m_{13}^2 + m_{23}^2)}{256\,\pi^3\, m_\tau^2\, m_{13}^2\, m_{23}^2} \right]$$

Simulation for 100 Events: Interference Terms

$$\frac{d^2\Gamma_{\text{mix}}^{(LL)(LL)}}{dm_{23}^2\, dm_{12}^2}$$



$$\frac{d^2\Gamma_{\text{mix}}^{(LL)(RR)}}{dm_{23}^2\, dm_{12}^2}$$



Remarks on Minimal Flavour Violation

Minimal Flavour Violation

- Symmetry principle to suppress FCNC
- Basic assumption: SM Yukawa couplings are only source of flavour violation

$\Rightarrow$ Automatically all FCNC are rendered small

$\Rightarrow$ "Reasonable" mass scales for new particles

- Can be formulated as EFT

Assumptions for Our Case

- Extended to lepton sector [Cirigliano et al.: hep-ph/0507001]
- Minimal field content: $SU(3)_L \times SU(3)_{E_R}$ flavour symmetry
- No right handed neutrinos (only SM fields)

$\Rightarrow$ Respect SM gauge group

Remarks on Minimal Flavour Violation

Minimal Flavour Violation

- Symmetry principle to suppress FCNC
- Basic assumption: SM Yukawa couplings are only source of flavour violation

⇒ Automatically all FCNC are rendered small

⇒ "Reasonable" mass scales for new particles

- Can be formulated as EFT

Assumptions for Our Case

- Extended to lepton sector [Cirigliano et al.: hep-ph/0507001]
- Minimal field content: $SU(3)_L \times SU(3)_{E_R}$ flavour symmetry
- No right handed neutrinos (only SM fields)

⇒ Respect SM gauge group

Minimal Flavour Violation: The Spurions

$SU(3)_L \times SU(3)_{E_R}$ Flavour Symmetry Breaking

- 2 spurions in minimal extension of SM: $\lambda \sim (\bar{3}, 3)$ and $g_\nu \sim (\bar{6}, 1)$

$$\lambda = \frac{1}{v}\,\mathrm{diag}(m_e, m_\mu, m_\tau) \qquad g_\nu = \frac{\Lambda_{\mathrm{LN}}}{v^2}\, U^*\,\mathrm{diag}(m_{\nu_1}, m_{\nu_2}, m_{\nu_2})\, U^\dagger$$

- $\lambda \sim (\bar{3}, 3)$ describes SM Yukawa interaction
- $g_\nu \sim (\bar{6}, 1)$ stems from dim-5 lepton number violating term

$$\mathcal{L}_{\mathrm{Maj}} = \frac{1}{2\Lambda_{\mathrm{LN}}}\left(N^T g N\right) \qquad \text{where} \qquad N = \left(T_3^{(R)} + \frac{1}{2}\right) H^\dagger L$$

$\Rightarrow$ Parametrised by lepton masses and PMNS matrix U

Comments

- Interested in 4-lepton processes $L^i L^j L^*_k L^*_l$, $L^i R^j L^*_k R^*_l$, etc.
- Consider minimal spurion insertion to form gauge invariant quantity

Minimal Flavour Violation: The Spurions

$SU(3)_L \times SU(3)_{E_R}$ Flavour Symmetry Breaking

- 2 spurions in minimal extension of SM: $\lambda \sim (\bar{3}, 3)$ and $g_\nu \sim (\bar{6}, 1)$

$$\lambda = \frac{1}{v}\,\mathrm{diag}(m_e, m_\mu, m_\tau) \qquad g_\nu = \frac{\Lambda_{\mathrm{LN}}}{v^2}\, U^*\,\mathrm{diag}(m_{\nu_1}, m_{\nu_2}, m_{\nu_2})\, U^\dagger$$

- $\lambda \sim (\bar{3}, 3)$ describes SM Yukawa interaction
- $g_\nu \sim (\bar{6}, 1)$ stems from dim-5 lepton number violating term

$$\mathcal{L}_{\mathrm{Maj}} = \frac{1}{2\Lambda_{\mathrm{LN}}}\left(N^T g N\right) \qquad \text{where} \qquad N = \left(T_3^{(R)} + \frac{1}{2}\right) H^\dagger L$$

