

Recent progress in hadronic τ decays

- More than 20 years ago, hadronic τ decays have been identified as an ideal system to study strong interactions.
- Investigations of hadronic τ decays already contributed tremendously for fundamental QCD parameters like α_s , the strange mass and non-perturbative condensates.
- The α_s determination from the τ hadronic width is amongst the most precise determinations of the strong coupling.
- The flavour-breaking difference of τ decay fractions can also be used to determine V_{us} and, with precise experimental data, even m_s and V_{us} simultaneously.

Consider the physical quantity R_τ : (Braaten, Narison, Pich 1992)

$$R_\tau \equiv \frac{\Gamma(\tau^- \rightarrow \text{hadrons } \nu_\tau(\gamma))}{\Gamma(\tau^- \rightarrow e^- \bar{\nu}_e \nu_\tau(\gamma))} = 3.640 \pm 0.010.$$

R_τ is related to the QCD correlators $\Pi^{T,L}(z)$: ($z \equiv s/M_\tau^2$)

$$R_\tau = 12\pi \int_0^1 dz (1-z)^2 \left[(1+2z) \text{Im}\Pi^T(z) + \text{Im}\Pi^L(z) \right],$$

with the appropriate combinations

$$\Pi^J(z) = |V_{ud}|^2 \left[\Pi_{ud}^{V,J} + \Pi_{ud}^{A,J} \right] + |V_{us}|^2 \left[\Pi_{us}^{V,J} + \Pi_{us}^{A,J} \right].$$

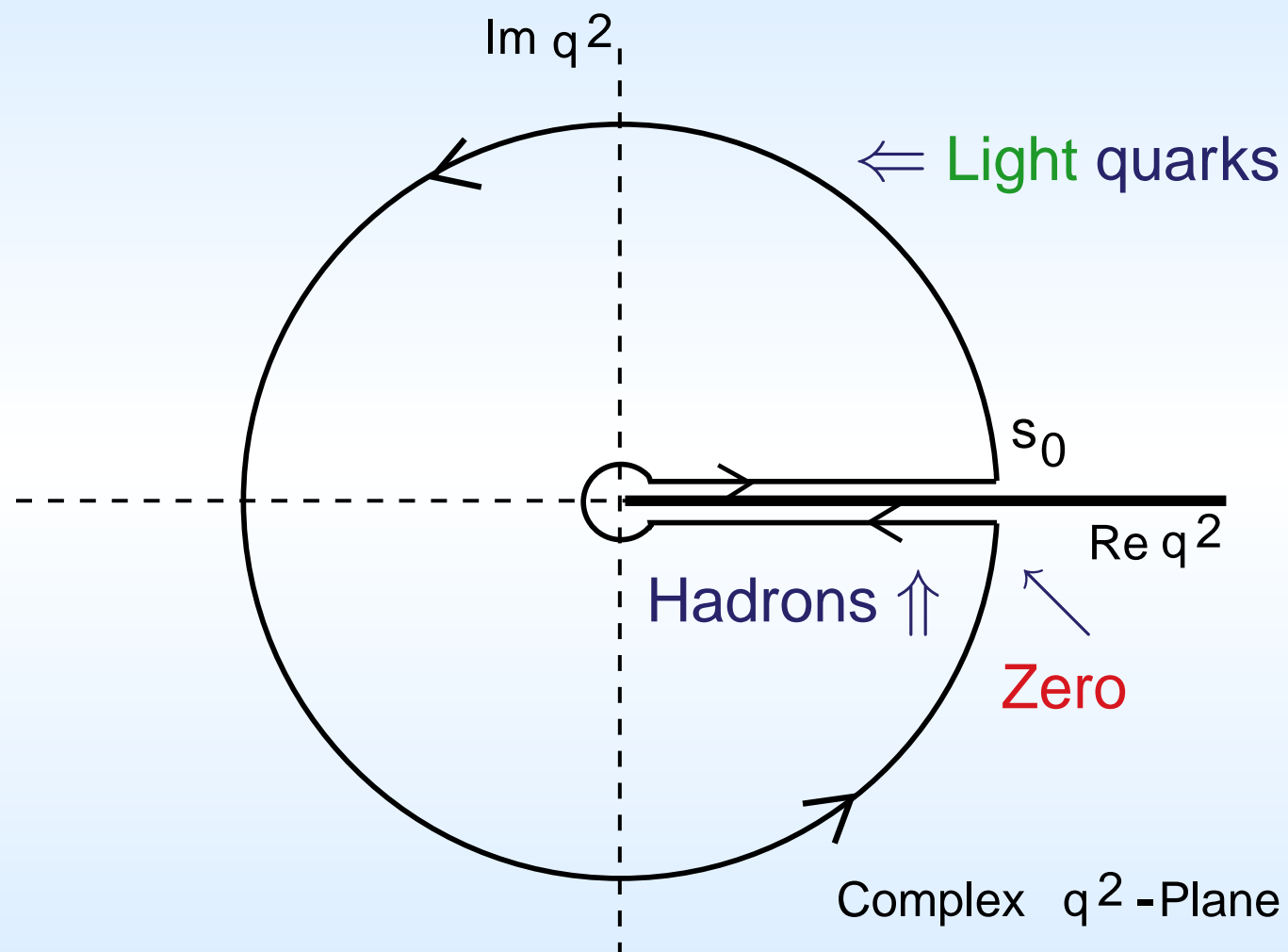
Additional information can be inferred from the **moments**

$$R_{\tau}^{kl} \equiv \int_0^1 dz (1-z)^k z^l \frac{dR_{\tau}}{dz} = R_{\tau,NS}^{kl} + R_{\tau,S}^{kl}.$$

Theoretically, R_{τ}^{kl} can be expressed as:

$$R_{\tau}^{kl} = N_c S_{EW} \left\{ (|V_{ud}|^2 + |V_{us}|^2) \left[1 + \delta^{kl(0)} \right] + \sum_{D \geq 2} \left[|V_{ud}|^2 \delta_{ud}^{kl(D)} + |V_{us}|^2 \delta_{us}^{kl(D)} \right] \right\}.$$

$\delta_{ud}^{kl(D)}$ and $\delta_{us}^{kl(D)}$ are corrections in the **Operator Product Expansion**, the most important ones being $\sim m_s^2$ and $m_s \langle \bar{q}q \rangle$.



The sensitivity to the **strange** quark mass can be enhanced by considering the flavour **SU(3)**-breaking difference:

(Pich, Prades; ALEPH 1998)

