Properties of non-supersymmetric heterotic vacua

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Aim of the Thesis

In my Master Thesis I am interested in the study of supersymmetry breaking in String Theory and its implications on the string spectrum at the onset of tachyonic instabilities. In particular, I shall focus on string models where supersymmetry is broken by the Scherk-Schwarz mechanism. This implies understanding how the entire tower of massive string states rearranges after supersymmetry is broken and how the masses of each string state vary as one moves on the classical moduli space (because of their dependence on the Scherk-Schwarz radius).

Supersymmetry

Supersymmetry is a remarkable symmetry that exchanges bosonic and fermionic degrees of freedom. Despite its theoretical appealing, it has some non-trivial phenomenological implications:

- A possible solution to the hierarchy problem.
- "Improved" unification of gauge interactions.
- A possible candidate for the Dark Matter.

Supersymmetry is a generic feature of all realistic string theories: it relates fermionic and bosonic fields and guarantees the stability of the vacuum.
In a theory with supersymmetry, bosonic and fermionic degrees of freedom are the same at any mass level.

So in trying to build a supersymmetric version of the standard model the simplest possibility is to add superpartners to all observed particles. However, the fact that superpartners have not been observed is an indication that, **if supersymmetry exists in Nature, it must be spontaneously broken**.

Motivation for String Theory

String Theory started in the late 1960s with the aim of organizing and explaining the observed spectrum of hadrons and their interactions.

In 1974 it was proposed to identify the massless spin-two particle in the string's spectrum with the **graviton** – the quantum of gravitational interaction – and String Theory also became a quantum theory of gravity.

String Theory became then the most promising candidate for a **quantum theory of gravity unified with the other forces**.

String Theory

String Theory replaces the concept of point particles with one-dimensional extended objects, called **strings**, whose internal quantum vibrational modes give rise to a certain number of massless states plus an **infinite tower of states with mass**:

$$M^2 \sim N + \tilde{N} - \Delta - \tilde{\Delta} \tag{1}$$

Physical states are characterised by the level-matching condition:

$$N - \Delta = \tilde{N} - \tilde{\Delta} \tag{2}$$

Superstring Theory

The main superstring theories that I analysed in my Master Thesis are the IIB theory and the $E_8 \times E_8$ heteoritc theory. Their one-loop partition functions are:

$$T_{IIB} = \int_{\mathcal{F}} \frac{d^2\tau}{\tau_2^2} \frac{1}{(\sqrt{\tau_2}\eta\bar{\eta})^8} |V_8 - S_8|^2 \tag{3}$$

$$T_{E_8 \times E_8} = \int_{\mathcal{F}} \frac{d^2\tau}{\tau_2^2} \frac{1}{(\sqrt{\tau_2}\eta\bar{\eta})^8} (V_8 - S_8)(\bar{O}_{16} + \bar{S}_{16})(\bar{O}_{16} + \bar{S}_{16}) \tag{4}$$

where:

- since a closed string sweeps a torus at one loop, we must complexify the Schwinger proper time and identify it with the Teichmüller parameter: $\tau = \tau_1 + i\tau_2$.
- The fundamental domain of integration $\mathcal{F} = \{ \tau \in \mathbb{H}, -\frac{1}{2} < \tau_1 \leq \frac{1}{2} , |\tau| > 1 \} \cup \{ \tau \in \mathbb{H}, 0 \leq \tau_1 \leq \frac{1}{2} , |\tau| = 1\}$, is obtained by considering only the contribution of inequivalent tori. The set of transformations of τ, which leaves the geometry invariant, forms the modular group $PSL(2, \mathbb{Z})$
- η is the Dedekind η-function: $\eta(q) = q^{1/24} \prod_{m=1}^{\infty}(1 - q^m)$
- We have used, more in general, the four characters of SO(2n) algebras :

$$O_{2n} = \frac{\theta_3^n + \theta_4^n}{2\eta^n} \quad V_{2n} = \frac{\theta_3^n - \theta_4^n}{2\eta^n} \quad S_{2n} = \frac{\theta_2^n + i^{-n}\theta_1^n}{2\eta^n} \quad C_{2n} = \frac{\theta_2^n - i^{-n}\theta_1^n}{2\eta^n} \tag{5}$$