$\Rightarrow$ Parametrised by lepton masses and PMNS matrix U

Comments

- Interested in 4-lepton processes $L^i L^j L_k^* L_l^*$, $L^i R^j L_k^* R_l^*$, etc.
- Consider minimal spurion insertion to form gauge invariant quantity

Symmetry Breaking

Spurions in 2-Lepton and 4-Lepton Operators

- Single right handed field requires λ insertion $\Rightarrow m_\ell/v$ suppression
- Tensor reduction for $L^i L^j L^*_k L^*_l$: $\bar{6} \times 6 = 1 + 8 + 27$

  with octet Δ^k_i and 27plet G^{kl}_{ij}
- 2-lepton operators only involve octet Δ^k_i

Constraints for Multiplets

- $\Delta = \Delta^\dagger$ and $G^{kl}_{ij} = G^{kl}_{ji} = G^{lk}_{ij}$, and $\sum_i G^{il}_{ij} = 0$

Examples of Spurion Insertions

- 4-Lepton operators $(L^* \Delta L)(L^* L)$ $(L^* \Delta L)(R^* R)$
- Purely right handed operator $(R^* R)(R^* \lambda g^\dagger g \lambda^\dagger R)$
- Radiative operator $(L^* \Delta \lambda^\dagger R)$ and $(R^* \lambda \Delta L)$

Symmetry Breaking

Spurions in 2-Lepton and 4-Lepton Operators

- Single right handed field requires λ insertion $\Rightarrow m_\ell/v$ suppression
- Tensor reduction for $L^i\, L^j\, L^*_k\, L^*_l$: $\quad \bar{6} \times 6 = 1 + 8 + 27$

  with octet Δ^k_i and 27plet G^{kl}_{ij}
- 2-lepton operators only involve octet Δ^k_i

Constraints for Multiplets

- $\Delta = \Delta^\dagger$ and $G^{kl}_{ij} = G^{kl}_{ji} = G^{lk}_{ij}$, and $\sum_i G^{il}_{ij} = 0$

Examples of Spurion Insertions

- 4-Lepton operators $(L^*\,\Delta\, L)(L^*\, L) \quad (L^*\,\Delta\, L)(R^*\, R)$
- Purely right handed operator $(R^*R)(R^*\lambda\, g^\dagger g\, \lambda^\dagger R)$
- Radiative operator $(L^*\,\Delta\,\lambda^\dagger R)$ and $(R^*\lambda\,\Delta\, L)$

Decomposition of the Spurions

Tensor Decomposition in Terms of PMNS matrix

- Singlet, no flavour transition: $\mathrm{tr}[g_\nu^\dagger g_\nu] = \frac{\Lambda_{\mathrm{LN}}^2}{v^4} \left(m_{\nu_1}^2 + m_{\nu_3}^2 + m_{\nu_3}^2 \right)$
- Octet: $\Delta = g_\nu^\dagger g_\nu - \frac{1}{3}\mathrm{tr}[g_\nu^\dagger g_\nu] = \frac{\Lambda_{\mathrm{LN}}^2}{v^4} U \left(\mathrm{diag}[m_{\nu_1}^2, m_{\nu_2}^2, m_{\nu_3}^2] - \bar{m}_\nu^2 \right) U^\dagger$
- 27plet: $$G_{ij}^{kl} = (g_\nu)_{ij} (g_\nu^*)^{kl} - \frac{1}{12} \left(\delta_i^k \delta_j^l + \delta_i^l \delta_j^k \right) \mathrm{tr}(g^\dagger g) - \frac{1}{5} \left(\delta_i^a \delta_b^l \delta_j^k + \delta_j^a \delta_b^l \delta_i^k + \delta_i^a \delta_b^k \delta_j^l + \delta_j^a \delta_b^k \delta_i^l \right) \Delta_a^b$$
- Combinations for Relevant Couplings
$$g_V^{(L_k L^i)(L_l L^j)} \to 2c_1 \Delta_i^k \delta_j^l + c_2 G_{ij}^{kl} \quad g_V^{(L_k L^i)(R_l R^j)} \to c_3 \Delta_i^k \delta_j^l \quad g_{\mathrm{rad}}^{(L_k R^i)} \to c_4 \Delta_i^k \lambda_i$$

