Applications of AdS/CFT
to very low-x physics

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Motivation

Deep inelasting scattering: Universal behaviour

Independence of species of incoming hadrons

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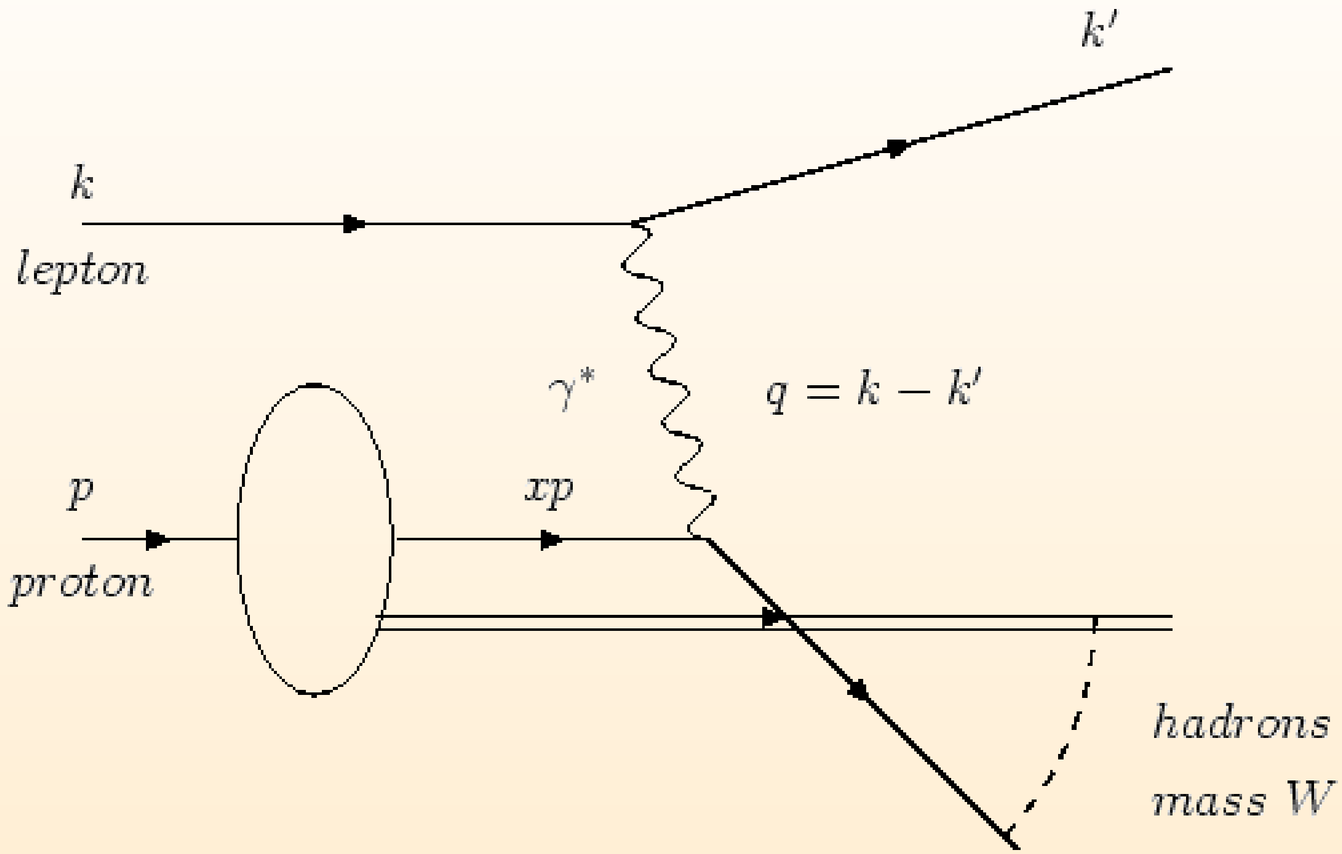
Independence of species of incoming hadrons

Gauge/gravity duality:

Map between strongly coupled quantum gauge theories and (classical) gravity

Useful approach for identifying universal behaviour

Deep-inelastic scattering



$$Q^2 = -q^2$$

Photon coherence length

Stodolsky, Wosiek Nucl.Phys. B413 (1994) 813;   Caldwell New J.Phys. 18 (2016) 073019

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In proton rest frame:

$$l = \frac{\hbar c}{\Delta E}$$

ΔE: Change in energy of the photon as it fluctuates into quarks and gluons

For $Q^2 = -q^2 \gg m^2{}_{\mathrm{parton}}$

$$\Delta E \simeq \frac{Q^2}{2\nu}\,, \qquad l \simeq \frac{2\nu\hbar c}{Q^2} \simeq \frac{\hbar c}{x M_{\mathrm{proton}}}$$

ν: Photon energy

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ν: Photon energy

Cross section is expected to increase with coherence length since there is more time for the photon to develop structure.

