A short introduction to the BAT limits & systematics tutorial

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Outline:
- Bayes‘ Theorem
- Setting limits
- Including systematic uncertainties
- Physics motivation
- The tutorial
- Documentation

Bayes‘ Theorem

Reminder:

Bayes‘ Theorem tells us how to update our knowledge of a *model* M with a *set of parameters* $\vec{\lambda}$ after conducting an experiment yielding a data set $\vec{x}$.

Posterior probability	$p(\vec{\lambda}\|\vec{x})$	Quantity you are really after: probability for the parameters given the data set.
Likelihood	$p(\vec{x}\|\vec{\lambda})$	Described by model, e.g., straight line fit. Has to be known for the analysis.
Prior probability	$p_0(\vec{\lambda})$	Describes prior knowledge about the parameters (and the model). Generates discussions.

Note: these quantities are probability densities for continuous parameters and probabilities for discrete parameters.

$$p(\vec{\lambda}|\vec{x}) = \frac{p(\vec{x}|\vec{\lambda})\, p_0(\vec{\lambda})}{\int p(\vec{x}|\vec{\lambda})\, p_0(\vec{\lambda})\, d\vec{\lambda}}$$

Setting limits I

Single parameter model:

Posterior probability (density) describes complete knowledge on the parameter but rather not handy. Summarize knowledge quoting

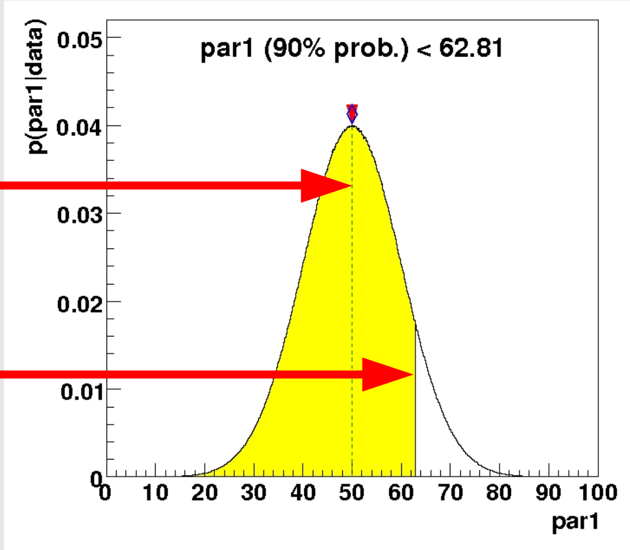
a *point estimate* (e.g., mean, mode, median),

or an *interval estimate* (e.g., smallest or 68% central interval, limit).

Examples:

Mean: $$\langle\lambda\rangle = \int_{\lambda_{min}}^{\lambda_{max}} \lambda \, p(\lambda|\vec{x}) \, d\lambda$$

90% prob. limit: $$0.9 = \int_{\lambda_{min}}^{\lambda_{90}} p(\lambda|\vec{x}) \, d\lambda$$



This means that the probability for the true value of the parameter to be less than λ_{90} is 90%. λ_{90} is the 0.9 quantile of the posterior probability.

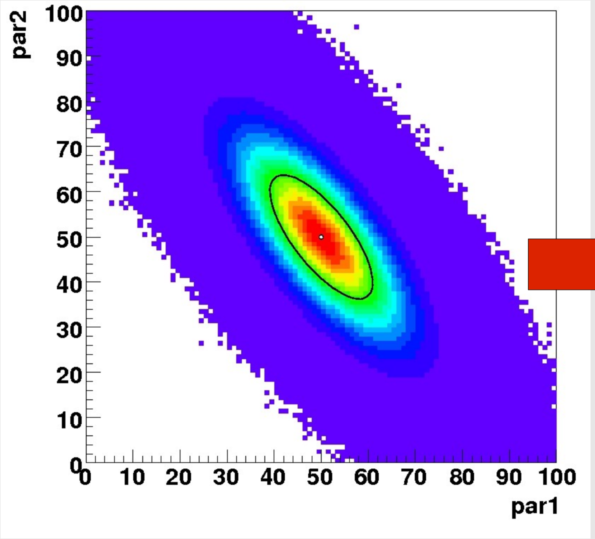
Setting limits II

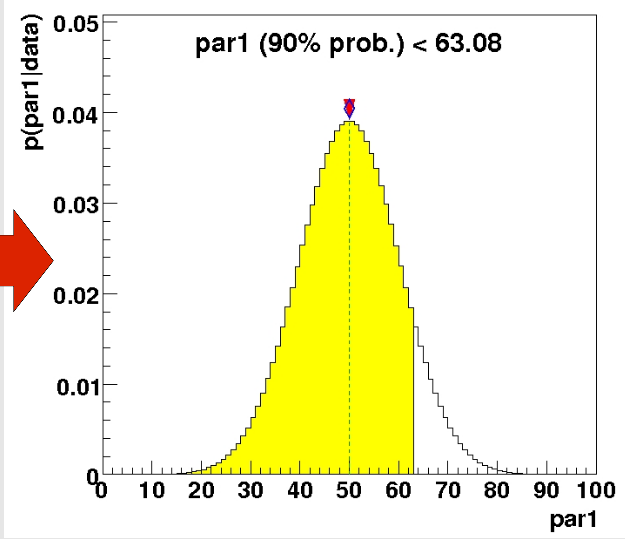
Multi-parameter model:

Posterior probability (density) is a multi-dimensional function. *Marginalize*, i.e., integrate over all parameters but the one under study:

$$p(\lambda_i|\vec{x}) = \int p(\vec{\lambda}|\vec{x}) \; d\prod_{i \neq j} \lambda_j$$

This is the posterior probability for λ_i given the data set taking into account all possible values of the other parameters (weighted!).





Use 1-d prescription to obtain limit on a single parameter.

Including systematics ...

... using nuisance parameters: This is just one way of doing it. There are many more...

- Add an extra *nuisance parameter* to the model for each source of systematic uncertainty.
- Parameterization of systematic uncertainty = prior of the extra parameter.
- Integrate over the parameter (marginalize).

Example:

Include jet energy scale as additional parameter and multiply to all energies, e.g.,

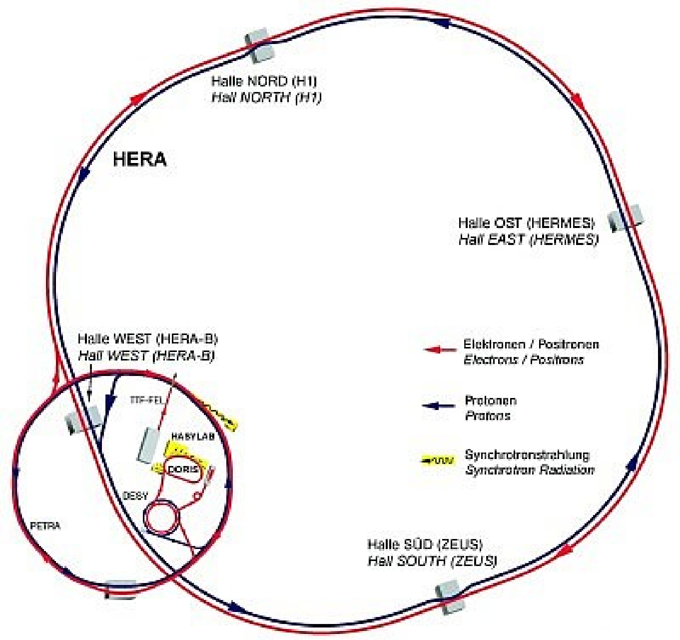
$$E_i \rightarrow E_i \cdot (1 + \delta_{JES})$$

The prior to δ_{JES} could, e.g., be a Gaussian distribution around 0 with a width of 0.05.

Physics motivation I

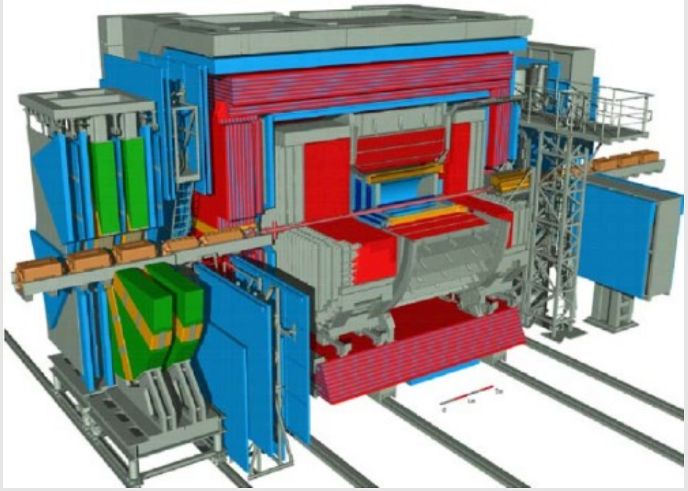
HERA:

- $e^{\pm}p$ collider at DESY (1992-2007)
- Center-of-mass energy of 320 GeV, polarized $e^{\pm}$
- Four experiments:
  H1, **ZEUS**, HERA-B, HERMES



ZEUS:

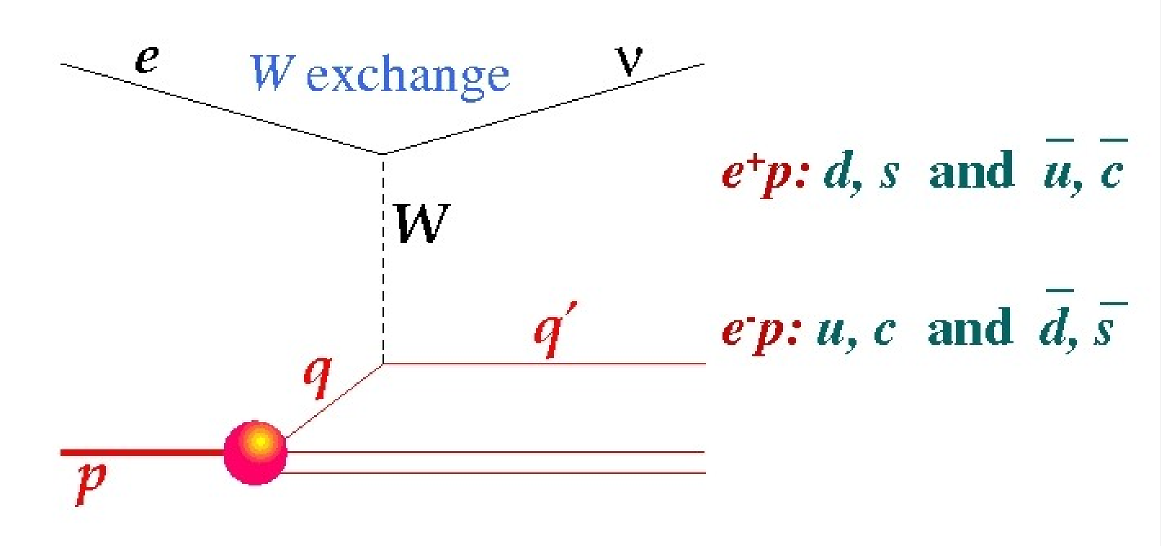
- One (of two) multi-purpose detectors with wide physics program
- Typical onion-shell structure



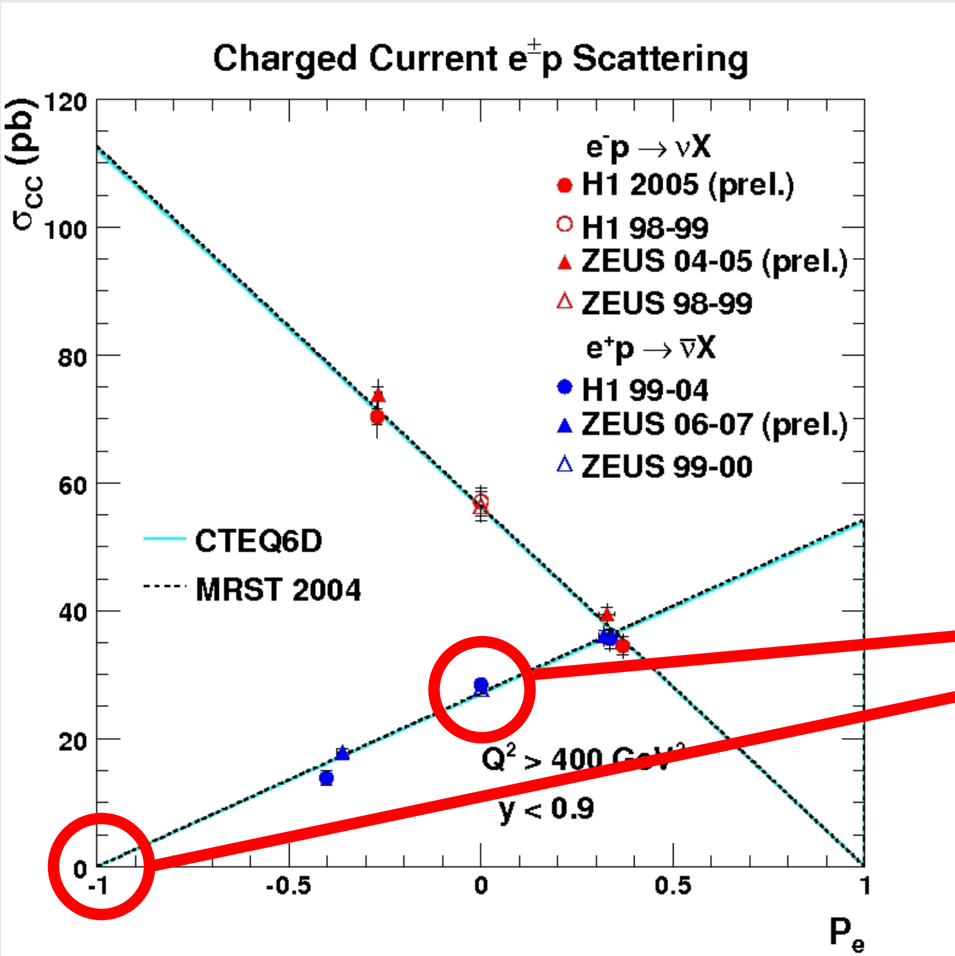
Physics motivation II

Charged-current reaction: Only studying positron scattering for now.

