Jets + Bremsstrahlung = OPE + ?

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Ringberg Workshop on New Physics, Flavors and Jets

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What does all this mean?

- Fermi motion
- Bremsstrahlung
- Nonperturbative ‘non-OPE’ effects
- Conventional OPE predictions for moments?
- Discussions and conclusions

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- Is there an effective field theory for SCET in reality?

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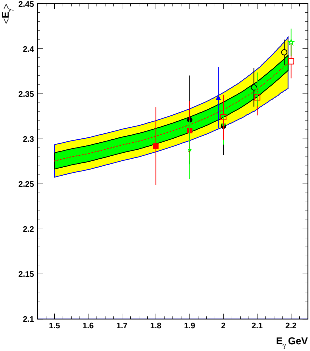
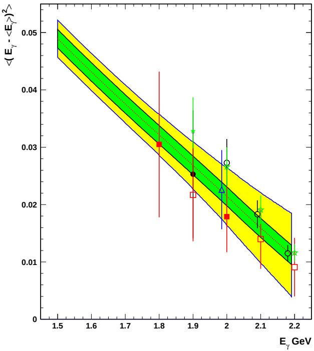
'Theoretical correlations'

Theory predictions (Wilsonian OPE) for photon energy moments in $b \to s + \gamma$ based on HQP from $b \to c\,\ell\nu$ data vs. experiment





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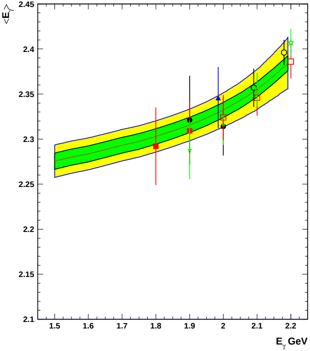
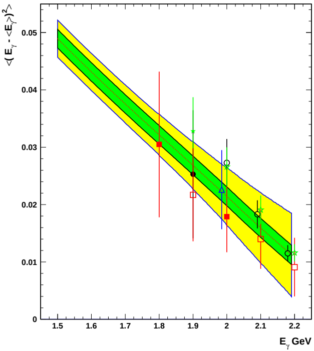




$b \to$ light q decays have some subtleties which represent the problems for applying the OPE

These motivated the theoretical study underlying this talk

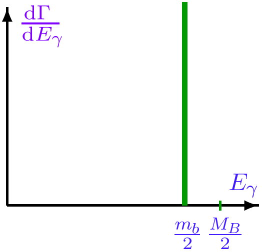
Fermi motion

Inclusive distributions are close to partonic?

$b \to s + \gamma$:

$E_\gamma \simeq \frac{m_b}{2}$ $\qquad \frac{\mathrm{d}\Gamma}{\mathrm{d}E_\gamma} \sim \delta\left(E_\gamma - \frac{m_b}{2}\right)$?

$B \to X_s + \gamma$: $\qquad E_\gamma = \frac{M_B^2 - M_X^2}{2M_B}$ – There should be a distribution...



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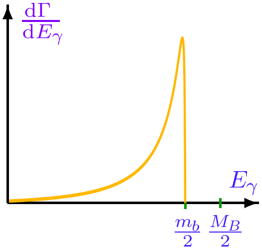
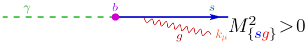


The parton line is modified by

- Perturbative bremsstrahlung lowers E_γ

$b \to \gamma + s_{\mathrm{virt}} \to \gamma + s + g$



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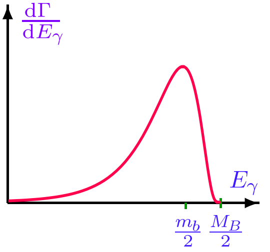

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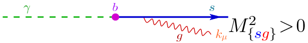

- Intrinsic motion of the heavy quark

  with $\vec{p}_b \neq 0 \quad E_\gamma = \frac{m_b}{2} + \frac{\vec{p}_b \vec{n}_\gamma}{2}$: *Doppler smearing*

  b-quark shivers in B-hadron, $\qquad \vec{p}_b \sim \Lambda_{\rm QCD}, \qquad \vec{v}_b \sim \frac{\Lambda_{\rm QCD}}{m_b}$

