Developing a Method for Measuring Plasma Radius using Schlieren Imaging

IMPRS Workshop

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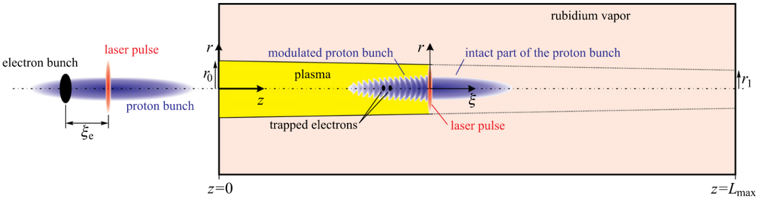
Advanced Wakefield Experiment (AWAKE) at CERN

Principle of AWAKE Experiment

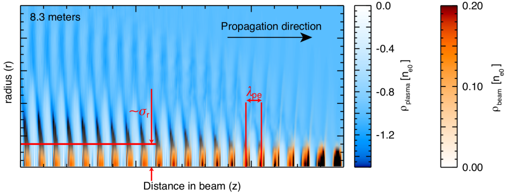


Physics of Plasmas 21, 123116 (2014); doi: 10.1063/1.4904365

Plasma Based Acceleration

- Propagation of a proton beam through rubidium plasma
- Seeding of self-modulation instability (SMI) through co-propagating laser
  → Generation of the wakefield
  → Acceleration of injected electrons

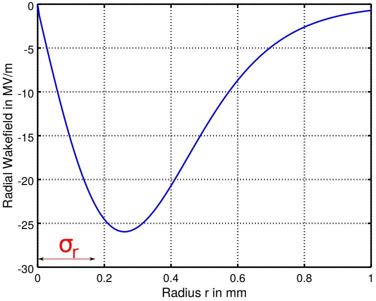
Formation of Micro Bunches



AWAKE Design Report

Plasma Radius Measurement at AWAKE

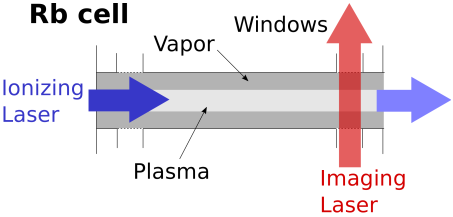
Radial wakefield at the front of the beam

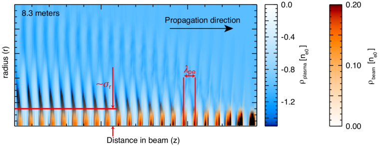


- Particle density $n_0 = 10^{15}\,\mathrm{cm}^{-3}$
- Beam size $\sigma_r = \sigma_z = 1/k_p \approx 0.17\,\mathrm{mm}$

$\Rightarrow$ Transverse component $\neq 0$ until $r \lesssim 1\,\mathrm{mm}$
$\Rightarrow$ **Require plasma radius $\mathbf{r_{plasma}} > 1\,\mathbf{mm}$**

Schlieren image through windows at the end of the cell
$\rightarrow$ Value should be larger than or equal to that along the cell





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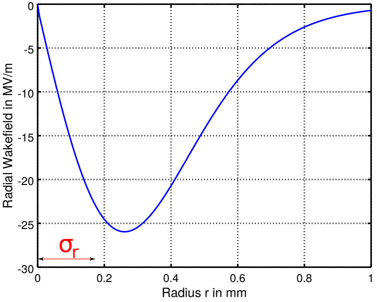

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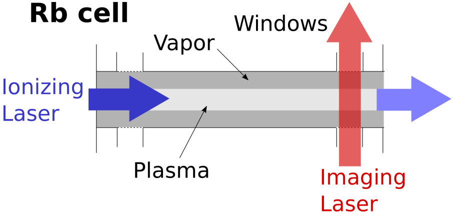

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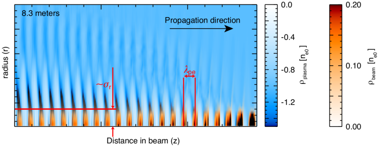