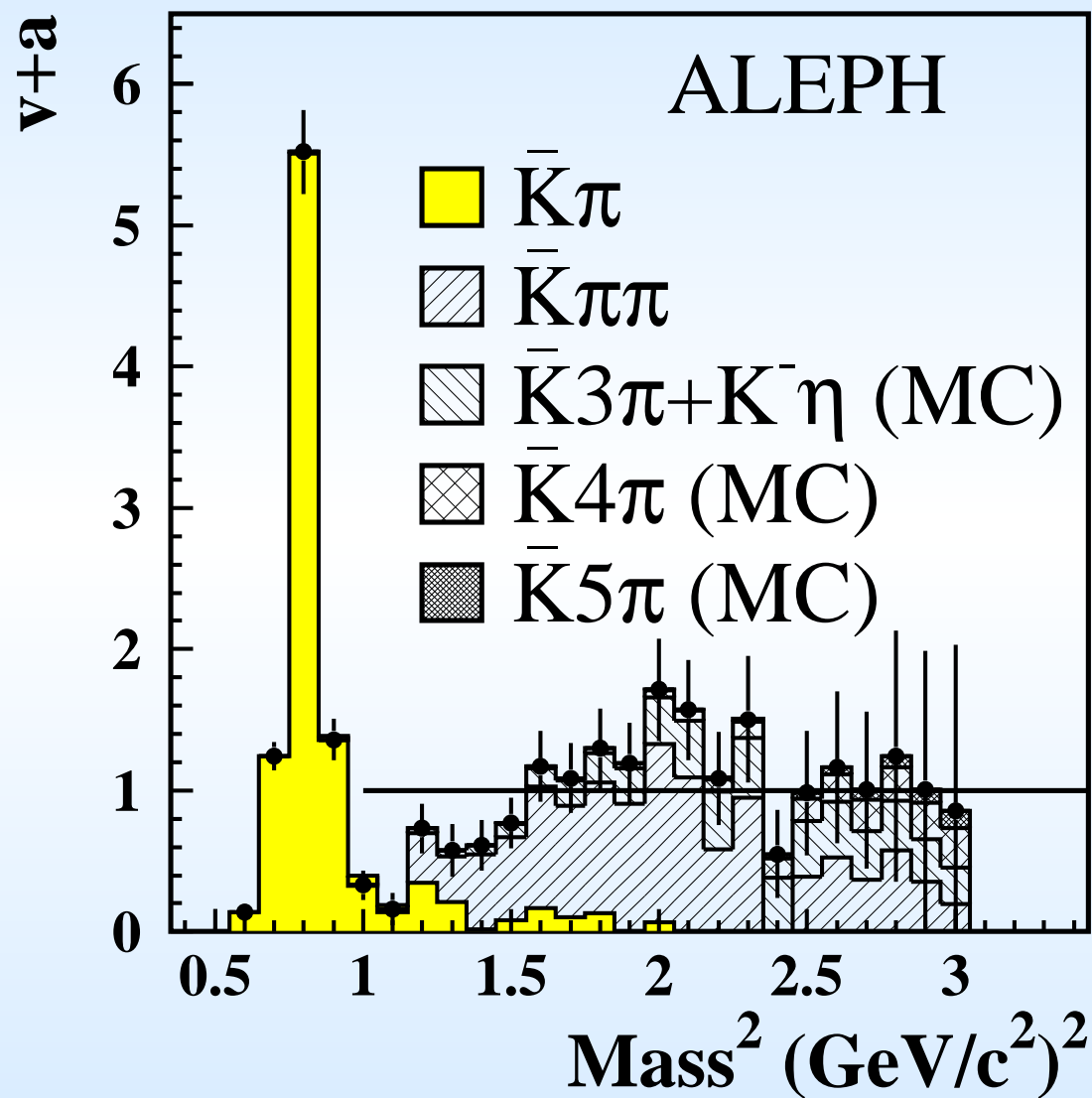
$$\delta R_\tau^{kl} \equiv \frac{R_{\tau,NS}^{kl}}{|V_{ud}|^2} - \frac{R_{\tau,S}^{kl}}{|V_{us}|^2} = 3 S_{EW} \sum_{D \geq 2} \left(\delta_{ud}^{kl(D)} - \delta_{us}^{kl(D)} \right).$$

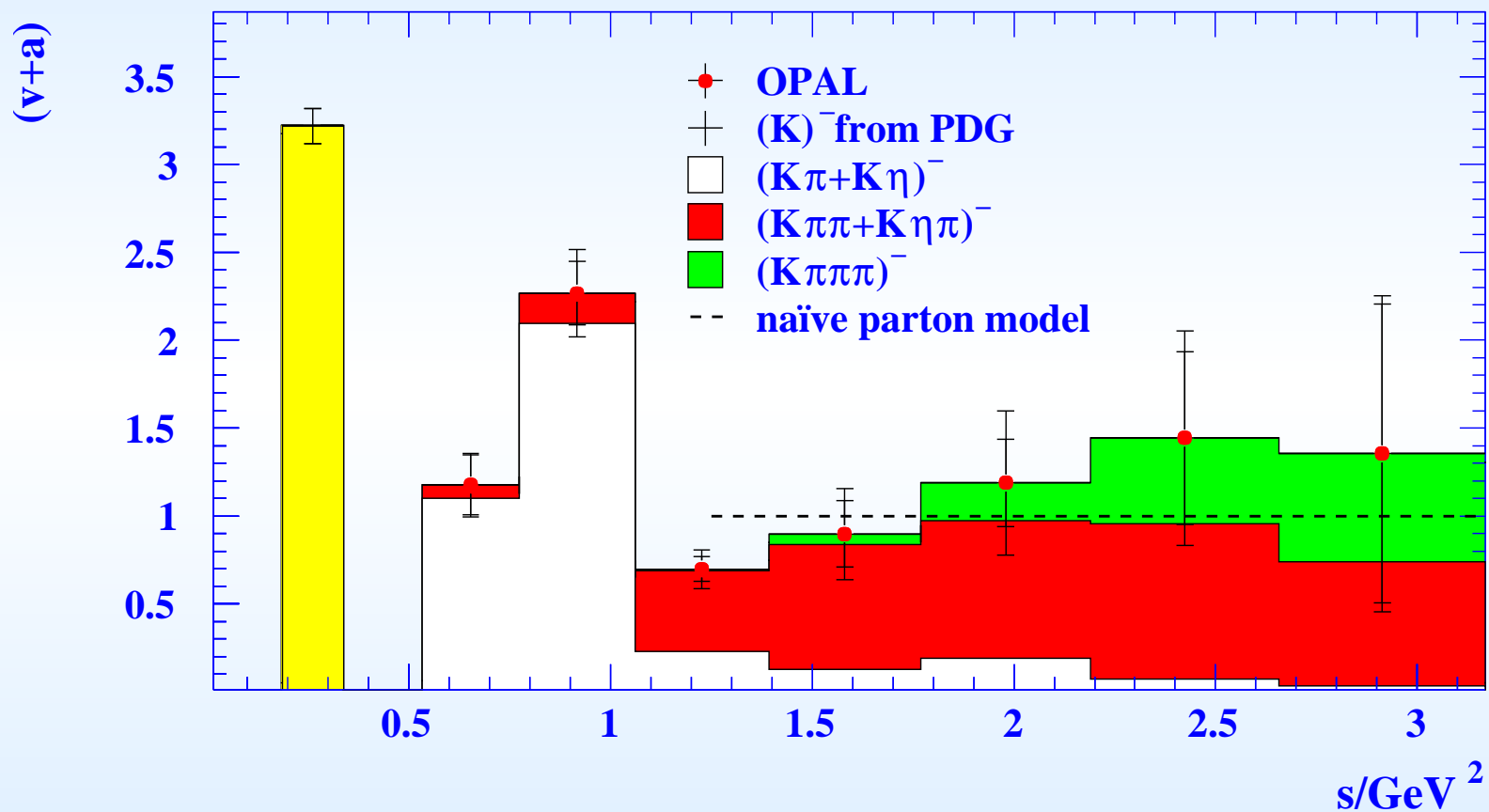
Flavour independent uncertainties drop out in the difference.

In previous analyses a sizeable part of the theoretical error was due to large α_s corrections in the **longitudinal** contribution.

This uncertainty could be greatly reduced by replacing badly behaved **scalar/pseudoscalar** correlators with phenomenology.

(Gámiz, MJ, Pich, Prades, Schwab 2003/04)





The perturbative part $\delta^{(0)}$ is related to the Adler function $D(s)$:

$$D(s) \equiv -s \frac{d}{ds} \Pi_V(s) = \frac{N_c}{12\pi^2} \sum_{n=0}^{\infty} a_\mu^n \sum_{k=1}^{n+1} k c_{n,k} \ln^{k-1} \left(\frac{-s}{\mu^2} \right)$$

where $a_\mu \equiv \alpha_s(\mu)/\pi$.