Superstring Theory

IIB massless spectrum: $g_{\mu\nu}$, $B_{\mu\nu}$, ϕ, $2\psi_L^{\mu}$, $2\lambda_R$, C_0, C_2, C_{4+}.
$E_8 \times E_8$ **massless spectrum**: $g_{\mu\nu}$, $B_{\mu\nu}$, ϕ, ψ_L^{μ}, λ_R, gauge bosons of $E_8 \times E_8$ group and their fermionic superpartners.
Superstring theories require a 10D space-time, but by compactifying extra-dimensions, we can obtain lower-dimensional theories. The **compactification** on a circle $\mathbf{S^1(R)}$ of radius R leads us to the following torus partition function for the two previous theories:

$$\mathcal{T}_{IIB} = \int_{\mathcal{F}} \frac{d^2\tau}{\tau_2^6} \frac{1}{(\eta\bar{\eta})^8} \, |V_8 - S_8| \sum_{m,n\in\mathbb{Z}} \Lambda_{m,n} \tag{6}$$

$$T_{E_8\times E_8} = \int_{\mathcal{F}} \frac{d^2\tau}{\tau_2^6} \frac{1}{(\eta\bar{\eta})^8} (V_8 - S_8)(\bar{O}_{16} + \bar{S}_{16})(\bar{O}_{16} + \bar{S}_{16}) \sum_{m,n\in\mathbb{Z}} \Lambda_{m,n} \tag{7}$$

where:

$$\Lambda_{m,n} = \sqrt{\tau_2} \cdot \sum_{m,n\in\mathbb{Z}} q^{\frac{1}{4}\left(\frac{m}{R}+nR\right)^2} \bar{q}^{\frac{1}{4}\left(\frac{m}{R}-nR\right)^2} \tag{8}$$

- **m** is the **Kaluza-Klein momentum number**
- **n** is the **winding number**, that counts how many times the closed string wraps around $S^1(R)$.

All the previous partition functions have two main properties:

- **Supersymmetry**: thanks to the theta-functions identities, **the superstring partition functions vanish identically**. This is a very important result because it implies that the theory is supersymmetric, namely it has the same bosonic and fermionic degrees of freedom at any mass level. Because of the spin-statistics relation, fermions enter the partition function with a minus sign while bosons enter with a plus sign.
- **Modular invariance**: this property eliminates the ultraviolet divergence that would have appeared as $\tau \to 0$. This is related to the celebrated finiteness of String Theory.

We are intrested in studying what happens to the string spectra when supersymmetry is broken: in fact, in this case, the partition functions will no longer vanish and we can have the onset of tachyonic instabilities. We shall break supersymmetry with the

Scherk-Schwarz mechanism

Scherk and Schwarz mechanism

The Scherk-Schwarz mechanism provides an elegant realization of supersymmetry breaking by compactification. It was introduced in field theory as a compactification of a D+E dimensional theory to D dimensions with a specific dependence of the fields upon the internal coordinates, given by some symmetry of the D+E dimensional theory. **(Scherk, Schwarz, 1979)**

Scherk and Schwarz mechanism

In Kaluza-Klein on a circle of radius R fields are taken to be periodic along the compact direction, thus we can write the Fourier expansion for bosonic field ϕ and fermionic field ψ as:

$$\phi(x^\mu, y) = \sum_{m=-\infty}^{+\infty} \phi_m(x^\mu) e^{imy/R} \tag{9}$$

$$\psi(x^\mu, y) = \sum_{m=-\infty}^{+\infty} \psi_m(x^\mu) e^{imy/R} \tag{10}$$

where $\mu = 0, .., 3$ and y is assumed to be compactified on the circle $S^1(R)$. In $D = 4$ the mass-spectra of the Kaluza-Klein excitations reads:

$$M_B^2 = \left(\frac{m}{R}\right)^2 \qquad\qquad M_F^2 = \left(\frac{m}{R}\right)^2 \tag{11}$$

Bosons and Fermions have the same masses, thus respecting supersymmetry.