AWAKE Design Report

$\Rightarrow$ Need to determine experimentally plasma radius

Principle of Schlieren Imaging

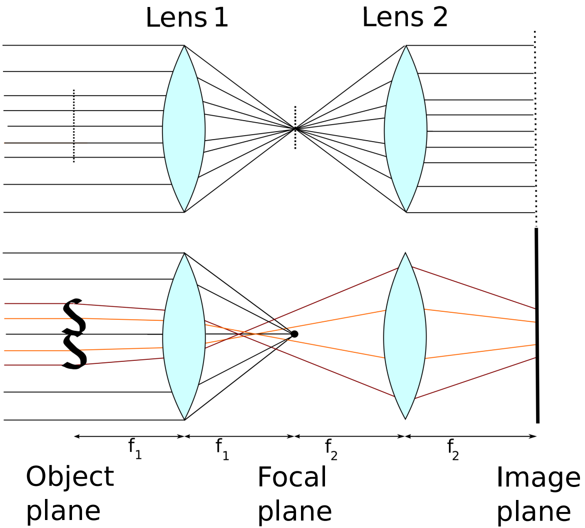
Principle of Schlieren Imaging

Without Object

All parallel incoming rays go through one point

Block non-deflected rays

→ only deflected rays reach screen



⇒ Intensity proportional to the **change** of index of refraction

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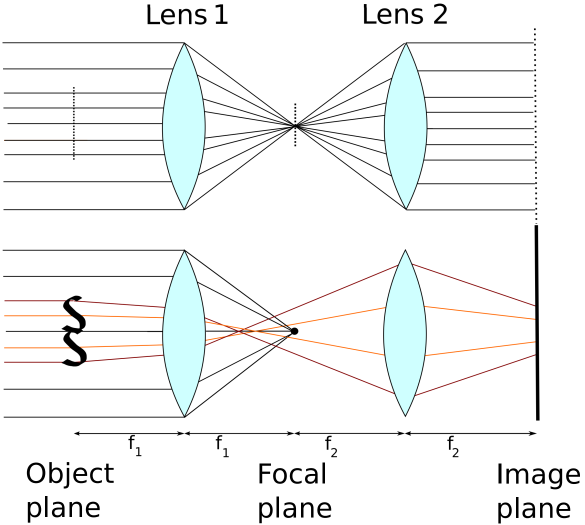

⇒ Intensity proportional to the **change** of index of refraction

⇒ Increasing contrast of the object by blocking background

Schlieren Image of Density Perturbation in Air

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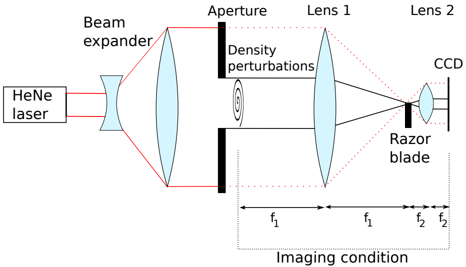
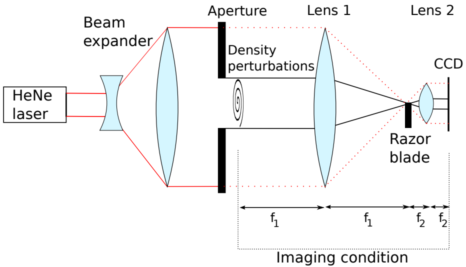
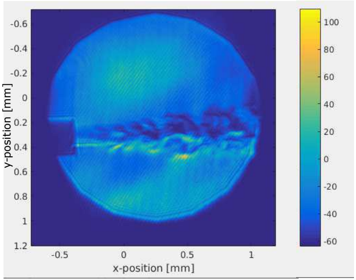


- Beam expander to increase spot size to image larger objects
- Aperture to suppress aberrations
- Lenses 1 and 2 act as thick lens to image object
- Razor blade to block undeflected rays $\rightarrow$ up and down deflection becomes visible

Schlieren Image of Density Perturbation in Air





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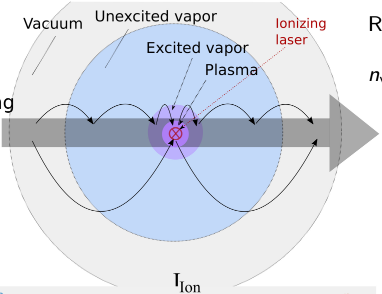
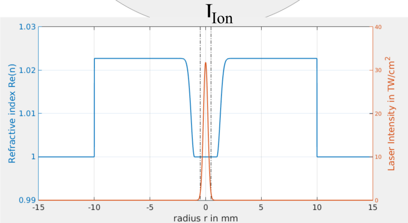
Schlieren image of floating air coming out of a compressed air can through a nozzle

