Introduction to F-theory and Dualities

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Take-Home Messages

- Theories can have **dualities** $\neq$ symmetries

  Different descriptions of the same theory

- F-theory **geometrizes** a lot of stuffs.

  including e.g. S-duality $\rightarrow$ strong coupling description

Electric-Magnetic Duality

Maxwell's equations

$$\nabla \cdot \mathbf{E} = 0$$
$$\nabla \cdot \mathbf{B} = 0$$
$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$
$$\nabla \times \mathbf{B} = \frac{\partial \mathbf{E}}{\partial t}$$

Invariant under

$$\mathbf{E} \to \mathbf{B}$$
$$\mathbf{B} \to -\mathbf{E}$$

i.e.

$$\begin{pmatrix} \mathbf{E} \\ \mathbf{B} \end{pmatrix} \to \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} \mathbf{E} \\ \mathbf{B} \end{pmatrix}$$

Explicit Dualization

$$\mathcal{L} = -\frac{1}{4g^2} F_{\mu\nu} F^{\mu\nu}, \qquad \text{Bianchi id: } \partial_\mu \tilde{F}^{\mu\nu} = 0, \quad \tilde{F}^{\mu\nu} = \epsilon^{\mu\nu\rho\sigma} F_{\rho\sigma}$$

Write a parent Lagrangian

$$\mathcal{L}_P = -\frac{1}{4g^2} F_{\mu\nu} F^{\mu\nu} + \frac{1}{2\pi} A^D_\nu \partial_\mu \tilde{F}^{\mu\nu}$$

Integrating over A_D gives back $\mathcal{L}$.

Integrating over $F^{\mu\nu}$ gives

$$\mathcal{L}^D = -\frac{g^2}{16\pi^2} F^D_{\mu\nu} F^{D\mu\nu}$$

Same form, different variable $\rightarrow$ **self-duality!** $g \rightarrow \frac{2\pi}{g}$.

Add a term $\frac{\theta}{2\pi} F_{\mu\nu} \tilde{F}^{\mu\nu}$ and define $\tau = \frac{\theta}{2\pi} + i\,\frac{2\pi}{g^2}$. Then $\tau \rightarrow -\frac{1}{\tau}$.

Non-abelian gauge theories

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + g[A_\mu, A_\nu]$$

Evidence for the duality to hold, but no Lagrangian prove.

Now g is really a coupling. $\tau = \frac{\theta}{2\pi} + i\,\frac{2\pi}{g^2}$

$\tau \to -1/\tau \qquad (g \to\sim 1/g)$ **strong-weak duality**.

$\frac{\theta}{2\pi} F_{\mu\nu}\tilde{F}^{\mu\nu} \qquad \theta \to \theta + 1 \qquad (\tau \to \tau + 1)$ is a symmetry.

$\Rightarrow$ All together

$$\tau \to \frac{a\tau + b}{c\tau + d}, \qquad \begin{pmatrix} \mathbf{E} \\ \mathbf{B} \end{pmatrix} \to \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} \mathbf{E} \\ \mathbf{B} \end{pmatrix}$$

where $ad - bc = 1$, i.e. $SL(2,\mathbb{Z})$. $\qquad \tau \to -1/\tau \;\leftrightarrow\; \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$

Type IIB String Theory

Low energy effective Lagrangian:

$$\mathcal{L} \sim \frac{1}{(\operatorname{Im}\tau)^2}\, \mathrm{d}\,\tau \wedge \star \mathrm{d}\,\bar{\tau} + \frac{1}{\operatorname{Im}\tau} G_3 \wedge \star \bar{G}_3 + C_4 \wedge H_3 \wedge F_3$$

$$\tau = C_0 + \frac{i}{g_s}, \qquad g_s = \text{string coupling}$$

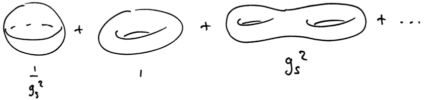
$$G_3 = F_3 - \tau H_3$$

Enjoys $SL(2,\mathbb{Z})$ **S-duality**.