Fermi motion

Phenomenological models incorporating Doppler smearing in the spectrum were called upon early on

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A C^2 M^2 1982

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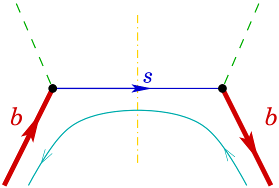
In fact the *light-cone* momentum component $\pi_0 - \vec{\pi}\vec{n} = \pi_\mu n^\mu$ enters, $\vec{n}^{\,2} = 1$

Comes from the s-quark propagator in the forward scattering amplitude

$$\bar{b} \, \ldots \, \frac{1}{m_b^2 - (p_b + \pi_b - q)^2 - \frac{i}{2}\sigma G} \, \ldots \, b \, ,$$



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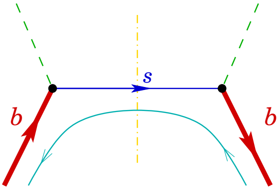

$\pi_\mu = iD_\mu - m_b v_\mu$ – nonrelativistic energy and momentum

$p_b - q$ is the s-quark parton momentum in the decay

Looks like an interaction of the b quark rather than a property of the s-quark jet...

NB : This applies to *soft gluons* only!

A peculiarity of the heavy-to-light decays: double-log Sudakov radiation

Emission resummation technique:

Catani-Mangano-Nason-Trentadue hep-ph/9604351

I will dwell on their different aspect

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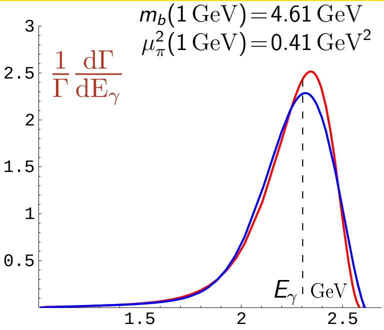
treating nonperturbative effects and the OPE

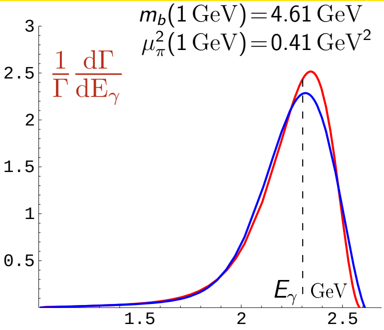
OPE



$m_b(1\,\mathrm{GeV}) = 4.61\,\mathrm{GeV}$
$\mu_\pi^2(1\,\mathrm{GeV}) = 0.41\,\mathrm{GeV}^2$
$\frac{1}{\Gamma}\frac{d\Gamma}{dE_\gamma}$
3
2.5
2
1.5
1
0.5
E_γ GeV
1.5
2
2.5

OPE:

soft modes $\Longrightarrow$ nonperturbative effects
hard gluons $\Longrightarrow$ Wilson coefficients
(perturbative spectrum)



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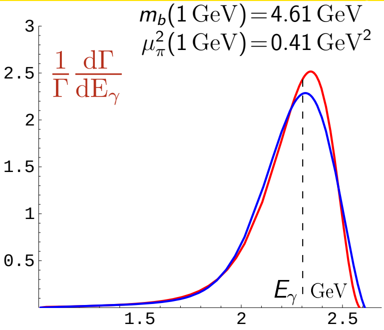

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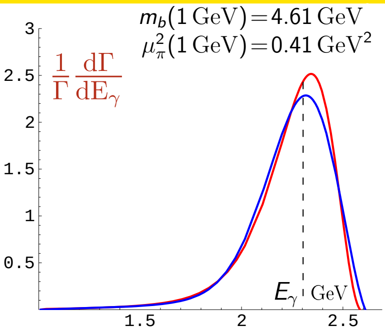

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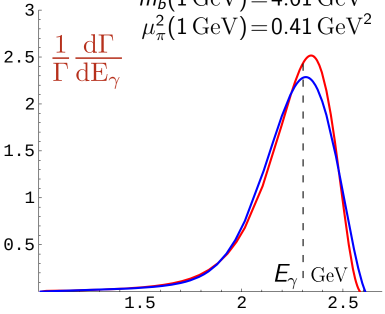

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$|\vec{k}| > \mu$ – short distances; then α_s enters at $Q^2 > \mu^2$ *'Kinetic scheme'*

