Factorization for Jet Production at the LHC: from PDFs to Initial State Jets

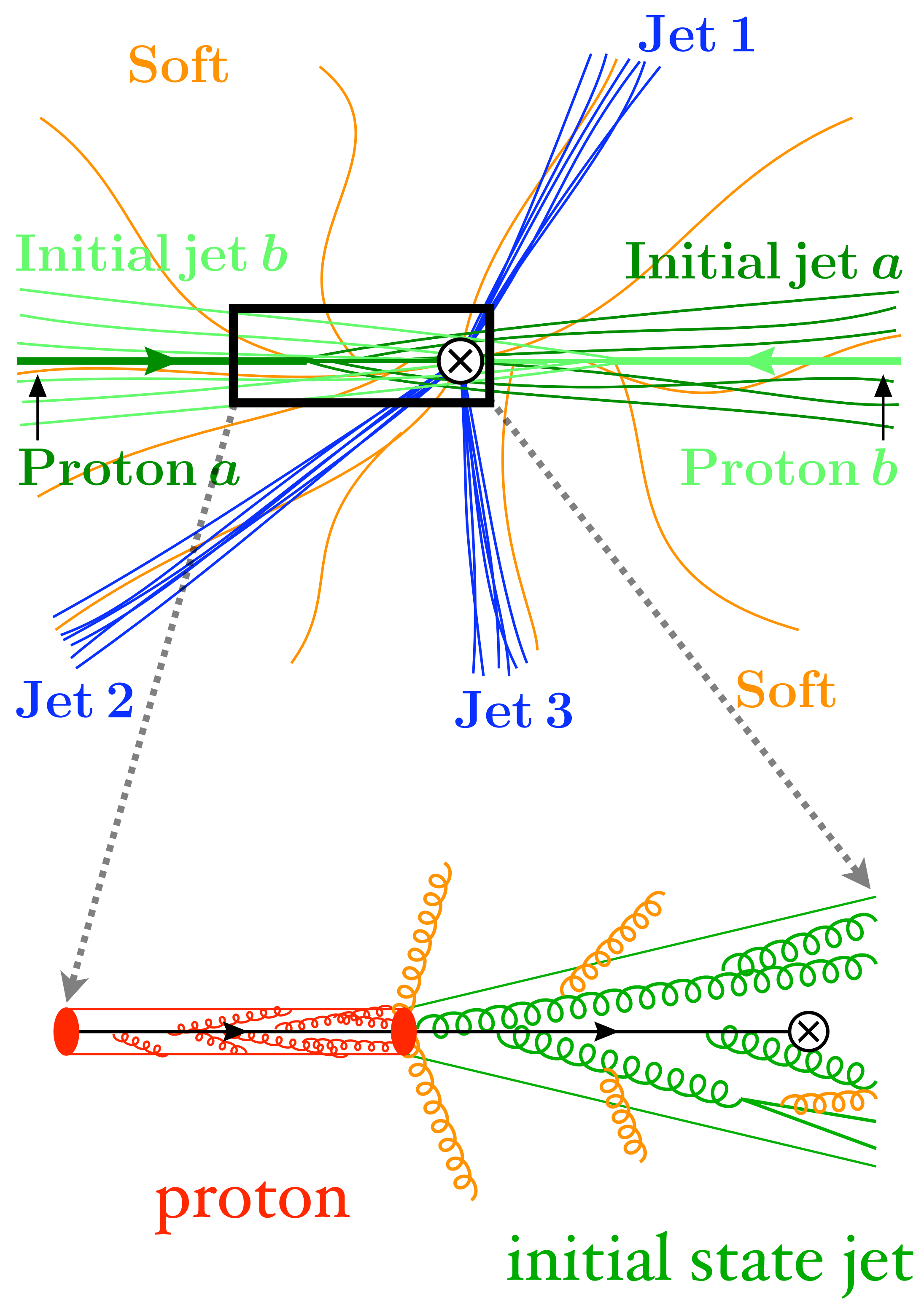
Iain Stewart, MIT

Ringberg Workshop
April 2009

based on work with
Frank Tackmann & Wouter Waalewijn

Goals

- Show certain LHC observables should be thought of as colliding partons in initial jets rather than in protons
- Universal Beam Functions describe this effect
- Allows factorization to be applied away from threshold and inclusive regions, eg. for $x \sim 10^{-1}$ with identified "exclusive jets"
- Sums large logs for initial state radiation
- Improve the accuracy of our description of LHC physics by deriving suitable factorization theorems



$$B(x, s, \mu) = \int dx' \; \mathcal{I}(s, x' - x, \mu) \; f(x', \mu)$$

Outline

- Final State Factorization:
  - Precision QCD with Hard, Jet, and Soft Functions
  - RGE, Sum large double logs
  - Simult. describe nonperturbative & perturbative effects
  - Smooth transitions between regions
- Initial State & Factorization:
  - Parton Distributions
  - Drell-Yan, Kinematics, & Scales
  - Jet Production with Beam Functions
  - Relation to Experimental Uncertainties at CDF (LHC) (underlying event)
- Beam Function
  - IR divergences and matching
  - UV divergences and RGE
  - quark, gluon, antiquark mixing

Final State Jets

$$Q^2 \gg m_X^2 \gg \frac{m_X^4}{Q^2} \gtrsim \Lambda_{\rm QCD}^2 \quad \textbf{is}$$

$$\mu_{\rm hard} \gg \mu_{\rm Jet} \gg \mu_{\rm soft} \gtrsim \Lambda_{\rm QCD}$$

SCET

Bauer, Fleming, Pirjol, Stewart

modes	$p^\mu = (+,-,\perp)$
n-collinear	$Q(\lambda^2, 1, \lambda)$
$\bar{n}$-collinear	$Q(1, \lambda^2, \lambda)$
usoft	$Q(\lambda^2, \lambda^2, \lambda^2)$

$B \to X_s \gamma$

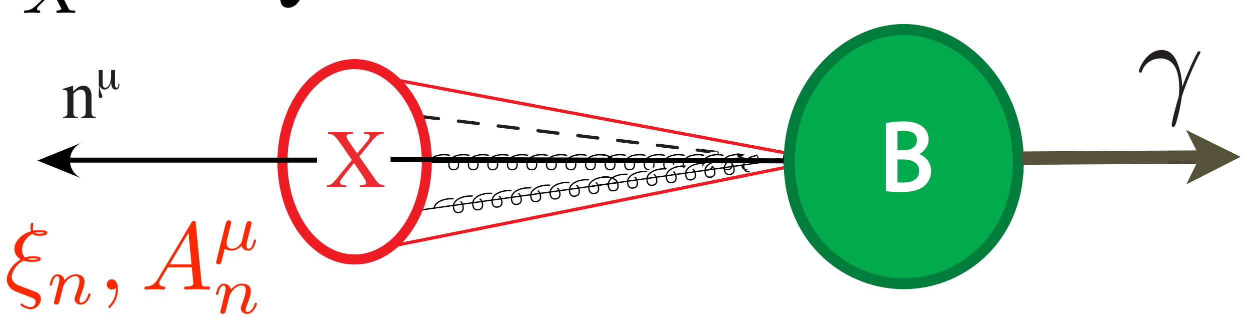
$Q = m_b$

Jet Invariant Mass

$m_X^2 \ll Q^2$

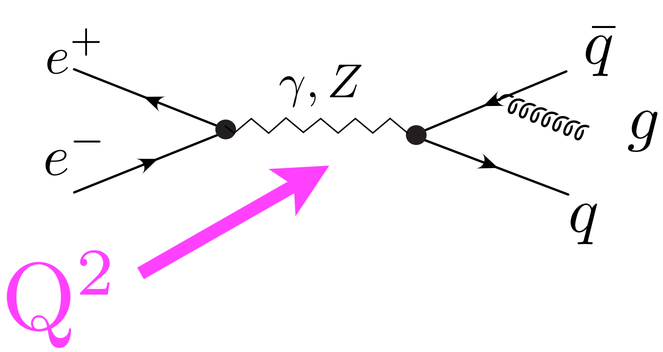
ultrasoft

q_{us}, A^μ_{us}, h_v

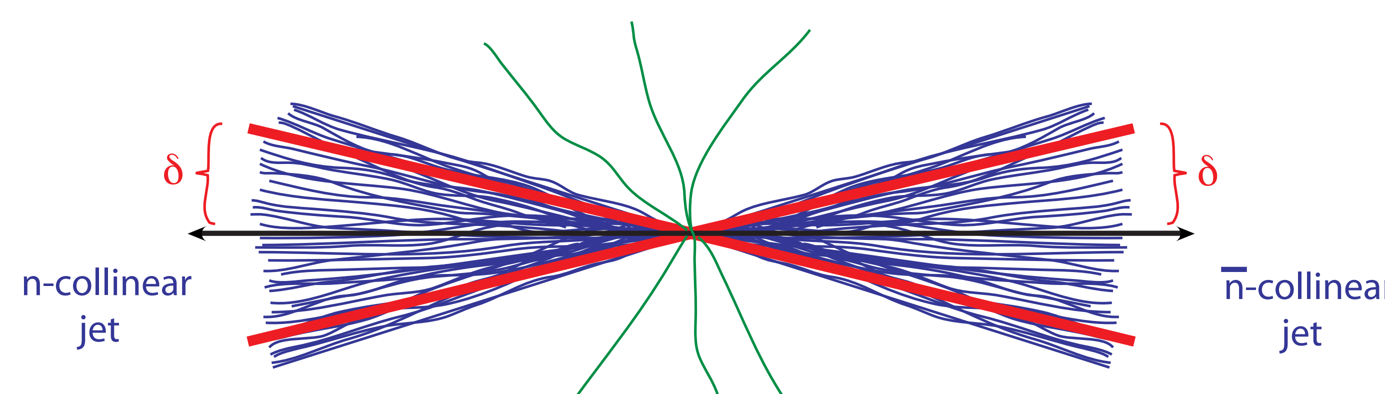


(talk by Tackmann here)

$e^+e^- \to 2$ jets



$Q = 14$ to 207 GeV



ξ_n, A_n^μ

q_{us}, A^μ_{us}

$\xi_{\bar{n}}, A_{\bar{n}}^\mu$

$e^+e^- \to 2$ jets

Korchemsky,Sterman;
Bauer,Lee,Manohar,Wise; Lee,Sterman; Mantry,Fleming,Hoang,I.S.;
Schwartz; Becher, Schwartz; Gehrmann et al.; Weinzierl;
Abbate, Fickinger, Hoang, Mateu, I.S. (talk by V. Mateu here)

IR safe observable: T = Thrust , $\tau = 1 - T$

$\tau = 0$ (dijet)

$\tau = \frac{1}{2}$ (multijet)

$\tau > 0$ singular non-singular

$$\frac{1}{\sigma}\frac{d\sigma}{d\tau} = \sum_{n,m} \alpha_s^n \frac{\ln^m \tau}{\tau} + \sum_{n,m} \alpha_s^n \ln^m \tau + \sum_{n,m} \alpha_s^n f_m(\tau)$$

$$+ f(\tau, \Lambda_{\rm QCD}/Q)$$

nonperturbative power corrections $\sim \left(\frac{\Lambda_{\rm QCD}}{Q\tau}\right)^k, \left(\frac{\Lambda_{\rm QCD}}{Q}\right)^k$

Energetic dijets

$m_X^2 + m_{\bar X}^2 = Q^2\tau$

$Q^2 \gg m_X^2 \gg \frac{m_X^4}{Q^2} \gtrsim \Lambda_{\rm QCD}^2$

$Q^2 \gg Q^2\tau \gg Q^2\tau^2 \gtrsim \Lambda_{\rm QCD}^2$

$\mu_{\rm hard} \gg \mu_{\rm Jet} \gg \mu_{\rm soft} \gtrsim \Lambda_{\rm QCD}$

logs are ratio of kinematic scales (from RGE in SCET)
LL, NLL, NNLL, N^3LL

Power Corrections:
$\frac{\Lambda_{\rm QCD}}{\mu_S}$, $\frac{\Lambda_{\rm QCD}}{\mu_h}$, $\frac{\mu_S^2}{\mu_J^2} \sim \tau$

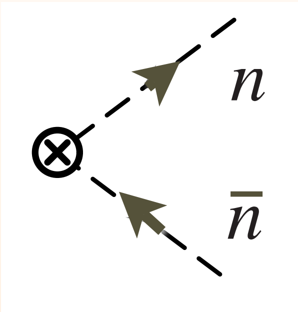
Multiple Regions:

i)	peak:	$\mu_h \gg \mu_J \gg \mu_S \sim \Lambda_{\rm QCD}$
ii)	tail:	$\mu_h \gg \mu_J \gg \mu_S \gg \Lambda_{\rm QCD}$
iii)	multi jet:	$\mu_h \sim \mu_J \sim \mu_S \gg \Lambda_{\rm QCD}$

Leading Order Factorization

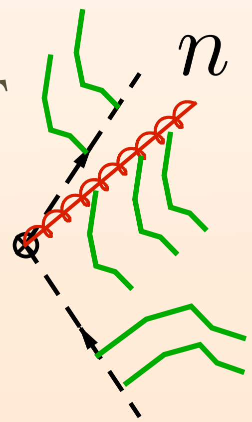
$\tau \ll 1$

Production Current:



$$\underbrace{\bar\psi\,\Gamma^\mu\psi}_{\mathcal{J}_i^\mu} \to (\bar\xi_n W_n)_\omega\,\Gamma^\mu (W_{\bar n}^\dagger \xi_{\bar n})_{\bar\omega} \Rightarrow (\bar\xi_n W_n)_\omega\,\underbrace{Y_n^\dagger \Gamma^\mu Y_{\bar n}}\,\underbrace{(W_{\bar n}^\dagger \xi_{\bar n})_{\bar\omega}}_{\chi_{n,-}}$$

SCET



$$Y(x) = P\exp\left(ig\int_0^\infty ds\, n\cdot A_{us}(x+ns)\right)$$

Soft Function

$\mathrm{S}_T(\ )$ symmetric projection

$$S_{\rm hemi}(\ell^+,\ell^-,\mu) = \frac{1}{N_c}\sum_{X_s}\delta(\ell^+ - k_s^{+a})\delta(\ell^- - k_s^{-b})\langle 0|\underbrace{\overline{Y}_{\bar n}\,Y_n}(0)|X_s\rangle\langle X_s|\underbrace{Y_n^\dagger\,\overline{Y}_{\bar n}^\dagger}(0)|0\rangle$$

usoft Wilson lines

Jet Function

$J_T(Q^2\tau)$ symmetric projection

$$J_n(Qr_n^+,\mu) = \frac{-1}{8\pi N_c Q}\,\mathrm{Disc}\int d^4x\, e^{ir_n\cdot x}\,\langle 0|\mathrm{T}\,\bar\chi_{n,Q}(0)\bar{\not n}\chi_n(x)|0\rangle$$

$$J_{\bar n}(Qr_{\bar n}^-,\mu) = \ldots$$

LO Thrust Factorization Theorem:

all orders in α_s

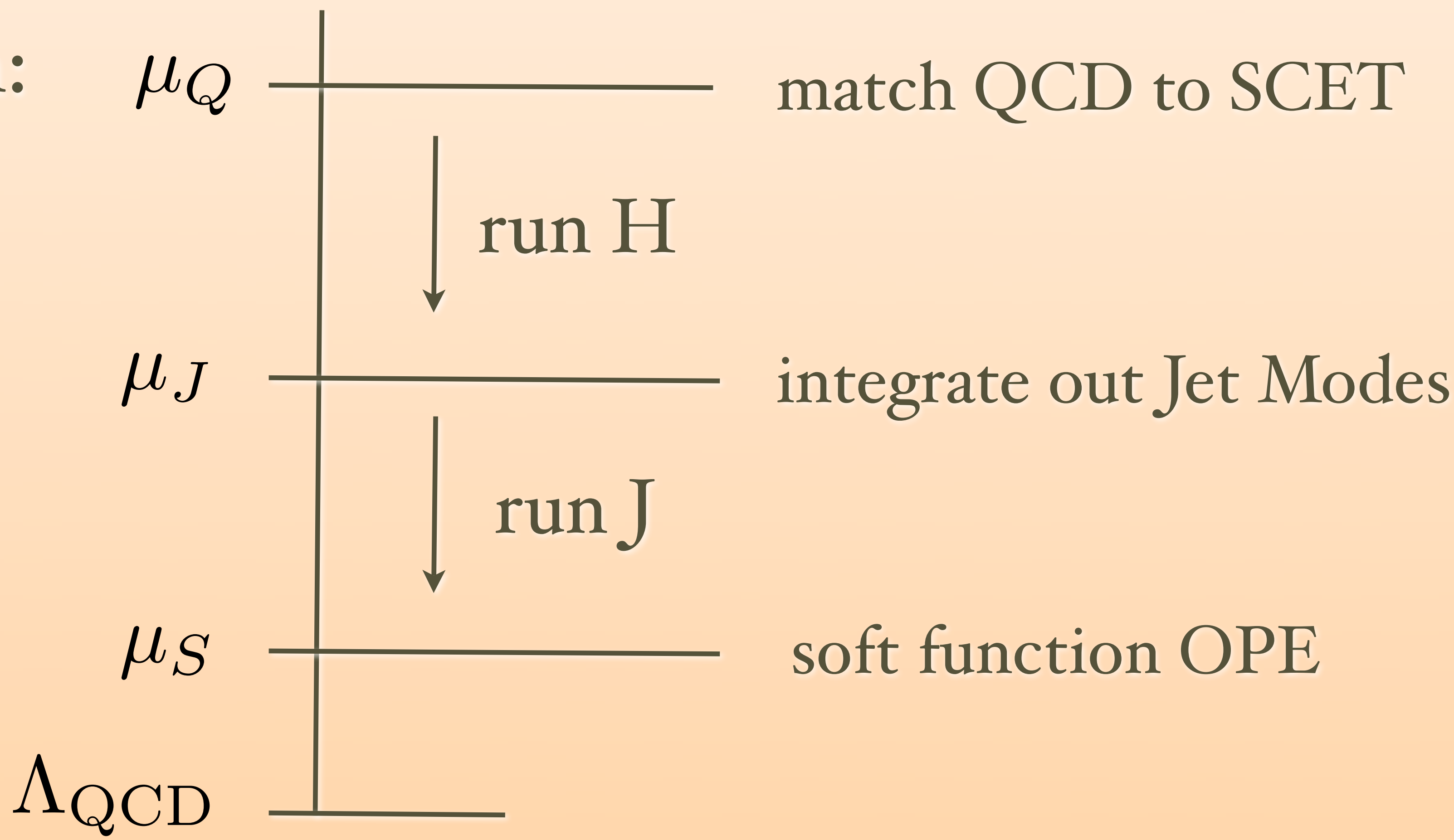
$$\frac{d\sigma}{d\tau} = \sigma_0 H(Q,\mu)\, Q \int d\ell\, J_T\Big(Q^2\tau - Q\ell, \mu\Big) S_T(\ell,\mu)$$

$p^2 \sim Q^2 \sim \mu_Q^2$ $\quad$ $p^2 \sim Q^2\tau \sim \mu_J^2$ $\quad$ $p^2 \sim Q^2\tau^2 \sim \mu_S^2$

Sum Large Logarithms

To minimize large logs we want to evaluate these functions at different scales

Match & Run:



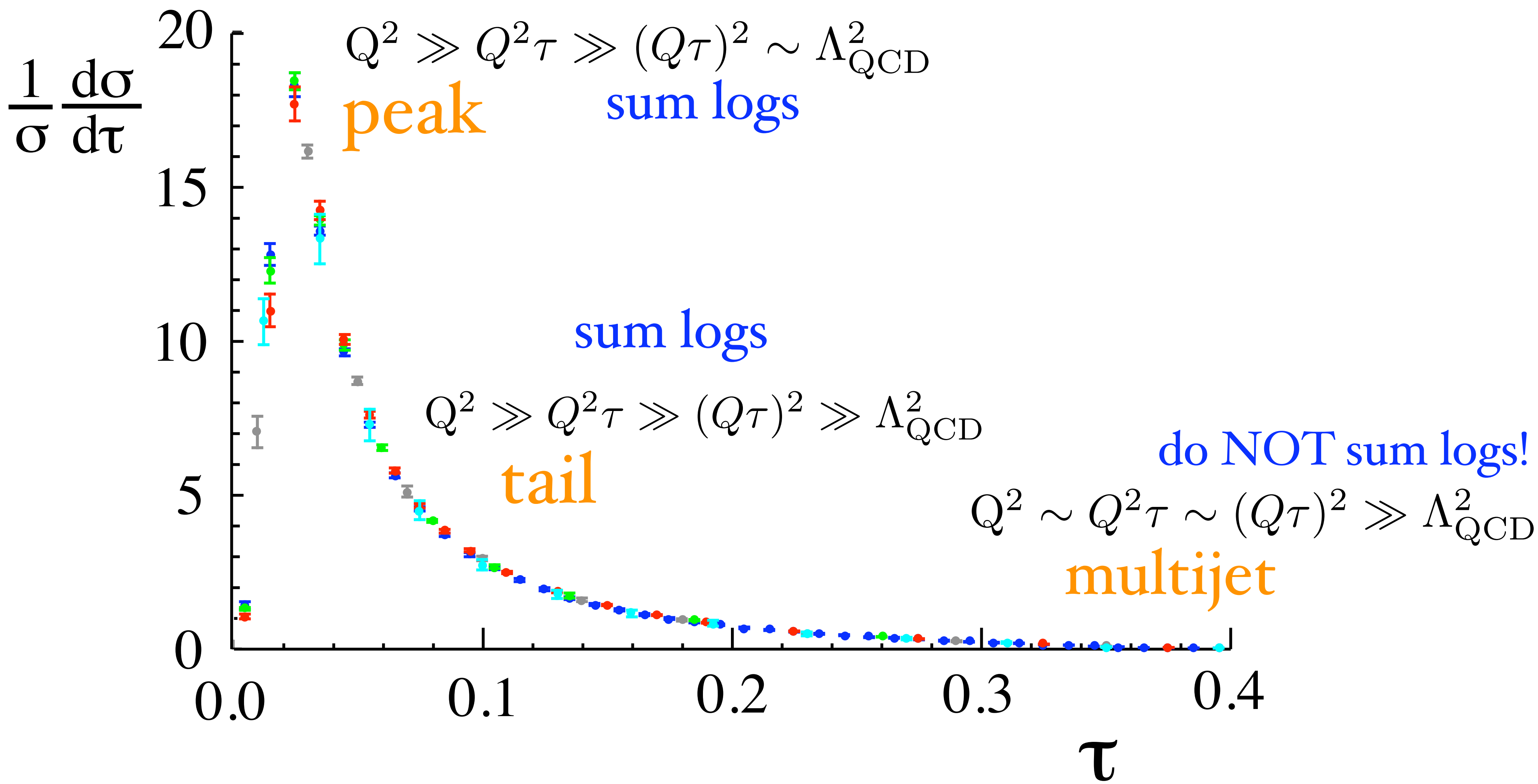
Factorization Thms

relative size of important terms is region dependent

	Peak:	Tail:	Multijet:
$\frac{1}{\sigma_0}\frac{d\sigma}{d\tau} = QH(Q,\mu)J_T(Q^2\tau - Q\ell,\mu)\otimes S_T(\ell,\mu)$	$\frac{\alpha_s^k \ln^j \tau}{\tau}\Big(\frac{\Lambda_{\rm QCD}}{Q\tau}\Big)^i$	$\frac{\alpha_s^k \ln^j \tau}{\tau}\Big(\frac{\Lambda_{\rm QCD}}{Q\tau}\Big)^{0,1}$	$\frac{\alpha_s^k \ln^j \tau}{\tau}$
$+H_i(Q,x_{i'},\mu)\otimes J_i(x_{i'},Q^2\tau - Q\ell)\otimes S_T(\ell,\mu)$	$f_{j'}(\tau)\alpha_s^k \ln^j \tau\Big(\frac{\Lambda_{\rm QCD}}{Q\tau}\Big)^i$	$f_{j'}(\tau)\alpha_s^k \ln^j \tau\Big(\frac{\Lambda_{\rm QCD}}{Q\tau}\Big)^{0,1}$	$f_{j'}(\tau)\alpha_s^k \ln^j \tau$

$$+\tilde H_i(Q,x_{i'},\mu)\otimes \tilde J_i(x_{i'},Q^2\tau,Q\ell_{j'})\otimes \tilde S_j(\ell_{j'},\mu)$$

residual error $\quad \frac{\delta\alpha_s}{\alpha_s} \sim \frac{\Lambda_{\rm QCD}}{Q} = 0.5\%$



Perturbative & NonPert. Soft Fn.

Hoang & I.S.

Ligeti, I.S., Tackmann

Factorization for Soft Function: simultaneously describe both the peak region (nonpert.), and tail regions (pert. & nonpert.)

$$S(\ell,\mu) = \int d\ell'\ S_{\rm part}(\ell-\ell',\mu)\ F(\ell')$$

$S_{\rm part}$: partonic soft function calculated at fixed order

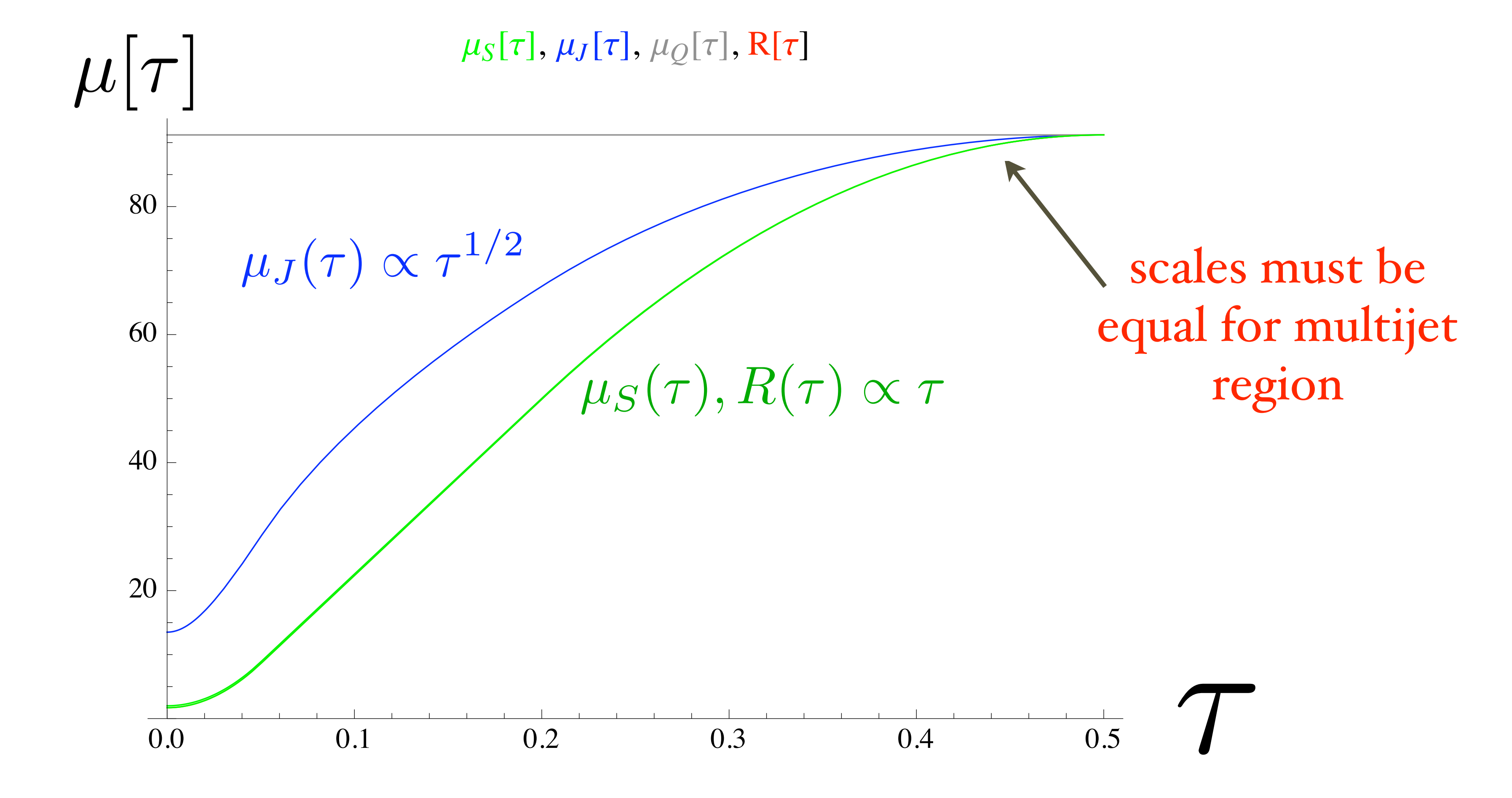
F: normalized model function (exponential fall off)

Correct μ dependence for $\overline{\rm MS}$

remove $u=1/2$ renormalon with gap $\Delta(R,\mu)$

R-RGE (talk by Jain)

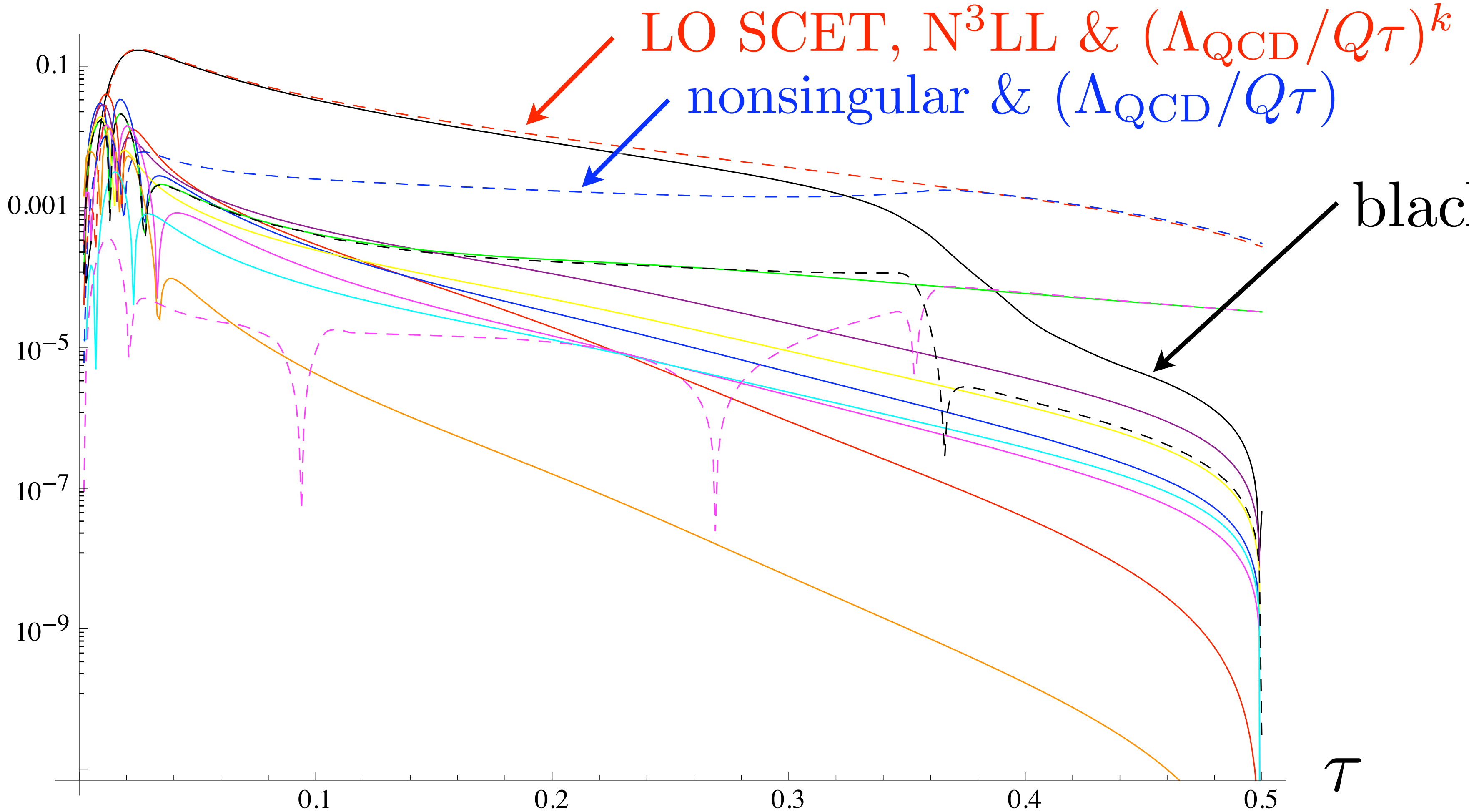
Multiple Regions



Profile functions, must satisfy multi region constraints cross section components

