Resummations in heavy-particle pair production

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Outline

- Pair production and resummation
- Top quarks near threshold (e^+e^-): ultrasoft correction
- Four fermion production near the W^+W^- pair production threshold:
  non-resonant background and invariant mass cuts with EFTs
  dominant NNLO correction
- SUSY particle pair production: colour flow and threshold resummation

Based on work with Y. Kiyo, A.P. Penin, K. Schuller (top)
S. Actis, P. Falgari, C. Schwinn, A. Signer, G. Zanderighi (*WW*)
P. Falgari, C. Schwinn (SUSY), in preparation

Pair production and resummation

$$e^+e^-,\, ij\,(q\bar{q}, qg, gg) \to HH' + X$$

near threshold: particle masses, spin, couplings

"Non-perturbative" despite small couplings:

- Strong Coulomb force: $g^2/v \sim 1$
- Sizeable decay widths of H, H': $\Gamma_H/m_H \sim g^2$
  Physical final states, "Dyson resummation", non-resonant backgrounds
- Soft gluon (photon) resummation, Sudakov logarithms: $g^2 \ln^2 v \sim 1$.

(Perturbative) Resummations in the frameworks of (P)NRQCD, SCET, unstable particle EFT.

Pair production and resummation

- $e^+e^- \longrightarrow t\bar{t}X$

  Determined by strong interactions + Coulomb force

- $e^+e^- \longrightarrow W^+[\rightarrow u\bar{d}]W^-[\rightarrow \mu^-\bar{\nu}_\mu]X$

  Determined by weak interaction + finite width ("four-fermion production")

	WW	$t\bar{t}$
α_{ew}, α_{em}	δ (def.)	δ^2
α_s	$\sqrt{\delta}$	δ (def.)
Γ/M	δ	δ^2
$v^2 \equiv (\sqrt{s} - [2M + i\Gamma])/M$	δ	δ^2
g^2/v (Coulomb)	$\sqrt{\delta}$	1

- $pp \longrightarrow t\bar{t}X, \tilde{g}\tilde{g}X, \tilde{q}\tilde{q}X$

  Soft gluon (photon) resummation + Coulomb force, maybe finite width

Top quarks near threshold

In the absence of electroweak corrections:

$$(q_\mu q_\nu - q^2 g_{\mu\nu})\,\Pi(q^2) = i\int d^4x\, e^{iq\cdot x}\,\langle 0|T(j_\mu(x) j_\nu(0))|0\rangle, \qquad j^\mu(x) = [\bar{t}\gamma^\mu t](x)$$

$$R \equiv \frac{\sigma_{t\bar{t}X}}{\sigma_0} = 12\pi e_t^2\,\mathrm{Im}\,\Pi(s)$$

Near threshold the relevant scales are $m_t \approx 175\,\mathrm{GeV}$, $m_t\alpha_s \approx 30\,\mathrm{GeV}$ and the ultrasoft scale $m_t\alpha_s^2 \approx 2\,\mathrm{GeV}$.

$$\mathcal{L}_{\mathrm{QCD}}\,[Q(h,s,p),\; g(h,s,p,us)] \qquad \mu > m_t$$

$$\downarrow$$

$$\mathcal{L}_{\mathrm{PNRQCD}}\,[Q(p),\; g(us)] \qquad \mu < m_t v$$

NNLO calculations completed in 1998/99 find large uncertainty [up $\pm 25\%$] in the cross section in the resonance peak region (MB, Signer, Smrinov; Hoang, Teubner; Melnikov,Yelkovsky; Yakovlev; Nagano et al.; Penin, Pivovarov). Maybe less after $\log(v)$ resummation [$\pm 3\%$] (Hoang et al., 2002)

The ultrasoft scale appears explicitly only at NNNLO. A complete calculation of the (non-logarithmic) NNNLO correction is therefore needed.

Matching/effective Lagrangian at NNNLO

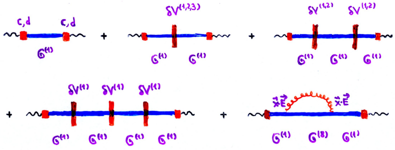
Matching of currents and interactions (potentials and ultrasoft)

$$j^i = c_v\, \psi^\dagger \sigma^i \chi + \frac{d_v}{6m_t^2} \psi^\dagger \sigma^i \mathbf{D}^2 \chi + \ldots$$

$$\mathcal{L}_{\text{QCD}} \to \mathcal{L}_{\text{PNRQCD}}$$

$$\begin{aligned}
\mathcal{L}_{\text{PNRQCD}} \quad = \quad & \psi^\dagger \left(iD_0 + i\frac{\Gamma_t}{2} + \frac{\boldsymbol{\partial}^2}{2m} + \frac{\boldsymbol{\partial}^4}{8m^3} \right) \psi + \chi^\dagger \left(iD_0 - i\frac{\Gamma_t}{2} - \frac{\boldsymbol{\partial}^2}{2m} - \frac{\boldsymbol{\partial}^4}{8m^3} \right) \chi \\
& + \int d^{d-1}\mathbf{r} \left[\psi^\dagger \psi \right] (x + \mathbf{r}) \left(-\frac{\alpha_s C_F}{r} + \delta V(r, \boldsymbol{\partial}) \right) \left[\chi^\dagger \chi \right] (x) \\
& - g_s \psi^\dagger (x) \boldsymbol{x} \boldsymbol{E}(t, \mathbf{0}) \psi(x) - g_s \chi^\dagger (x) \boldsymbol{x} \boldsymbol{E}(t, \mathbf{0}) \chi(x)
\end{aligned}$$