Assumptions for PMNS Matrix

- Use PDG Parametrization
- $\sin^2\theta_{13} \sim \Delta m^2_{\mathrm{sol}}/\Delta m^2_{\mathrm{atm}} \ll 1$ and $\theta_{23} = 45°$
- normal neutrino hierarchy $(m_{\nu_1} \sim m_{\nu_2} \ll m_{\nu_3})$
- inverted hierarchy $(m_{\nu_1} \sim m_{\nu_2} \gg m_{\nu_3})$

Results for Relevant Combinations (limit of small mixing angle)

- Octet

$$\Delta^\mu_\tau = \mathcal{O}\left(\frac{\Lambda^2_{LN}}{v^4}\,\Delta m^2_{\mathrm{atm}}\right)$$

- 27plet in the limit of vanishing Majorana phases
  - In the case of normal neutrino hierarchy

$$G^{\mu\mu}_{\tau\mu} \simeq \frac{\Lambda^2_{\mathrm{LN}}}{v^4}\,\frac{\Delta m^2_{\mathrm{atm}}}{20}$$

  - In the case of inverted neutrino hierarchy

$$G^{\mu\mu}_{\tau\mu} \simeq -\frac{\Lambda^2_{\mathrm{LN}}}{v^4}\,\frac{\Delta m^2_{\mathrm{atm}}}{20}$$

Summary

- LFV provides important test of SM against NP
- Here: Made a model-independent analysis of $\tau \to \ell\ell\ell$ decays
- Classified different operators in EFT
- Calculated Dalitz distributions for individual structures including interference terms

Conclusion

- Dalitz distributions provide a tool to distinguish NP models complementary to e.g. $\Gamma(\tau \to 3\ell)/\Gamma(\tau \to \ell\gamma)$
- $\mathcal{O}(100)$ measured events may be sufficient to distinguish effects from radiative and four-lepton-operators
- Experimental bounds on NP could be improved by taking into account different alternatives for decay distributions
- MFV analysis provides relation between general coupling and PMNS matrix parameters

Summary

- LFV provides important test of SM against NP
- Here: Made a model-independent analysis of $\tau \to \ell\ell\ell$ decays
- Classified different operators in EFT
- Calculated Dalitz distributions for individual structures including interference terms

Conclusion

- Dalitz distributions provide a tool to distinguish NP models complementary to e.g. $\Gamma(\tau \to 3\ell)/\Gamma(\tau \to \ell\gamma)$
- $\mathcal{O}(100)$ measured events may be sufficient to distinguish effects from radiative and four-lepton-operators
- Experimental bounds on NP could be improved by taking into account different alternatives for decay distributions
- MFV analysis provides relation between general coupling and PMNS matrix parameters

NP scenarios

- Typical SUSY scenario: photon-dipole operator dominates four-lepton operators [hep-ph/0404211]:

$$\frac{\Gamma(\tau \to 3\mu)}{\Gamma(\tau \to \mu\gamma)} \simeq \frac{\alpha_{\rm em}}{3\pi}\left(\ln\frac{m_\tau^2}{m_\mu^2} - \frac{11}{4}\right) = \mathcal{O}(10^{-3})$$

- Higgs-mediated $\tau \to \mu$ in decoupling limit [$\cos(\beta - \alpha) = 0$, $m_{A^0} \gg M_Z$)], $\tan\beta$ & off-diagonal slepton mass-matrix element $\delta_{3\ell} = \tilde{m}_{3\ell}^2/\tilde{m}^2$ large [hep-ph/0505046]:

$$\frac{\Gamma(\tau \to \ell\mu\mu)}{\Gamma(\tau \to \ell\gamma)} \leq \frac{3 + 5\delta_{\ell\mu}}{36} \sim \mathcal{O}(0.1)$$

- LHT [$M_{\tilde{l}} \sim \mathcal{O}(1\text{ TeV})$]: Z_0 and box-diagram dominate radiative operators [hep-ph/0609095,hep-ph/0612327,hep-ph/0702136]:

$$\frac{\Gamma(\tau \to 3\mu)}{\Gamma(\tau \to \mu\gamma)} = \mathcal{O}(1)$$

⇒ Sub-dominance of radiative dipole operator: Expect a rather flat Dalitz distribution for $\tau \to 3\mu$