Scherk and Schwarz mechanism

The Scherk-Schwarz mechanism amounts instead to allowing different boundary conditions for fermions and bosons. We can assign to bosons and fermions periodic and anti-periodic boundary conditions, respectively. Since fermionic fields appears always as bilinears in every Lagrangian, this choice is compatible with the symmetries of the Action. Therefore we require:

$$\phi(x^\mu, y + 2\pi R) = \phi(x^\mu, y) \tag{12}$$

$$\psi(x^\mu, y + 2\pi R) = -\psi(x^\mu, y) \tag{13}$$

Due to the periodicity condition (12) and (13), the fields are now Fourier expanded as:

$$\phi(x_\mu, y) = \sum_{m=-\infty}^{+\infty} \phi_m(x^\mu) e^{imy/R} \tag{14}$$

$$\psi(x_\mu, y) = \sum_{m=-\infty}^{+\infty} \psi_m(x^\mu) e^{i(m+\frac{1}{2})y/R} \tag{15}$$

Thus the tower of Kaluza-Klein states have masses:

$$M_B^2 = \left(\frac{m}{R}\right)^2, \qquad M_F^2 = \frac{(m+1/2)^2}{R^2}, \qquad m \in \mathbb{Z} \tag{16}$$

This shift on the fermionic Fourier modes with respect to the bosons ones thus breaks partially or totally supersymmetry. Compared to Field Theory, String Theory offers more possibilities, since one has also the option of affecting the windings.

Scherk-Schwarz mechanism

The torus partition function for **IIB** theory with **supersymmetry breaking "à la Scherk-Schwarz" on a circle** is given by:

$$\mathcal{T} = \int_{\mathcal{F}} \frac{d^2\tau}{\tau_2^6 (\eta\bar{\eta})^8} \sum_{m,n\in\mathbb{Z}} \Big[(|V_8|^2 + |S_8|^2)\Lambda_{m,2n} - (\bar{V}_8 S_8 + V_8 \bar{S}_8)\Lambda_{m+1/2,2n} + (|O_8|^2 + |C_8|^2)\Lambda_{m,2n+1} - (\bar{O}_8 C_8 + O_8 \bar{C}_8)\Lambda_{m+1/2,2n+1} \Big] \quad (17)$$

(Rohm,1984)

Furthermore, Scherk-Schwarz deformation can be also realized via **freely-acting orbifolds**, projecting the IIB superstring spectrum with the $\mathbb{Z}_2$ generator $(-1)^F\delta$, where:

- $F = F_L + F_R$ is the total space-time fermion number
- δ is the shift operator on the circle $X \to X + \pi R$

An **orbifolds** $\mathcal{O}$ is a space that can locally be defined as the quotient of a certain manifold M by the action of a discrete group G: $\mathcal{O} = M/G$. Using the projection operator $P = \frac{1+(-1)^F\delta}{2}$ and requiring modular invariance, we have: **(Kounnas, Rostand,1990)**

$$\mathcal{T} = \int_{\mathcal{F}} \frac{d^2\tau}{\tau_2^6 (\eta\bar{\eta})^8} \sum_{m,n\in\mathbb{Z}} \Big[(|V_8|^2 + |S_8|^2)\Lambda_{2m,n} + (|O_8|^2 + |C_8|^2)\Lambda_{2m,n+1/2} - (V_8\bar{S}_8 + S_8\bar{V}_8)\Lambda_{2m+1,n} - (O_8\bar{C}_8 + C_8\bar{O}_8)\Lambda_{2m+1,n+1/2} \Big] \quad (18)$$