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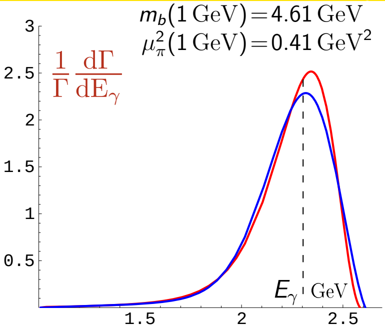

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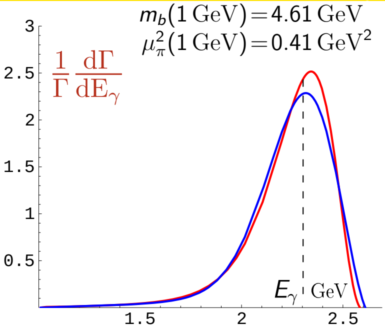

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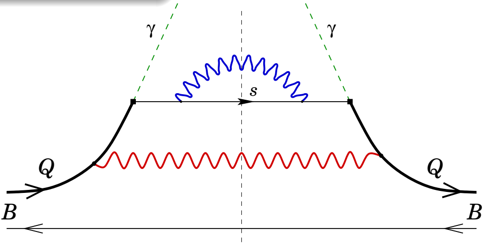
OPE expands in the fields' momenta $\Longrightarrow$ have to assume that all the components of the momentum are literally small for the n/p fields

Soft gluons $|k_\mu| \lesssim \mu_{\rm hadr}$ are included into HQ distribution function $F(x)$ (Fermi motion). Other, hard gluons are in the Wilson coefficients (kernel)



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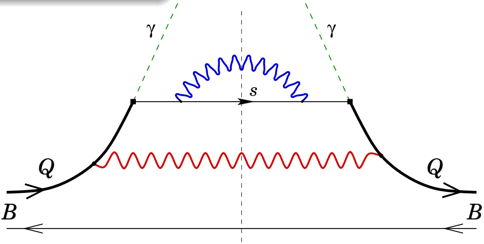

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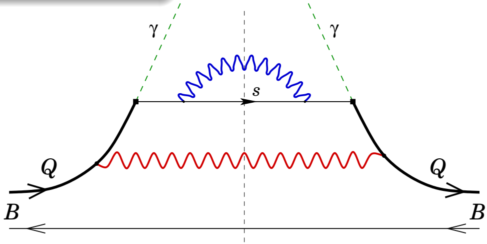

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$\dfrac{\mathrm{d}\Gamma_{\rm pert}}{\mathrm{d}E}$ accounts for all other modes

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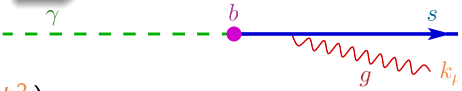

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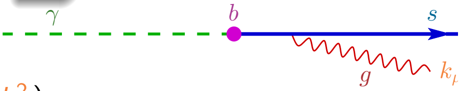

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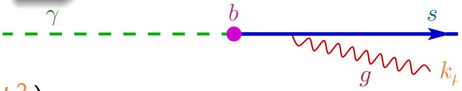

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at least the total decay width is not affected

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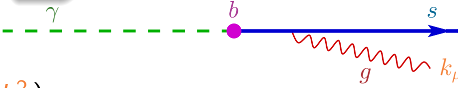

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No virtual contributions to cancel...

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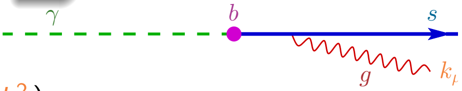

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Additional nonperturbative terms on top of the OPE?

A priori this cannot be excluded. OPE controls what it can, the effect of 'all-soft' physics. If there are nonperturbative effects from large-frequency modes, they would be additional contributions

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Through 2004 the V_{ub} (inclusive) programs at B factories were steered to exclude $b \to c\,\ell\nu$ information and only use $b \to s + \gamma$ spectrum for $b \to u\,\ell\nu$ description...