Loop Contributions

- Operators bi-linear in lepton fields, contributing at loop level, e.g. in [hep-ph/9510309, hep-ph/0404211, hep-ph/0507001]
- We neglect possible form factor effects for decays into virtual photons from long-distance lepton or quark loops: In the most general case, the $\tau \to \ell\gamma^*$ vertex could be parametrized as

$$\frac{e}{4\pi}\frac{v}{\Lambda^2}\sum_{h,s}\bar{\ell}_h\left\{g_{\mathrm{rad}}^{(s,h)}(q^2)\,(-i\sigma_{\mu\nu})\,q^\mu + m_\tau\, f_{\mathrm{rad}}^{(s,h)}(q^2)\left(\gamma_\nu - \frac{q_\nu}{q^2}\,\not{q}\right)\right\}\tau_s$$

where $g_{\mathrm{rad}}^{(s,h)}(0) \equiv g_{\mathrm{rad}}^{(s,h)}$ and $f_{\mathrm{rad}}^{(s,h)}(0) = 0$, see e.g. [hep-ph/9710389]

Back

Radiative Operators

- Photon exchange, where $(s,h) = (L,R)$ respectively (R,L)

$$\frac{e}{4\pi}\frac{v}{\Lambda^2}\sum_{h,s} g_{\text{rad}}^{(s,h)} \left(\bar{\ell}_h(-i\sigma_{\mu\nu})\tau_s\right) F^{\mu\nu}$$

$$\rightarrow \quad \frac{e}{4\pi}\frac{v}{\Lambda^2}\sum_{h,s} g_{\text{rad}}^{(s,h)} \left(\bar{\ell}_h(-i\sigma_{\mu\nu})\tau_s\right) \frac{q^\nu}{q^2}\bar{\ell}'\gamma^\mu\ell''$$

- Z_0 exchange

$$\frac{v}{\Lambda^2}\frac{1}{v^2}(\bar{\ell}\sigma_{\mu\nu}\tau)q^\nu(\bar{\ell}'\gamma^\mu(g_V + g_A\gamma_5)\ell'')$$

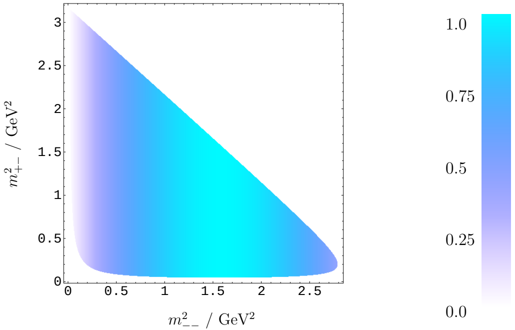
Where $(g_A)\, g_V$: (axial)vector couplings of Z_0 to leptons

$\Rightarrow$ Z_0 exchange suppressed by small Yukawa coupling of tau

$$\frac{Z_{\text{contr.}}}{\gamma_{\text{contr.}}} \propto \frac{q^\nu}{v^2}\frac{q^2}{q^\nu} \propto \frac{m_\tau^2}{v^2}$$

Back

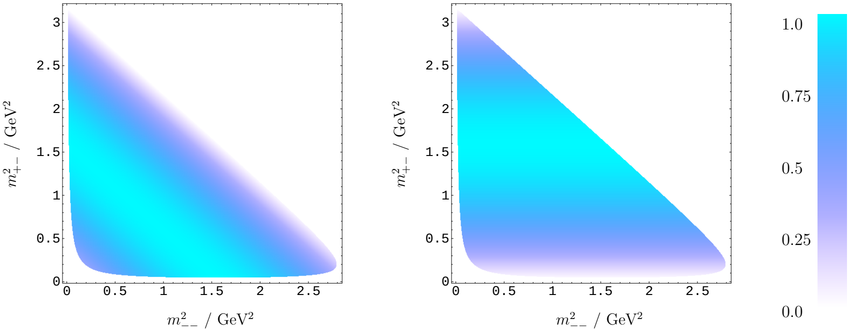
Vector Current Purely Left Handed



Decay Distribution

$$\frac{d^2\Gamma_V^{(LL)(LL)}}{dm_{23}^2\, dm_{12}^2} = \frac{|g_V^{(L_e L^\tau)(L_\mu L^\mu)}|^2}{\Lambda^4} \, \frac{m_\tau^4 - (2m_{12}^2 - m_\tau^2 - 2m_\mu^2)^2}{512\,\pi^3\, m_\tau^3}$$