Resumming the Log's with the scale choice $\mu^2 = -s \equiv Q^2$:

$$D(Q^2) = \frac{N_c}{12\pi^2} \sum_{n=0}^{\infty} c_{n,1} a^n(Q^2)$$

As a consequence, only the coefficients $c_{n,1}$ are independent:

$$c_{0,1} = c_{11} = 1, \quad c_{2,1} = 1.640, \quad c_{3,1} = 6.371,$$

$$c_{4,1} = 49.076 !! \quad (\text{Baikov, Chetyrkin, Kühn 2008})$$

Fixed order perturbation theory amounts to choose $\mu^2 = M_\tau^2$:

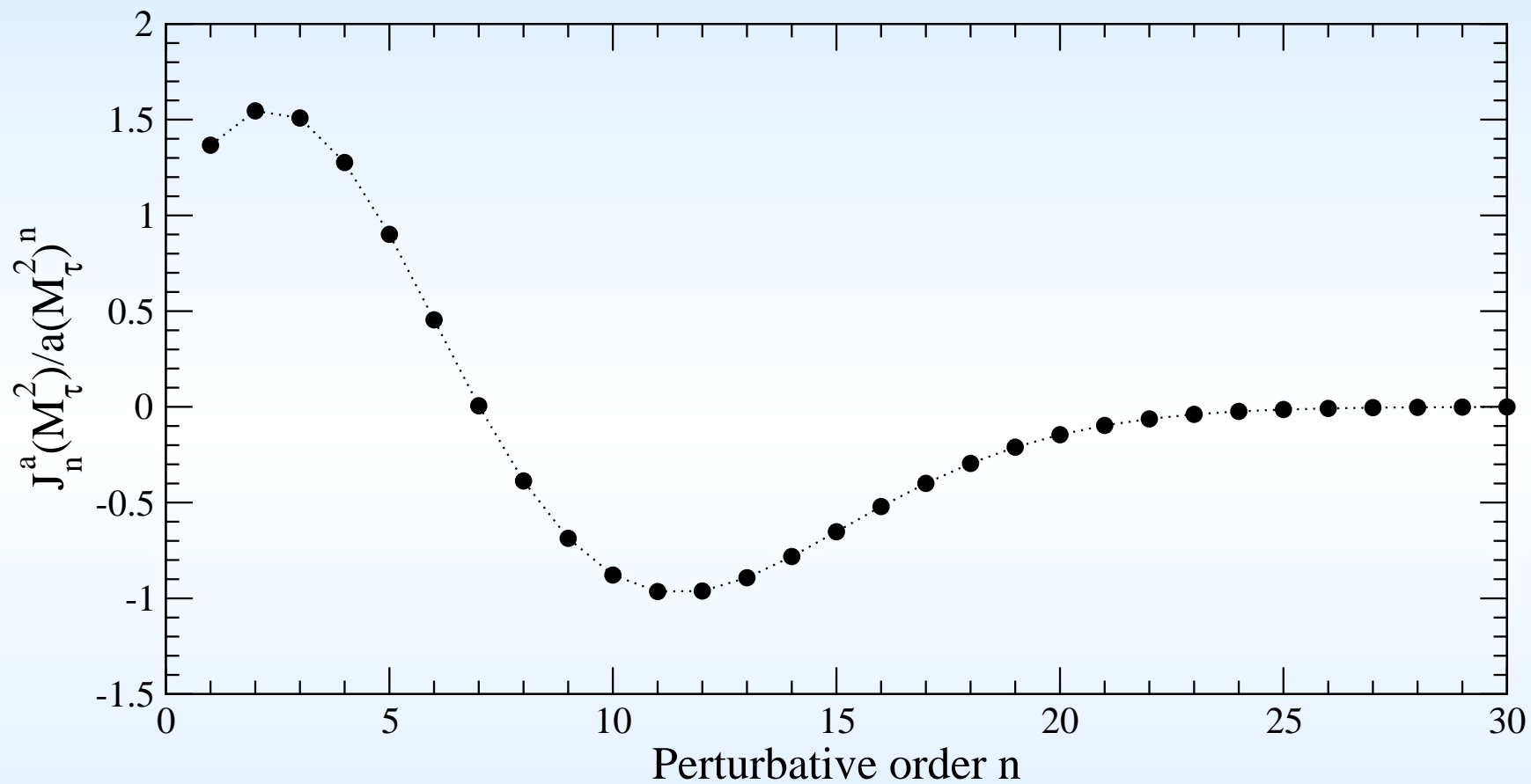
$$\delta_{\text{FO}}^{(0)} = \sum_{n=0}^{\infty} a^n(M_\tau^2) \sum_{k=1}^{n+1} k c_{n,k} J_{k-1} = \sum_{n=0}^{\infty} [c_{n,1} + g_n] a^n(M_\tau^2)$$

A given perturbative order n depends on all coefficients $c_{m,1}$ with $m \leq n$, and on the coefficients of the QCD β -function.

Contour improved perturbation theory employs $\mu^2 = -M_\tau^2 x$:
(Pivovarov; Le Diberder, Pich 1992)

$$\delta_{\text{CI}}^{(0)} = \sum_{n=0}^{\infty} c_{n,1} J_n^a(M_\tau^2) \quad \text{with}$$

$$J_n^a(M_\tau^2) = \frac{1}{2\pi i} \oint_{|x|=1} \frac{dx}{x} (1-x)^3 (1+x) a^n(-M_\tau^2 x)$$



$$\alpha_S(M_\tau) = 0.34.$$

Employing $\alpha_s(M_\tau) = 0.34$, the numerical analysis results in:

$$\delta_{\text{FO}}^{(0)} = \overset{a^1}{0.108} + \overset{a^2}{0.061} + \overset{a^3}{0.033} + \overset{a^4}{0.017} (+\overset{a^5}{0.009}) = 0.220 \text{ (0.229)}$$

$$\delta_{\text{CI}}^{(0)} = 0.148 + 0.030 + 0.012 + 0.009 (+0.004) = 0.198 \text{ (0.202)}$$

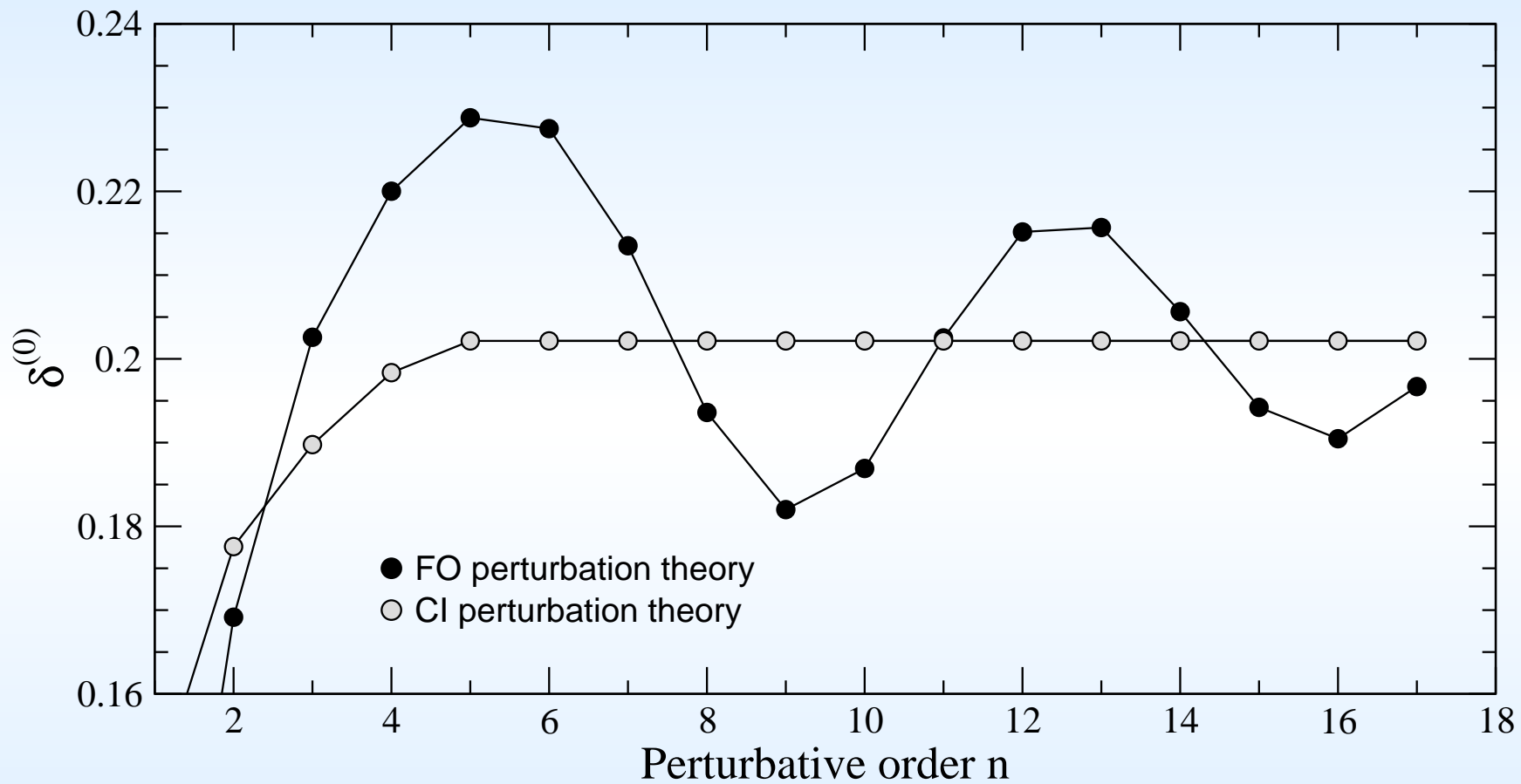
Contour improved **PT** appears to be better convergent.