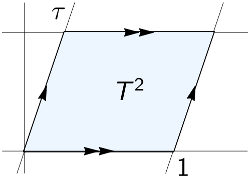
$$\tau \;\to\; \frac{a\,\tau + b}{c\,\tau + d}, \qquad \begin{pmatrix} F_3 \\ H_3 \end{pmatrix} \to \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} F_3 \\ H_3 \end{pmatrix}$$

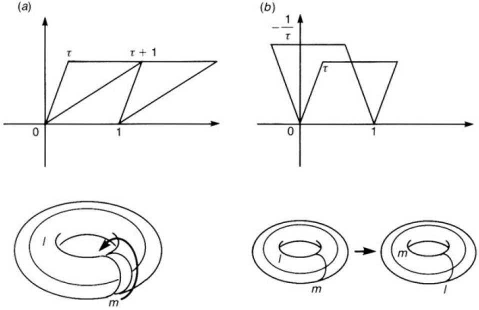
$\Rightarrow$ Strong/weak duality.

String Theory defined **perturbatively** in g_S:



Torus: SL(2,$\mathbb{Z}$)-invariant object





$$\tau \;\rightarrow\; \frac{a\,\tau + b}{c\,\tau + d}: \qquad \text{equivalent tori.}$$

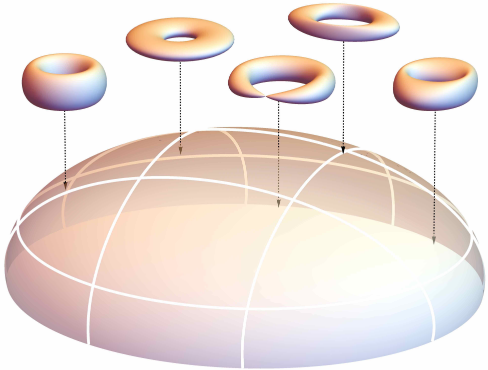
Idea: identify τ (torus) with τ (string theory).

$\Rightarrow$ Realize S-duality **geometrically**!

F-theory: T^2-fibration

Attach a torus at every point of the space

Torus (2D compact space)



base: 6D compact space

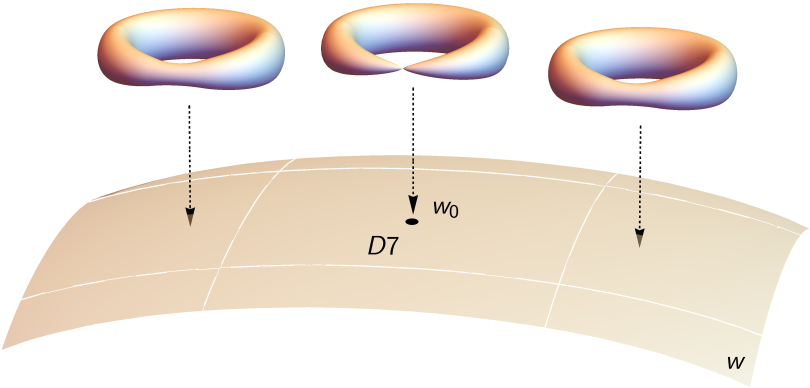
8D compact space (Calabi-Yau 4-fold CY_4)

Mathematical description

$$y^2 = x^3 + f(u)\,x + g(u), \qquad \Delta = 4f^3 + 27g^2 = 0$$

D7-branes

Torus becomes singular ⇔ D7-brane



D7-branes ⇔ Gauge theories

Different singularities ⇔ Different gauge groups

Physics $\leftrightarrow$ Geometry

Can read off more than gauge group from singularities.

Object	$\mathbb{C}$-codimension	Type IIB
Gauge Theory	1	stack of branes
Matter	2	2 branes intersecting
Couplings	3	3 branes intersecting

Advantages

- Describes strong coupling regimes, i.e. **non-perturbative physics** ($\rightarrow$ top yukawa possible)
- Give consistent **global models** (global constraints, such as tadpole cancellation, are automatically satisfied)

4D effective actions

- Need to compactify on a CY_4 instead of a CY_3.
- But 12D theory not defined
- Use duality with M-theory