# Quantum Gravity, Unification and $\mathbf{E}_{10}$

*100 Jahre Max-Planck-Institut*

*12 October 2017, München*

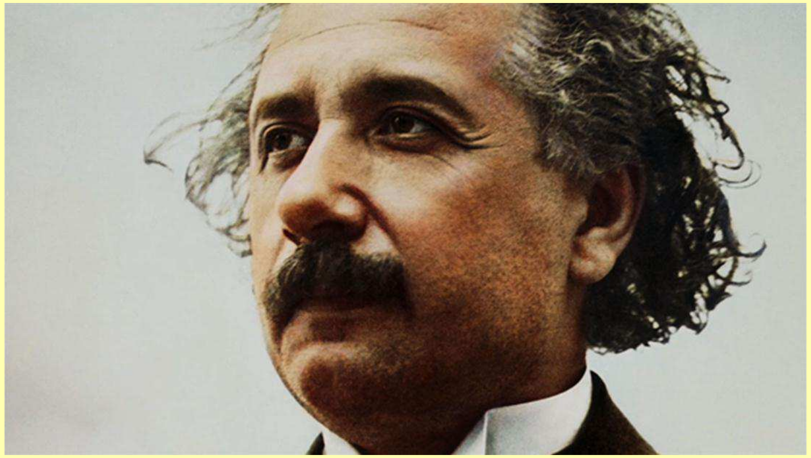
*Hermann Nicolai*

*MPI für Gravitationsphysik, Potsdam*

*(Albert Einstein Institut)*

*See also:* ***"Gravity's Quantum Side"***
*January/February issue of the CERN Courier (2017)*

# 100 Years of MPI: revolutionizing physics!



# The Phenomenal Success of QM and GR

- Quantum Mechanics (QM) is the most successful and best tested theory in all of science:
  - Overwhelming evidence over a vast range of phenomena
  - Standard Model (SM) of Particle Physics
- ... and General Relativity (GR) is catching up:
  - Numerous tests over large range of distances
  - Direct detection of gravitational waves

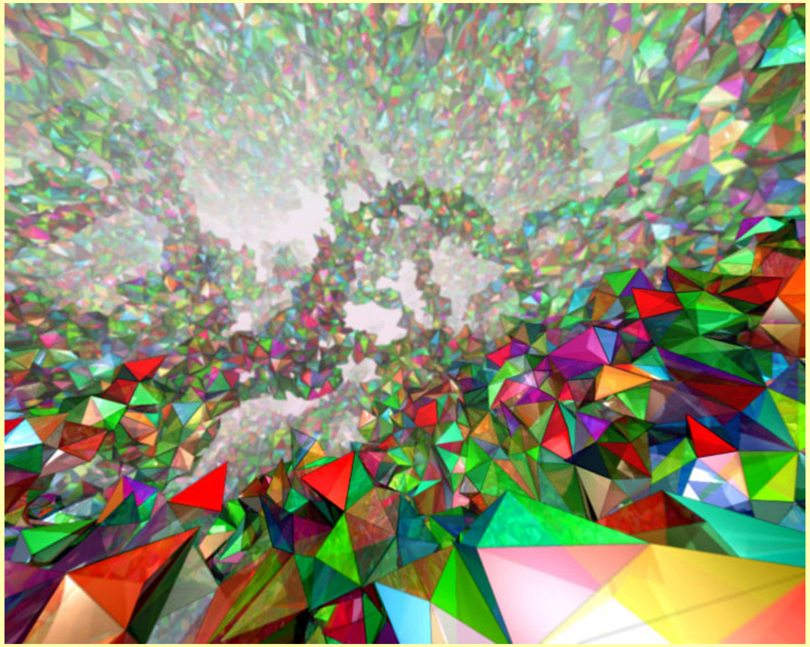
So far experimental searches have found *no deviation whatsoever* from these theories, despite a great effort.

YET: both the Standard Model of Particle Physics and General Relativity are incomplete theories, despite their overwhelming success in explaining phenomena over a vast range of distances and energy scales.