The **difference** between both approaches amounts to **0.022!**

From the **uniform** convergence of $\delta_{\text{FO}}^{(0)}$, and the **assumption** that the series is not yet **asymptotic**, one may also infer

$$c_{5,1} \approx 283,$$

leading to a difference of $\delta_{\text{FO}}^{(0)} - \delta_{\text{CI}}^{(0)} = 0.027$.



$$c_{n,1} = 0 \quad \text{for} \quad n \geq 6$$

To further investigate the **difference** between **CI** and **FOPT**, let us **consider** the Borel-transformed Adler function.

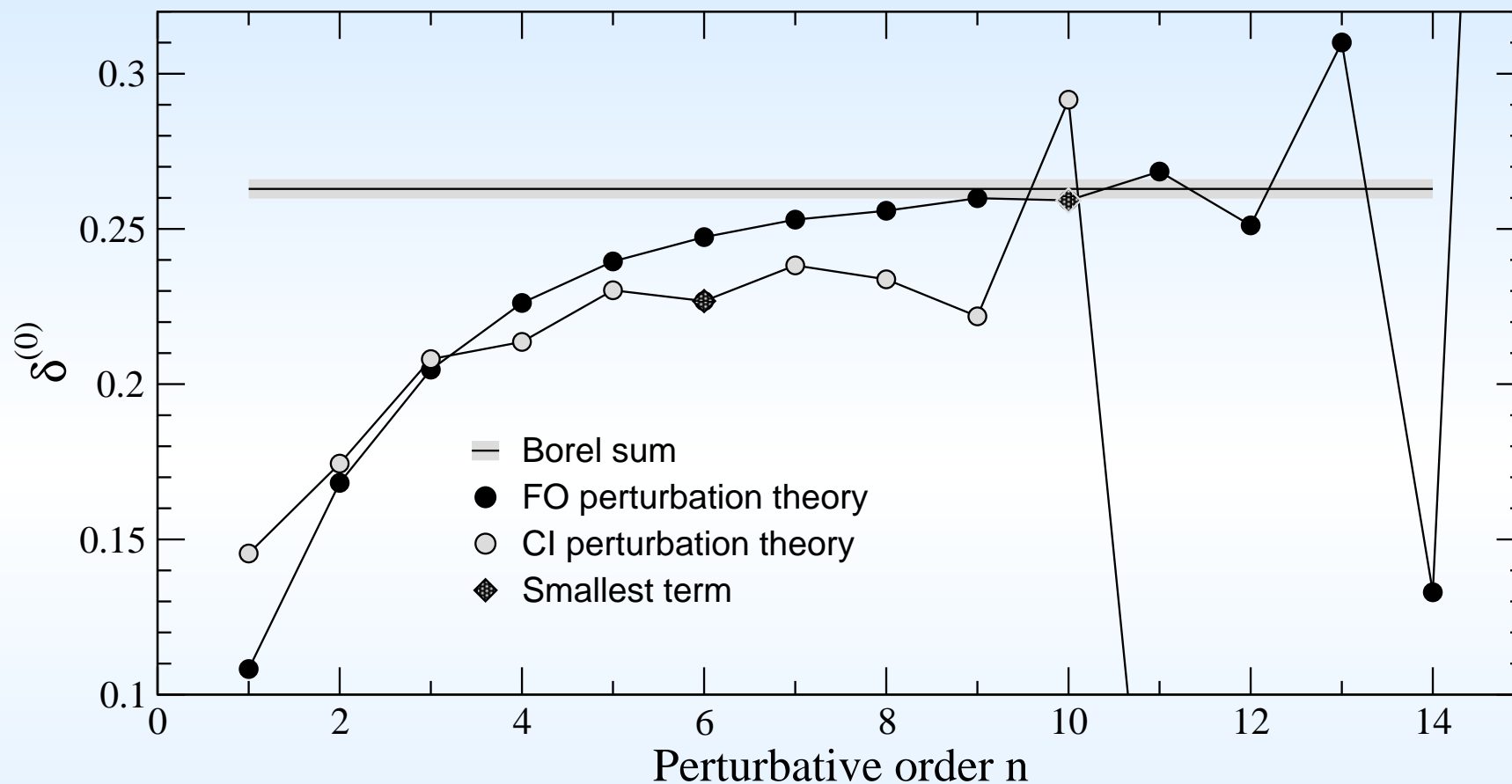
$$4\pi^2 D(s) \equiv 1 + \widehat{D}(s) \equiv 1 + \sum_{n=0}^{\infty} r_n \alpha_s(s)^{n+1},$$

where $r_n = c_{n+1,1} / \pi^{n+1}$. The Borel-transform reads:

$$\widehat{D}(\alpha_s) = \int_0^{\infty} dt e^{-t/\alpha_s} B[\widehat{D}](t); \quad B[\widehat{D}](t) = \sum_{n=0}^{\infty} r_n \frac{t^n}{n!}.$$

Generally, the Borel-transform $B[\widehat{D}]$ develops **poles** and **cuts** at **integer** values p of $u \equiv \beta_1 t / (2\pi)$. (Except at $u=1$.)

The **poles** at **negative** p are called **UV** renormalon **poles** and the ones at **positive** p **IR** renormalons.



$$\tilde{\delta}^{(0)} = 0.263 \pm i 0.009, \quad \delta_{\text{FO}}^{(0)} = 0.259, \quad \delta_{\text{CI}}^{(0)} = 0.227.$$

(Ball, Beneke, Braun 1995)

Quite generally, $\delta_{\text{FO}}^{(0)}$ can be written as:

$$\delta_{\text{FO}}^{(0)} = \sum_{n=1}^{\infty} [c_{n,1} + g_n] a(M_{\tau}^2)^n.$$

The large-order behaviour of the two components reads:

$$c_{n+1,1} = \left(\frac{\beta_1}{2}\right)^n n! \left[\frac{4}{9} e^{-5/3} (-1)^n \left(n + \frac{7}{2}\right) + \frac{e^{10/3}}{2^n} + \dots \right],$$

$$g_{n+1} = \left(\frac{\beta_1}{2}\right)^n n! \left[-\frac{4}{9} e^{-5/3} (-1)^n \left(n + \frac{16}{2}\right) - \frac{e^{10/3}}{2^n} + \dots \right].$$

Thus, large cancellations of the leading UV ($u = -1$) and IR ($u = 2$) renormalon contributions are seen to take place.