- Almost everything needed at NNNLO known (Manohar, 1997; Wüster, 2003, Luke, Savage, 1997; Marquard et al., 2006, 2009; MB, Signer, Smirnov, 1999; Kniehl et al., 2001, 2002; Wüster, 2003).
  Important missing piece: non-fermionic and singlet pieces of three-loop $c_v^{(3)}$.



where

$$G_c^{(1,8)}(\boldsymbol{r},\boldsymbol{r}',E) = \frac{my}{2\pi} e^{-y(r+r')} \sum_{l=0}^{\infty} (2l+1)(2yr)^l (2yr')^l P_l\left(\frac{\boldsymbol{r}\cdot\boldsymbol{r}'}{rr'}\right) \sum_{s=0}^{\infty} \frac{s! L_s^{(2l+1)}(2yr) L_s^{(2l+1)}(2yr')}{(s+2l+1)!(s+l+1-\lambda)}$$

$$y = \sqrt{-m(E+i\epsilon)},\ \lambda = \frac{m\alpha_s}{2y} \times \{C_F\ (\text{singlet});\ C_F - C_A/2\ (\text{octet})\}$$

The ultrasoft contribution is ($D = d-1$)

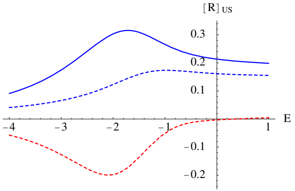
$$\delta G_{\rm us} = (-i)(ig_s)^2 C_F \int \frac{d^d k}{(2\pi)^d} \frac{-i}{k^2} \left(\frac{k^i k^j}{k_0^2} - \delta^{ij}\right) \int \prod_{n=1}^{6} \frac{d^D p_n}{(2\pi)^D}\, i\tilde{G}^{(1)}(p_1,p_2;E)\, i\tilde{G}^{(8)}(p_3,p_4;E+k^0)\, i\tilde{G}^{(1)}(p_5,p_6;E)$$

$$\times\, i\left[\frac{2p_3^i}{m_t}(2\pi)^D \delta^{(D)}(p_3-p_2) + (ig_s)^2 \frac{C_A}{2} \frac{2(p_2-p_3)^i}{(p_2-p_3)^4}\right] i\left[-\frac{2p_4^j}{m_t}(2\pi)^D \delta^{(D)}(p_4-p_5) + (ig_s)^2 \frac{C_A}{2} \frac{2(p_4-p_5)^j}{(p_4-p_5)^4}\right]$$

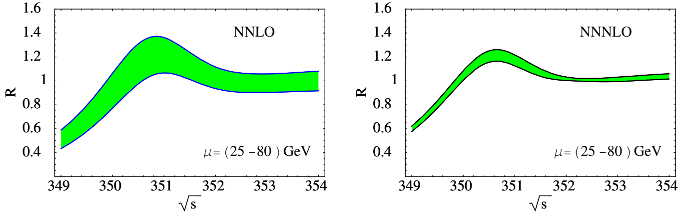
Difficulty is the extraction of the divergence in dim. reg.

Results

Non-logarithmic ultrasoft correction
is very large [up to 30%]



Scale dependence at NNNLO versus NNLO:



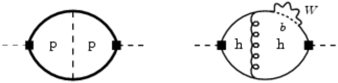
(Non-fermionic c_3 terms shift the NNNLO result, but do not change the scale uncertainty.)

“Electroweak effects”

The QCD-only result usually discussed is far from reality

- Significant “non-resonant” background from off-shell top decay (unless tight invariant mass cuts are applied), not described by NRQCD
- Initial state radiation
- Finite-width divergences (overall divergence, already at NNLO):

$$[\delta^{us} G(E)]_{\text{overall}} \propto \frac{\alpha_s}{\epsilon} \cdot E$$



With $E = \sqrt{s} - 2m_t + i\Gamma$ the divergence survives in the imaginary part, and is

$$\text{Im}\,[\delta^{us} G(E)]_{\text{overall}} \propto m_t \times \frac{\alpha_s \alpha_{ew}}{\epsilon}$$

The systematic study of a realistic cross section prediction has only just started (Hoang, Reisser, 2004; Actis et al., 2008; Hoang, Reisser, Ruiz-Femenia (in preparation); MB, Jantzen, Ruiz-Femenia (work in progress))

Four-fermion production near the *WW* threshold

- Consider

$$e^- e^+ \to \mu^- \bar{\nu}_\mu u \bar{d} X$$

near $\sqrt{s} = 160\,\text{GeV}$. Dominated by nearly on-shell $W^- W^+$. Large sensitivity to M_W.
ILC with GIGAZ option: $\delta M_W \approx 6\,\text{MeV}$ experimentally (Wilson, 2001).
Rule of thumb: $\delta\sigma \approx 1\% \Leftrightarrow \delta M_W \approx 15\,\text{MeV}$.