# Why Quantum Gravity?

- **Singularities in General Relativity (GR)**
  - Black holes: gravitational collapse generically unavoidable
  - Singularity theorems: space and time ‘end’ at the singularity
  - Cosmological (big bang) singularity: what ‘happened’ at $t = 0$?
  - Structure of space-time at the smallest distances?
- **Singularities in Quantum Field Theory (QFT)**
  - Perturbation theory: UV divergences in Feynman diagrams
  - Can be removed by infinite renormalizations order by order
  - Standard Model (or its extensions) unlikely to exist as rigorous QFT(s)
  - Therefore must look for an UV completion of the theory!
- **Difficulties probably have common origin:**
  - Elementary Particles as exactly *pointlike* excitations
  - Space-time as a continuum (differentiable manifold)

$\Rightarrow$ expect *something* to happen at the Planck scale!



- **Granular structure of quantum space-time?**
- **A 'gas' of strings and branes?**
- **Spin networks and spin foams?**
- **De-emergence of space and time?**

## Gravity and Matter [→ Hermann Weyl (1918)]

**Einstein's equations according to Einstein:**

$$\underbrace{R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R}_{\text{Marble}} = \underbrace{\kappa T_{\mu\nu}}_{\text{Timber?}}$$

**Question: can we understand the r.h.s. geometrically?**

- ***Kaluza-Klein theories and higher dimensions?***
- ***Supersymmetry ⇒ fermionic dimensions and geometry?***

**Gravity vs. Quantum Mechanics: do we need to change the basic rules of quantum mechanics?**

- ***Black hole evaporation and information loss?***
- ***Emergent space and time vs. quantum non-locality?***

# A Basic Fact

**Perturbative quantum gravity is non-renormalizable**

$$\Gamma^{(2)}_{div} = \frac{1}{\varepsilon}\frac{209}{2880}\frac{1}{(16\pi^2)^2}\int dV\, C_{\mu\nu\rho\sigma}C^{\rho\sigma\lambda\tau}C_{\lambda\tau}{}^{\mu\nu}$$

[Goroff& Sagnotti(1985); van de Ven(1992)]

**Two possible conclusions:**

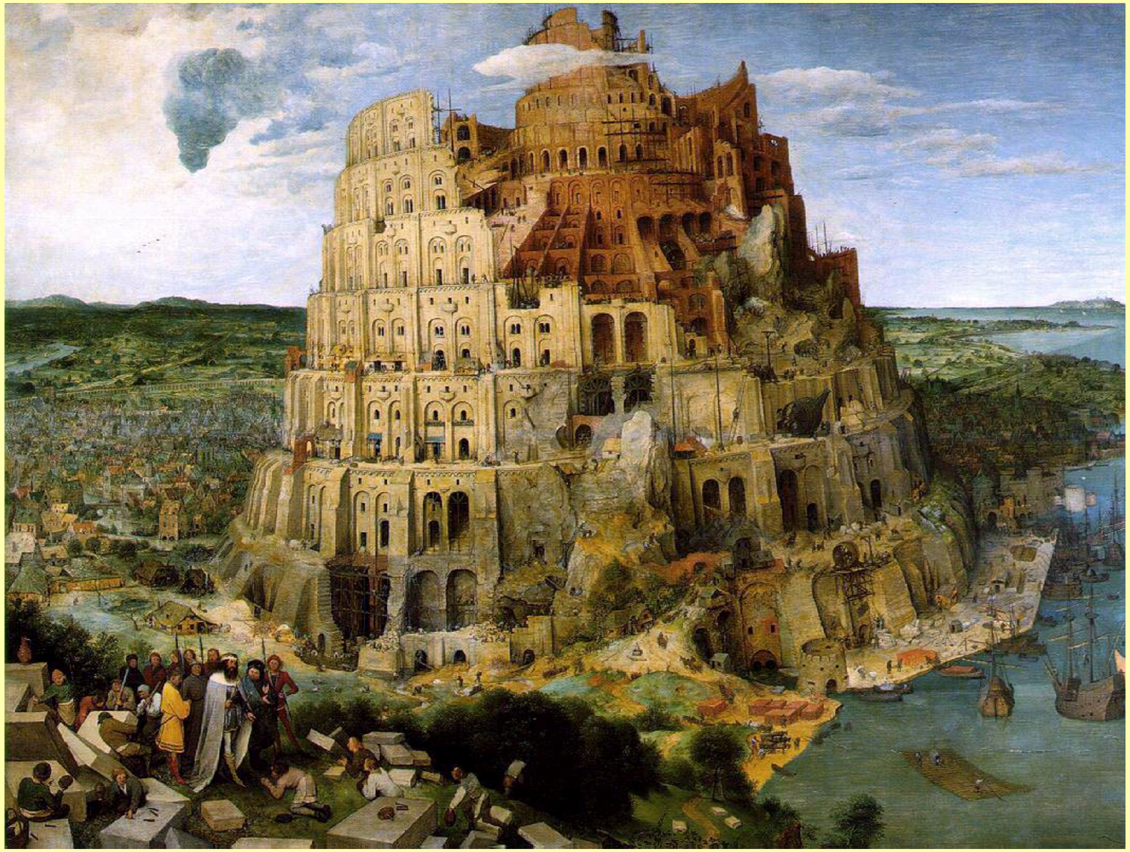
- **UV divergences are artefacts of perturbative treatment ⇒ disappear upon a proper *non-perturbative* quantization of Einstein's theory; or**
- **Consistent quantisation of gravity requires a radical modification of Einstein's theory at short distances, in particular inclusion of supersymmetric matter.**