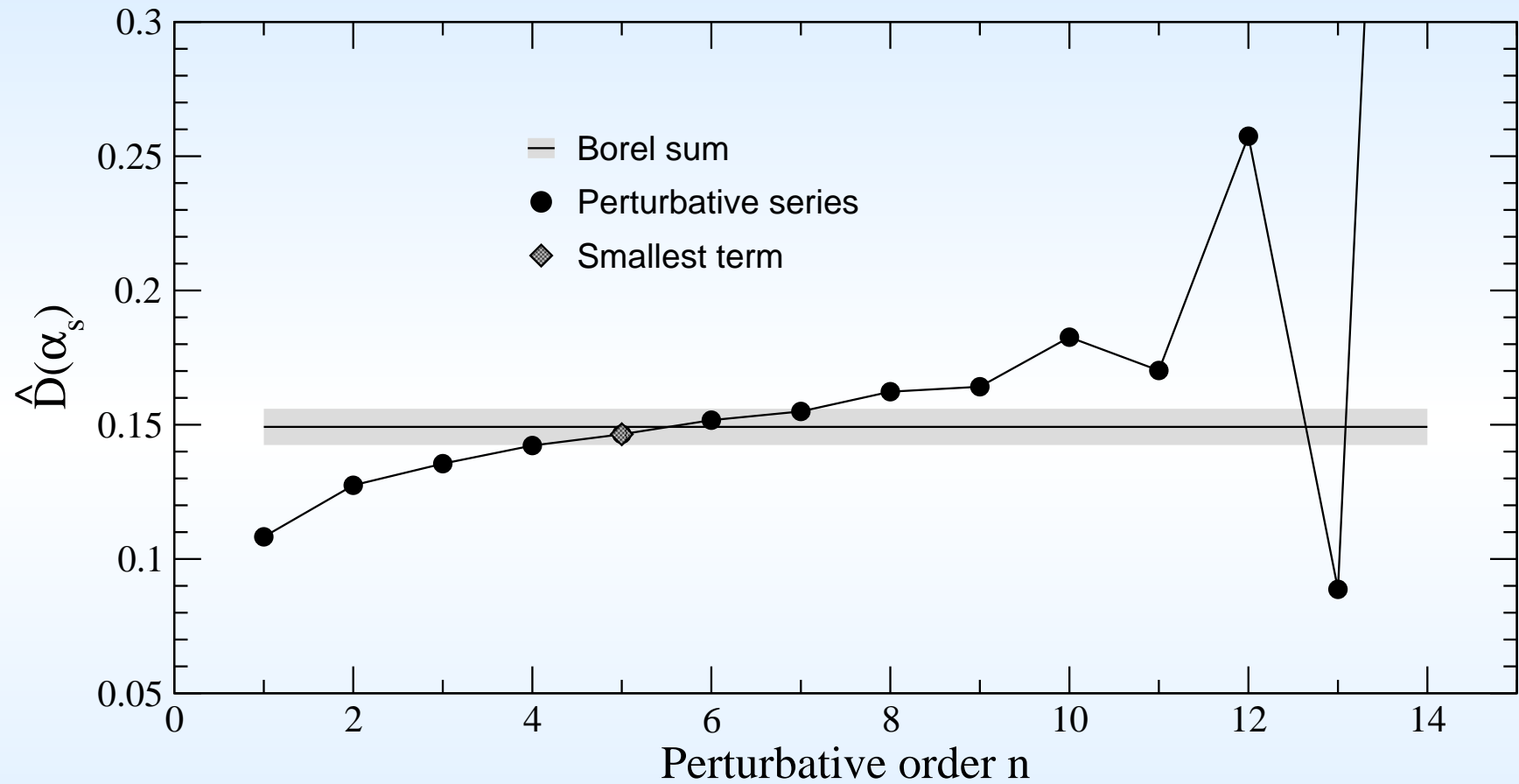
To proceed, model $B[\widehat{D}](u)$: (Beneke, MJ 2008)

$$B[\widehat{D}](u) = B[\widehat{D}_1^{\text{UV}}](u) + B[\widehat{D}_2^{\text{IR}}](u) + B[\widehat{D}_1^{\text{IR}}](u) \\ + d_0^{\text{PO}} + d_1^{\text{PO}} u,$$

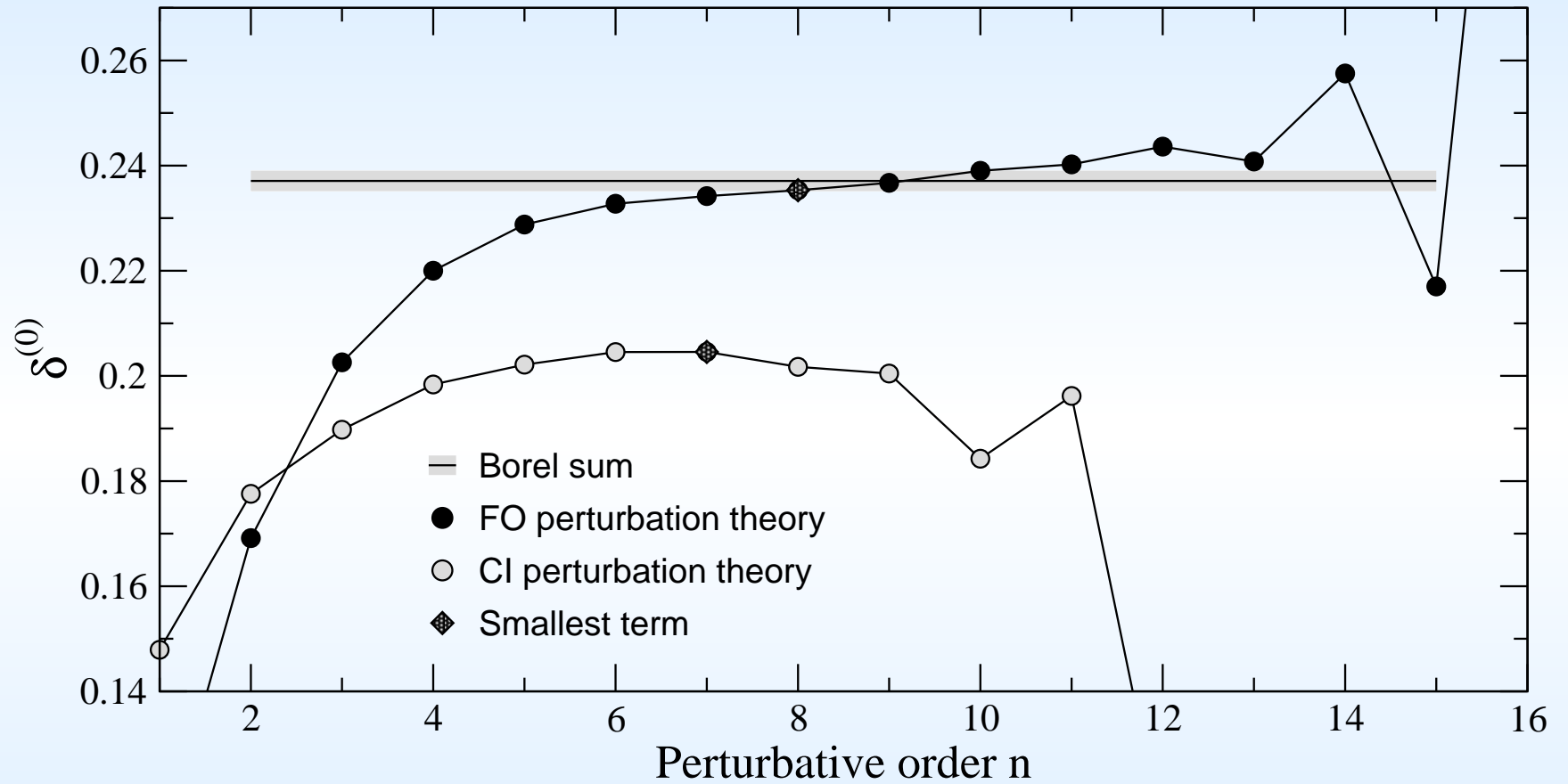
where

$$B[\widehat{D}_p](u) = \frac{d_p}{(p \pm u)^{1+\gamma}} [1 + b_1(p \pm u) + b_2(p \pm u)^2].$$