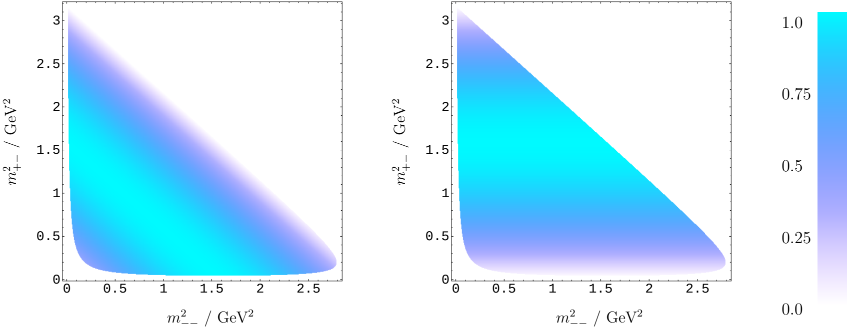
Vector Current



Decay Distribution for Left Figure: Left Handed Electron

$$\left.\frac{d^2\Gamma_V^{(LL)(RR)}}{dm_{23}^2 dm_{12}^2}\right|_{e_L} = \frac{|g_V^{(L_e L^\tau)(R_\mu R^\mu)}|^2}{\Lambda^4} \frac{m_\tau^4 - (2m_{13}^2 - m_\tau^2 - 2m_\mu^2)^2}{512\,\pi^3\, m_\tau^3}$$

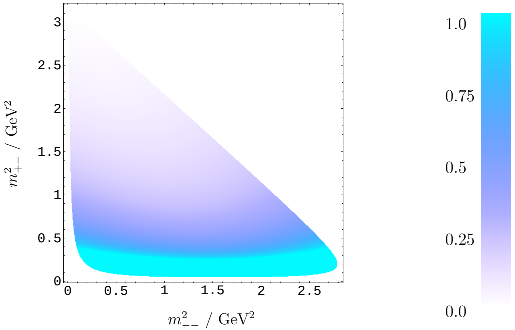
Vector Current



Decay Distribution for Right Figure: Right Handed Electron

$$\left.\frac{d^2\Gamma_V^{(LL)(RR)}}{dm_{23}^2\, dm_{12}^2}\right|_{e_R} = \frac{|g_V^{(L_\mu L^\tau)(R_e R^\mu)}|^2}{\Lambda^4}\,\frac{(m_\tau^2 - 2m_\mu^2)^2 - (2m_{23}^2 - m_\tau^2 - 2m_\mu^2)^2}{512\,\pi^3\, m_\tau^3}$$

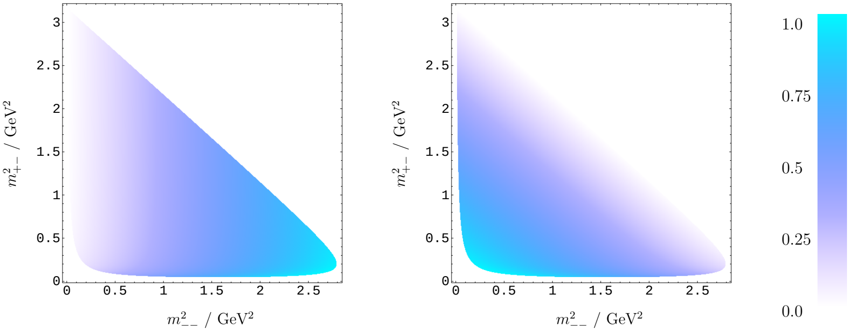
Radiative Transition



Decay Distribution for Radiative Mediated Decay

$$\frac{d^2\Gamma^{(LR)}_{\mathrm{rad}}}{dm^2_{23}\,dm^2_{12}} = \alpha^2_{\mathrm{em}} \frac{|g^{(L_e R^\tau)}_{\mathrm{rad}}|^2 v^2}{\Lambda^4} \left[\frac{m^2_\mu (m^2_{23} - m^2_\tau)^2}{64\pi^3 m^3_\tau m^4_{23}} + \frac{m^4_{12} + m^4_{13} - 2m^4_\mu}{128\pi^3 m^3_\tau m^2_{23}} + \frac{m^2_\tau - m^2_{23}}{128\pi^3 m^3_\tau} \right]$$