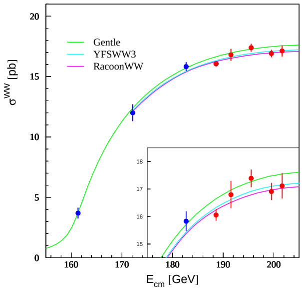
- Extract cross section from the forward-scattering amplitude

$$\hat{\sigma} = \frac{1}{s}\,\text{Im}\,\mathcal{A}(e^- e^+ \to e^- e^+)_{|\mu^- \bar{\nu}_\mu u \bar{d}}$$

- "QCD-style" calculation of the short-distance cross section with massless electrons in the $\overline{\text{MS}}$ scheme, then

$$\sigma(s) = \int_0^1 dx_1 dx_2 f_{e/e}(x_1) f_{e/e}(x_2)\, \hat{\sigma}(x_1 x_2 s).$$

$\overline{\text{MS}}$ electron distribution function depends on m_e, but not on $\sqrt{s}$, M, Γ.



Unstable particle EFT

For simplicity, consider SM with $\alpha_s = 0$.
Integrate out short-distance fluctuations, such that only virtualities $k^2 \ll M_W^2$ are left.

- Fields
  - No top, Z, Higgs.
  - Two non-relativistic spin-1 fields $\Omega^i_\mp$.
  - Photon and light fermion fields (soft and collinear).
- Lagrangian

$$\mathcal{L}_{\rm eff} = \mathcal{L}_{\rm SCET}(\text{collinear, anti-collinear,soft}) + \sum_{a=\mp} \left[\Omega_a^{\dagger i} \left(iD_s^0 + \frac{\vec{\partial}^2}{2M_W} - \frac{\Delta}{2} \right) \Omega_a^i + \Omega_a^{\dagger i} \frac{(\vec{\partial}^2 - M_W \Delta)^2}{8M_W^3} \Omega_a^i \right] + \int d^3 r \left[\Omega_-^{\dagger i} \Omega_-^i (x + \vec{r}) \right] \left(-\frac{\alpha_{\rm QED}}{r} \right) \left[\Omega_+^{\dagger j} \Omega_+^j \right] (x) + \ldots$$

- Matching conditions on the complex pole. Pole mass scheme $\Delta = -i\Gamma$.
  Effective theory propagator accomplishes "Dyson resummation".

Unstable particle EFT

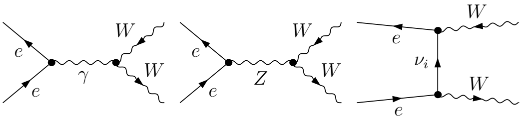
General formula for the forward-scattering amplitude including non-resonant production
(MB,Chapovsky, Signer, Zanderighi, 2003)

$$i\,\mathcal{A} = \sum_{k,l} \int d^4x \, \langle e^+e^- | T(i\mathcal{O}_p^{(k)}(0) i\mathcal{O}_p^{(l)}(x)) | e^+e^- \rangle + \sum_k \langle e^+e^- | i\mathcal{O}_{4e}^{(k)}(0) | e^+e^- \rangle.$$

The local four-electron operator includes off-shell WW or single W intermediate states.

Contrary to pure QCD $t\bar{t}$ case, the initial state is not sterile.

Matching of the leading production operator



$$\mathcal{O}_p^{(0)} = \frac{\pi\alpha_{ew}}{M_W^2} \left(\bar{e}_{c2,L}\gamma^{[i}n^{j]}e_{c1,L}\right)\left(\Omega_-^{\dagger i}\Omega_+^{\dagger j}\right)$$

At LO in the expansion around threshold, only the t-channel diagram contributes.

Hard (non-resonant) N$^{1/2}$LO and N$^{3/2}$LO corrections

- Contribution to the matching coefficient of

$$\mathcal{O}_{4e}^{(k)} = \frac{C_{4e}^{(k)}}{M_W^2}(\bar{e}_{c1}\Gamma_1 e_{c2})(\bar{e}_{c2}\Gamma_2 e_{c1})$$

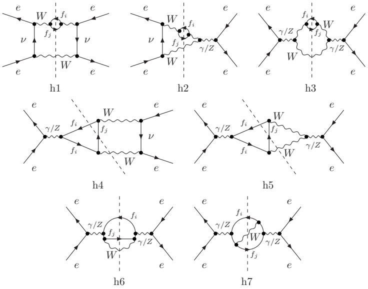
  Computed in standard PT with propagator $-ig_{\mu\nu}/(p^2 - M_W^2)$ since W lines are hard (formally off-shell).

- Two-loop cut diagrams result in

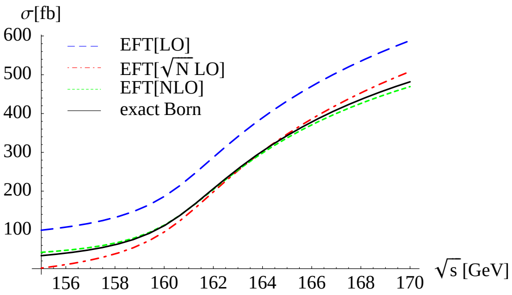
$$\sigma_{LR}^{(1/2+3/2)} = \frac{4\alpha^3}{27 s_w^6 s}\left[K_{h1} + K_{h2}\,\frac{E}{M_W}\right]$$

  Contribution from the diagrams h4, h5 and with a single W turns out to be very small.