There is an important relation between the two partition functions: **halving the radius R** of the circle in orbifold description (18), we recover exactly the eq (17).
From (18), we can see that **all the space-time fermions acquire a mass term**; furthermore, **tachyonic instabilities can arise** for the following values of the radius:

$$M^2 = -\frac{1}{2} + \frac{R^2}{16} < 0 \quad \rightarrow \quad R < 2\sqrt{2} \tag{19}$$

For the heterotic string we have more possibilities. We can use an orbifold construction to break the supersymmetry of the $E_8 \times E_8$ theory, with $P = \frac{1+(-1)^{F_{s.t}+F_{\mathbf{1}}+F_{\mathbf{2}}}\delta}{2}$.

- $F_{s.t}$ counts the space-time fermions
- F_1 counts the spinorial representations of the first $(\bar{O}_{16} + \bar{S}_{16})$
- F_2 counts the spinorial representations of the second $(\bar{O}_{16} + \bar{S}_{16})$.

$$\begin{aligned} \mathcal{T} = \frac{1}{2(\eta\bar{\eta})^8} \sum_{m,n\in\mathbb{Z}} \Big[& (V_8 - S_8)(\bar{O}_{16} + \bar{S}_{16})(\bar{O}_{16} + \bar{S}_{16})\Lambda_{m,n} + \\ & + (V_8 + S_8)(\bar{O}_{16} - \bar{S}_{16})(\bar{O}_{16} - \bar{S}_{16})(-1)^m \Lambda_{m,n} + \\ & + (O_8 - C_8)(\bar{V}_{16} + \bar{C}_{16})(\bar{V}_{16} + \bar{C}_{16})\Lambda_{m,n+1/2} - \\ & - (O_8 + C_8)(\bar{V}_{16} - \bar{C}_{16})(\bar{V}_{16} - \bar{C}_{16})(-1)^m \Lambda_{m,n+1/2} \Big] \end{aligned} \tag{20}$$

This is a **tachyon-free theory**.

Supersymmetry breaking of $E_8 \times E_8$ theory

We can breake the supersymmetry of $E_8 \times E_8$ theory with another orbifold construction using the projection operator:

$$P = \frac{1 + (-1)^{F_{s.t}}\, \delta}{2}$$

We obtain the following partition function:

$$\mathcal{T} = \frac{1}{2(\eta\bar{\eta})^8} \sum_{m,n\in\mathbb{Z}} \Big[V_8\, \Lambda_{2m,n} - S_8\, \Lambda_{2m+1,n} + O_8\, \Lambda_{2m+1,n+1/2} - \\ - C_8 \Lambda_{2m,n+1/2} \Big] (\bar{O}_{16} + \bar{S}_{16})(\bar{O}_{16} + \bar{S}_{16}) \tag{21}$$

We can see that all the massless fermions of the $E_8 \times E_8$ supersymmetric theory become massive whereas the bosonic fields remain massless.
Tachyon instabilities can arise from the term:

$$\frac{1}{(\eta\bar{\eta})^8} O_8\, \bar{O}_{16}\, \bar{O}_{16}\, \Lambda_{2m+1,n+1/2} \tag{22}$$

Therefore, tachyon fields appear for the subsequent values of R:

$$M^2 = -\frac{1}{2} + \frac{1}{4}\left(\frac{1}{R} - \frac{R}{2}\right)^2 < 0 \qquad \Longrightarrow \qquad \sqrt{6 - 4\sqrt{2}} < R < \sqrt{6 + 4\sqrt{2}} \tag{23}$$

Misaligned and asymptotic supersymmetry

Most of presently known string vacua are classically unstable when space-time supersymmetry is absent: string spectra are typically plagued by the presence of tachyonic modes that suggest the onset of an instability.