Knife Edge as Block in Focal Plane

$\rightarrow$ Density perturbation becomes visible

Refractive Indices of Vapor and Plasma



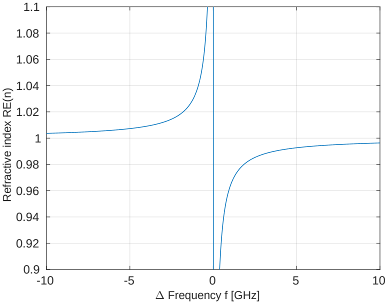


Refractive index of vapor

$$n_{vapor}(r) = \mathrm{Re}\left(\sqrt{1 + \frac{N_i(r)\, e^2}{\epsilon_0\, m_e} \sum_{j\neq i} \frac{f_{ij}}{({\omega_{ij}}^2 - \omega^2 - \frac{i\,\omega}{\tau_{ij}})}}\right)$$

$$\rightarrow n_{vapor} \approx 1 + 2 \cdot 10^{-2} \text{ for } d\omega = -5\,\text{GHz}$$

Refractive index of unexcited Rb vapor around D2 line



$\Delta n_{vacuum,plasma}$ very small

$\Rightarrow$ Schlieren effect comes
not from the appearance of plasma
but from the disappearance of vapor

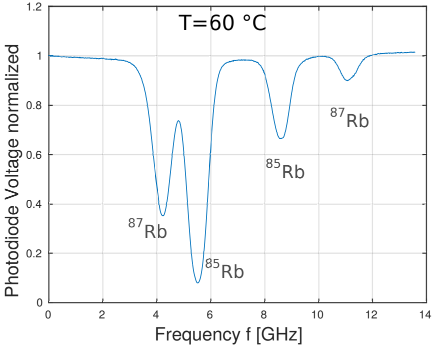
Refractive index of plasma

$$n_{plasma} = \sqrt{1 - \frac{n_{pe}\, e^2}{\epsilon_0\, m_e}\, \frac{1}{\omega^2}}$$

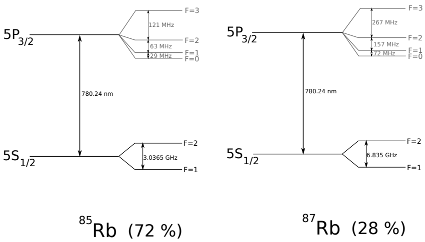
$$\rightarrow n_{plasma} \approx 1 - 3 \cdot 10^{-7} \text{ for } d\omega = -5\,\text{GHz}$$

Absorption Spectroscopy for Laser Wavelength Determination

Rb Spectroscopy around D2 Transition Line



Atomic Level Structure of Rubidium



Choosing a Laser Wavelength using Spectroscopy

Lower temperature

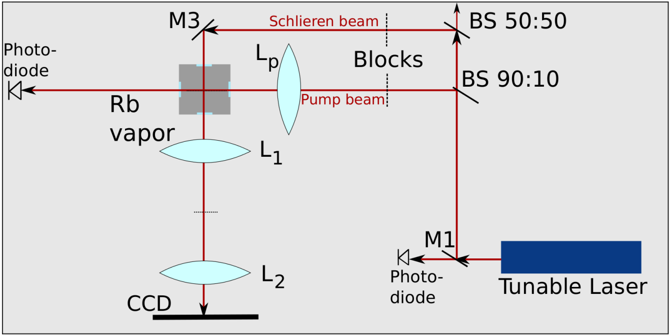
→ Less Doppler broadening → Resolution of hyperfine structure

→ Wavelength determination with absorption spectroscopy

Higher temperature

→ Higher density → Stronger signal at Schlieren imaging

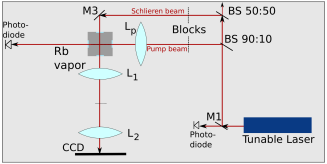
Set Up at MPP for Imaging an Excitation Column