- Leading term is N$^{1/2}$LO (loop suppression, but no threshold suppression $\propto \sqrt{\delta}$ as in the potential region)
- Three-loop diagrams give another N$^{3/2}$LO term $\propto \alpha^4$ but energy-independent.



Born cross section



	$\sigma(e^- e^+ \to \mu^- \bar{\nu}_\mu u\bar{d})$(fb)				
$\sqrt{s}$ [GeV]	EFT(LO)	EFT($\sqrt{N}$LO)	EFT(NLO)	EFT($N^{\frac{3}{2}}$ LO)	exact Born
155	101.61	1.62	43.28	31.30	34.43(1)
158	135.43	39.23	67.78	62.50	63.39(2)
161	240.85	148.44	160.45	160.89	160.62(6)
164	406.8	318.1	313.5	318.8	318.3(1)
167	527.8	442.7	420.4	429.7	428.6(2)
170	615.5	533.9	492.9	505.4	505.1(2)

Radiative corrections

Radiative corrections correspond to cuts involving loops on each side of the cut or five-particle $\mu^- \bar{\nu}_\mu u \bar{d} \gamma$ cuts. Up to NLO:

- Two-loop $\Delta^{(2)} = M_W(\Pi^{(2,0)} + \Pi^{(1,1)}\Pi^{(1,0)}) = -i\Gamma_W^{(1)}$, i.e. one-loop EW correction to on-shell W decay in the pole mass renormalization scheme.
- One-loop EW correction to the LO production operator

$$\mathcal{O}_p^{(1)} = \frac{\pi\alpha_{ew}}{\hat{M}_W^2} \left[C_{p,LR}^{(1)} \left(\bar{e}_L \gamma^{[i} n^{j]} e_L \right) + C_{p,RL}^{(1)} \left(\bar{e}_R \gamma^{[i} n^{j]} e_R \right) \right] \left(\Omega_-^{\dagger i} \Omega_+^{\dagger j} \right)$$

- Up to two insertions of the Coulomb potential interaction.
- Soft and collinear photon corrections to the EFT forward-scattering amplitude.
- Resummation of large collinear logarithms $\ln(M_w/m_e)$ from initial-state radiation.

Up to NLO QCD affects mainly the hadronic partial W decay width. Mixed three-loop QCD/EW hard effects are small and will be neglected.

NLO Result (MB, Falgari, Schwinn, Signer, Zanderighi, 2007)

Comparison with of Born, EFT, full four fermion (*Denner, Dittmaier, Roth, Wieders, 2005*) and DPA NLO calculations, ISR resummed. Same input parameters.

	$\sigma(e^- e^+ \to \mu^- \bar{\nu}_\mu u \bar{d}\, X)$(fb)			
$\sqrt{s}$ [GeV]	Born (SM)	EFT	full ee4f	DPA
161	107.06(4)	117.38(4)	118.12(8)	115.48(7)
170	381.0(2)	399.9(2)	401.8(2)	402.1(2)

Sensitivity to M_W and theoretical uncertainty

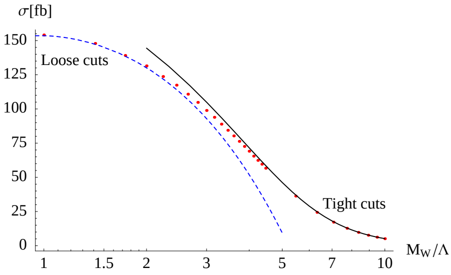
(Variation of cross section normalized to standard input)

- At the point of maximal sensitivity large uncertainty from current implementation of ISR ($\delta M_W \approx 30\,\text{MeV}$)
- Uncertainties from $\text{N}^{3/2}$LO radiative effects are estimated:
  10 MeV from hard corrections included in full four-fermion calculation
  4 MeV from Coulomb times hard + soft
- Experimental accuracy (6 MeV) can be reached by NLL ISR implementation and inclusion of $\text{N}^{3/2}$LO – use existing full NLO $4f$ calculation plus dominant NNLO terms from EFT approach.
- Perhaps also need a less inclusive treatment.

EFT and cuts

Cuts are not straightforward in the EFT approach: may introduce new scales regions.
Example: Invariant mass cuts $|M^2_{u\bar{d}} - M^2_W|, |M^2_{\mu\bar{\nu}_\mu} - M^2_W| < \Lambda^2$

- Loose cut: $\Lambda \sim M_W$
  No modification of potential loops (momenta always within the cut by power counting).
  Cut affects the matching coefficient of the four-electron operator (non-resonant terms).
- Tight cut: $\Lambda \sim \sqrt{M_W \Gamma_W}$
  Four-electron operator (non-resonant terms) does not contribute at all.
  Cut affects loop calculations in the effective theory.