$$\left|\frac{1}{Q\sigma}\frac{d\sigma_{\mathrm{i}}}{d\tau}\right|$$

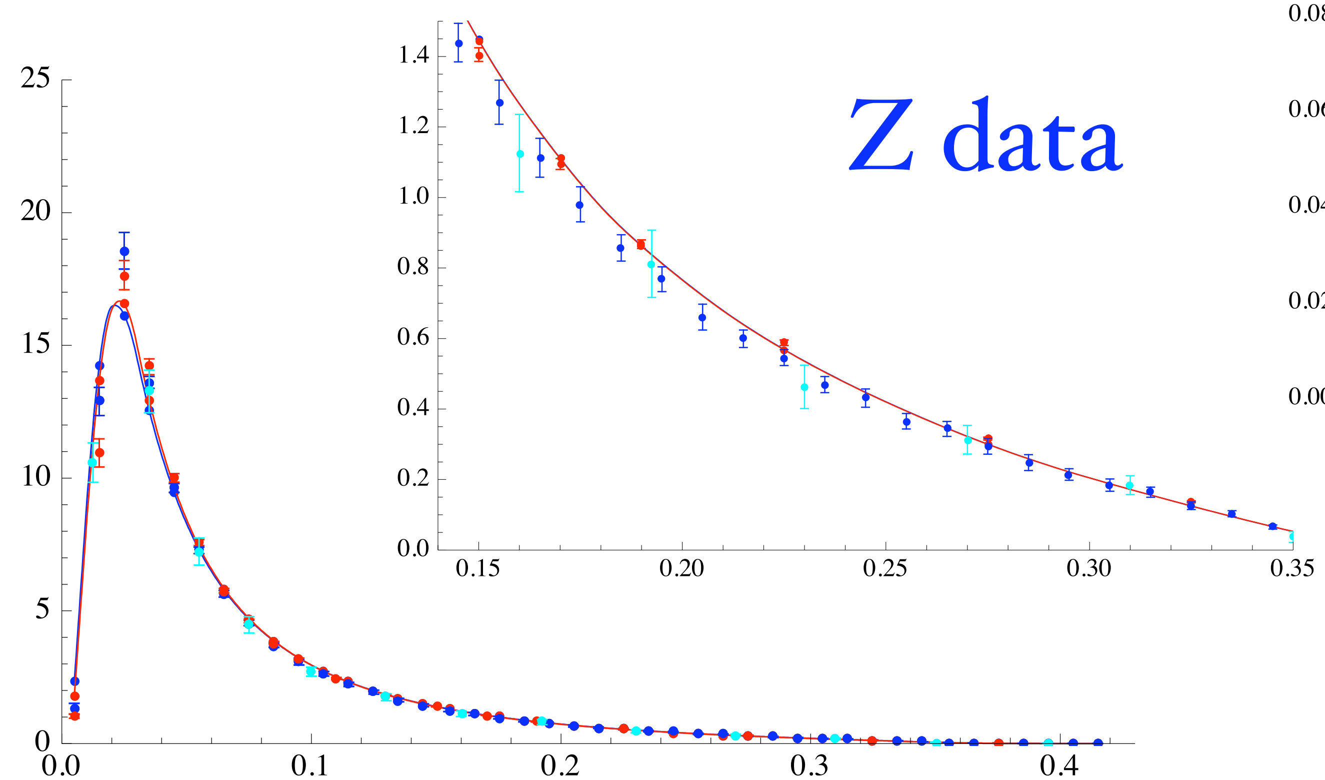
AFHMS

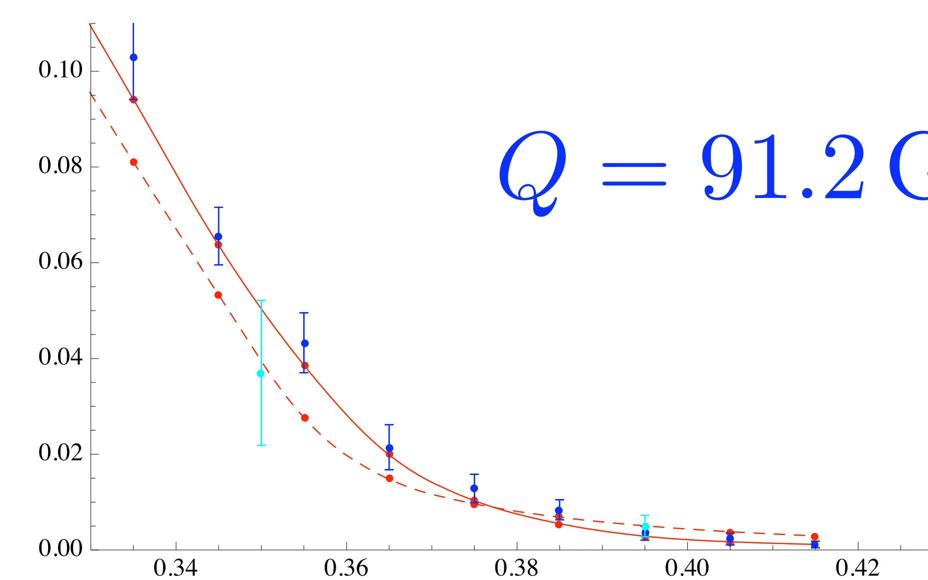


Sample Fit results:

$$\frac{1}{\sigma}\frac{d\sigma}{d\tau}$$

Z data





$Q = 91.2\,\mathrm{GeV}$

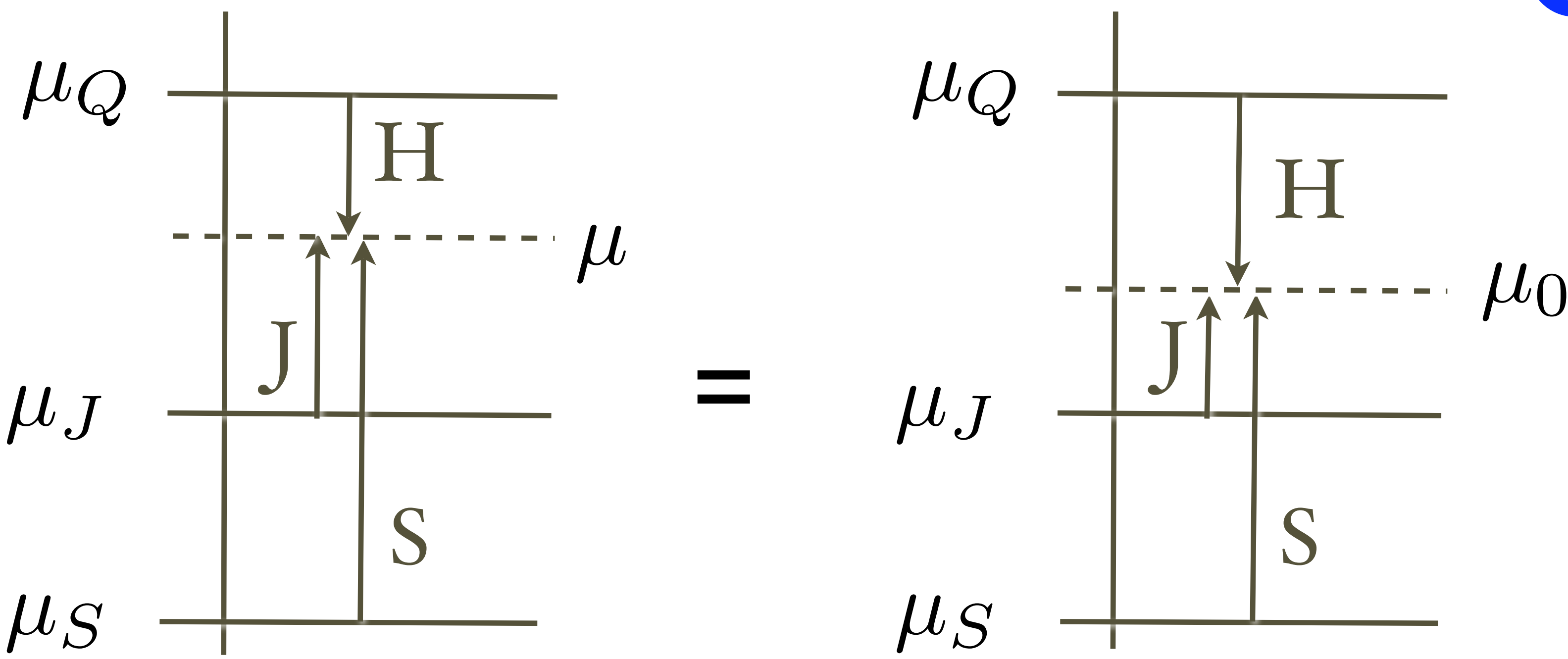
3 parameters:

$\alpha_s(m_Z)$, Ω_1, c_2

(see talk by V. Mateu)

RGE Constraints on Factorization

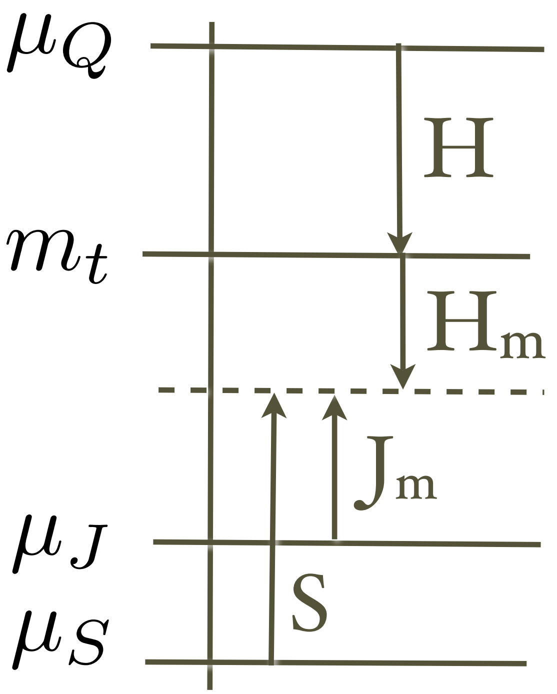
top-down = bottom-up



$$U_H^{1/2}(Q,\mu,\mu_0)\delta(M_X^2) = \int d\ell\ U_J(M_X^2 - Q\ell,\mu,\mu_0)U_S(\ell,\mu,\mu_0)$$

Non-trivial application: massive top jet production

Fleming,Hoang,Mantry,I.S.



$$U_{H_m}^{1/2}(\tfrac{Q}{m},\mu,\mu_0)\delta(\hat{s}) = \int d\ell\ U_{J_m}(\hat{s} - \tfrac{Q}{m}\ell,\mu,\mu_0)U_S(\ell,\mu,\mu_0)$$

$\hat{s} \equiv (m_X^2 - m_t^2)/m_t$

Constrains type of objects that can consistently appear in the factorization theorem

Initial State Hadrons & Factorization

For many processes of interest at the LHC there is no proof of factorization.

$$d\sigma = \sum_{i,j} d\sigma_{ij}^{\text{part}}(x_a, x_b, \mu, \ldots) \otimes f_i(x_a, \mu) f_j(x_b, \mu)$$

parton distribution functions

Strict interpretation:

- $d\sigma_{ij}^{\text{part}}$ should be computed in fixed order pert. theory, behaves like a hard Wilson coefficient
- parton distributions are only nonperturbative input

Looser interpretation:

- $d\sigma_{ij}^{\text{part}}$ computed as best we can (log resummation, further factorization,...)
- parton distributions are only nonperturbative input

Factorization Paradigm:

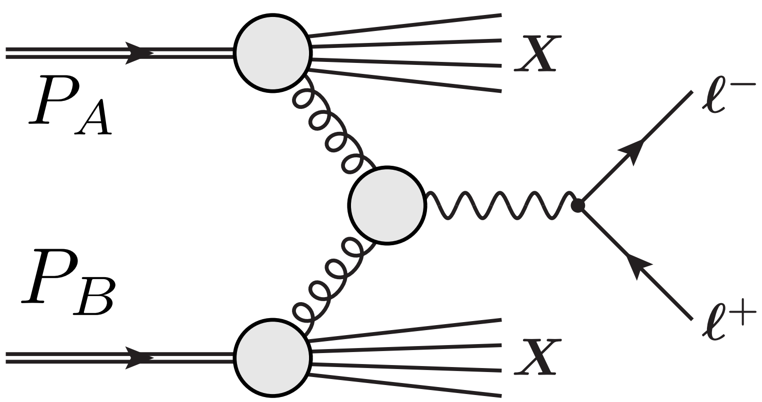
$d\sigma =$ [initial state parton shower] $\otimes$ [hard scattering fixed order perturbative computation] $\otimes$ [final state parton showers] $\otimes$ [hadronization model, underlying event, pileup]

(with parton distributions)

Drell-Yan $pp \to X\ell^+\ell^-$

(Collins, Soper, Sterman)

- Factorization has been proven rigorously for inclusive Drell-Yan



X = anything = hard (sum over all final states)

$$Q^2 = M^2 = \text{(dilepton invariant mass)}$$

$$\frac{d\sigma}{dM^2} = \int dx_a dx_b \sum_{ij} \left[\frac{d\hat{\sigma}_{ij}}{dM^2}(x_a, x_b, \mu)\right] f_i(x_a,\mu)\, f_j(x_b,\mu)$$

$$p_a^- = x_a P_a^-$$
$$p_b^+ \quad x_b P_b^+$$

Kinematics:

$$S = (P_A + P_B)^2 = E_{\rm cm}^2 \qquad \frac{M^2}{E_{\rm cm}^2} \le x_a x_b \le 1$$

- A different Factorization Thm holds near threshold $M \to E_{\rm cm}$

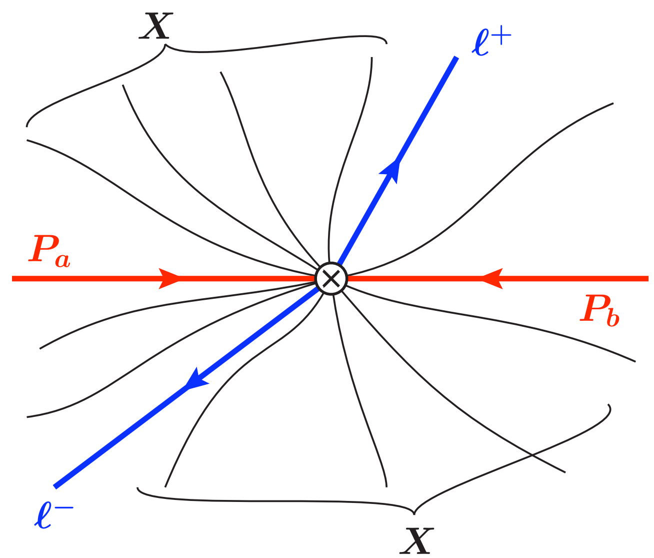
$$E_X = E_{\rm cm} - q^0 \le E_{\rm cm} - M = E_0^{\rm soft}$$

$$x_a, x_b \to 1$$

X = soft

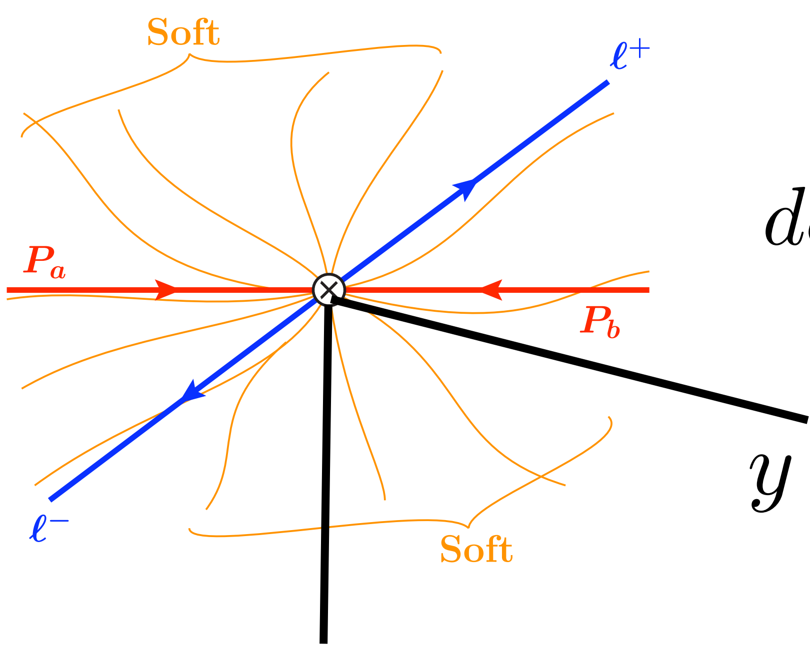
Sterman; Catani,Trentadue; Idilbi,Ji,Yuan; Becher,Neubert,Xu

$$\frac{1}{\sigma_0}\frac{d\sigma}{dM^2}\Big|_{\rm thresh} = H(M,\mu) \int dx_a dx_b\; S\Big[M\Big(1 - \frac{M^2}{x_a x_b E_{\rm cm}^2}\Big), \mu\Big]\; f(x_a,\mu) f(x_b,\mu)$$