**No approach to quantum gravity can claim complete success that does not explain *in detail* the ultimate fate of this divergence and other divergences!**

# Approaches to Quantum Gravity

- Supergravity, Superstrings and M Theory
- AdS/CFT and Holography
- Path integrals: Euclidean, Lorentzian, matrix models,...
- Canonical Quantization (metric formalism)
- Loop Quantum Gravity
- Discrete Quantum Gravity: Regge calculus, (C)DT
- Discrete Quantum Gravity: spin foams, group field theory,...
- Non-commutative geometry and non-commutative space-time
- Asymptotic Safety and RG Fixed Points
- Causal Sets, emergent (Quantum) Gravity
- Cellular Automata ('computing quantum space-time')

## The steady progress of Quantum Gravity?



# Wheeler-DeWitt Equation (1962)

**'Schrödinger equation of quantum gravity'**

$$\mathcal{H}(\mathbf{x})\Psi \equiv 16\pi G\hbar^2 G_{mn\,pq}(\mathbf{x})\frac{\delta^2\Psi[g]}{\delta g_{mn}(\mathbf{x})\delta g_{pq}(\mathbf{x})} + \frac{\sqrt{g}}{16\pi G}\left(R(\mathbf{x}) - 2\Lambda\right)\Psi[g] = 0$$

**with 'wave function of the universe' $\Psi[g]$?**

- **Numerous mathematical/technical issues:**
  - **WDW equation is mathematically *very* ill-defined $\Rightarrow$ non-renormalizable infinities in a different guise?**
  - **Recent attempts to overcome difficulties are based on 'new variables' (LQG, spin foams, GFT,...)**
- **Numerous conceptual issues:**
  - **Interpreting $\Psi[g]$ : Copenhagen *vs.* Everett?**
  - **Quantum cosmology.**
  - **Emergence of time from 'timeless' WDW equation?**
- **Which is the 'correct' Hamiltonian operator $\mathcal{H}(\mathbf{x})$?**

# Searching for Symmetries of QG

- **Supersymmetry:** matter (fermions) ***vs.*** forces (bosons)

$$\{Q^i_\alpha\,,\,\bar{Q}_{j\dot\beta}\} = 2\,\delta^i_j\,\sigma^\mu_{\alpha\dot\beta}P_\mu + \cdots \qquad (i,j,\dots = 1,...,N)$$

**Global (rigid) supersymmetry:** $N \leq 4\,,\; s \leq 1$

**Local supersymmetry → supergravity:** $N \leq 8\,,\; s \leq 2$

**Supersymmetry can merge space-time and internal symmetries, *but only for* $N > 1$.**

- **Duality Symmetry** [Dirac(1931)]

$$\mathbf{E} + i\mathbf{B} \;\rightarrow\; e^{i\omega}(\mathbf{E} + i\mathbf{B})$$

  – **Invariance of Maxwell's equations in empty space, but can be generalized to matter couplings.**

  – **Dualities are essential and ubiquitous in supergravity and superstring theory.**

# The Superworld

**Basic strategy: render gravity perturbatively consistent (i.e. finite) by modifying GR at short distances.**

- **(Partial) cancellation of UV infinities**
- **The *raison d'etre* for matter to exist?**
- **Maximally symmetric point field theories**
  - $D = 4, N = 8$ **Supergravity**
  - $D = 11$ **Supergravity**
- **Superstrings:**
  - ***no point-like interactions*** $\Rightarrow$ ***no UV singularities?***
  - ***IIA, IIB and heterotic (D=10)***
- **Supermembranes and Matrix theory**
- **M theory: the unknown completion of physics**