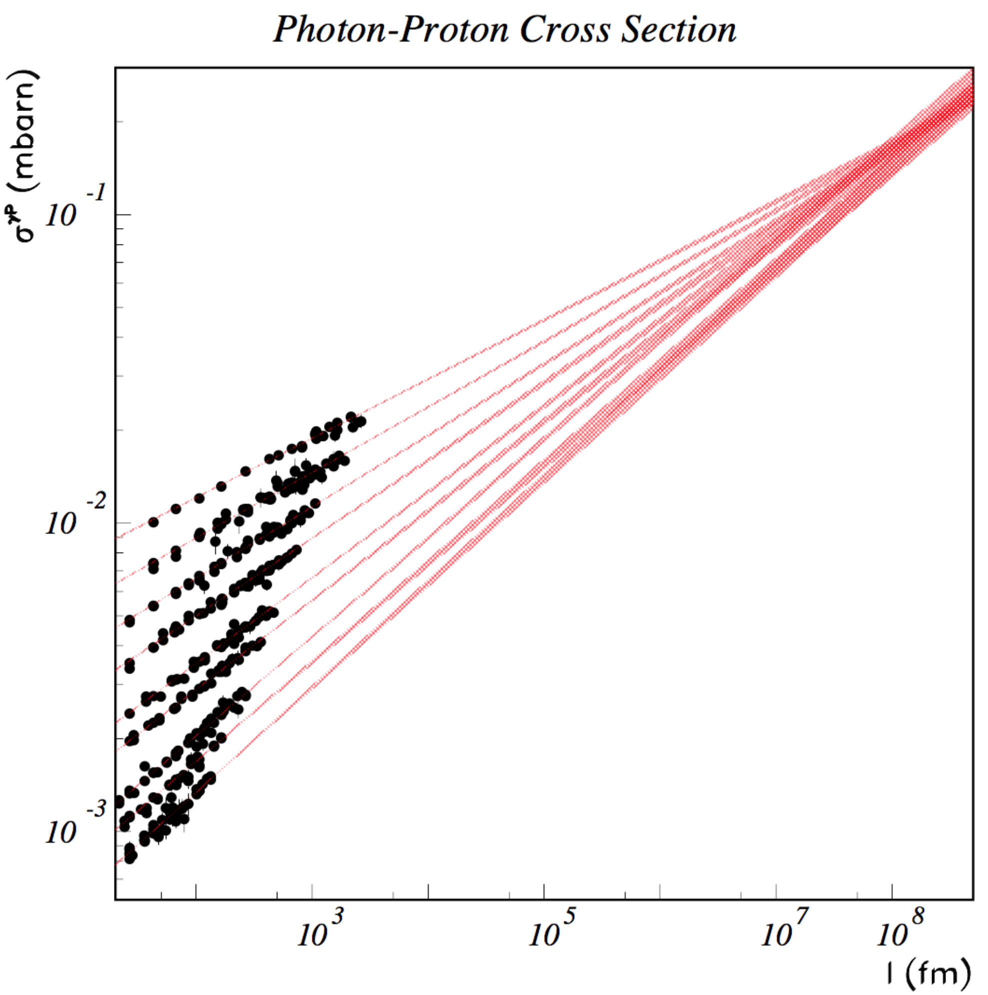

Figure 5: *Extrapolation of the fit functions to larger l for values of Q^2 in the range $3.5 \leq Q^2 \leq 90$ GeV^2. The width of the bands represent the 68 % credible intervals for the functions from the data fits described in the text. The values of Q^2 range from 3.5 GeV^2 (top curve) to 90 GeV^2 (bottom curve) with the intermediate curves corresponding to intermediate Q^2 values.*

Caldwell New J.Phys. 18 (2016) 073019

Dependence of cross section on coherence length

At large l, cross-sections are expected to merge and level off

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Example for universal behaviour

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Description within gauge/gravity duality?