Inclusive Drell-Yan

$$d\sigma = d\hat{\sigma} \otimes f \otimes f$$



Threshold Drell-Yan

$$d\sigma = H \; S \otimes f \otimes f$$

$$y = 2$$

$$y = 0$$

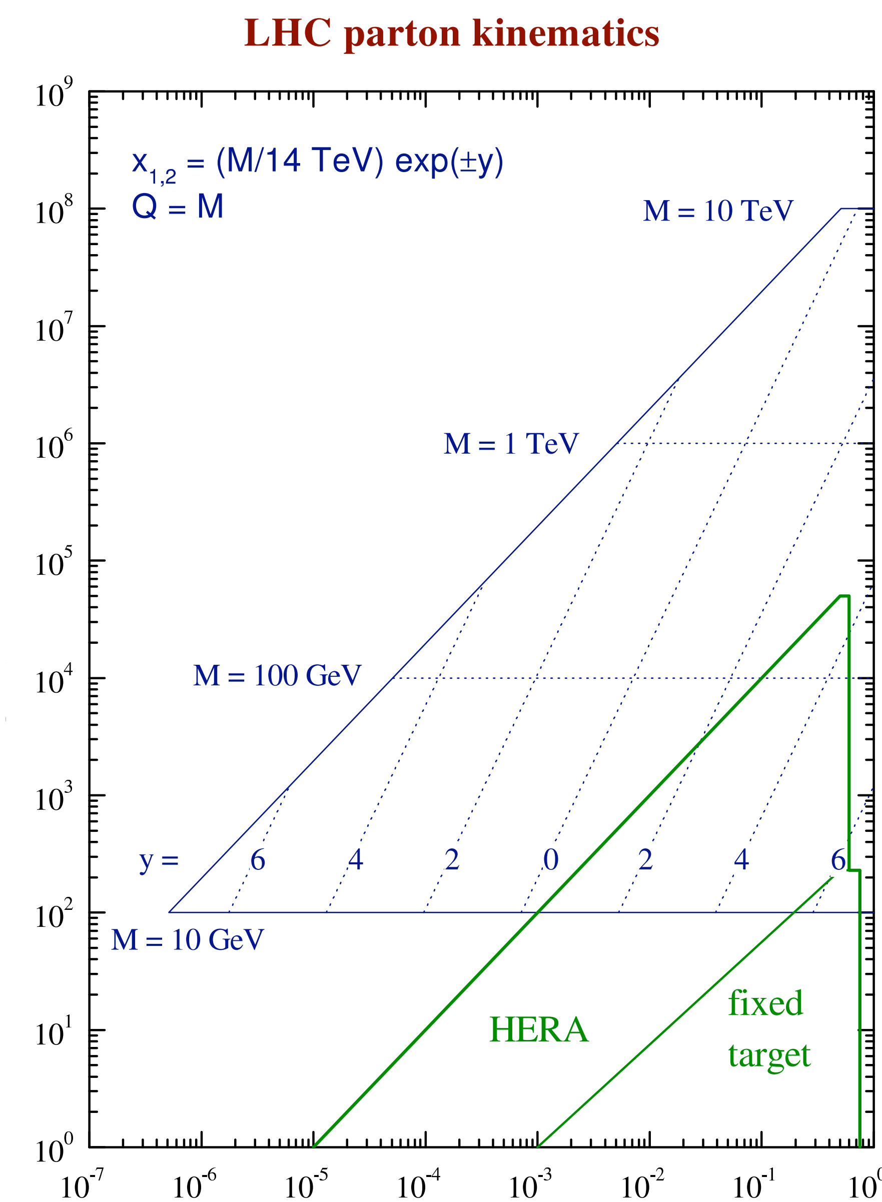
$(\ell^+\ell^-)$ rapidity y

tree level:

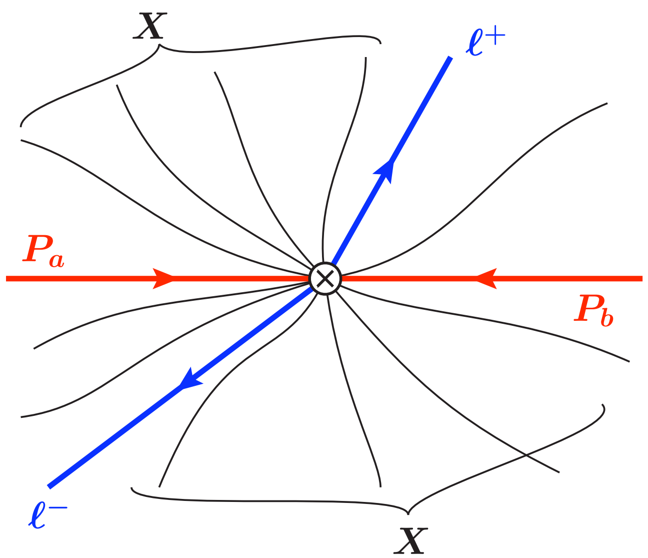
$$x_a = \frac{M}{E_{\rm cm}}\, e^{y}$$

$$x_b = \frac{M}{E_{\rm cm}}\, e^{-y}$$

$M^2(\mathrm{GeV}^2)$

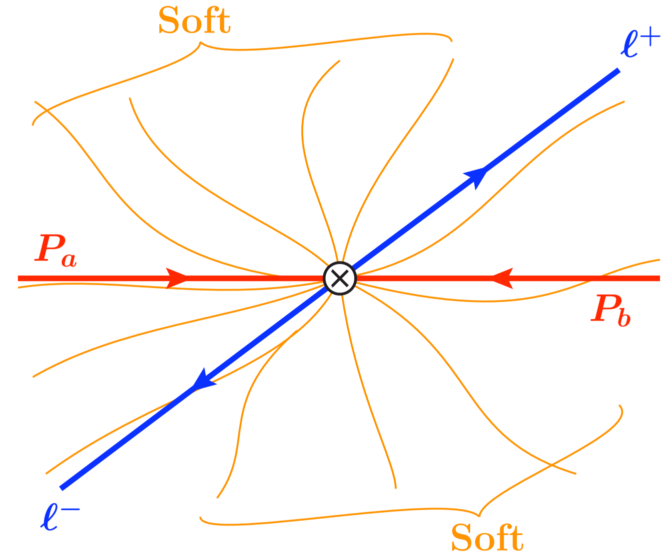


x



Inclusive Drell-Yan

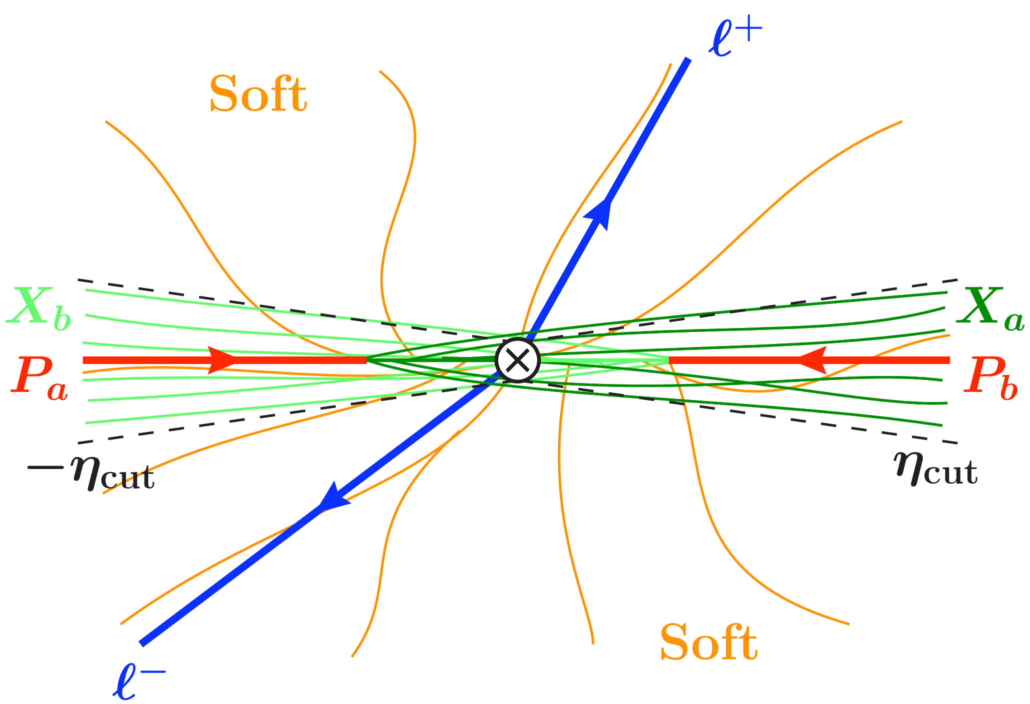
$$d\sigma = d\hat{\sigma} \otimes f \otimes f$$



Threshold Drell-Yan

$$d\sigma = H \; S \otimes f \otimes f$$

Add a rapidity cutoff, $\eta_{\rm cut}$, and demand that no jets are observed with small rapidities $\mathrm{E}(\eta < \eta_{\rm cut}) \leq E_0$ (soft scale μ_s)



$$d\sigma = H \times S \otimes B \otimes B$$

Can now have $x < 10^{-1}$

Large energy $E_{\rm cm}(1-x)$ goes into a cone around the beam

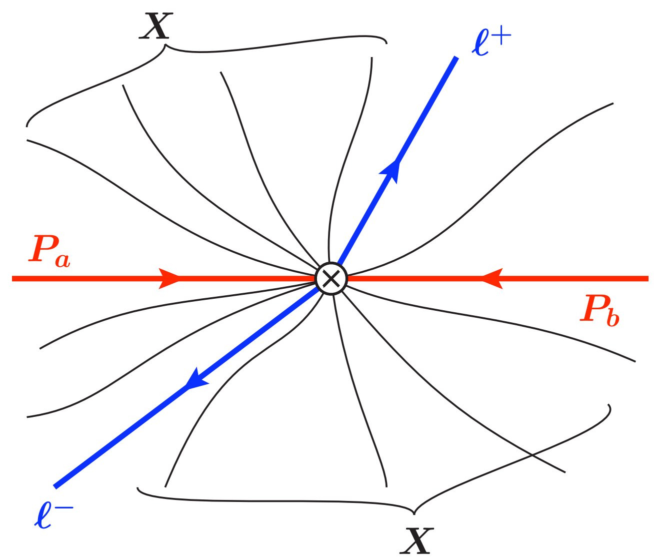
New scale is introduced, $\mu_b = e^{-\eta_{\rm cut}} \hat{Q}$

($\hat{Q} = M$ here)

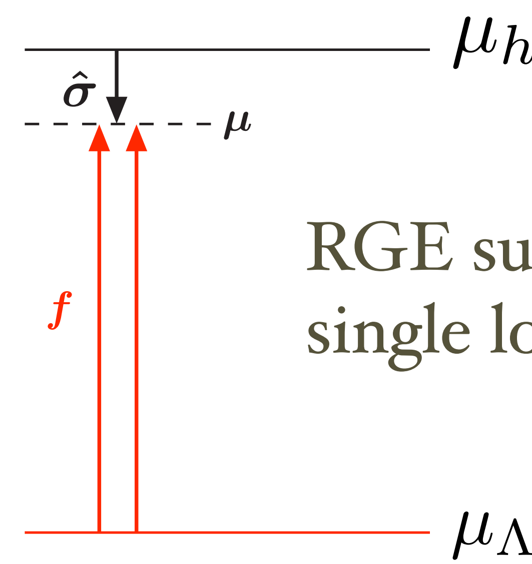
$\eta_{\rm cut} \to 0:$ inclusive DY

$\eta_{\rm cut} \to \infty:$ threshold DY

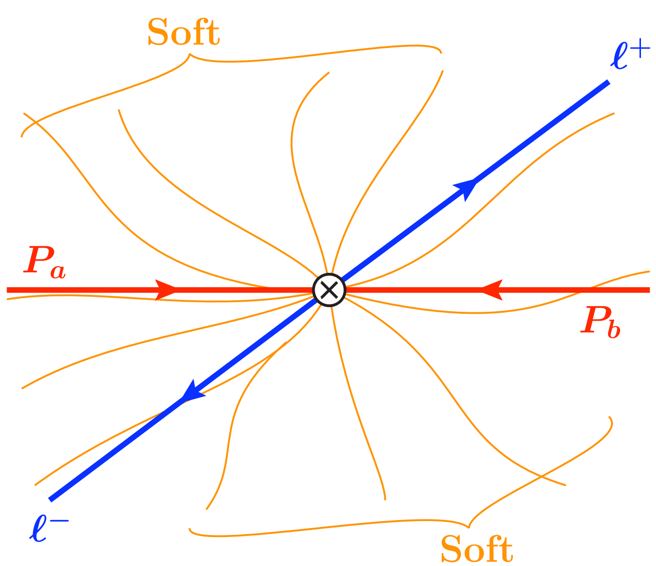
Beam functions, B, describe proton and initial state jets