Red dots: Born cross section ($\sqrt{s} = 161$ GeV, WHIZARD)

Cuts for WW

W mass measurement uses almost the inclusive cross section. Cut for the cross section measurement at $\sqrt{s} = 161$ GeV used at LEP:

Cut	$\sigma_{\text{Born}}(e^- e^+ \to \mu^- \overline{\nu}_\mu u\bar{d})$(fb)	$\sigma_{\text{cut}}/\sigma_{\text{tot}}$
–	154.18(5)	
$\lvert\vec{p}_\mu\rvert > 20$ GeV	153.71(5)	99.69(5) %
$M_{\mu\nu} > 55$ GeV, 40 GeV $< M_{jj} < 120$ GeV	150.61(5)	97.68(5) %
$\theta_{\mu j} > 15$ degrees	149.35(5)	96.87(5) %
$\lvert \cos\theta_\nu \rvert < 0.95$	148.28(5)	96.17(5) %
all	140.03(5)	90.82(5) %

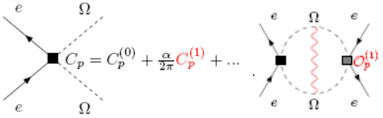
2nd and 3rd column: Effect of LEP phase-space cuts on the Born cross section at $\sqrt{s} = 161$ GeV

Strategy: Use full NLO computation à la Denner-Dittmaier (complex mass scheme) including all cuts + EFT calculation of the leading NNLO terms without cuts ($\approx 7\%$ error on a small correction).

Beyond NLO (I)

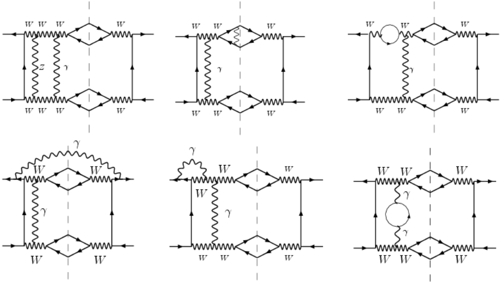
Dominant NNLO = N$^{3/2}$LO in EFT counting

- NLO correction to non-resonant four-electron operator – already included in full NLO (non-resonant Born terms were N$^{1/2}$LO).
- Interference of Coulomb exchange with tree-level higher-dimensional production operators – already included in full NLO.

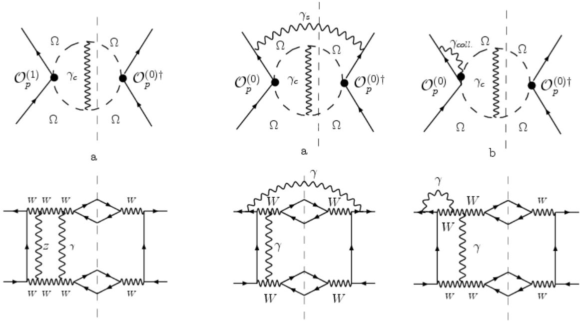


N$^{3/2}$LO terms from true NNLO diagrams contain at least one Coulomb photon:

- Mixed hard/Coulomb corrections
- Interference of Coulomb exchange with soft and collinear radiative corrections
- Correction to the Coulomb potential itself.



Beyond NLO (II)

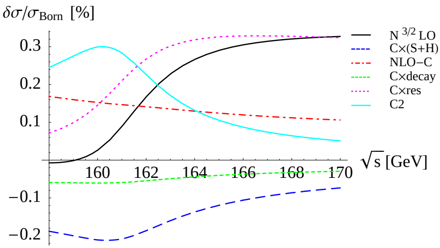


Result for [hard+soft+collinear]×Coulomb, to be convoluted with electron structure functions:

$$\hat{\sigma}^{C}_{LR}(s) = \frac{16\pi\alpha_{ew}^2\alpha}{27 s M_W^2}\left[\left(\frac{9}{2}+\frac{\pi^2}{4}+\mathrm{Re}\,c^{(1),\mathrm{fin}}_{p,LR}\right)\mathrm{Im}\,G^{(0)}_{\mathrm{C}}(0,0;\mathcal{E}_W) + 2\,\mathrm{Im}\int_0^\infty dk\,\frac{G^{(0)}_{\mathrm{C}}(0,0;\mathcal{E}_W-k)}{[k]_{M_W+}}\right]$$

$$\longrightarrow -\frac{\alpha_{ew}^2\,\alpha^2}{27\,s}\left\{\left(9+\frac{\pi^2}{2}+2\,\mathrm{Re}\,c^{(1),\mathrm{fin}}_{p,LR}\right)\mathrm{Im}\left[\ln\left(-\frac{\mathcal{E}_W}{M_W}\right)\right]+2\,\mathrm{Im}\left[\ln^2\left(-\frac{\mathcal{E}_W}{M_W}\right)\right]\right\}.$$

Result (Actis, MB, Falgari, Schwinn, 2008)



In total a small correction:
$[\delta M_W]_{\text{BeyondNLO}} \approx (3-5)\,\text{MeV}$

Partonic cross section known accurately enough.