Set Up Including Spectroscopy and Schlieren Imaging

- Excitation of atoms through pump beam tuned to a wavelength close to transition line D2 (but **no** ionization)
- Absorption spectroscopy to determine laser wavelength at lower temperature
- Schlieren Imaging of excitation column

Procedure for Schlieren Images of the Excitation Column



Various blocking of the two beams
→ Images in 4 configurations for image processing

Example at a detuning frequency $df = 5\,\text{GHz}$

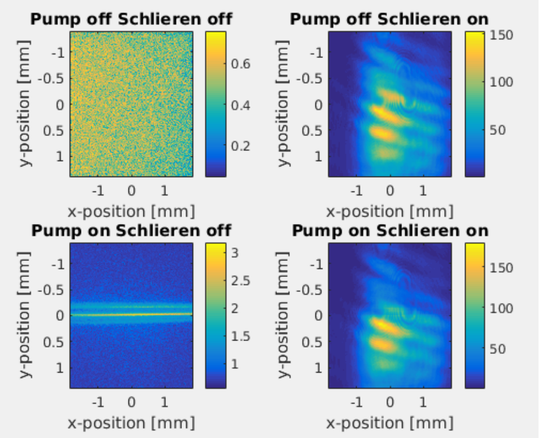


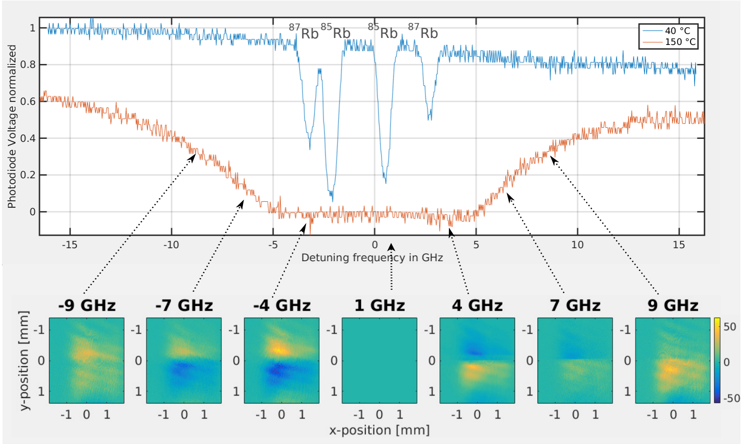
Image Processing

- subtraction of noise from the other images
- subtraction of image
  - without pump beam
  - without Schlieren beam

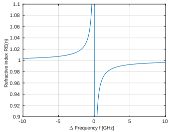
  from image with pump and Schlieren beam

⇒ Excitation column becomes visible

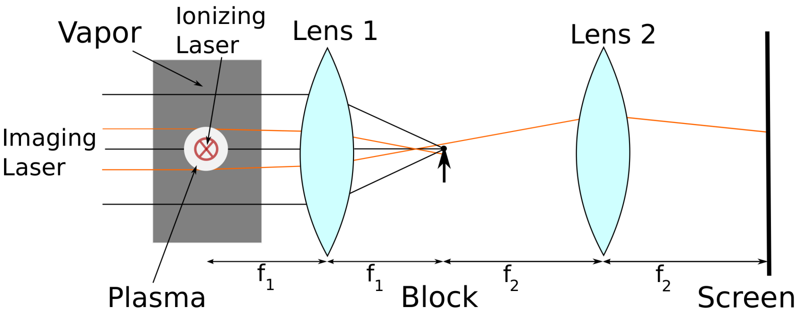
Schlieren Images of Excitation Column at Different Detuning Frequencies around Transition Line D2



- at $df \lesssim \pm 3$ GHz no transmission of the Schlieren beam
- at $df \gtrsim \pm 9$ GHz no significant phase shift any more
- change of sign of Δn around transition line D2 visible



Plasma Radius Measurement at CERN



Specification ionizing laser at CERN

Power P	4 TW
Duration $\Delta\tau$	100 fs
Wavelength λ	(780 ± 10) nm

$\rightarrow$ Intensity $I \gg 1.7 \cdot 10^{12}\,\mathrm{Wcm}^{-2}$ for $r \leq 1\,\mathrm{mm}$ and length $z = 10\,\mathrm{m}$