$$d\sigma = d\hat{\sigma} \otimes f \otimes f$$

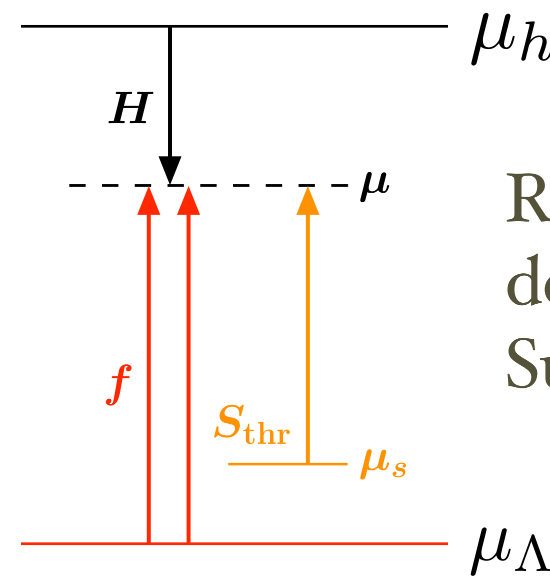


RGE sums up single logs in pdf

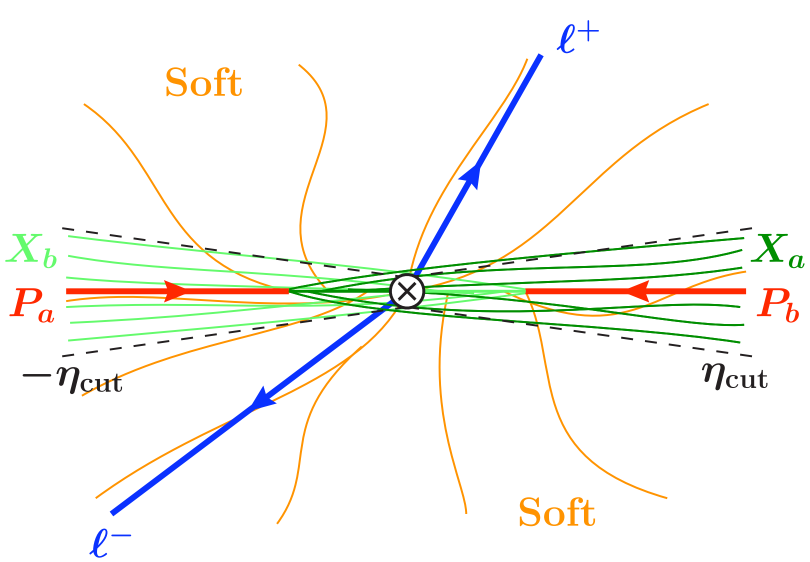


$$d\sigma = H\, S \otimes f \otimes f$$

$$U_H \delta = U_S \otimes U_{ff}$$



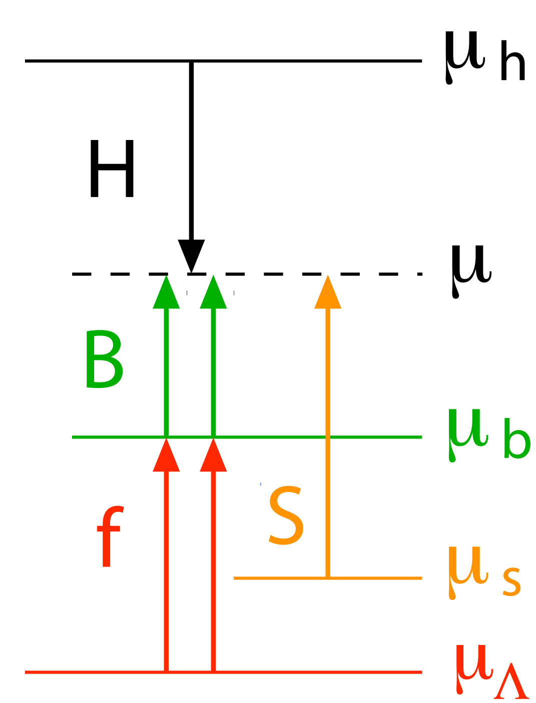
RGE sums double Sudakov logs



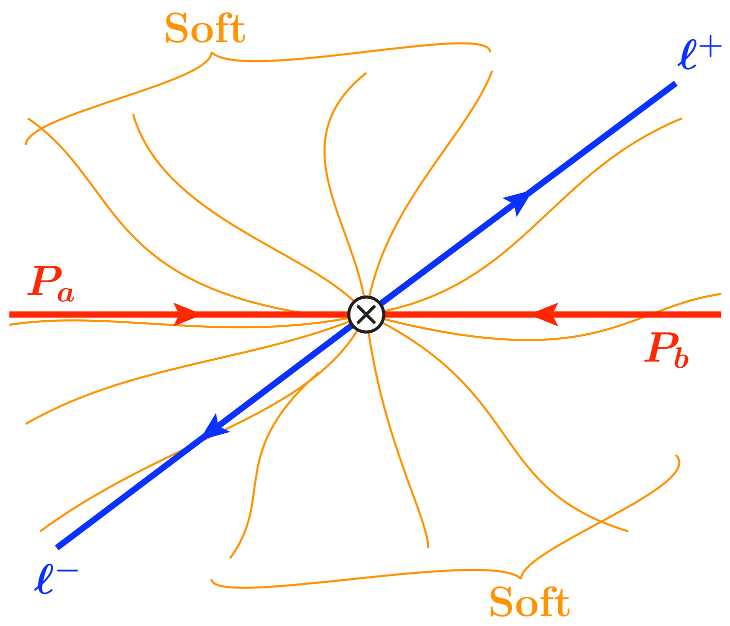
$$d\sigma = H \times S \otimes B \otimes B$$

$$U_H^{1/2} \delta = U_S \otimes U_B$$

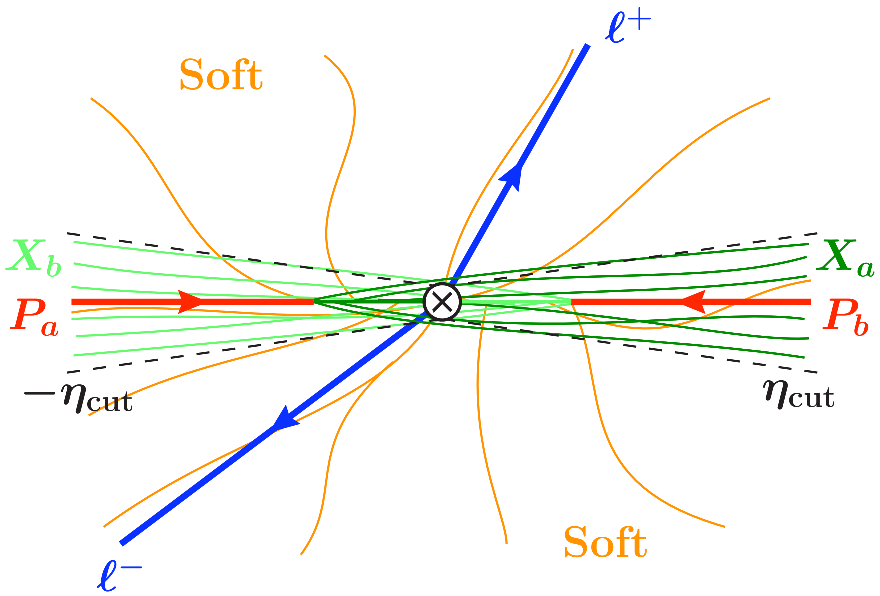
can't have just pdfs !



RGE sums double Sudakov logs

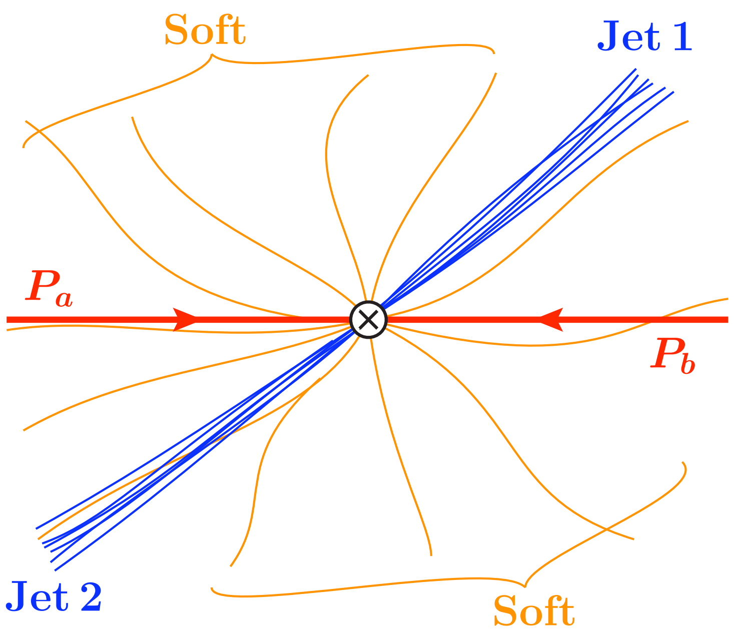


$$d\sigma = H \; S \otimes f \otimes f$$



$$d\sigma = H \times S \otimes B \otimes B$$

Let's add Final State Jets



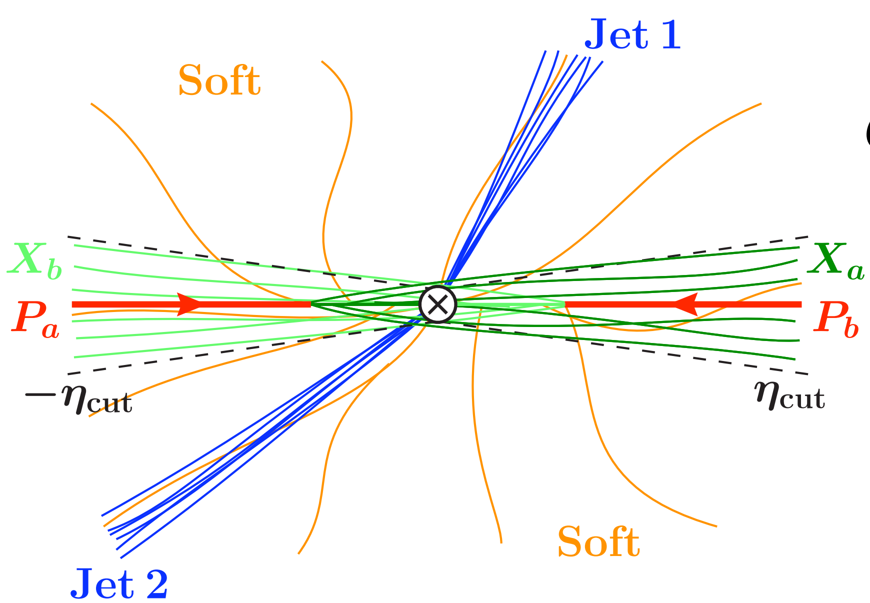
Threshold Jet Production

$$d\sigma = H\ S \otimes J \otimes J \otimes f \otimes f$$

$$M_{JJ} \to E_{\rm cm}\ , \quad x_{a,b} \to 1$$

Kidonakis, Orderda, Sterman

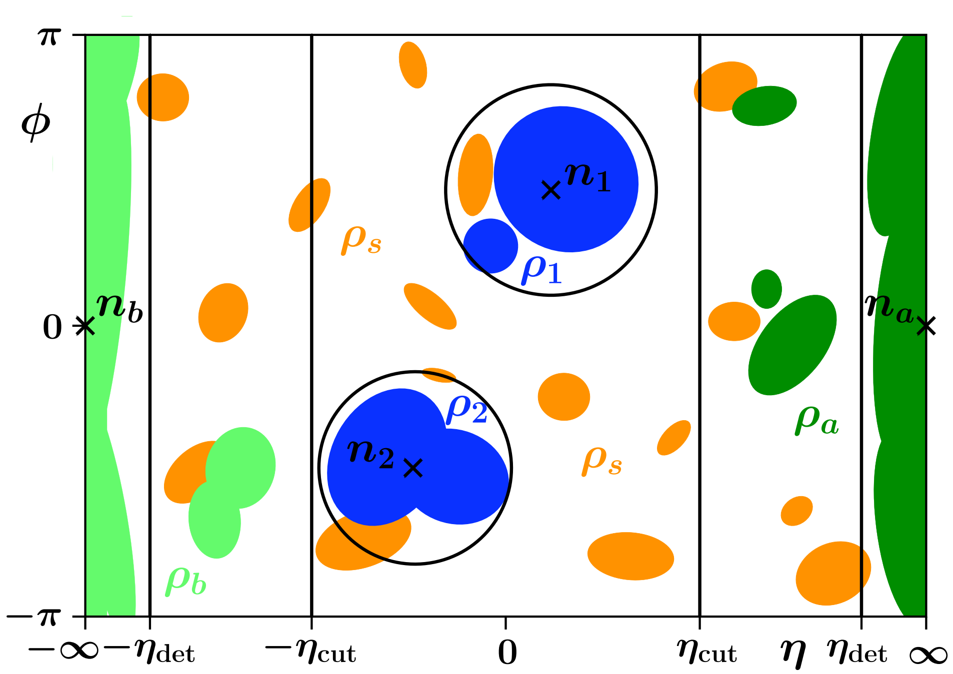
"Exclusive" Jet Production

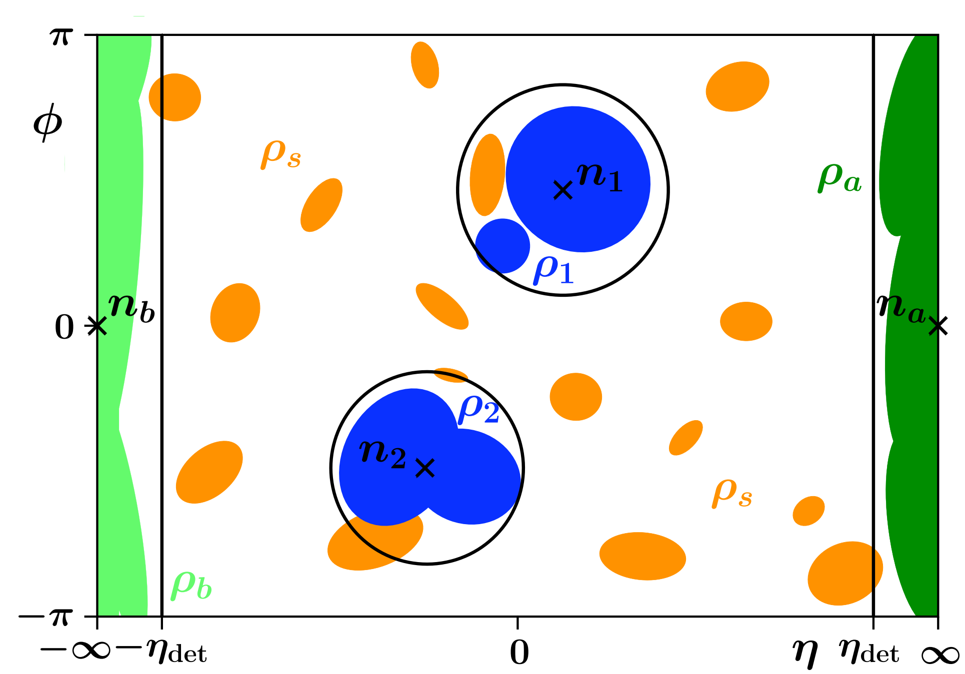


$$d\sigma = H \times S \otimes J \otimes J \otimes B \otimes B$$

I.S., Tackmann, Waalewijn

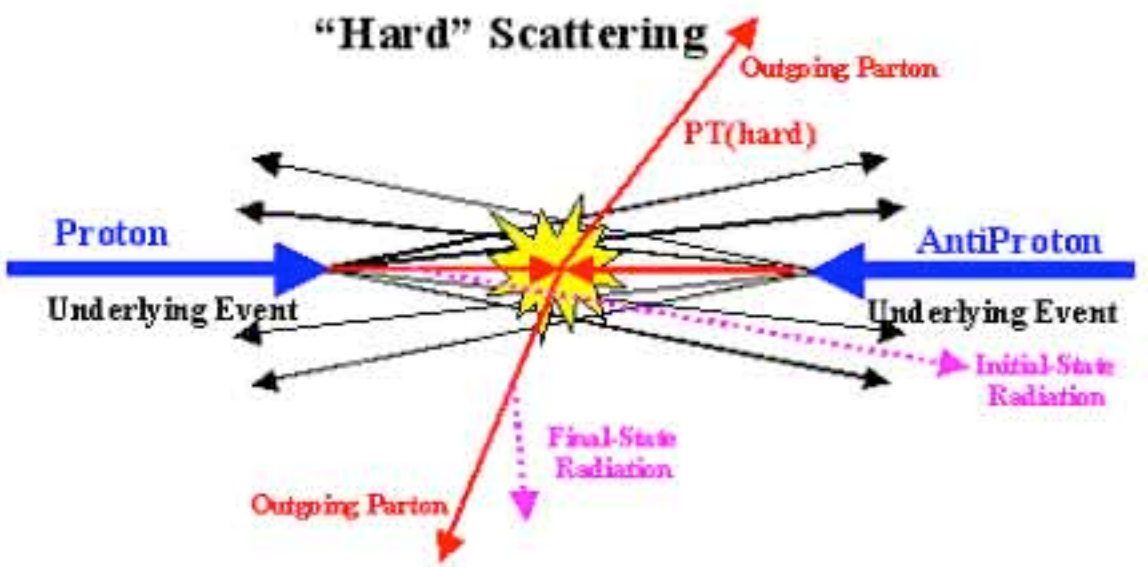
Require:
final state jets (defined by jet algorithm) are well separated from each other, from the beam direction, and from soft radiation ($p_T^{\rm min}$, $E_{\rm soft}<E_0$)





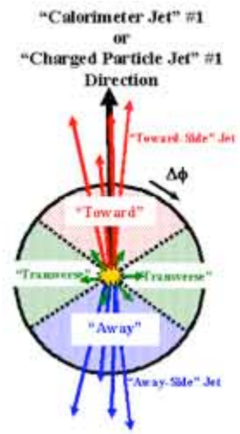
Comparison to concepts used at CDF/Pythia

R.D. Field

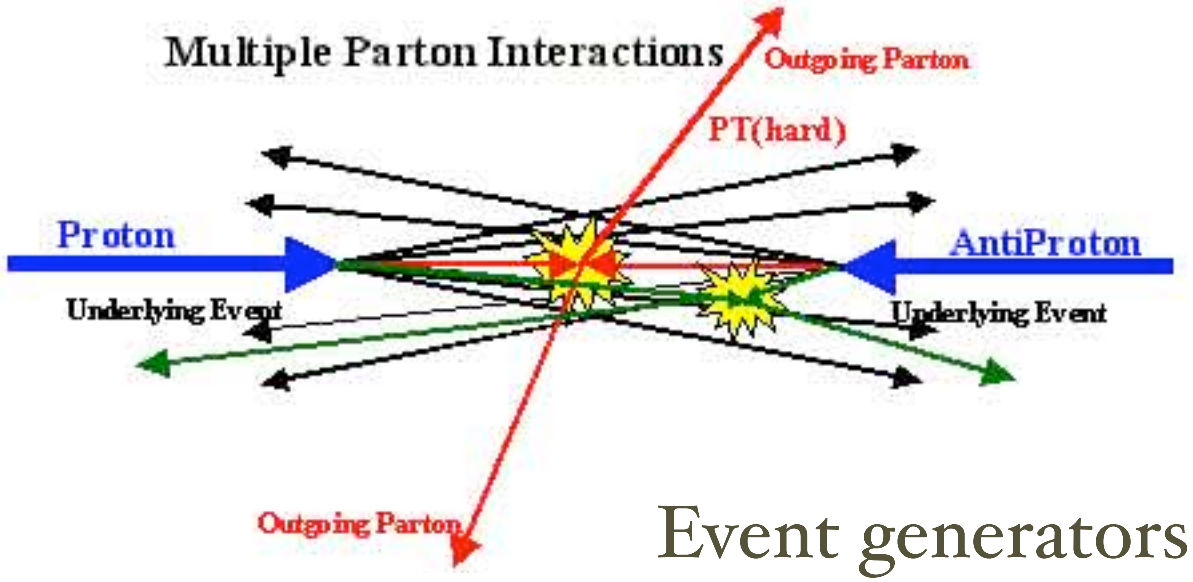


"Underlying event is everything but the outgoing hard jets (and accompanying radiation). It consists of particles arising from the beam-beam remnants and multiple parton interactions."

measured at mid-rapidity, transverse to central jets



[This radiation should be described by soft functions in Factorization Theorems.]