## Supersymmetry in the light of LHC

- Absence of ***low energy*** $(N=1)$ supersymmetry: should we altogether abandon the idea of supersymmetry?
- Nevertheless: probably needed for consistent quantisation of gravity (cancellation of infinities,...).
- Key problem: how is supersymmetry broken?
  - Can be arranged in supersymmetric ***field*** theories and $(N=1)$ supergravity models, though not very compellingly.
  - Problem is more acute/difficult and of more fundamental significance in superstring theory.
  - Supersymmetry ***not*** compatible with $\Lambda > 0$.
- Space-time supersymmetry ***vs.*** emergent space-time: Distinction between space-time bosons and fermions may become meaningless in 'pre-geometric' regime.

... and NO, Loop Quantum Gravity *cannot* step in as a replacement if superstring theory fails to live up to its promises for unification and quantum gravity:

- still struggling to recover classical space-time and the classical Einstein equations (after $>$ 30 years).
- cannot recover standard and well tested results of QFT and quantum gauge theories, both perturbative (QCD) and non-perturbative (confinement).
- in particular: cannot recover gauge anomalies = a whiff of trans-Planckian physics filtering down to Standard Model physics (fermion representations).

# $N = 8$ Supergravity

**Unique theory (modulo 'gauging'), *most symmetric* known field theoretic extension of Einstein's theory!**

$$1\times[2] \oplus 8\times\left[\tfrac{3}{2}\right] \oplus 28\times[1] \oplus 56\times\left[\tfrac{1}{2}\right] \oplus 70\times[0]$$

- ***Diffeomorphisms and local Lorentz symmetry***
- $N = 8$ ***local supersymmetry***
- $SU(8)$ ***R symmetry (local or rigid)***

**In addition: 'hidden' duality symmetry $E_{7(7)}$ combining 28 electric $\oplus$ 28 dual magnetic vectors** [Cremmer,Julia]

**70 scalar fields described by 56-bein** $\mathcal{V}(x) \in E_{7(7)}/SU(8)$

$$\mathcal{V}(x) \;\rightarrow\; \mathcal{V}'(x) = g\mathcal{V}(x)h(x)\;, \qquad g \in E_{7(7)}\,,\; h(x) \in SU(8)$$

## $N = 8$ Supergravity: main issues

- **Is the theory UV finite to all orders?**
- **Is there any relation to 'real physics'?**

- On-shell superspace [Brink,Howe(1979)] enables construction of possible counterterm candidates [Kallosh(1981); Howe,Stelle,Townsend(1981)] But: absence of *off-shell* formulation impedes quantization.
- Perturbation theory compatible with continuous $E_{7(7)} \Rightarrow$ counterterms must respect full $E_{7(7)}$.[Bossard,Hillmann,HN(2010)]

  *But:* expect non-perturbative effects to break $E_{7(7)}$ to discrete subgroup. [Hull,Townsend(1995)]
- No counterterms known that are *fully* compatible with $N = 8$ SUSY *and* non-linear $E_{7(7)}$. [Kallosh,HN,Roiban,Yamada, work in progress]

## $N = 8$ Supergravity: new perspectives

**Very recent work has shown that $N = 8$ supergravity**

- **is much more finite than expected (behaves like $N = 4$ super-Yang-Mills up to four loops)**

  [Bern,Carrasco,Dixon,Johansson, Roiban, PRL103(2009)081301]

- **... and could thus be finite to all orders!**
- **However: efforts towards five loops still pending.**
- **Thus:** $D_{cr} = 4 + \frac{6}{L}\,(= \frac{26}{5})$ ***vs.*** $D_{cr} = 2 + \frac{14}{L}\,(= \frac{24}{5})$ **???**

**Even if theory *is* finite, no obvious reason **why** it should be $\rightarrow$ unknown symmetry structures?**

**If no new spin-$\frac{1}{2}$ degrees of freedom are found at LHC, the following curious fact could also become relevant:**