- Positron scatters off quark via W^+: $e^+ + p \rightarrow \nu + q' + X$
- Signature:
  - missing E_T (neutrino),
  - at least one jet (quark + proton remnant)
- Vertex:
  - V-A structure
  - ν only couples to left-handed W
- *Expect CC-cross-section to linearly depend on the polarization*



Physics motivation III



Tutorial:
Redo the analysis to find a limit on the cross-section at polarization of -1

Model:

- 2 Parameters:
  cross-section at P= 0
  cross-section at P=-1
- *Finite cross-section at P=-1 could come from right-handed W*

$$\frac{d^2\sigma^{CC}_{e\pm p}}{dx dQ^2} = \frac{1}{128\pi x}\left[\frac{g_R^4}{(M_{W,R}^2+Q^2)^2}(1\mp P) + \frac{g_L^4}{(M_{W,L}^2+Q^2)^2}(1\pm P)\right]\tilde{\sigma}^{CC}_{e\pm p}$$

A tutorial on BAT

Charged current cross-section analysis - a BAT tutorial

Physics motivation

The charged current (CC) deep inelastic scattering cross-section in e^+p interactions, σ_{CC}, was measured with a polarized pos
predicts that the CC cross-section depends linearly on the polarization P, i.e., $\sigma_{CC} = \sigma_{CC}(P) \propto P$, and vanishes for a polariza
left-handed particles. A limit on a right-handed contribution to the cross-section is set in the following analysis, i.e., on the qua
the original analysis can be found here.

Tutorial

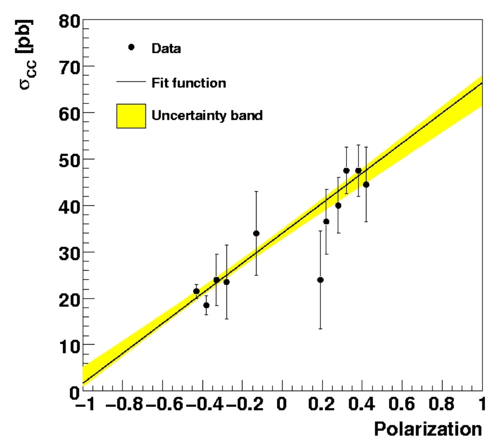
This tutorial shows how to implement a Bayesian model into BAT. In the first part a straight line fit is done in order to estimate t
how prior knowledge from previous measurements can be used to improve the results. Sources of systematic uncertainties a
Finally, the result obtained is combined with a second meausrement of cross-section.

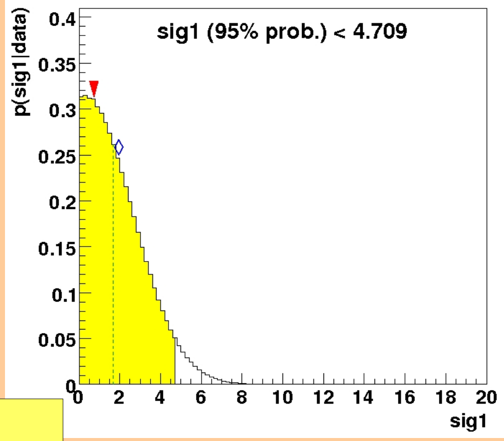
The tutorial is split into seven steps:

- Step 1 - Getting started
- Step 2 - Reading in the data
- Step 3 - Defining the model
- Step 4 - Calculating the limit
- Step 5 - Including prior knowledge
- Step 6 - Including systematic uncertainties
- Step 7 - Combining results

A proposal for additional studies is also given.

Step 1 - Getting started





Tutorial is available here:
http://www.mppmu.mpg.de/bat/?page=tutorials

Documentation:

- A. Caldwell, D. Kollar, K. Kröninger,
  BAT – The Bayesian analysis toolkit,
  Comp. Phys. Comm. **180** (2009) 2197 [arXiv:0808.2552]

- BAT webpage:
  http://www.mppmu.mpg.de/bat/

- Introduction to BAT:
  http://www.mppmu.mpg.de/bat/?page=documentation

- Talks on BAT:
  http://www.mppmu.mpg.de/bat/?page=meetings

- Tutorials on BAT
  http://www.mppmu.mpg.de/bat/?page=tutorials