If one insists that a non-supersymmetric vacuum ought to be predictive, at least classically, one is bound to consider only string configurations where tachyonic excitations are absent. While modular invariance eliminates the ultraviolet divergence that would have appeared as $\tau \to 0$, **infra-red (IR) finiteness** of the one-loop vacuum energy of closed oriented strings is a subtler issue. Although IR divergences are associated to massless or tachyonic fields only, modular invariance relates the IR to the UV, and the distribution of states determine the IR behavior. This relation is an important but not particularly well-understood issue.

Although space-time supersymmetry imposes a perfect equilibrium between bosonic and fermionic degrees of freedom at every energy level and automatically yields a finite (actually, vanishing) one-loop partition function, this is no longer evident when supersymmetry is absent or spontaneously broken.

In the late 90's/early 2000, people have studied conditions for **classical stability**. These studies have resulted in the following conditions:

asymptotic supersymmetry

$$" \quad \sum_{\{m_B^2, m_F^2\}} d_B(m_B^2) - d_F(m_F^2) = 0 \quad " \tag{24}$$

(Kutasov, Seiberg, 1991)

- presence of space-time fermions in the spectrum
- exact cancellation, independently of the mass, of overall number of fermions against the spacetime bosons

A more refined analysis of the distribution of bosonic and fermionic excitations at all energy levels led to the discovery of a hidden (supersymmetry) that governs the arrangement of bosonic and fermionic states:

misaligned supersymmetry

(Dienes, 1994)

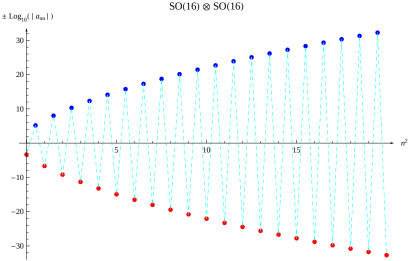

Figure: SO(16)$\times$ SO(16) theory. We plot $\pm\, log_{\mathbf{10}} \left|d(m^{\mathbf{2}})\right|$ versus mass$^{\mathbf{2}}$

Misaligned and asymptotic supersymmetry are incorporated in the behavior:

$$\sum_{\{m^2\}} d(m^2)\, e^{-4\pi m^2 \tau_2} = C\, \tau_2^{\frac{d-2}{2}} + \sum_{\zeta^*(\rho)=0} C_\rho\, \tau_2^{\frac{d-\rho}{2}} \tag{25}$$

valid for small τ_2.

Since:

$$\sum_{\zeta^*(\rho)=0} C_\rho\, \tau_2^{\frac{2-\rho}{2}} = \tau_2^{3/4} \sum_{m=1}^{\infty} C_m \cos\left(\frac{1}{2}\gamma_m \log\tau_2 + \phi_m\right) \tag{26}$$

we have that **the graded density of states oscillates with frequencies dictated by the zeroes of the Riemann ζ-function**.

(Angelantonj, Cardella, Elitzur, Rabinovici, 2010)

Necessary and sufficient conditions for classical stability

Asymptotic and misaligned supersymmetry are actually **necessary conditions for classical stability**.

Are they also sufficient conditions?

NO! Asymptotic supersymmetry still persists when tachyons are present: the Scherk-Schwarz mechanism is a **continuous deformation** of the spectrum and degrees of freedom are not generated or eliminated, the masses are only shifted.

The analysis for misaligned supersymmetry is subtler.

However, I have shown that this **misaligned supersymmetry is generally present also when tachyonic fields appear in the spectrum**.