# A strange coincidence with SM physics

$SO(8) \to SU(3)\times U(1)$ **breaking and 'family color locking'**

$$
\begin{array}{rlll}
(u\,,\,c\,,\,t)_L & : & \mathbf{3}_c \times \bar{\mathbf{3}}_f \to \mathbf{8} \oplus \mathbf{1}\,, & Q = \frac{2}{3} - q \\
(\bar{u}\,,\,\bar{c}\,,\,\bar{t})_L & : & \bar{\mathbf{3}}_c \times \mathbf{3}_f \to \mathbf{8} \oplus \mathbf{1}\,, & Q = -\frac{2}{3} + q \\
(d\,,\,s\,,\,b)_L & : & \mathbf{3}_c \times \mathbf{3}_f \to \mathbf{6} \oplus \bar{\mathbf{3}}\,, & Q = -\frac{1}{3} + q \\
(\bar{d}\,,\,\bar{s}\,,\,\bar{b})_L & : & \bar{\mathbf{3}}_c \times \bar{\mathbf{3}}_f \to \bar{\mathbf{6}} \oplus \mathbf{3}\,, & Q = \frac{1}{3} - q \\
(e^-, \mu^-, \tau^-)_L & : & \mathbf{1}_c \times \mathbf{3}_f \to \mathbf{3}\,, & Q = -1 + q \\
(e^+, \mu^+, \tau^+)_L & : & \mathbf{1}_c \times \bar{\mathbf{3}}_f \to \bar{\mathbf{3}}\,, & Q = 1 - q \\
(\nu_e\,,\,\nu_\mu\,,\,\nu_\tau)_L & : & \mathbf{1}_c \times \bar{\mathbf{3}}_f \to \bar{\mathbf{3}}\,, & Q = -q \\
(\bar{\nu}_e\,,\,\bar{\nu}_\mu\,,\,\bar{\nu}_\tau)_L & : & \mathbf{1}_c \times \mathbf{3}_f \to \mathbf{3}\,, & Q = q
\end{array}
$$

$N = 8$ **Supergravity and Standard Model assignments agree if spurion charge is chosen as** $q = \frac{1}{6}$ [Gell-Mann (1983)]

$\to$ **realized at SU(3) $\times$ U(1) stationary point** [Warner,HN(1985)]

**Mismatch $\pm\frac{1}{6}$ can be fixed by deforming U(1), but deformation *not contained* in SU(8) R symmetry.** [Meissner,HN:1412.1715]

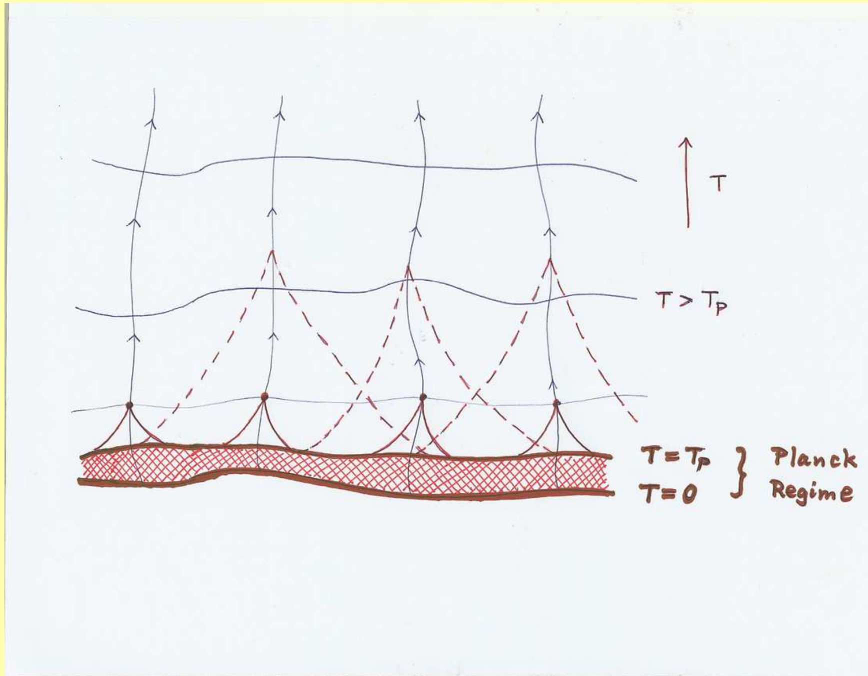
# Beyond $N = 8$ Supergravity

- **Take the hints from unexpected ('hidden') $E_{7(7)}$ duality symmetry of $N = 8$ supergravity**
- **$\rightarrow$ can duality symmetry supersede space-time symmetries, and in particular supersymmetry?**
- **'Dimensional reduction' $\equiv$ metamorphoses space-time symmetries into internal symmetries:**

$$\cdots \subset E_6 \subset E_7 \subset E_8 \subset E_9 \subset E_{10}$$

  **with the $\infty$-dimensional 'prolongations' $E_9$ and $E_{10}$**
- **$E_{10}$ = maximally extended hyperbolic Kac–Moody algebra – a symmetry after reduction to $D = 1$?**
- **$\Rightarrow$ 'De-Emergence' of space (and time) ?!?**