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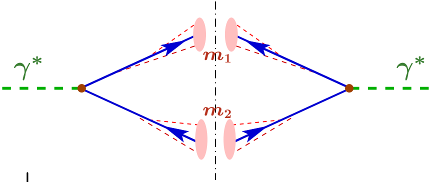
Through 2004 the V_{ub} (inclusive) programs at B factories were steered to exclude $b \to c\,\ell\nu$ information and only use $b \to s+\gamma$ spectrum for $b \to u\,\ell\nu$ description...

Extra effects here would lead to certain paradoxes

Consider $\sigma(e^+e^- \to \mathrm{hadrons})$ from this perspective

no OPE for jets

Look at it now as a decay of the photon with the mass $\sqrt{s}$ into two jets:



$$\Gamma \sim 1 - \frac{m_1^2+m_2^2}{s} + \frac{(m_1^2+m_2^2)^2}{s^2} + \ldots$$

If there were $\delta m_{\rm jet}^2$, we would normally have the phase-space–related $1/s$, $1/s^2$, ... corrections in the total rate, not only in jet distributions where no Euclidean OPE applies

However, for the *total cross section* we know from the OPE that

$$\delta_{\rm IR} R(s) \sim \frac{1}{s^3} + \alpha_s \frac{1}{s^2} + \ \ldots$$

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This is a consequence of analyticity and unitarity for the polarization operator of the currents

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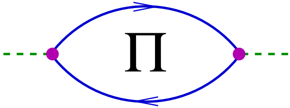

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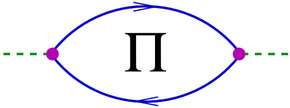

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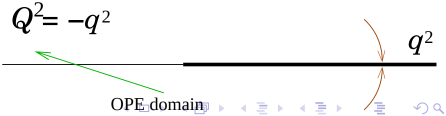
If we know the asymptotics of $D(Q^2)$ in deep Euclid, we know power corrections in $R(s)$ at large s:



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The answer:

No, integer moments of jet mass^2 are not affected !

Kinematics and the OPE

$Q \longrightarrow q + \varphi$

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Usual nonperturbative (OPE) window is $M_X^2 \lesssim \mu_{\rm hadr} m_b$

larger $M_X^2 \gg \mu_{\rm hadr} m_b$ come from hard bremsstrahlung

Scale separation

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Physics: this is not the bound-state dynamics!

Travel distance

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much larger than $\mu_{\rm hadr}^{-1}$

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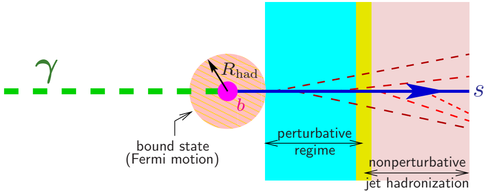

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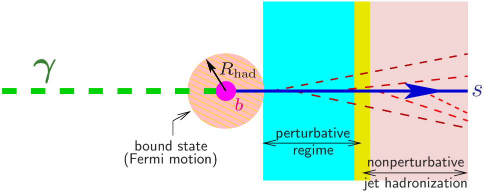

In the total width virtual corrections cancel these radiation effects no matter what α_s coupling is

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How to show this?

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illustrate in the framework of

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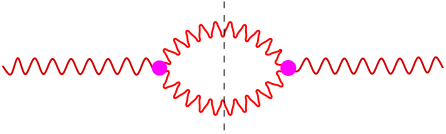

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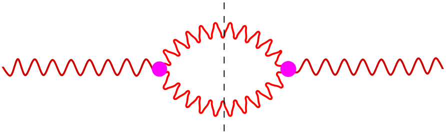

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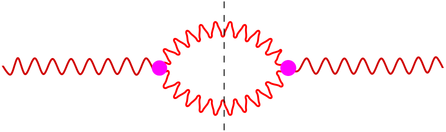

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in jet physics the nonperturbative effects are given by the log-moments of $\delta\rho$

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$$\mathrm{d}W_{\mathrm{brem}} = C_F \int \frac{\mathrm{d}\omega}{\omega} \underbrace{\int \frac{\mathrm{d}\lambda^2}{\lambda^2}\, \rho(\lambda^2) \int \frac{\mathrm{d}k_\perp^2}{k_\perp^2+\lambda^2}}_{\int \mathrm{d}k_\perp^2 \;\frac{1}{\pi}\frac{\alpha_s^\epsilon(k_\perp^2)}{k_\perp^2}} \mathrm{d}W_{\mathrm{born}}$$