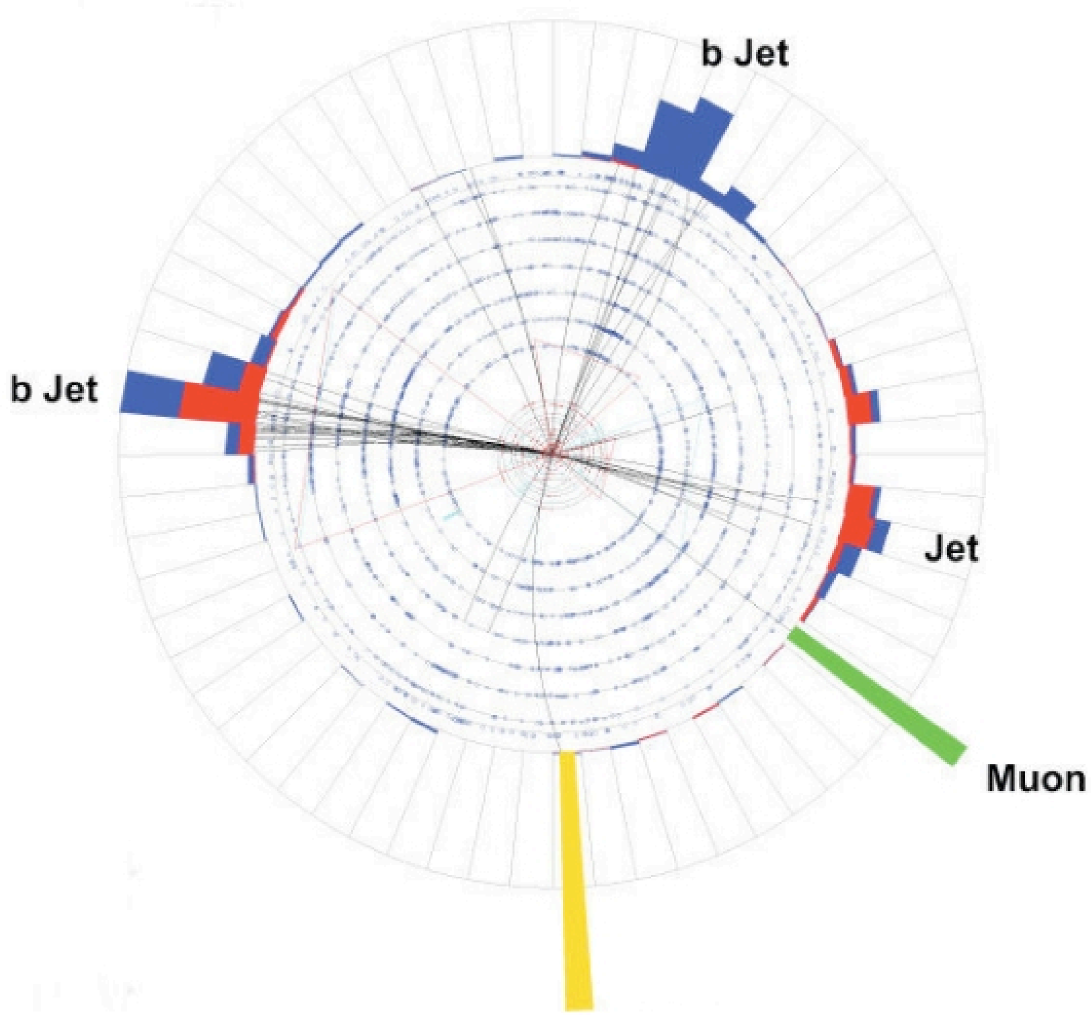
Event generators such as Pythia implement initial state showers & model the underlying event with a treatment of the beam-remnants and multiple interactions.

[Consistent with soft radiation convoluted with the beam function.]

Derivation of Factorization Theorem for Exclusive Jet Production

Energy Flow Operator

- Lets derive the factorization theorem to see where B comes from



- Most observables only depend on the energy distribution $\rho_X(\Omega)$
- For example:
  X has n particles with $p_i = (E_i, \vec{p}_i)$

$$\rho_X(\Omega) = \sum_i E_i\, \delta(\Omega - \Omega_i)$$

- Energy flow operator:

$$\mathcal{E}(\Omega)|X\rangle = \rho_X(\Omega)|X\rangle$$

[Korchemsky, Oderda, Sterman; Lee, Sterman, Bauer, Fleming;

We follow: Bauer, Hornig, Tackmann]

Energy Flow & Factorization

- QCD cross section:

$$\frac{d\sigma}{dO} = \frac{1}{2E_{\rm CM}^2}\sum_X \left|\mathcal{M}(pp \to X)\right|^2 \delta[O - f_O(X)] \times (2\pi)^4\delta^4(P_a + P_b - P_X)$$

- Match onto SCET

$$\mathcal{M}(pp \to X) = \langle X|\mathcal{Q}|pp\rangle$$

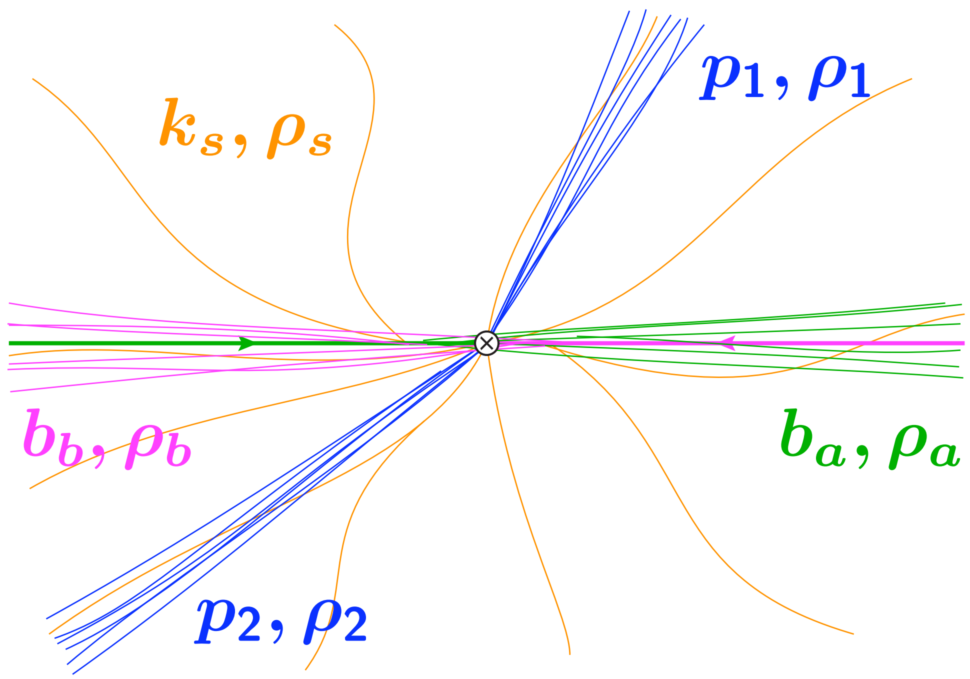
- Use energy flow to do sum over the final states

$$\frac{d\sigma}{dO} = \frac{1}{2E_{\rm CM}^2}\sum_X \langle pp|\mathcal{Q}^\dagger|X\rangle\langle X|\mathcal{Q}|pp\rangle\,\delta(O - f_O[\rho_X]) \times (2\pi)^4\delta^4(P_a + P_b - P_X)$$
$$= \frac{1}{2E_{\rm CM}^2}\int \mathcal{D}\rho \int d^4x\,\langle pp|\mathcal{Q}^\dagger(x)\,\delta[\rho - \mathcal{E}]\mathcal{Q}(0)|pp\rangle\,\delta(O - f_O[\rho])$$

Soft-Collinear Factorization

- SCET Lagrangian:

$$\mathcal{L} = \mathcal{L}_{n_a} + \mathcal{L}_{n_b} + \sum_i \mathcal{L}_{n_i} + \mathcal{L}_s$$



- Decouple collinear and soft

$$\xi_{n,p}(x) \to Y_n(x)\, \xi^{(0)}_{n,p}(x)$$
$$A^{\mu}_{n,p}(x) \to Y_n(x)\, A^{(0)\mu}_{n,p}(x)\, Y_n^{\dagger}(x)$$

- Energy flow factorizes

$$\mathcal{E}(\Omega) = \mathcal{E}_a(\Omega) + \mathcal{E}_b(\Omega) + \sum_i \mathcal{E}_i(\Omega) + \mathcal{E}_s(\Omega)$$

$$\int \mathcal{D}\rho\, \delta[\rho - \mathcal{E}] = \int \mathcal{D}\rho_a\, \delta[\rho_a - \mathcal{E}_a]\, \mathcal{D}\rho_b\, \delta[\rho_b - \mathcal{E}_b] \prod_i \mathcal{D}\rho_i\, \delta[\rho_i - \mathcal{E}_i]$$
$$\times\, \mathcal{D}\rho_s\, \delta[\rho_s - \mathcal{E}_s]$$

Step 11

$$\frac{d\sigma}{dO} = \frac{1}{2E_{\mathrm{CM}}^2} \int \mathcal{D}\rho \, d^4x \, \langle pp | \mathcal{Q}^\dagger(x) \, \delta[\rho - \mathcal{E}] \mathcal{Q}(0) | pp \rangle \, \delta[O - f_O[\rho]]$$

$$= \int d^4 b_{a,b} \, \mathcal{D}\rho_{a,b} \, d^4 p_{1,2} \, \mathcal{D}\rho_{1,2} \, d^4 k_s \, \mathcal{D}\rho_s \, \delta[O - f_O[\rho]]$$

$$\times H(\hat{s}, \dots) \, B(b_a, \rho_a) \, B(b_b, \rho_b) \, J(p_1, \rho_1) \, J(p_2, \rho_2) \, S(k_s, \rho_s)$$

$$\times (2\pi)^4 \delta^4(P_a + P_b - b_a - b_b - p_1 - p_2 - k_s)$$

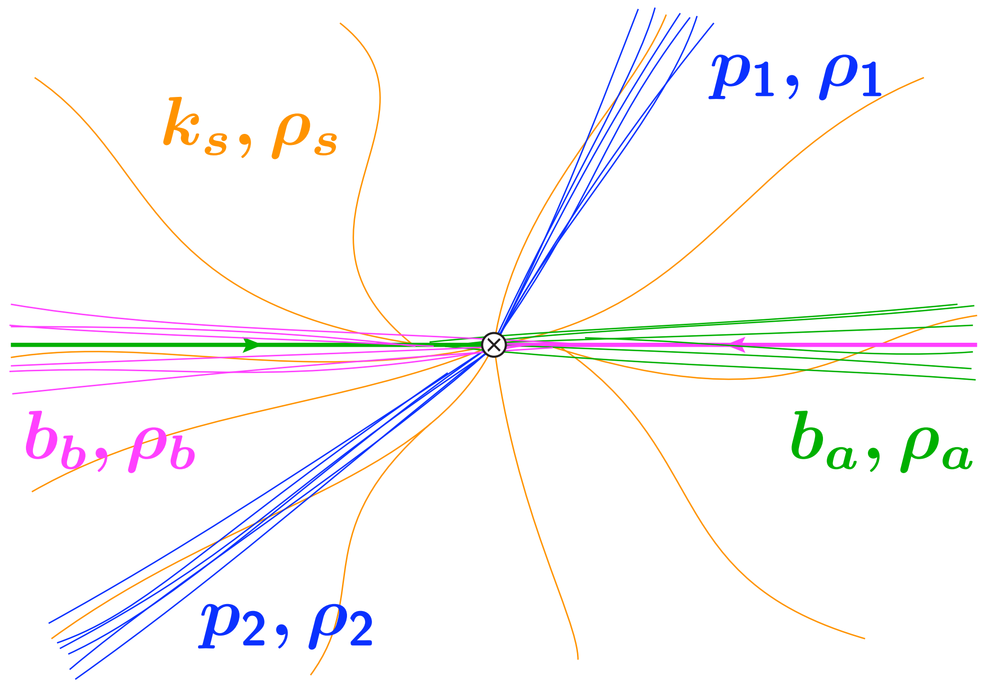
with

$H(\hat{s}, \dots)$ = hard function

$B(b_a, \rho_a)$ = beam distribution

$J(p_i, \rho_i)$ = jet distribution

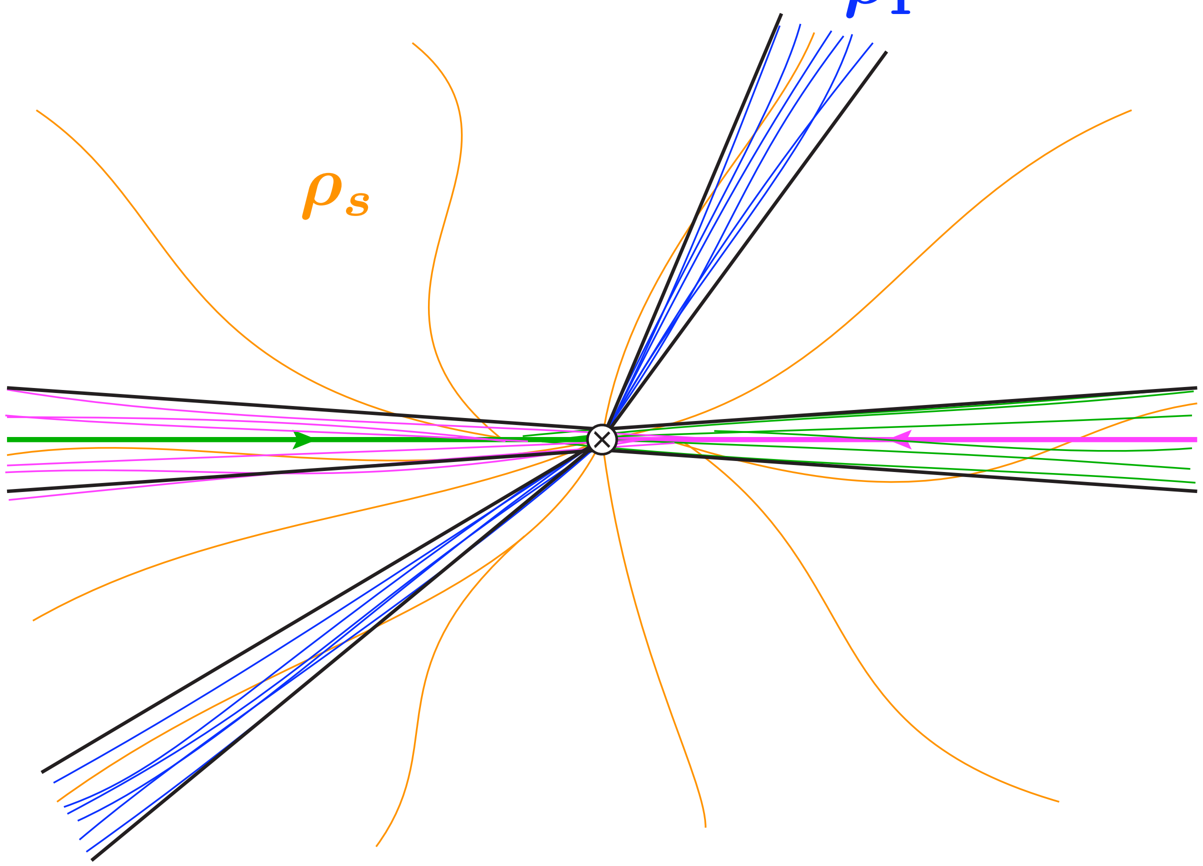
$S(k_s, \rho_s)$ = soft distribution



- These distributions are not inclusive, they depend on ρ_i

Jet Kinematics

- $P^\mu_{i\triangleleft}[\rho] \equiv P^\mu_{\triangleleft}(\vec{n}_i, R)[\rho]$ = momentum in cone of radius R around $\vec{n}_i$



$$P^\mu_1 = P^\mu_{1\triangleleft}[\rho]$$
$$= P^\mu_{1\triangleleft}[\rho_1] + P^\mu_{1\triangleleft}[\rho_s]$$
$$= q^\mu_1 + \ell^\mu_1$$

$$B^\mu_a = P^\mu_{a\triangleleft}[\rho_a] + P^\mu_{a\triangleleft}[\rho_s]$$
$$= q^\mu_a + \ell^\mu_a$$