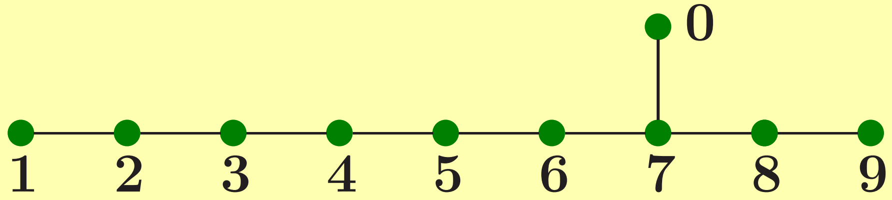
# Another hint: BKL and Spacelike Singularities




**For $T \to 0$ spatial points decouple and the system is effectively described by a continuous *superposition of one-dimensional systems* $\to$ effective dimensional reduction to $D = 1$!** [Belinski,Khalatnikov,Lifshitz (1972)]

## A candidate symmetry: $G = E_{10}$?

$E_{10}$ is the 'group' associated with the Kac-Moody Lie algebra $\mathfrak{g} \equiv \mathfrak{e}_{10}$ defined via the Dynkin diagram



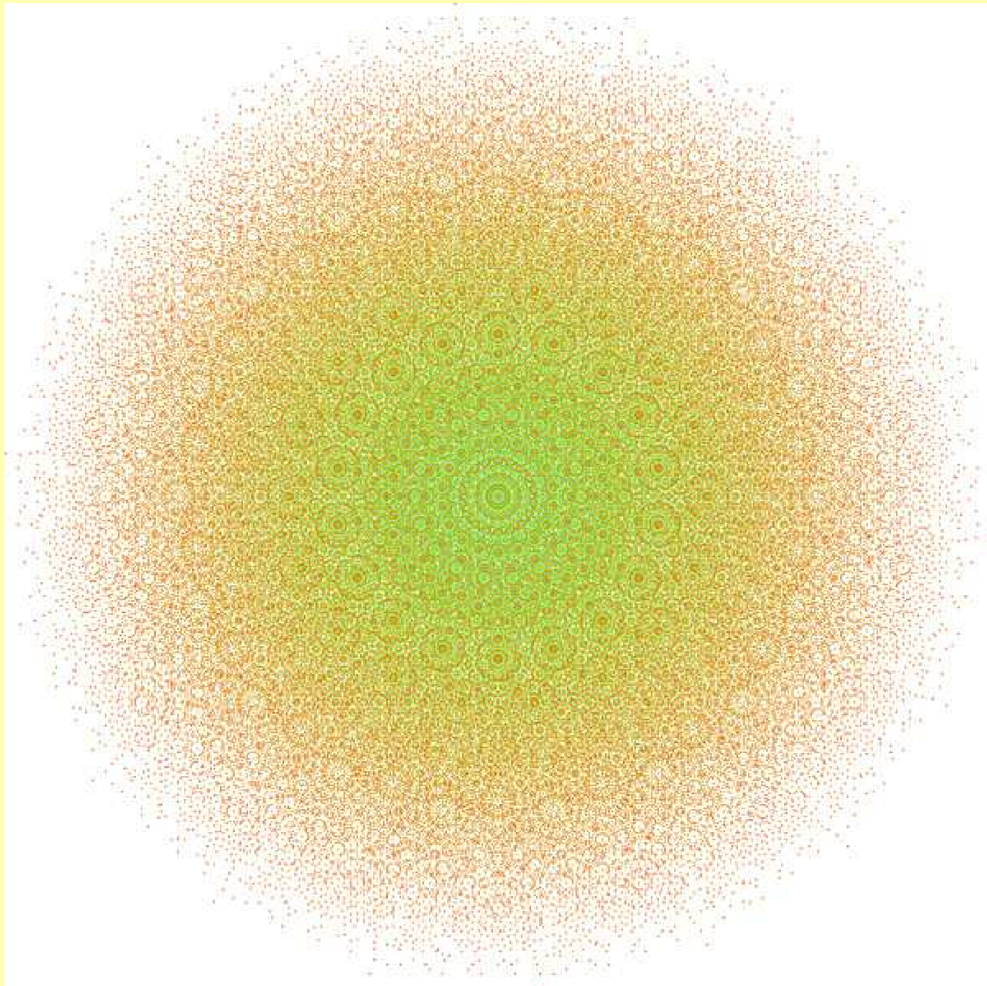

Defined by generators $\{e_i, f_i, h_i\}$ and relations via Cartan matrix $A_{ij}$ ('Chevalley-Serre presentation')