Soft / collinear bremsstrahlung is driven by $\alpha_s^\epsilon(k_\perp^2)$

Ok, for Γ_{tot} virtual corrections cancel the effect due to $k_\perp^2 \lesssim \mu_{\mathrm{hadr}}^2$

Moments of M_X^2 : applying such a result we get, following KS

$$\int \frac{\mathrm{d}k_\perp^2}{k_\perp^2}\, k_\perp^{2n}\, \alpha_s(k_\perp^2)$$

saturated at perturbative $k_\perp \sim m_b$, but the domain $k_\perp^2 \lesssim \mu_{\mathrm{hadr}}^2$ yields a nonperturbative piece $\sim \mu_{\mathrm{hadr}}^{2n}$

Do explicitly:

$$\langle M_X^2\rangle^{\rm pert} = C_F \int \mathrm{d}M_X^2 \int \frac{\mathrm{d}\omega}{\omega}\, \vartheta(\omega-\mu) \int \frac{\mathrm{d}\lambda^2}{\lambda^2}\, \rho(\lambda^2) \times$$

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Product of independent integrals over $k_\perp^2$ and λ^2 !

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Product of independent integrals over $k_\perp^2$ and λ^2 ! No $\alpha_s(k_\perp^2)$, minimal scale Q^2 is determined by the kinematics, it is μ at worst,

$$\text{for} \qquad \int \frac{{\rm d}\lambda^2}{\lambda^2}\,\rho(\lambda^2) = \lim_{Q^2\to\infty}\delta\alpha_s^{\epsilon}(Q^2) = 0$$

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instead of

$$\frac{C_F}{M_X^2} \int \frac{\mathrm{d}\omega}{\omega}\, \vartheta(\omega-\mu) \underbrace{\int \frac{\mathrm{d}\lambda^2}{\lambda^2}\, \rho(\lambda^2)\, \frac{\frac{2\omega}{m_b} M_X^2}{\lambda^2 + \frac{2\omega}{m_b} M_X^2}}_{\frac{1}{\pi}\, \alpha_s^{\epsilon}\left(\frac{2\omega}{m_b} M_X^2\right)}$$

The radiation is now driven by a *different* effective coupling $\tilde{\alpha}_s(k_\perp^2)$:

$$\delta\tilde{\alpha}_s(Q^2)=\pi\int_0^{Q^2}\frac{\mathrm{d}\lambda^2}{\lambda^2}\,\rho(\lambda^2) \qquad \text{vs.} \qquad \delta\alpha_s^{\epsilon}(Q^2)=\pi\int_0^{\infty}\frac{\mathrm{d}\lambda^2}{\lambda^2}\,\frac{Q^2}{\lambda^2+Q^2}\,\rho(\lambda^2)$$

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The difference is significant: Integer moments of $\delta\tilde{\alpha}_s(Q^2)$ all vanish, while those of $\delta\alpha_s^{\epsilon}(Q^2)$ are 'positive'

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Effective α_s becomes observable-dependent since using it implies we forcibly interpret the process in the massless one-gluon language

Physics behind:

The coupling grows from $\alpha_s(E_{\rm jet})$ or from $\alpha_s(M_X)$ due to the final-state interaction, viz. gluon (jet) splitting. FSI do not change truly inclusive characteristics

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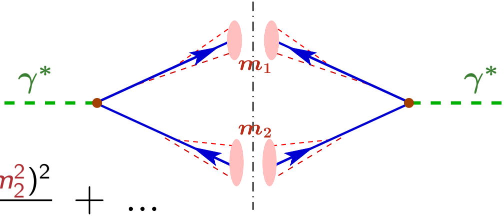
KLN implies that 'rescattering' does not affect the `total` probabilities and other truly local observables. That is what we get

Go back to $\sigma(e^+e^- \to \text{hadrons})$

$$\delta_{\rm IR} R(s) \sim \frac{1}{s^3} + \alpha_s \frac{1}{s^2}$$



$$\Gamma \sim 1 - \frac{m_1^2+m_2^2}{s} + \frac{(m_1^2+m_2^2)^2}{s^2} + \ldots$$

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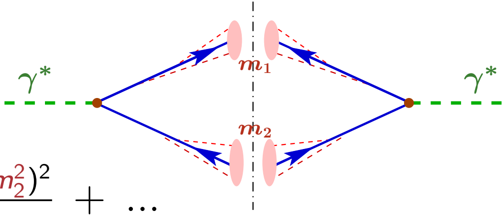