	$\sigma(e^-e^+ \to \mu^-\bar{\nu}_\mu u\bar{d}X)$(fb)				
$\sqrt{s}$ [GeV]	Born	Born (ISR)	NLO	$\hat{\sigma}^{(3/2)}$	$\sigma^{(3/2)}_{\text{ISR}}$
158	61.67(2)	45.64(2) [−26.0%]	49.19(2) [−20.2%]	−0.001 [−0.00%]	0.000 [+0.00%]
161	154.19(6)	108.60(4) [−29.6%]	117.81(5) [−23.6%]	0.147 [+0.10%]	0.087 [+0.06%]
164	303.0(1)	219.7(1) [−27.5%]	234.9(1) [−22.5%]	0.811 [+0.27%]	0.544 [+0.18%]
167	408.8(2)	310.2(1) [−24.1%]	328.2(1) [−19.7%]	1.287 [+0.31%]	0.936 [+0.23%]
170	481.7(2)	378.4(2) [−21.4%]	398.0(2) [−17.4%]	1.577 [+0.33%]	1.207 [+0.25%]

Soft gluon resummation in hadronic production (MB, Falgari, Schwinn)

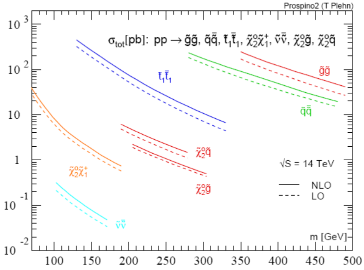
Partonic cross sections $i + j \longrightarrow HH' + X$ contain

$$\left[\alpha_s \ln^2(1-z)\right]^n, \qquad z = (m_H + m_H')^2/\,\hat{s}$$

which should be resummed, if the total hadronic cross section is dominated by the partonic threshold.

($t\bar{t}$: Catani et al., 1996; Bonciani et al, 1998, Kidonakis et al, 2001; Moch, Uwer, 2008; Sparticle pairs: Kulesza, Motyka, 2008; Langenfeld, Moch, 2009)

- Not clear why the partonic threshold should be relevant at LHC energies for $t\bar{t}$.
  Even for sparticles really only for $m_H \geq 3$ TeV.
- Main reason is probably empirical observation of improved scale dependence.
- Anyway an interesting problem due to colour exchange.



Soft gluon resummation for the total cross section

Formalism for $2 \to 2$ scattering processes with massless coloured particles was set up by (Kidonakis, Sterman; 1997).

But threshold (Sudakov) resummation for total $t\bar{t}$ cross section (or any heavy coloured particle pairs) was in fact never performed accurately.

In the following aim at

- Complete treatment of colour exchange
  Separate short-distance coefficients in each independent colour channel
- Proof of factorization of soft gluons from Coulomb exchange
- Threshold resummation for heavy particles with sizeable decay width
  (not discussed in this talk)

Solution of RGE equation done in moment space using the formalism developed by (Becher, Neubert, 2007) for Drell-Yan production.

Hard amplitude

Hard sub-process always $2 \to 2$. Operators with more fields are power-suppressed in $(1-z)$.

$$\mathcal{A}(pp' \to HH'X) = \sum_{\ell} C^{(\ell)}_{\{a;\alpha\}}(\mu)\, \langle HH'X|\mathcal{O}^{(\ell)}_{\{a;\alpha\}}(\mu)|pp'\rangle_{\text{EFT}}$$

$$\mathcal{O}^{(\ell)}_{\{a;\alpha\}}(\mu) = \left[\phi_{c;a_1\alpha_1}\,\phi_{\bar{c};a_2\alpha_2}\,\psi^{\dagger}_{a_3\alpha_3}\,\psi'^{\dagger}_{a_4\alpha_4}\right](\mu).$$

- Collinear fields (initial state)

$$\Phi_c \in \{W_c^{\dagger}\xi_c,\ \bar{\xi}_c W_c,\ \mathcal{A}_c^{\mu\perp} = g_s^{-1}(W_c^{\dagger}[iD_c^{\mu\perp}W_c])\}$$

  and non-relativistic scalar, spinor, vector, ... fields (final state)

- Not useful to perform a spin decomposition of the operators, since anomalous dimensions are spin independent.
- Soft gluons interact with everything, and “in between” Coulomb exchange.

Soft-gluon decoupling

From the initial state:

$$\mathcal{L}_c = \bar{\xi}_c \left(in\cdot D + i\not{D}_{\perp c} \frac{1}{i\bar{n}\cdot D_c} i\not{D}_{\perp c} \right) \frac{\not{\bar{n}}}{2}\, \xi_c - \frac{1}{2}\,\mathrm{tr}\left(F_c^{\mu\nu} F^c_{\mu\nu} \right)$$

by the SCET field redefinitions (Bauer, Pirjol, Stewart, 2001)
$\xi_c(x) = S_n^{(3)}(x_-)\, \xi_c^{(0)}(x)$, $A^A_{c\mu}(x) = S_n^{(8)}(x_-)\, A^{A(0)}_{c\mu}(x)$, such that $n\cdot D \to n\cdot D_c$.