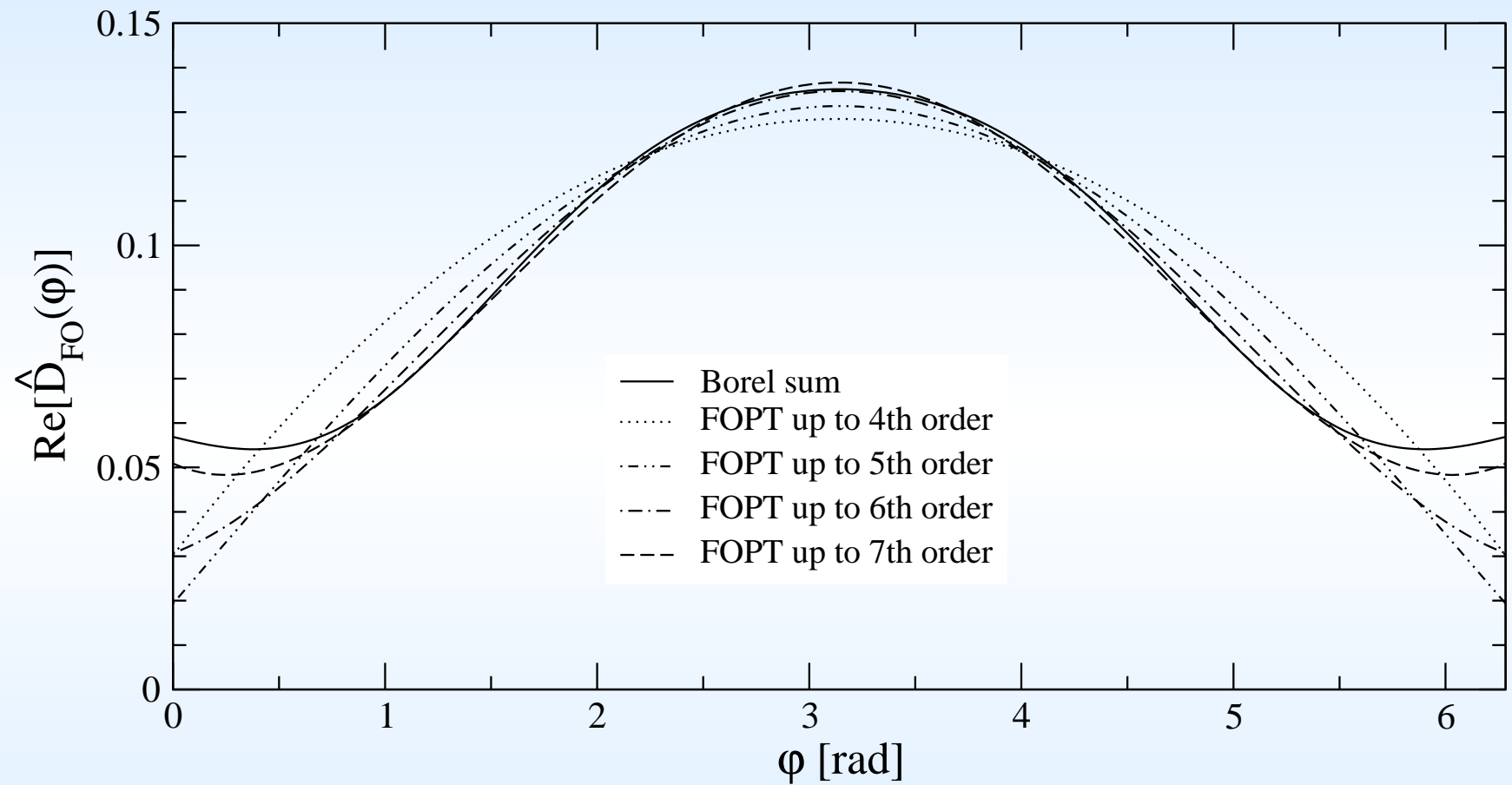
- ☞ Our main model incorporates the leading UV pole ($u = -1$), as well as the two leading IR renormalons ($u = 2, 3$).
- ☞ It should reproduce the exactly known $c_{n,1}$, $n \leq 4$.
- ☞ For both UV and IR, the residues d_p are free while γ , $b_{1,2}$ depend on anomalous dimensions and β -coefficients.



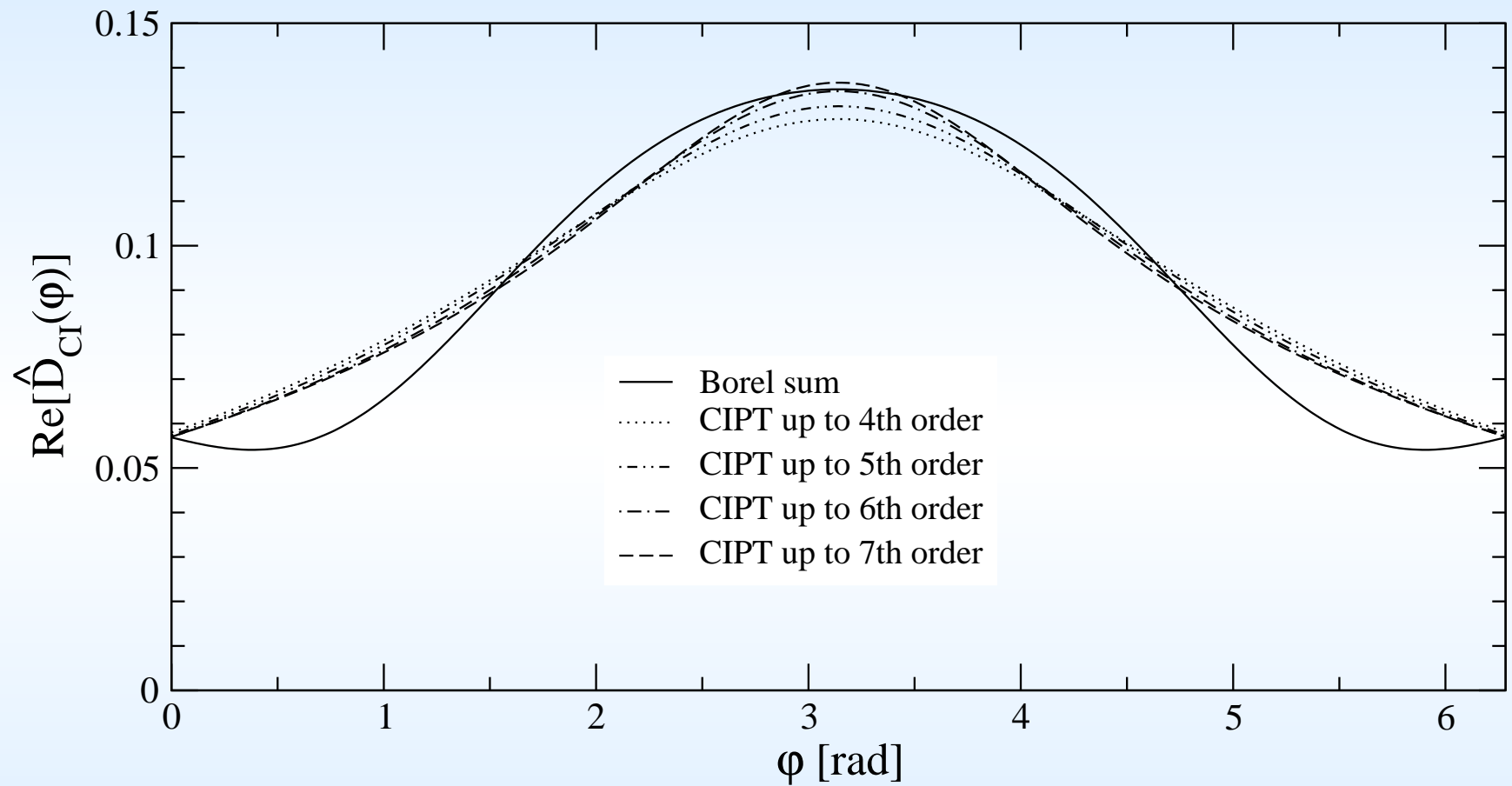
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Employing the **hadronic** decay rate into **light** quarks