$$\begin{aligned} [h_i, h_j] &= 0, & [e_i, f_j] &= \delta_{ij} h_i, \\ [h_i, e_j] &= A_{ij} e_j, & [h_i, f_j] &= -A_{ij} f_j, \\ (\mathrm{ad}\, e_i)^{1-A_{ij}} e_j &= 0 & (\mathrm{ad}\, f_i)^{1-A_{ij}} f_j &= 0. \end{aligned}$$

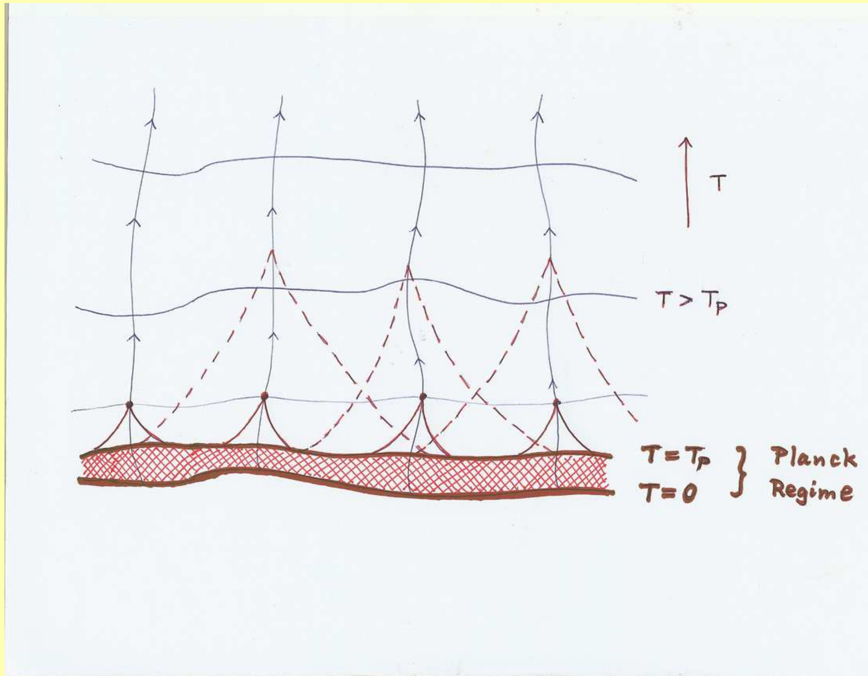
$\mathfrak{e}_{10}$ is the free Lie algebra generated by $\{e_i, f_i, h_i\}$ modulo these relations $\rightarrow$ infinite dimensional as $A_{ij}$ is ***indefinite*** $\rightarrow$ Lie algebra of ***exponential growth*** !

# A candidate symmetry: $G = E_{10}$?

[Image credit: Teake Nutma]



# $E_{10}$: The Basic Picture




**Conjecture:** **for $0 < T < T_{PL}$ space-time 'de-emerges', and space-time based (quantum) field theory is replaced by quantised 'spinning' $E_{10}/K(E_{10})$ $\sigma$-model.**

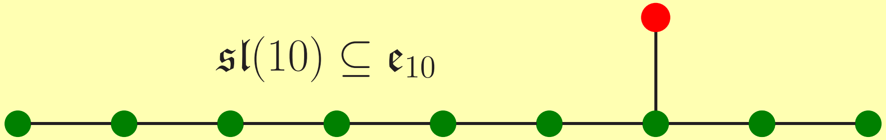
[Damour,Henneaux,HN, Kleinschmidt: since 2002]