- Expanding:

$$P^-_i = \bar{n}_i \cdot P_i = q^-_i \qquad P^+_i = n_i \cdot P_i = q^+_i + \ell^+_i$$
$$B^-_a = \bar{n}_a \cdot B_a = q^-_a \qquad B^+_a = n_a \cdot B_a = q^+_a + \ell^+_a$$

Factorization

$$
\begin{aligned}
d\sigma = \int & d^4 b_{a,b}\, \mathcal{D}\rho_{a,b}\, d^4 p_{1,2}\, \mathcal{D}\rho_{1,2}\, d^4 k_s\, \mathcal{D}\rho_s \\
& \times H(\hat{s},\dots)\, B(b_a,\rho_a)\, B(b_b,\rho_b)\, J(p_1,\rho_1)\, J(p_2,\rho_2)\, S(k_s,\rho_s) \\
& \times (2\pi)^4 \delta^4(P_a + P_b - b_a - b_b - p_1 - p_2 - k_s) \\
& \times d\ell^+_{a,b,1,2}\, \delta(\ell^+_{a,b} - P^+_{\triangleleft a,b}[\rho_s])\, \delta(\ell^+_{1,2} - P^+_{\triangleleft 1,2}[\rho_s]) \\
& \times dq^+_{a,b}\, \delta(q^+_{a,b} - P^+_{\triangleleft}[\rho_{a,b}])\, dB^+_{a,b}\, \delta(B^+_{a,b} - q^+_{a,b} - \ell^+_{a,b}) \\
& \times dq^+_{1,2}\, \delta(q^+_{1,2} - P^+_{\triangleleft}[\rho_{1,2}])\, dP^+_{1,2}\, \delta(P^+_{1,2} - q^+_{1,2} - \ell^+_{1,2}) \\
& \times dP^-_{1,2}\, \delta(P^-_{1,2} - P^-_{\triangleleft}[\rho_{1,2}]) \\
= \int & d\ell^+_{a,b,1,2} \quad S_{\triangleleft}(\ell^+_a, \ell^+_b, \ell^+_1, \ell^+_2, E_0) \\
& \times dx_{a,b}\, dB^+_{a,b} \quad B(p^-_a(B^+_a - \ell^+_a), x_a)\, B(p^-_b(B^+_b - \ell^+_b), x_b) \\
& \times d^4 p_{1,2}\, J_{\triangleleft}(p^-_1, P^+_1 - \ell^+_1)\, J_{\triangleleft}(p^-_2, P^+_2 - \ell^+_2) \\
& \times H(x_a, x_b, p^-_1, p^-_2)\, dO\, \delta[O - f_O(x_a, x_b, B^+_a, B^+_b, p^-_1, p^-_2)] \\
& \times (2\pi)^4 \delta^4(\tfrac{1}{2} x_a E_{\rm CM}\, n_a + \tfrac{1}{2} x_b E_{\rm CM}\, n_b - p_1 - p_2)
\end{aligned}
$$

Final Formula

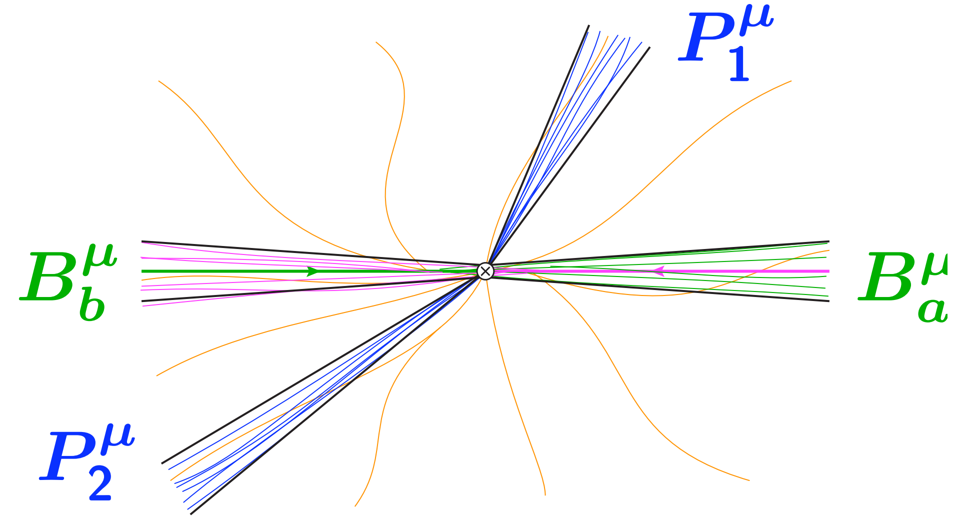
Same H as for threshold Factorization Theorem

[Talks here by Neubert, Becher, Manohar]

$$\frac{\mathbf{d}\sigma}{\mathbf{d}O} = \int \mathbf{d}\ell^+_{a,b,1,2} \quad S_{\triangleleft}(\ell^+_a, \ell^+_b, \ell^+_1, \ell^+_2, E_0)$$
$$\times\, \mathbf{d}x_{a,b}\, \mathbf{d}B^+_{a,b} \quad B(p_a^-(B_a^+ - \ell_a^+), x_a)\, B(p_b^-(B_b^+ - \ell_b^+), x_b)$$
$$\times\, \mathbf{d}^4 p_{1,2}\, J_{\triangleleft}(p_1^-, P_1^+ - \ell_1^+)\, J_{\triangleleft}(p_2^-, P_2^+ - \ell_2^+)$$
$$\times\, H(x_a, x_b, p_1^-, p_2^-)\, \delta[O - f_O(x_a, x_b, B_a^+, B_b^+, p_1^-, p_2^-)]$$
$$\times\, (2\pi)^4 \delta^4(\tfrac{1}{2} x_a E_{\rm CM}\, n_a + \tfrac{1}{2} x_b E_{\rm CM}\, n_b - p_1 - p_2)$$

- Pick an observable, e.g.

$$M_{JJ}^2 = 2P_1 \cdot P_2 = \frac{P_1^- \, P_2^-}{p_1^- \, p_2^-}\, x_a\, x_b\, E_{\rm CM}^2$$



Does not depend on $B^+_{a,b}$ *but*

Can only integrate up to upper cutoff given by $\eta_{\rm cut}$ restriction

$$\int_0^{(\hat{Q} e^{-\eta_{\rm cut}})} dB_a^+ \; B_a(p_a^-(B_a^+ - \ell_a^+), x_a)$$

Beam Function Factorizes: $B(x, s, \mu) = \int dx' \; \mathcal{I}(s, x' - x, \mu)\; f(x', \mu)$ (next)

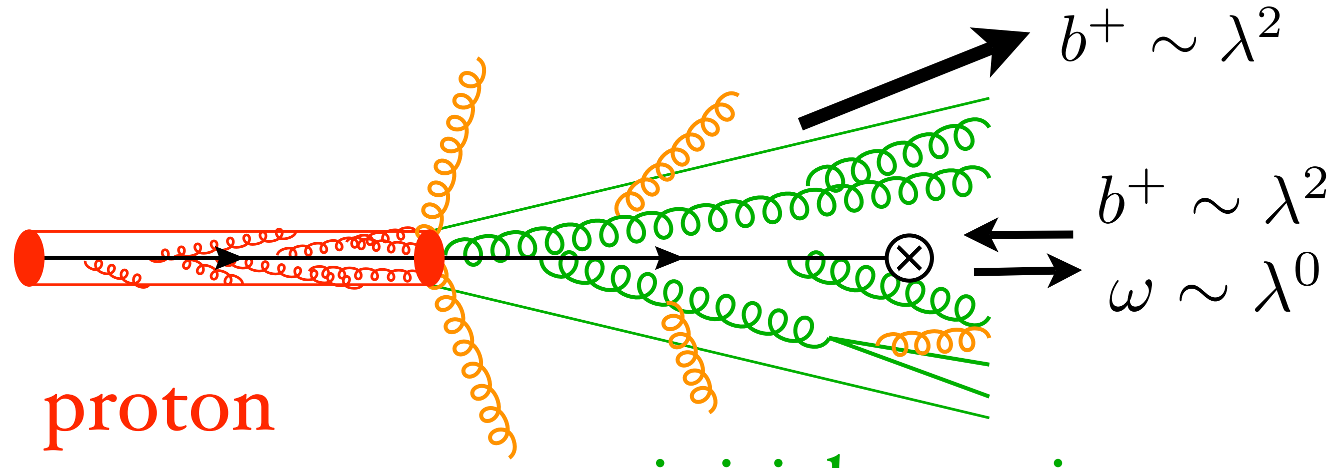
Beam Functions

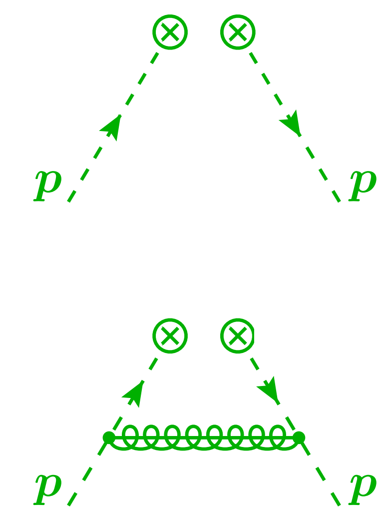
Quark Beam Function

$B_q(s, x, \mu)$

$\chi_n = W^\dagger \xi_n$

$$B_q\Big(b^+\omega, \frac{\omega}{P_a^-}, \mu\Big) = \frac{1}{\omega}\int \frac{dx^-}{4\pi} e^{ib^+x^-/2} \Big\langle p_n(P_a^-)\Big|\bar{\chi}_n\Big(x^-\frac{n}{2}\Big)\frac{\bar{n}\!\!\!/}{2}\big[\delta(\omega - \bar{\mathcal{P}})\chi_n(0)\big]\Big|p_n(P_a^-)\Big\rangle$$





$, \cdots$

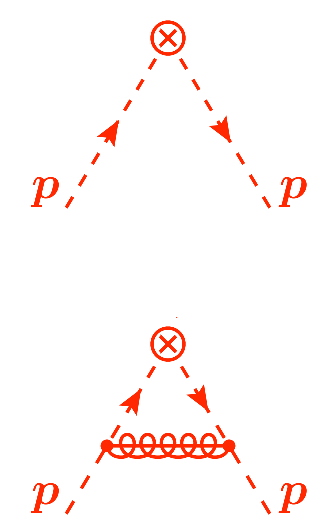
like jet function in initial state, BUT also transitions to pdf

Quark Parton Distribution

$f_q(x, \mu)$

$$f_q\Big(\frac{\omega}{P_a^-}, \mu\Big) = \theta(\omega)\Big\langle p_n(P_a^-)\Big|\bar{\chi}_n(0)\frac{\bar{n}\!\!\!/}{2}\big[\delta(\omega - \bar{\mathcal{P}})\chi_n(0)\big]\Big|p_n(P_a^-)\Big\rangle$$

(Fourier transform of standard definition)



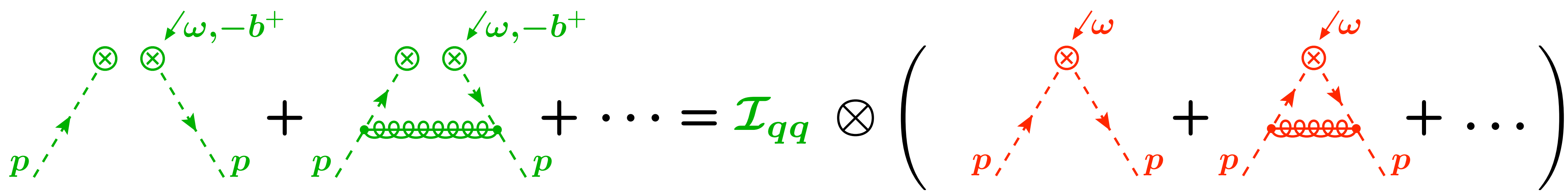
$, \cdots$

Beam Function Factorization

perturbatively calculable
initial state jet coefficient

- B factorizes (matching SCET$_I$ $\longrightarrow$ SCET$_{II}$)

$$B_q(b^+\omega, z; \mu) = \sum_{i=q,g,\bar{q}} \int \mathrm{d}z' \, \mathcal{I}_{qi}(b^+\omega, z - z'; \mu) \, f_i(z'; \mu)$$



Beam Function at Tree Level

partonic matching:

$$\langle q(p) \big| \bar{\xi}_n(x^- n/2) \frac{\bar{n}\!\!\!/}{2} \Big[\delta(\omega - \overline{\mathcal{P}}) \xi_n(0) \Big] \big| q(p) \rangle = \bar{u}(p) \frac{\bar{n}\!\!\!/}{2} u(p) \; e^{i x^- p^+/2} \, \delta(\omega - p^-) = e^{i x^- p^+/2} \, \delta(1 - \omega/p^-)$$

$$\Longrightarrow \quad \hat{B}_q^{\rm tree}(b^+\omega, \omega/p^-) = \delta(b^+\omega) \, \delta(1 - \omega/p^-)$$

hadronic results:

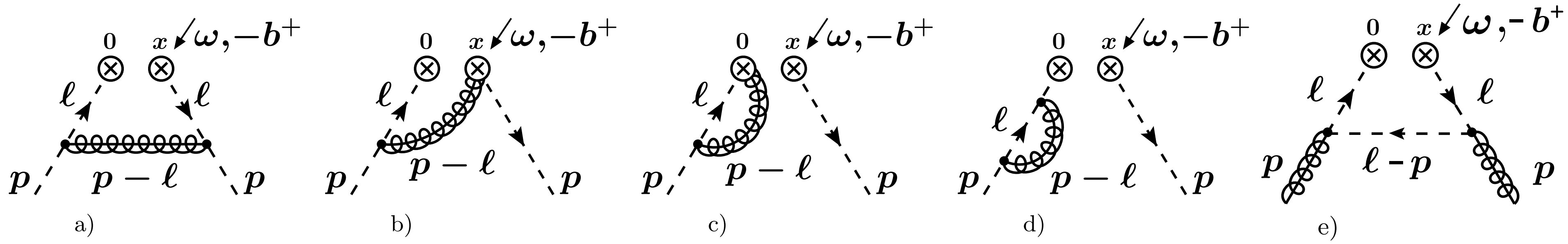
$$\mathcal{I}_{qq}^{\rm tree}(b^+\omega, x - z) = \delta(b^+\omega) \, \delta(x - z)$$

$$\boxed{B_q(b^+\omega, x) = \delta(b^+\omega) f_q(x, \mu)}$$

Tree level results
not effected by
beam vs. pdf

IR divergences and One-Loop Matching

Renormalized Beam Function Graphs

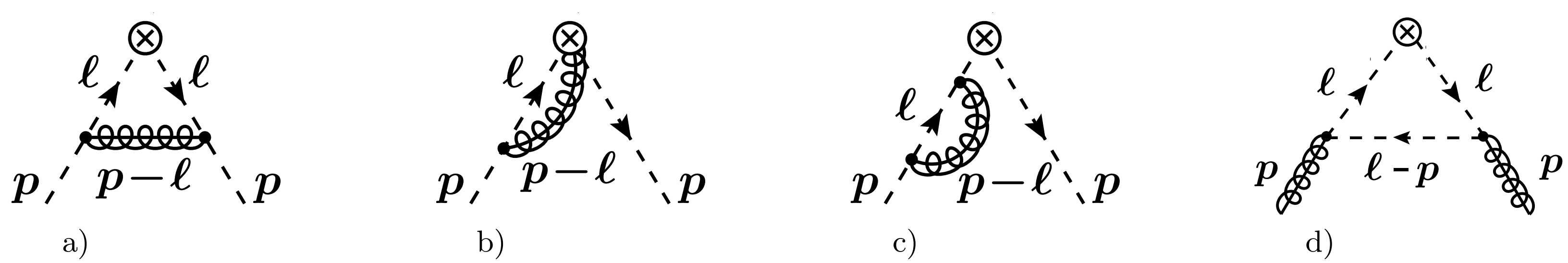


$$\hat{B}_q^{[q]}(s,z,\mu) = \delta(s)\delta(1-z) + \frac{C_F\alpha_s(\mu)}{\pi}\bigg\{\delta(s)\Big[\frac{1}{2}\ln\Big(\frac{\mu^2}{p_{\rm IR}^2 z}\Big)\Big(\frac{1+z^2}{1-z}\Big)_+ + \ldots\Big] + \frac{1}{\mu^2}\mathcal{L}_0\Big(\frac{s}{\mu^2}\Big)\Big[\frac{z}{(1-z)_+} + \ldots\Big] + \ldots\bigg\},$$

$$\mathcal{L}_n(x) = \Big[\frac{\theta(x)\ln^n(x)}{x}\Big]_+$$

$$\hat{B}_q^{[g]}(s,z,\mu) = \frac{-\alpha_s(\mu)}{4\pi}\theta(z)\theta(1-z)(1-2z+2z^2)\bigg\{\delta(s)\ln\frac{z p_{\rm IR}^2}{\mu^2} + \ldots\bigg\}$$

Renormalized PDF Graphs



$$\hat{f}_q^{[q,1]}(z,\mu) = \frac{C_F\alpha_s(\mu)}{\pi}\Big[\frac{1}{2}\ln\Big(\frac{\mu^2}{p_{\rm IR}^2}\Big)\Big(\frac{1+z^2}{1-z}\Big)_+ - (1-z)\theta(z)\theta(1-z) + \ldots\Big]$$

$$\hat{f}_q^{[g,1]}(z,\mu) = \frac{-\alpha_s(\mu)}{4\pi}\theta(z)\theta(1-z)\Big[[(1-z)^2+z^2]\ln\Big(\frac{(1-z)p_{\rm IR}^2}{\mu^2}\Big) + \ldots\Big]$$

IR Matches up

Difference gives matching results:

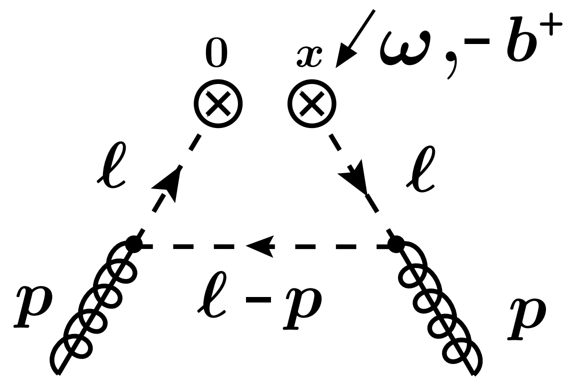
quark pdf into quark beam function, nontrivial corrections

$$\mathcal{I}_{qq}(s,1-z,\mu) = \delta(s)\delta(1-z) + \frac{C_F\alpha_s(\mu)}{\pi}\theta(z)\Big\{\delta(s)\Big[z\mathcal{L}_1(1-z)+\dots\Big] + \frac{1}{2}\,\frac{1}{\mu^2}\mathcal{L}_0\Big(\frac{s}{\mu^2}\Big)(1+z^2)\,\mathcal{L}_0(1-z)+\dots\Big\},$$

$$\mathcal{I}_{qg}(s,1-z,\mu) = \frac{\alpha_s(\mu)}{4\pi}\theta(z)\theta(1-z)\Big[[(1-z)^2+z^2]\frac{1}{\mu^2}\mathcal{L}_0\Big(\frac{s}{\mu^2}\Big)+\dots\Big]$$

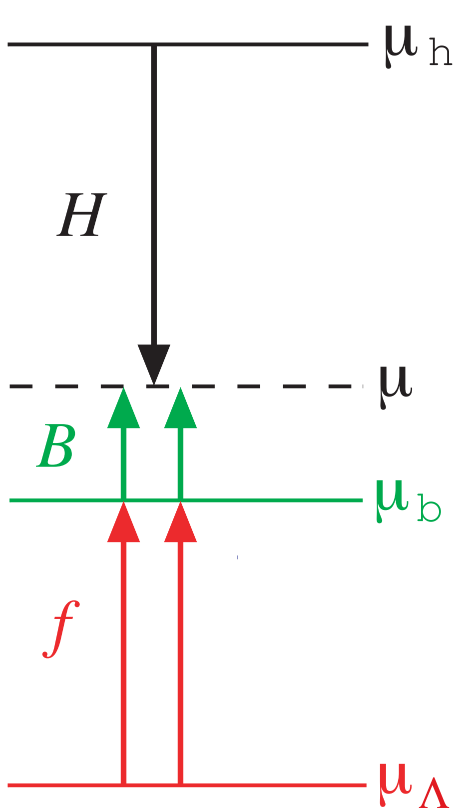
gluon pdf into quark beam function (!)

These mixing effects and radiative corrections are not accounted for by PDF evolution

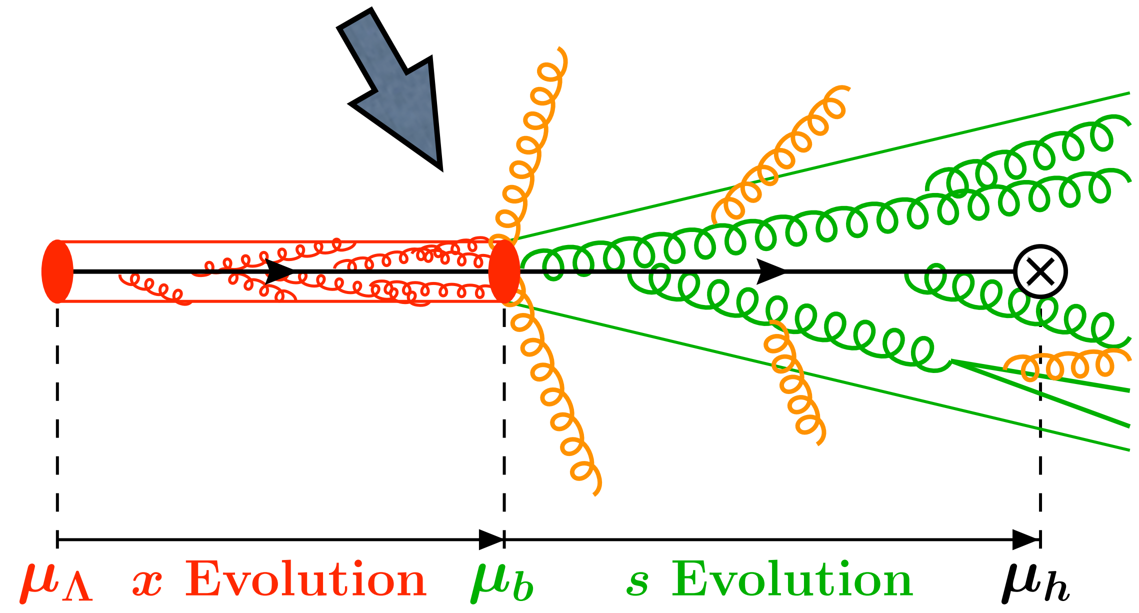


e)

UV divergences and RGE



at μ_b the proton breaks apart producing an initial state jet



single logs

double logs

$$\mu \frac{d}{d\mu} B_q(s,z,\mu) = \int_0^s ds' \, \gamma^{B_q}(s,s',\mu) B_q(s',z,\mu)$$

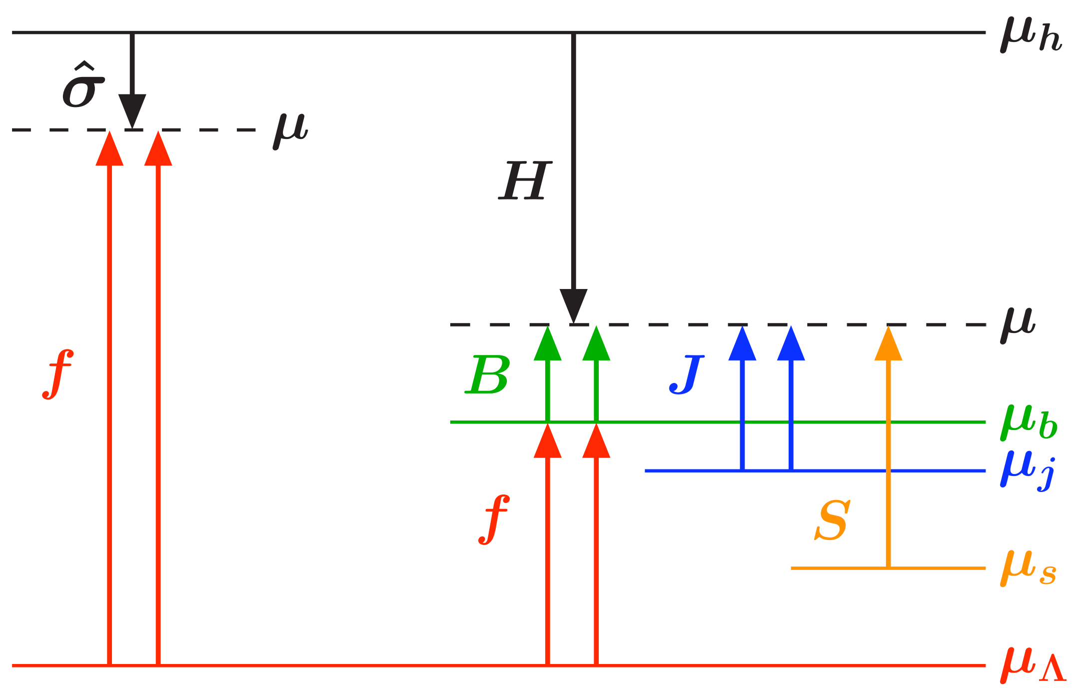
diagonal, no mixing at one-loop

Constraints on invariant mass of the real radiation yield $\gamma^B(s,s';\mu) \propto \ln(\mu/s) + \ldots$, which sum double Sudakov logs

LL solution: $\mathrm{B}_q^{\mathrm{LL}}(s,z,\mu) = \frac{1}{\mu^2} R^{\mathrm{LL}}(s/\mu^2) f_q(z,\mu)$

compare to pdf: $\mu \frac{d}{d\mu} f_q(x,\mu) = \frac{\alpha_s(\mu)}{\pi} \int_x^1 \frac{dx'}{x'} P_{qj}\left(\frac{x}{x'}\right) f_j(x',\mu)$ mixing at one-loop

Full RGE for "Exclusive" dijet production in pp



$\mu_h \sim$ scale of hard interaction

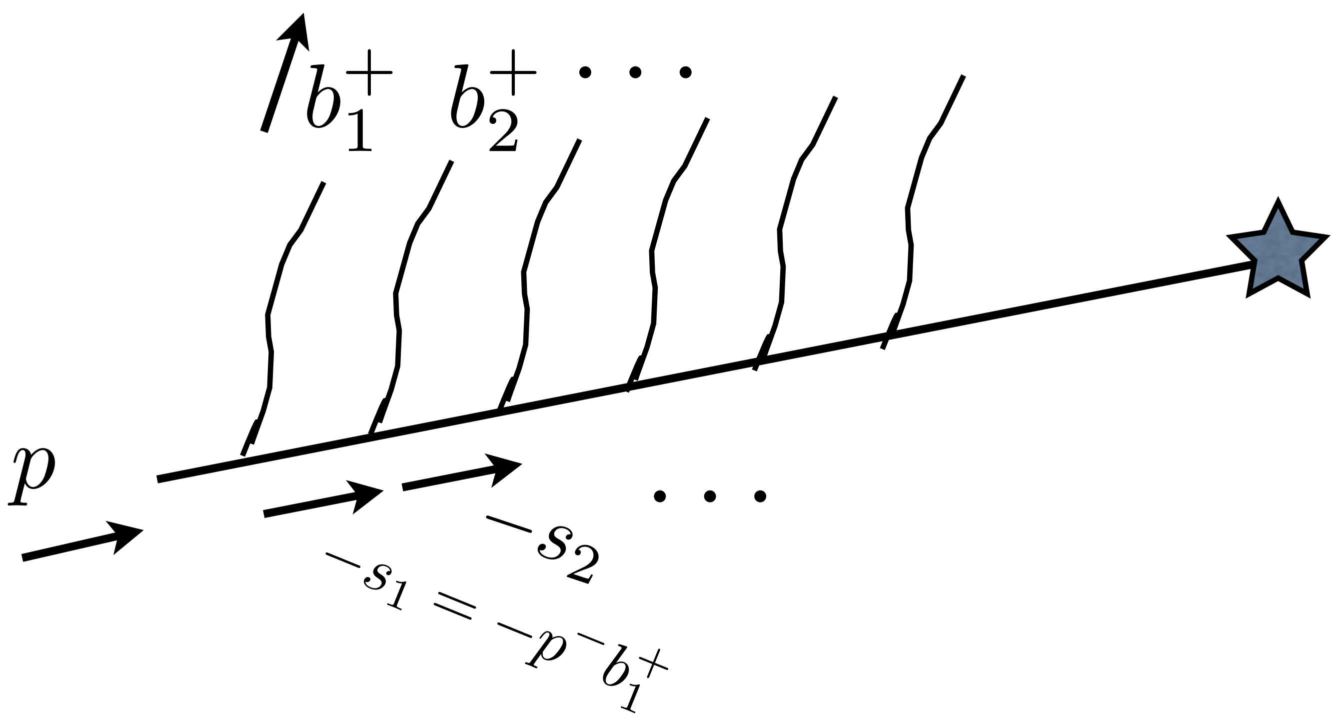
$\mu_j \sim$ inv. mass of final state jet

$\mu_b \sim$ inv. mass of initial state jet

$\mu_s \sim$ energy of soft radiation

$\mu_\Lambda \sim$ low scale (Λ_{QCD})

- Phenomenology, work in progress
- Compare RGE to Initial State Shower
  - agree on strong ordering, space-like shower, single branch
  - RGE's aim to sum different large logs. Needs more study.



Conclusions and Outlook

- Factorization Theorem for "Exclusive" N-jet production at the LHC
- Depends on initial state radiation, described by universal Beam Functions for quarks, gluons, antiquarks

$$\boldsymbol{\sigma} = \boldsymbol{B} \otimes \boldsymbol{B} \otimes \Big(\sum_{\boldsymbol{N} \text{ jets}} \boldsymbol{H_N} \otimes \underbrace{\boldsymbol{J} \otimes \ldots \otimes \boldsymbol{J}}_{\boldsymbol{N} \text{ times}} \otimes \boldsymbol{S_N} \Big)$$

Allows $x < 10^{-1}$ in fact.thm.

- ▸ PDF $\boldsymbol{f}$ replaced by $\boldsymbol{B}$
- ▸ $\boldsymbol{H_N}$ as in threshold resummation

- Beam function factorizes at scale $\mu_b \simeq \hat{Q} e^{-\eta_{\rm cut}}$

$$B_q(x, s, \mu) = \int dx' \, \mathcal{I}_{qi}(s, x' - x, \mu) \, f_i(x', \mu)$$

- Evolution in x below μ_b; Evolution in s above μ_b

  Parton distributions enter at $\mu_b \ll \hat{Q}$

  Mixing below μ_b (pdf rge), at μ_b (matching), but not above μ_b (beam rge)

- Future Applications: * new classes of factorization theorems
  * improve initial state shower Monte Carlo?