$$R_{\tau, V+A} = N_c |V_{ud}|^2 S_{EW} \left[1 + \delta^{(0)} + \delta_{V+A}^{NP} \right]$$

with $\delta_{V+A}^{NP} = (-7.1 \pm 3.1) \cdot 10^{-3}$, one finds

$$\delta^{(0)} = \frac{R_{\tau, V+A}}{3 |V_{ud}|^2 S_{EW}} - 1 - \delta_{V+A}^{NP} = 0.2042(38)(33)$$

The **first** uncertainty is due to $R_{\tau, V+A}$, while the **remaining** error is **dominated** by δ_{V+A}^{NP} .

Scanning over **plausible** models and adjusting α_s such as to reproduce $\delta^{(0)}$, one finally obtains

$$\alpha_s(M_\tau) = 0.3156(30)(51) \Rightarrow \alpha_s(M_Z) = 0.1180(8)$$

☞ Baikov, Chetyrkin, Kühn (0801.1821): Average FOPT & CIPT

$$\alpha_s(M_\tau) = 0.332(16) \Rightarrow \alpha_s(M_Z) = 0.1202(19)$$

☞ Davier et al. (0803.0979): CIPT analysis

$$\alpha_s(M_\tau) = 0.344(9) \Rightarrow \alpha_s(M_Z) = 0.1212(11)$$

☞ Beneke, MJ (0806.3156): FOPT and Renormalon model

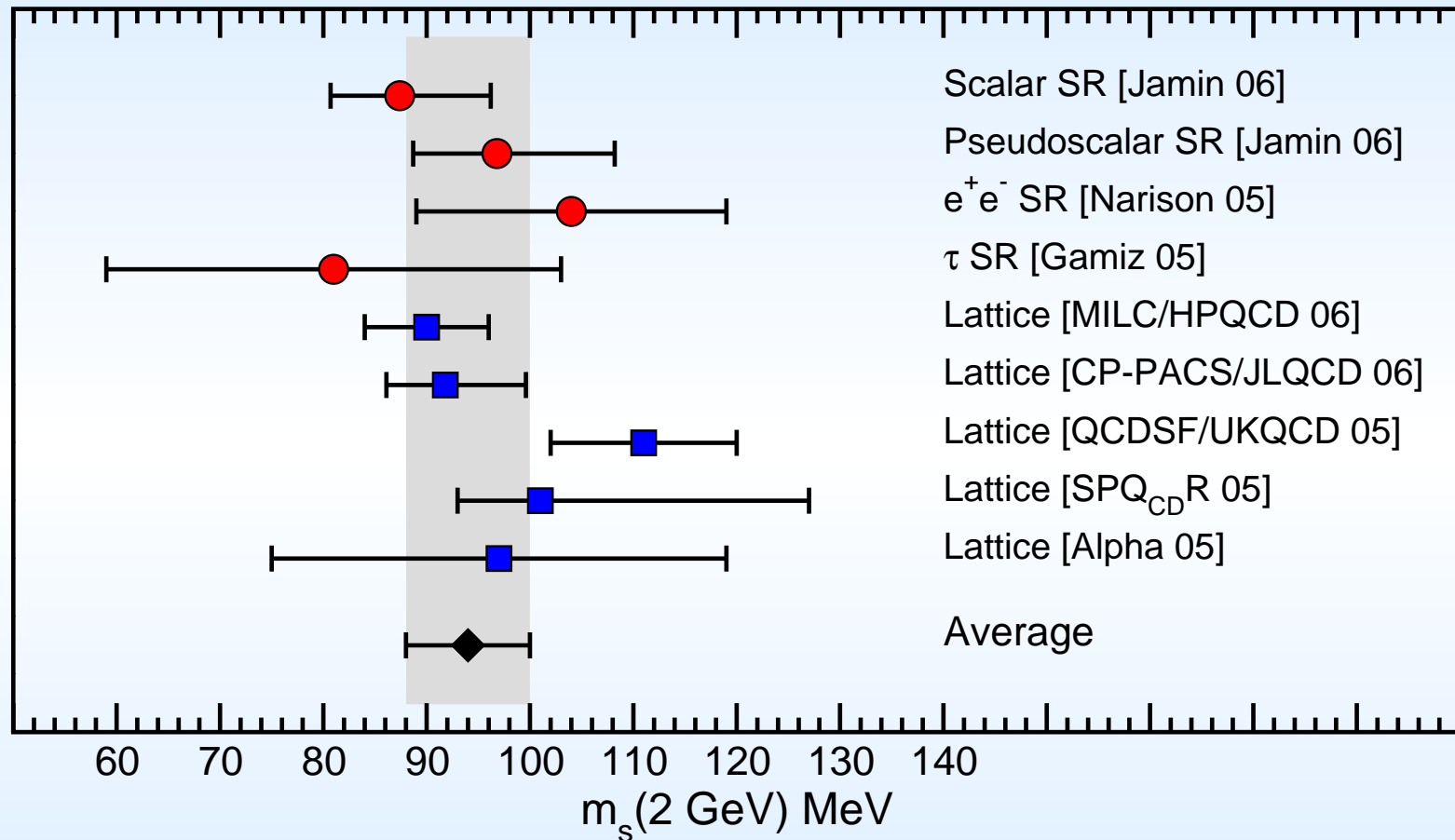
$$\alpha_s(M_\tau) = 0.3156(59) \Rightarrow \alpha_s(M_Z) = 0.1180(8)$$

☞ Maltman, Yavin (0807.0650): alternative weight functions

$$\alpha_s(M_\tau) = 0.3209(12) \Rightarrow \alpha_s(M_Z) = 0.1187(16)$$

☞ Menke (0904.1796): CIPT analysis

$$\alpha_s(M_\tau) = 0.342(11) \Rightarrow \alpha_s(M_Z) = 0.1213(12)$$



$$\Rightarrow \text{Average: } m_s(2 \text{ GeV}) = 94 \pm 6 \text{ MeV}$$

Given m_s , we are in a position to predict δR_τ^{kl} from theory. Theoretically, the uncertainty is **smallest** for the $(0,0)$ moment:

$$\delta R_{\tau,th} = \text{Pheno} \quad m_s^2 \\ = 0.155 + 0.082 + 0.003 = 0.240 \pm 0.032 .$$

Let us now reconsider the **equation** for δR_τ : (Gámiz et al. 2003/04)

$$|V_{us}| = \sqrt{\frac{R_{\tau,S}}{\underbrace{R_{\tau,NS}/|V_{ud}|^2}_{\approx 3.66} - \delta R_{\tau,th}}}$$

Thus the **theoretically** derived quantity $\delta R_{\tau,th}$ only gives a **small** correction to **experimentally** measured quantities.