$\rightarrow$ Excitation **and** ionization

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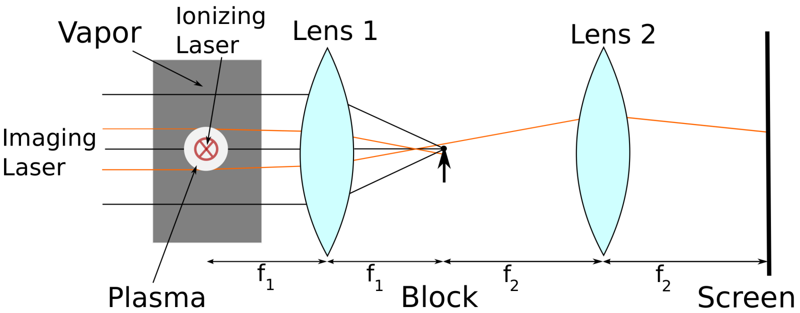

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$\rightarrow$ Excitation **and** ionization

$\Rightarrow$ Is it possible to see plasma formation in the middle of excitation?

Conclusion

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- Development of a method to determine the radius at AWAKE in the case of ionized vapor

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- Calculation of the Schlieren images with a plasma column (not shown here)
  $\rightarrow$ Comparison with measurement

Conclusion

- Development of a method to determine the radius at AWAKE in the case of ionized vapor
- Schlieren imaging to show an excitation column
  → Schlieren imaging at CERN to show a plasma column
- Calculation of the Schlieren images with a plasma column (not shown here)
  → Comparison with measurement

Thank you!

AWAKE

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(Werner-Heisenberg-Institut)

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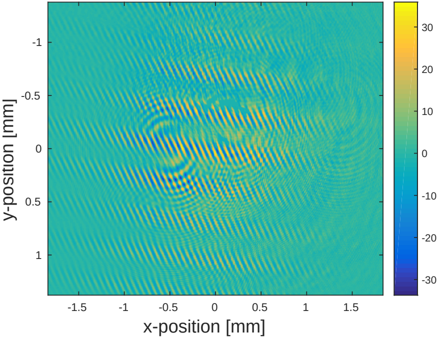
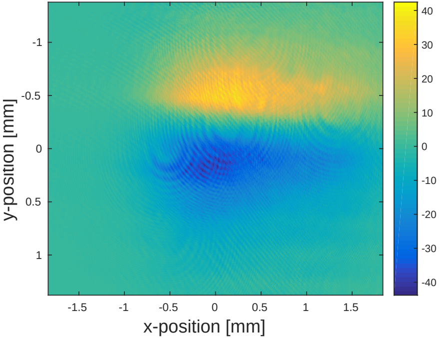
UNIVERSITÄT DÜSSELDORF

Schlieren vs Shadowgraphy of Excited Atoms

Without Blocking

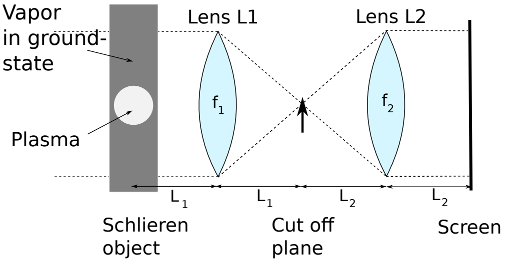


With Knife Edge



Signal at $\Delta f = -9\,\text{GHz}$

Formulas of Fourier Optics



Propagation over z along optical axis [1]

$$S_0(\vec{k}) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} u_0(\vec{r}) \, \exp(-i\,\vec{k}\vec{r}) \; d^2\vec{r}$$

$$u_1(\vec{r}, z) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} S_0(\vec{k}) \, \exp\left(i\,z\,\sqrt{k_0^2 - \vec{k}^2}\right) \exp(i\,\vec{k}\vec{r})\, d^2\vec{k}$$

Phase Shift through Object

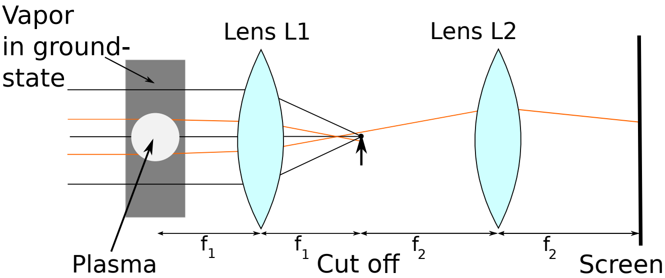
$u_1(\vec{r}) = u_0(\vec{r}) \cdot \exp(\mathrm{i}\,\Phi)$ with Φ phase shift through object