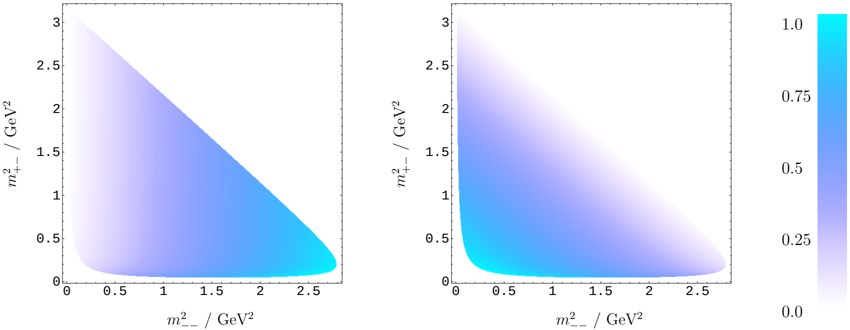
Interference Terms



Decay Distribution for Left Figure: $\propto \mathrm{Re}[g_{\mathrm{V}}^{(L_e L^\tau)(L_\mu L^\mu)}\, g_{\mathrm{rad}}^{*(L_e R^\tau)}]$

$$\frac{d^2\Gamma^{(LL)(LL)}_{\mathrm{mix}}}{dm^2_{23}\,dm^2_{12}} = \alpha_{\mathrm{em}}\, \frac{2\, v\, \mathrm{Re}[g_{\mathrm{V}}^{(L_e L^\tau)(L_\mu L^\mu)}\, g_{\mathrm{rad}}^{*(L_e R^\tau)}]}{\Lambda^4} \left[\frac{m^2_{12} - 2m^2_\mu}{128\pi^3 m^2_\tau} + \frac{m^2_\mu}{128\pi^3 m^2_{23}}\right]$$

Interference Terms



Decay Distribution for Right Figure: $\propto \mathrm{Re}[g_{\mathrm{V}}^{(L_e L^\tau)(R_\mu R^\mu)}\; g_{\mathrm{rad}}^{*(L_e R^\tau)}]$

$$\frac{\mathrm{d}^2\Gamma_{\mathrm{mix}}^{(LL)(RR)}}{\mathrm{d}m_{23}^2\,\mathrm{d}m_{12}^2} = \alpha_{\mathrm{em}}\,\frac{2\,v\,\mathrm{Re}[g_{\mathrm{V}}^{(L_e L^\tau)(R_\mu R^\mu)}\; g_{\mathrm{rad}}^{*(L_e R^\tau)}]}{\Lambda^4}\left[\frac{m_{13}^2 - 2m_\mu^2}{128\pi^3 m_\tau^2} + \frac{m_\mu^2}{128\pi^3 m_{23}^2}\right]$$

Results for Relevant Combinations (limit of small mixing angle)

- Octet

$$\Delta_\tau^\mu = \mathcal{O}\left(\frac{\Lambda_{LN}^2}{v^4}\,\Delta m^2_{\rm atm}\right) \qquad \Delta_\tau^e = \mathcal{O}\left(\frac{\Lambda_{LN}^2}{v^4}\,\Delta m^2_{\rm atm}\sin\theta_{13}\right)$$

- 27plet in the limit of vanishing Majorana phases
  - In the case of normal neutrino hierarchy

$$G^{\mu\mu}_{\tau\mu} \simeq \frac{\Lambda^2_{\rm LN}}{v^4}\,\frac{\Delta m^2_{\rm atm}}{20}$$

$$G^{e\mu}_{\tau\mu} \simeq \frac{\Lambda^2_{\rm LN}}{v^4}\,\frac{\sqrt{\Delta m^2_{\rm atm}}}{2}\left(\frac{3\cos\delta - 7i\sin\delta}{5}\,\sin\theta_{13}\,\sqrt{\Delta m^2_{\rm atm}} + \sin 2\theta_{12}\,\frac{\Delta m^2_{\rm sol}}{4m_{\nu_{1,2}}}\right)$$

  - In the case of inverted neutrino hierarchy

$$G^{\mu\mu}_{\tau\mu} \simeq -\frac{\Lambda^2_{\rm LN}}{v^4}\,\frac{\Delta m^2_{\rm atm}}{20} \qquad G^{e\mu}_{\tau\mu} \simeq \frac{\Lambda^2_{\rm LN}}{v^4}\,\frac{7\Delta m^2_{\rm atm}}{10\sqrt{2}}\,e^{i\delta}\,\sin\theta_{13}$$