Together with the experimental results $R_{\tau,NS} = 3.474 \pm 0.011$ as well as $R_{\tau,S} = 0.1595 \pm 0.0039$, V_{us} can be determined:

$$|V_{us}| = 0.2160 \pm 0.0027_{\text{exp}} \pm 0.0011_{\text{th}} = 0.2160 \pm 0.0029$$

The uncertainty on V_{us} is dominated by the experimental error on $R_{\tau,S}$. The theoretical error by the perturbative expansion.

In the near future, it should be possible to further reduce the uncertainty with the τ -data sets from BABAR and BELLE.

If the experimental value $B(\tau \rightarrow K \nu_\tau) = (0.695 \pm 0.023)\%$ is replaced by the theoretical prediction $(0.715 \pm 0.004)\%$ based on $K_{\mu 2}$ decays, one finds $|V_{us}| = 0.2169 \pm 0.0028$.

Viabale information can be obtained from the decay spectra for exclusive τ -decay channels.

A first step in this direction is a reliable description of the

$\tau \rightarrow \nu_\tau K \pi$ decay spectrum:

(MJ, Pich, Portolés 2006/08)

(Boito, Escribano, MJ 2008)

$$\frac{d\Gamma_{K\pi}}{d\sqrt{s}} = \frac{G_F^2 |V_{us}|^2 M_\tau^3}{32\pi^3 s} \left(1 - \frac{s}{M_\tau^2}\right)^2 \times$$

$$\left[\left(1 + 2 \frac{s}{M_\tau^2}\right) q_{K\pi}^3 |F_+^{K\pi}(s)|^2 + \frac{3\Delta_{K\pi}^2}{4s} q_{K\pi} |F_0^{K\pi}(s)|^2 \right].$$

To this end the $K\pi$ vector and scalar form factors $F_+^{K\pi}(s)$ and $F_0^{K\pi}(s)$ are required as an input.

A description of the $K\pi$ vector form factor can be obtained within chiral perturbation theory with resonances ($R\chi$ PT):

$$F_{+}^{K\pi}(s) = \frac{M_{K^*}^2 e^{\frac{3}{2}\text{Re}[\tilde{H}_{K\pi}(s) + \tilde{H}_{K\eta}(s)]}}{M_{K^*}^2 - s - iM_{K^*}\Gamma_{K^*}(s)}.$$

where

$$\Gamma_{K^*}(s) = \frac{G_V^2 M_{K^*} s}{64\pi F_{\pi}^4} [\sigma_{K\pi}^3(s) + \sigma_{K\eta}^3(s)].$$

The parameters of this model, namely M_{K^*} and G_V , can be fitted from experimental data for p -wave $K\pi$ scattering.

Also a second resonance contribution can easily be included.

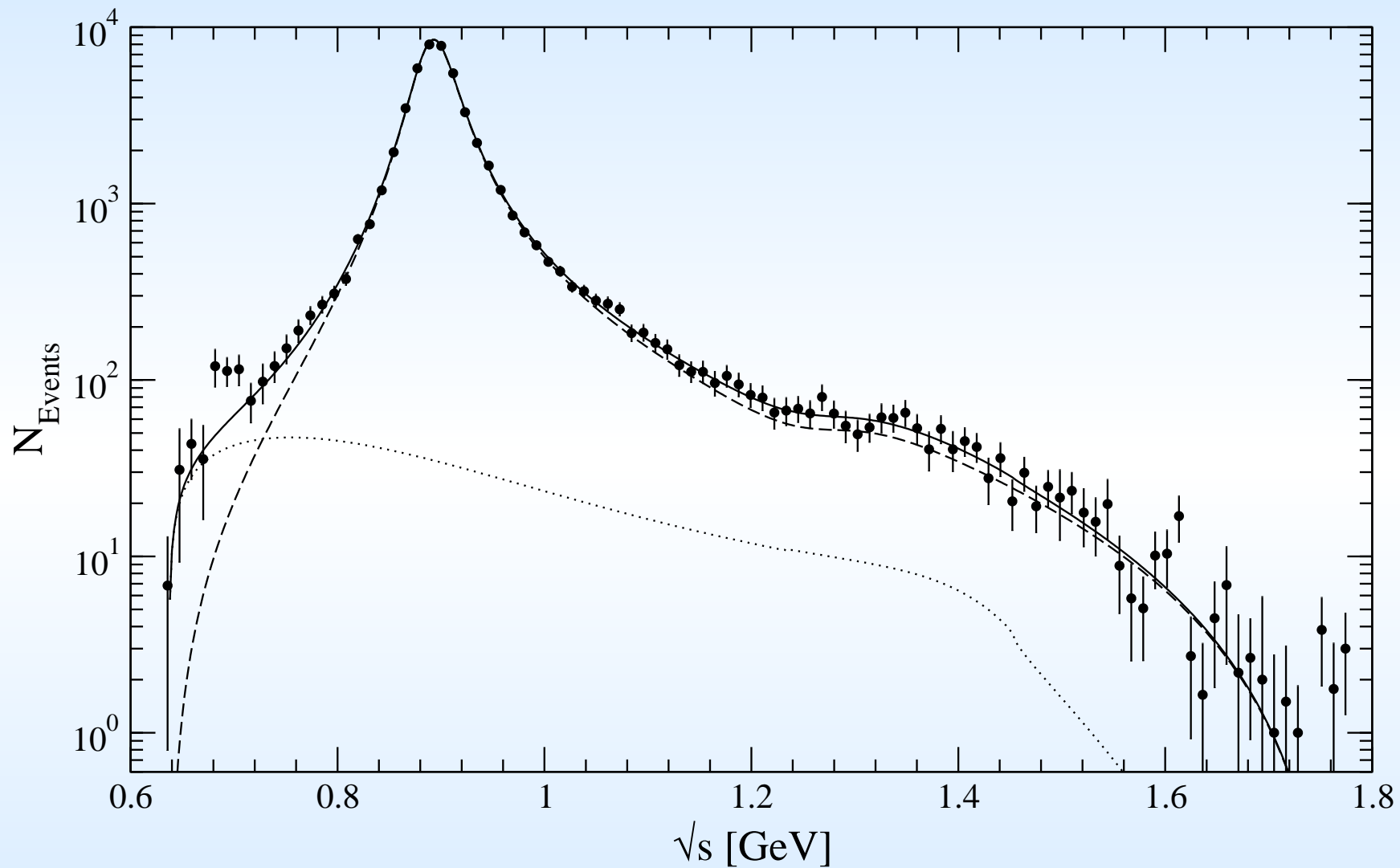
As a prediction of the model, we obtain the slope and the curvature of the vector form factor $F_+^{K\pi}(s)$:

$$\lambda'_+ = (24.7 \pm 0.8) \cdot 10^{-3}, \quad \lambda''_+ = (12.0 \pm 0.2) \cdot 10^{-4}.$$

Collaboration	$\lambda'_+ [10^{-3}]$	$\lambda''_+ [10^{-3}]$
ISTRA 04	23.2 ± 1.6	0.84 ± 0.41
KTEV 04	20.64 ± 1.75	3.20 ± 0.69
NA48 04	28.0 ± 2.4	0.2 ± 0.5
KLOE 06	25.5 ± 1.8	1.4 ± 0.8

The scalar form factor $F_0^{K\pi}(s)$ can be obtained from a dispersion relation analysis of S-wave $K\pi$ scattering data.

(MJ, Oller, Pich 2000/02)



$$M_{K^*} = 892.0 \pm 0.9 \text{ MeV}, \quad \Gamma_{K^*} = 46.2 \pm 0.4 \text{ MeV}$$

- FOPT appears to provide the **more reliable** approach to the perturbative series for $\delta^{(0)}$ while CIPT misses **cancellations**.

$$\Rightarrow \alpha_s(M_Z) = 0.1180 \pm 0.0008$$

- It is possible to determine the **CKM**-element V_{us} from the **SU(3)**-breaking difference of the hadronic τ -decay rate.

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Thank You for Your attention !