[1] HECHT, E.: *Optics (4th ed.)*. Addison Wesley, 1987

GOAL: Plasma Radius Measurement using Schlieren Imaging

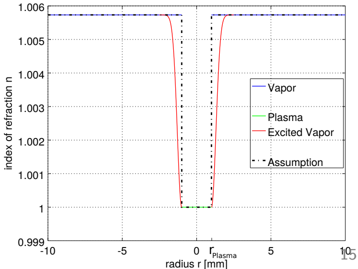
Parameters

- Beam size
  $\sigma_{beam} = 5\,\text{mm}$
- Plasma radius
  $r_{plasma} = 1\,\text{mm}$
- Focal lengths
  $f_1 = 500\,\text{mm}$,
  $f_2 = 100\,\text{mm}$
- Laser detuning
  $\Delta\omega = 20\,\text{GHz}$



Index of refraction

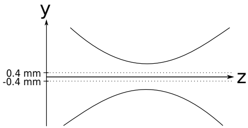
- for vapor
  $$n(r) = RE\left(\sqrt{1 + \frac{N_i(r)\,e^2}{\epsilon_0\, m_e}\sum_{j\neq i}\frac{f_{ij}}{({\omega_{ij}}^2 - \omega^2 - \frac{i\,\omega}{\tau_{ij}})}}\right)$$
- for plasma
  $$n = \sqrt{1 - \frac{\omega_{pe}^2}{\omega^2}}$$

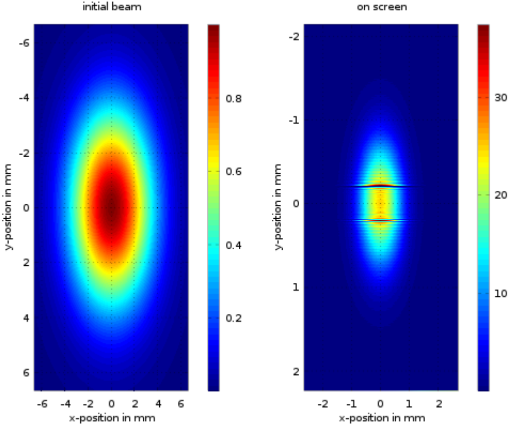


Gaussian Beam - Plasma Column - No Cut Off

Parameters:

- Standard derivation of Gaussian beam: $\sigma_r = 5\,\mathrm{mm}$
- Focal lengths and propagation distances $f1 = L1 = 500\,\mathrm{mm}$, $f2 = L2 = 100\,\mathrm{mm}$
- Radius of plasma column: $r_{plasma} = 1\,\mathrm{mm}$





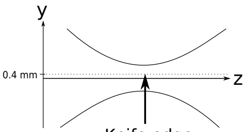
Contours of the plasma column due to diffraction

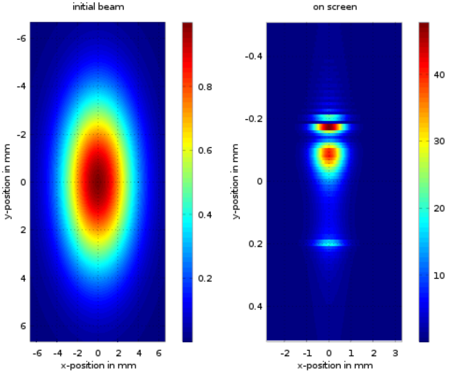
⇒ Information about the size
⇒ No information about the shape

Gaussian Beam - Plasma Column - Horizontal Knife Edge

Parameters:

- Standard derivation of Gaussian beam: $\sigma_r = 5\,\text{mm}$
- Focal lengths and propagation distances $f1 = L1 = 500\,\text{mm}$, $f2 = L2 = 100\,\text{mm}$
- Radius of plasma column: $r_{plasma} = 1\,\text{mm}$
- Position horizontal knife-edge: $y = 0.04\,\text{mm}$





Half of the plasma column and its contour is imaged

$\Rightarrow$ Information about the size

$\Rightarrow$ Information about the shape