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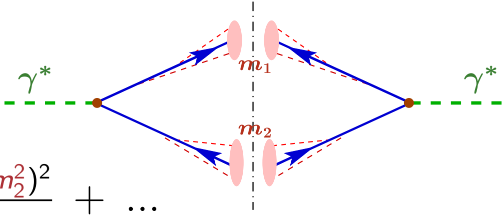

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We got this for inclusive jet moments as well

My conclusions

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The Wilsonian OPE nonetheless is not affected. The usual moments are given by the local heavy quark expectation values, plus the perturbative corrections originating only from the short-distance domain

μ-dependence of the moments is given by $\alpha_s^{\epsilon}(\mu)$

My conclusions

This appears to be a rather general feature of sufficiently inclusive jet physics. Inclusive B decays are only more transparent since admit OPE allowing to isolate and to take care of the effects of (truly) soft physics

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At the same time, non-integer moments do get affected by nonperturbative collinear emissions *These are not bogus!*


RECYCLE

Quite general reasons suggest it may be problematic to construct a real effective field theory like HQET for the light-like jet physics. Its Lagrangian may hardly be anything but the original QCD one: the notion of "fast jet" is meaningful only in the context of having a spectator around (or another jet, initial color source, ...). For a given quark alone one can always take a boost to view it as having 'normal' energy or momentum

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Jet bremsstrahlung adds a more specific doubt, it has to do with what one expects from an effective field theory. Like $m_Q \to \infty$ in HQET, we need to take here the limit of jet energy to infinity. This may be meaningful provided increasing m_Q resulted in a pure perturbative convolution. It would then have been accounted for by perturbative 'renormalization' of the initial jet (cf. HQET). However, increasing the physical high mass scale — say, passing from m_Q to $2m_Q$ in $b \to s+\gamma$ — brings in new emissions of strongly coupled collinear gluons with $k_\perp \lesssim \Lambda_{\rm QCD}$ but energy between $m_Q/2$ and m_Q, that are nonperturbative. Their effect generally is not suppressed, rather logarithmically enhanced. If one has to involve an unsuppressed nonperturbative matching, an effective theory may lose its primary textbook motivation

The standard expression used for the resummed distribution in the Mellin space

$$\Delta_N = \int_0^1 \mathrm{d}z\, z^{N-1} \Delta(z)$$

has the form

$$\ln \Delta_N(Q^2) = \frac{C_F}{\pi} \int_0^1 \mathrm{d}x\, \frac{x^N - 1}{1-x} \left[\int_{Q^2(1-x)^2}^{Q^2(1-x)} \frac{\mathrm{d}k_\perp^2}{k_\perp^2}\, \alpha_s^\epsilon(k_\perp^2) - B\, \alpha_s^\epsilon(Q^2(1-x)) - D\, \alpha_s^\epsilon(Q^2(1-x)^2) \right]$$

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$$\left. - B\, \alpha_s^\epsilon(Q^2(1-x)) - D\, \alpha_s^\epsilon(Q^2(1-x)^2) \right]$$

While correctly describing the contribution of multiple emissions with different momenta, $\alpha_s(k_\perp^2)$ does not count precisely the contributions of a particular gluon "virtuality" μ^2 in the emission: it yields $\ln\left(\frac{Q^2}{\mu^2} + \frac{1}{1-x}\right)$ instead of $\ln \frac{Q^2}{\mu^2} + \ln \frac{1}{1-x}$; a different effective coupling $\tilde{\alpha}_s$ must be there. This is reshuffled into B and D through explicit integration using the logarithmic running of α_s with the scale, to any particular N^{k}LO, but potentially introduces large higher-order corrections

This is related to running of the coupling, and therefore is complementary to the Catani-Mangano-Nason-Trentadue analysis of resummation

I think it is advantageous to pass here to the proper effective coupling. This usually significantly reduces higher-order corrections and yields accurate numerical estimates already with much simpler quasi-LO calculations, at least for sufficiently inclusive distributions