# Main Points and Challenges

- $E_{10}$ **encompasses previously known dualities and is *more powerful* than maximal supersymmetry**
  **$\rightarrow$ can $E_{10}$ replace SUSY as a unifying principle?**

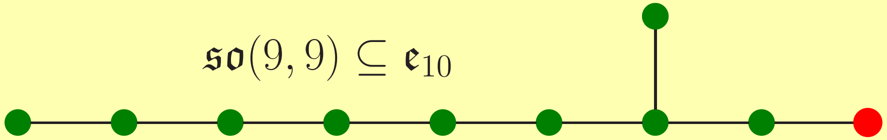
# $E_{10}$ Versatility

$\mathfrak{sl}(10) \subseteq \mathfrak{e}_{10}$



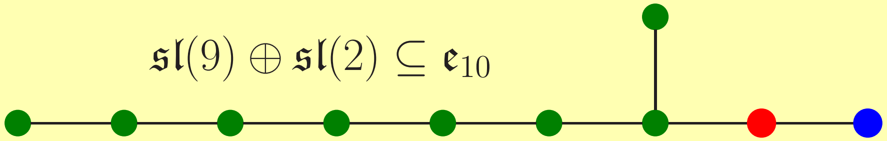
$D = 11$ **SUGRA**

$\mathfrak{so}(9,9) \subseteq \mathfrak{e}_{10}$



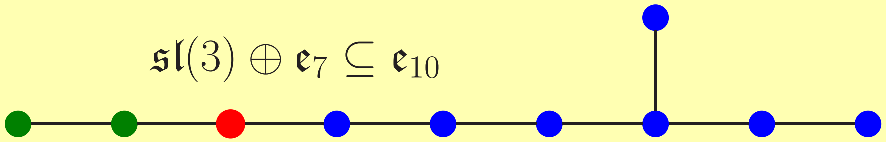
**mIIA** $D = 10$ **SUGRA**

$\mathfrak{sl}(9) \oplus \mathfrak{sl}(2) \subseteq \mathfrak{e}_{10}$



**IIB** $D = 10$ **SUGRA**

$\mathfrak{sl}(3) \oplus \mathfrak{e}_7 \subseteq \mathfrak{e}_{10}$



$\mathcal{N} = 8,$ $D = 4$ **SUGRA**

# Main Points and Challenges

- $E_{10}$ encompasses previously known dualities and is ***more powerful*** than maximal supersymmetry $\rightarrow$ **can $E_{10}$ replace SUSY as a unifying principle?**
- **A very concrete proposal for the (de-)emergence of space and time in quantum gravity: 'spreading' coordinate dependence over $E_{10}$ Lie algebra!**
- **Maximal compact subgroup ('R-symmetry')** $K(E_{10}) \subset E_{10}$ $\Rightarrow$ **governs fermionic sector.**
- $K(E_{10})$ **is required to rectify** $\pm\frac{1}{6}$ **mismatch between supergravity and SM fermions, and also contains *chiral* transformations and family symmetries.**
- **A new and very different paradigm for getting SM physics from Planck scale theory?**

# Outlook

- **Incompleteness of the SM and GR are strongest arguments in favor of quantizing gravity.**
- **Main Question: how are short distance singularities resolved in GR *and* QFT, and how can this resolution be reconciled with classical Einstein equations in continuum space-time?**
  - **Dissolving pointlike interactions (strings, branes,...)**
  - **Cancellation of UV infinities (e.g. $N = 8$ supergravity)?**
  - **Fundamental discreteness (LQG, discrete gravity)?**
  - **Other mechanism (e.g. AS, non-commutative space-time)?**
- **Symmetry enhancement may be the best strategy!**

# Outlook

- **Incompleteness of the SM and GR are strongest arguments in favor of quantizing gravity.**
- **Main Question: how are short distance singularities resolved in GR *and* QFT, and how can this resolution be reconciled with classical Einstein equations in continuum space-time?**
  - **Dissolving pointlike interactions (strings, branes,...)**
  - **Cancellation of UV infinities (e.g. $N = 8$ supergravity)?**
  - **Fundamental discreteness (LQG, discrete gravity)?**
  - **Other mechanism (e.g. AS, non-commutative space-time)?**
- **Symmetry enhancement may be the best strategy!**

**... there is still a long way to go !**