From the final state:

$$\mathcal{L}_{\mathrm{PNRQCD}} = \psi^\dagger \left(iD_s^0 + \frac{\vec{\partial}^2}{2m_H} + \frac{i\Gamma_H}{2} \right) \psi + \psi'^\dagger \left(iD_s^0 + \frac{\vec{\partial}^2}{2m_{H'}} + \frac{i\Gamma_{H'}}{2} \right) \psi' + \int d^3\vec{r}\, \left[\psi^\dagger \mathbf{T}^{(R)a} \psi \right](\vec{r})\, \left(\frac{\alpha_s}{r} \right) \left[\psi'^\dagger \mathbf{T}^{(R')a} \psi' \right](0)$$

by the PNRQCD field redefinition $\psi_a(x) = S_v^{(R)}(x^0)_{ab}\, \psi_b^{(0)}(x)$, such that $D_s^0 \to \partial^0$.

S_v drops out from the Coulomb interaction, since $S_v^{(R)\dagger} \mathbf{T}^{(R)a} S_v^{(R)\dagger} = [S_{\mathrm{ad}}^T]^{ab}\, \mathbf{T}^{(R)b}$ in any rep R;
S_{ad} is real and independent of $\vec{r}$.
Proves decoupling of soft gluon and Coulomb resummation (valid for S-wave production only)!

Factorization formula and soft functions

Soft, collinear, potential fields are decoupled: $\sigma_{pp'} = f_p \star f_{p'} \star |C^2| \times J \star W$

$$\hat{\sigma}_{pp'}(\hat{s},\mu) = \frac{1}{2\hat{s}N_{pp'}} \sum_{i,i'} C^{(i)}_{pp'} C^{(i')*}_{pp'} \;\; c^{(i)}_{\{a\}} c^{(i')*}_{\{b\}} \int d\omega \; J_{\{k\}}(E-\tfrac{\omega}{2}) \; W^{\{k\}}_{\{ab\}}(\omega,\mu)$$

$$= \frac{1}{2\hat{s}N_{pp'}} \sum_{i,i'} \sum_{R_\alpha} C^{(i)}_{pp'} C^{(i')*}_{pp'} \int d\omega \; J_{R_\alpha}(E-\tfrac{\omega}{2}) \; W^{R_\alpha}_{ii'}(\omega,\mu)$$

$$W^{\{k\}}_{\{ab\}}(\omega,\mu) = \int \frac{dz_0}{4\pi} e^{i\omega z_0/2} \, W^{\{k\}}_{\{ab\}}(z_0,\vec{0},\mu)$$

$$W^{\{k\}}_{\{a,b\}}(z,\mu) = \langle 0|\overline{\mathrm{T}}[S^{\dagger}_{n,ib_1} S^{\dagger}_{\bar{n},jb_2} S_{v,b_4k_4} S_{v,b_3k_3}](z)\mathrm{T}[S_{\bar{n},a_2j} S_{n,a_1i} S^{\dagger}_{v,k_1a_3} S^{\dagger}_{v,k_2a_4}](0)|0\rangle$$

The PNRQCD Lagrangian can be diagonalized in colour space by decomposing the colour representation $R\otimes R' = \sum_\alpha R_\alpha$ into irreducible representations.
The soft functions simplify to those for a single heavy particle in rep R_α.

$$W^{R_\alpha}_{ii'}(\omega,\mu) = P^{R_\alpha}_{\{k\}} c^{(i)}_{\{a\}} W^{\{k\}}_{\{ab\}}(\omega,\mu) c^{(i')*}_{\{b\}} = c^{R_\alpha(i)}_{\{a,A\}} W^{R_\alpha}_{\{ab,AB\}}(\omega,\mu) c^{R_\alpha(i')*}_{\{b,B\}}$$

$$W^{R_\alpha}_{\{ab,AB\}}(z,\mu) \equiv \langle 0|\overline{\mathrm{T}}[S^{\dagger}_{n,ib_1} S^{\dagger}_{\bar{n},jb_2} S^{R_\alpha}_{v,BK}](z)\mathrm{T}[S_{\bar{n},a_2j} S_{n,a_1i} S^{R_\alpha\dagger}_{v,KA}](0)|0\rangle$$

Soft function

Example: $8+8 \longrightarrow 3+\bar{3}$ ($gg \longrightarrow t\bar{t}$)
Colour conservation implies that only $1, 8_S, 8_A$ from $8+8$ contribute. An orthogonal basis is

$$c^{S(1)}_{a_1 a_2} = \frac{1}{\sqrt{D_A}}\delta_{a_1 a_2} \qquad c^{8(2)}_{a_1 a_2 A} = \frac{1}{2\sqrt{D_A B_F}} d^{A a_2 a_1} \qquad c^{8(3)}_{a_1 a_2 A} = \frac{1}{\sqrt{N_c D_A}} F^A_{a_2 a_1}$$

Soft function is diagonal to all orders in PT in this basis.