Asymptotic and misaligned supersymmetry are not sufficient conditions for stability

IIB theory

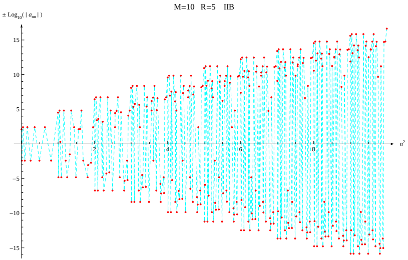

Figure: IIB theory $R = 5 \rightarrow$ **tachyons not present**. We plot $\pm \log_{\mathbf{10}} \left| d(m^{\mathbf{2}}) \right|$ versus $mass^{\mathbf{2}}$.

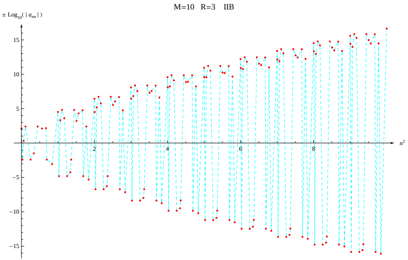

Figure: IIB theory $R = 3 \rightarrow$ **tachyons not present**. We plot $\pm \log_{\mathbf{10}} \left| d(m^{\mathbf{2}}) \right|$ versus $mass^{\mathbf{2}}$.

For these two specific values of the radius we do not have tachyons, as we expected, since **for the IIB theory we should have tachyon instabilities only for $R < 2\sqrt{2}$**.

IIB theory

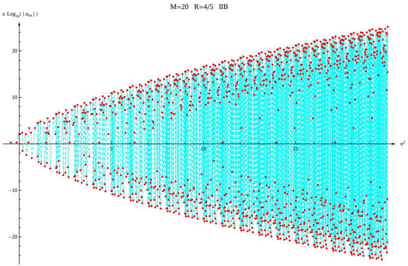

Figure: IIB theory $R = \frac{4}{5} \rightarrow$ **tachyons present**. We plot $\pm \log_{10} \left| d(m^2) \right|$ versus $mass^2$.

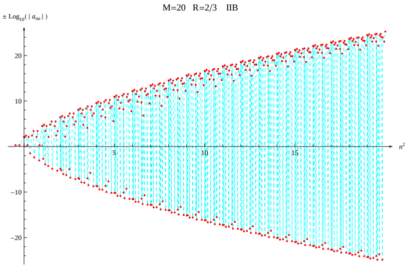

Figure: IIB theory $R = \frac{2}{3} \rightarrow$ **tachyons present**. We plot $\pm \log_{10} \left| d(m^2) \right|$ versus $mass^2$.

These are the new results of my Thesis

Also when tachyon instabilities appear, the oscillatory behaviour typical of misaligned supersymmetry is present, confirming that misaligned supersymmetry cannot be a sufficient condition for stability.

Heterotic model with no physical tachyons after supersymmetry breaking

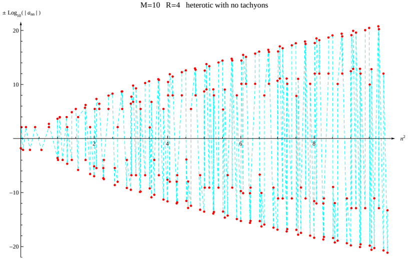

Figure: Heterotic theory with **no tachyons present** $R = 4$. We plot $\pm \log_{10} \left| d(m^2) \right|$ versus $mass^2$.

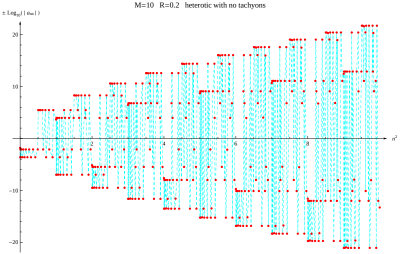

Figure: Heterotic theory with **no tachyons present** $R = 0.2$. We plot $\pm \log_{10} \left| d(m^2) \right|$ versus $mass^2$.