$$W^S_{11}(z,\mu) = \frac{1}{N_C^2-1}\langle 0|\,\mathrm{tr}\,\overline{\mathrm{T}}[S_n^\dagger S_{\bar{n}}](z)\mathrm{T}[S_{\bar{n}}^\dagger S_n](0)|0\rangle$$

$$W^8_{22}(z,\mu) = \frac{N_C}{(N_C^2-1)(N_C^2-4)}\langle 0|\mathrm{Tr}[\overline{\mathrm{T}}[S_n^\dagger D^a S_{v,ac} S_{\bar{n}}](z)\mathrm{T}[S_{\bar{n}}^\dagger S^\dagger_{v,cb} D^b S_n](0)]|0\rangle$$

$$W^8_{33}(z,\mu) = \frac{1}{N_C(N_C^2-1)}\langle 0|\mathrm{Tr}[\overline{\mathrm{T}}[S_n^\dagger F^a S_{v,ac} S_{\bar{n}}](z)\mathrm{T}[S_{\bar{n}}^\dagger S^\dagger_{v,cb} F^b S_n](0)]|0\rangle$$

Consider all $2 \rightarrow 2$ processes with 3, $\bar{3}$ and 8.
General one-loop soft function (in Catani's colour-operator notation):

$$\mathbf{W}^{(1)\,R_\alpha}(L) = \frac{\alpha_s}{4\pi}\left[((\mathbf{T}_1+\mathbf{T}_2)\cdot\mathbf{T}_3 - 2\mathbf{T}_1\cdot\mathbf{T}_2)\left(\frac{2}{\epsilon^2}+\frac{2}{\epsilon}L+L^2+\frac{\pi^2}{6}\right)+\mathbf{T}_3^2\left(\frac{1}{\epsilon}+L+2\right)\right]$$
$$= \frac{\alpha_s}{4\pi}\left[\left((\mathbf{T}_1^2+\mathbf{T}_2^2\right)\left(\frac{2}{\epsilon^2}+\frac{2}{\epsilon}L+L^2+\frac{\pi^2}{6}\right)+\mathbf{T}_3^2\left(\frac{1}{\epsilon}+L+2\right)\right]$$

Resummation

Resummation of soft gluon threshold blogs via renormalization group equations:

$$\frac{d}{d\ln\mu} C^i_{pp'}(4\bar{m}^2,\mu) = \left[\frac{1}{2}\left(\Gamma^R_{\rm cusp} + \Gamma^{R'}_{\rm cusp}(\alpha_s)\right)\delta_{ij}\ln\left(-\frac{4\bar{m}^2}{\mu^2}\right) + \gamma^C_{ij}(\alpha_s)\right] C^j_{pp'}(4\bar{m}^2,\mu)$$

$$\frac{d}{d\log\mu}\mathbf{W}^{R_\alpha}(\omega,\mu) = -\int_0^\omega d\omega\left(\frac{1}{\omega-\omega'}\right)_{[\mu]}\left[\mathbf{\Gamma}^{R_\alpha}\mathbf{W}^{R_\alpha}(\omega',\mu) + \mathbf{W}^{R_\alpha}(\omega',\mu)\mathbf{\Gamma}^{R_\alpha\,\dagger}(\alpha_s)\right]$$
$$-\boldsymbol{\gamma}^{S,R_\alpha}\mathbf{W}^{R_\alpha}(\omega,\mu) - \mathbf{W}^R\boldsymbol{\gamma}^{S,R_\alpha\,\dagger}(\alpha_s)$$

$$\gamma^C_{ij} = \gamma^S_{ji} - \delta_{ij}\left(\gamma^{\phi,r} + \gamma^{\phi,r'}\right)$$

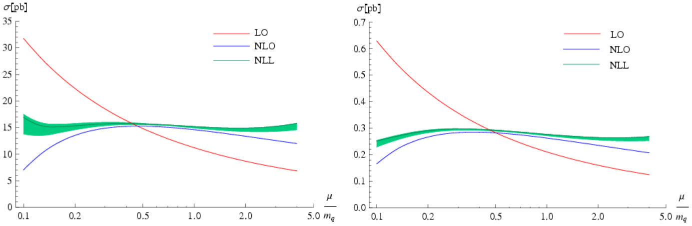
Since the soft functions are diagonal in a suitable basis, can use the formalism developed for Drell-Yan (and Higgs) production by (Becher, Neubert, 2007).
See Thomas' talk.

Squark-antisquark production, numerical results

Results at NLL for $pp \to$ squark+antisquark $+ X$ at $\sqrt{s} = 14$ TeV

NLL = Tree C and W, 1-loop anomalous dim. + LO Coulomb Green function + matching to NLO fixed order (Prospino (Plehn), fitting functions from (Langenfeld, Moch, 2009))

Scale dependence at LO, NLO, NLL



($m_{\tilde{q}} = 500$ GeV (left), 1 TeV (right); $m_{\tilde{g}}/m_{\tilde{q}} = 1.2$)

Resummation is a small effect at the natural scale, but shows less scale variation. Green band: variation of the soft scale.

Conclusion

1) The NNNLO QCD prediction for the top-antitop cross section near threshold is (nearly) complete.

$$\frac{\delta\sigma}{\sigma} \approx 5\%$$

2) Four-fermion production in e^+e^- can be done with EFT methods including the dominant NNLO terms. Enough for

$$\delta M_W < 3\,\text{MeV}$$

from threshold scan.

3) General framework for soft gluon resummation for coloured particle pair production (total cross section).
For squark-antisquark production find (once again) improvement of scale dependence at NLL. Few percent corrections beyond NLO.