Heterotic model with physical tachyons after supersymmetry breaking

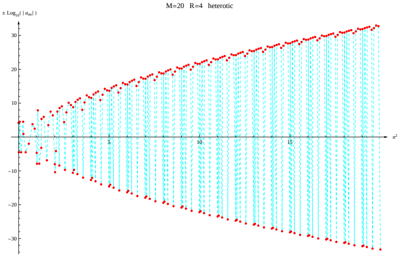

Figure: Heterotic theory other model $R = 4 \rightarrow$ **tachyons not present**. We plot $\pm \log_{10} \left| d(m^2) \right|$ versus *mass*^{2}.

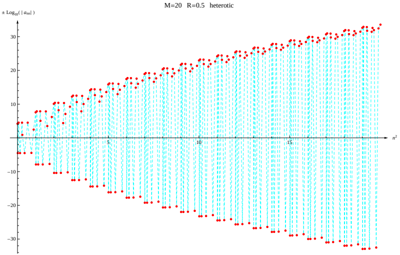

Figure: Heterotic theory other model $R = 0.5 \rightarrow$ **tachyons not present**. We plot $\pm \log_{10} \left| d(m^2) \right|$ versus *mass*^{2}.

Heterotic model with physical tachyons after supersymmetry breaking

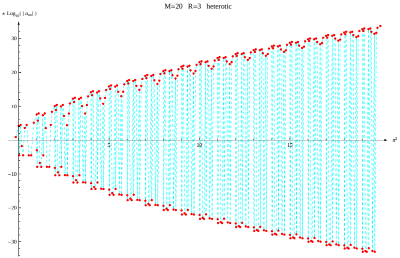

Figure: Heterotic theory other model $R = 3 \rightarrow$ **tachyons present**. We plot $\pm \log_{10} \left| d(m^2) \right|$ versus $mass^2$.

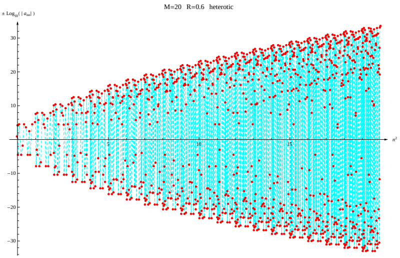

Figure: Heterotic theory other model $R = 0.6 \rightarrow$ **tachyons present**. We plot $\pm \log_{10} \left| d(m^2) \right|$ versus $mass^2$.

These are the new results of my Thesis

Conclusion

- I have studied ways to break supersymmetry in String Theory
- I have analysed properties of the string spectrum in classically stable/unstable vacua and found that asymptotic and misaligned supersymmetry are only necessary (but not sufficient) conditions for the absence of tachyons.

Outlook

An outlook of future study would be to understand how classically stable vacua can be destabilised by quantum corrections to the vacuum energy which, in the absence of supersymmetry, are not longer vanishing. In fact, studying these radiative corrections is an interesting open problem.

Asymptotic behavior of IIB theory

- $\mathbf{R} \to \infty$: the standard supersymmetric IIB theory in 10D is formally recovered.
- $\mathbf{R} \to \mathbf{0}$: the Scherk-Schwarz partition function for the IIB theory becomes the 0B partition function in 9D.
  Thus, as the radius decreases, the bosonic excitations become dominant over the fermionic ones, as we can see in the following graphs made for $R = 2/45$.

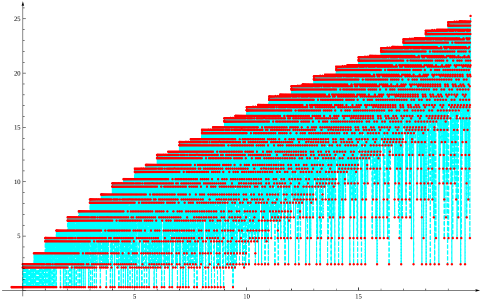

Figure: IIB theory $R = 2/45$. We plot $\pm \log_{10} \left| d(m^2) \right|$ versus $mass^2$