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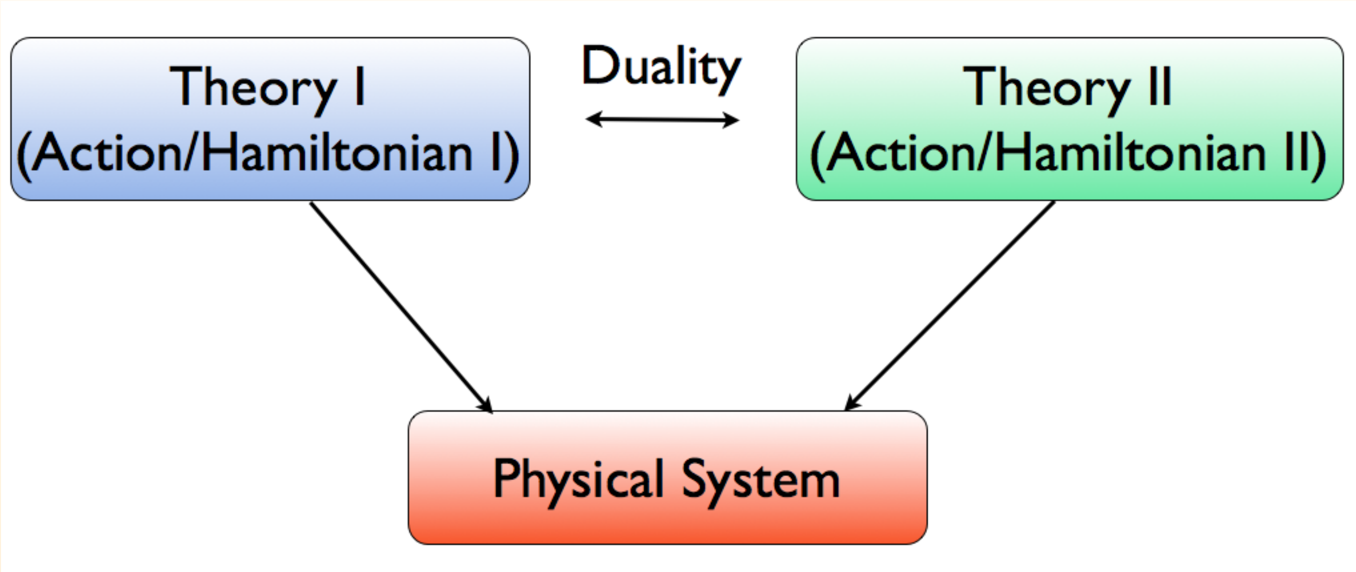
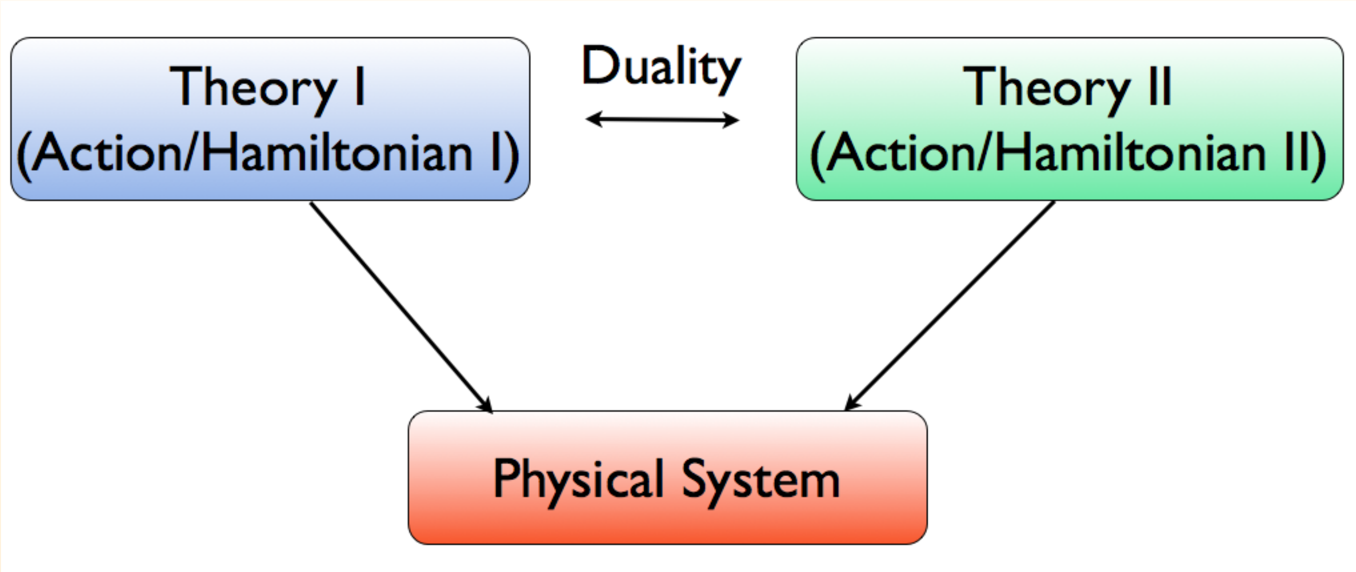
Description within gauge/gravity duality?

- Brief review of gauge/gravity duality
- Examples for universal behaviour from gauge/gravity duality

Duality:


Theory I
(Action/Hamiltonian I)
Duality
Theory II
(Action/Hamiltonian II)
Physical System

Duality:



- Gauge/gravity duality:

  Quantum field theory at strong coupling

  $\Leftrightarrow$ Gravity theory at weak coupling

AdS/CFT Correspondence

(Maldacena 1997, AdS: Anti-de Sitter space, CFT: conformal field theory)

Best understood example of gauge/gravity duality:

- Conformal field theory in four dimensions

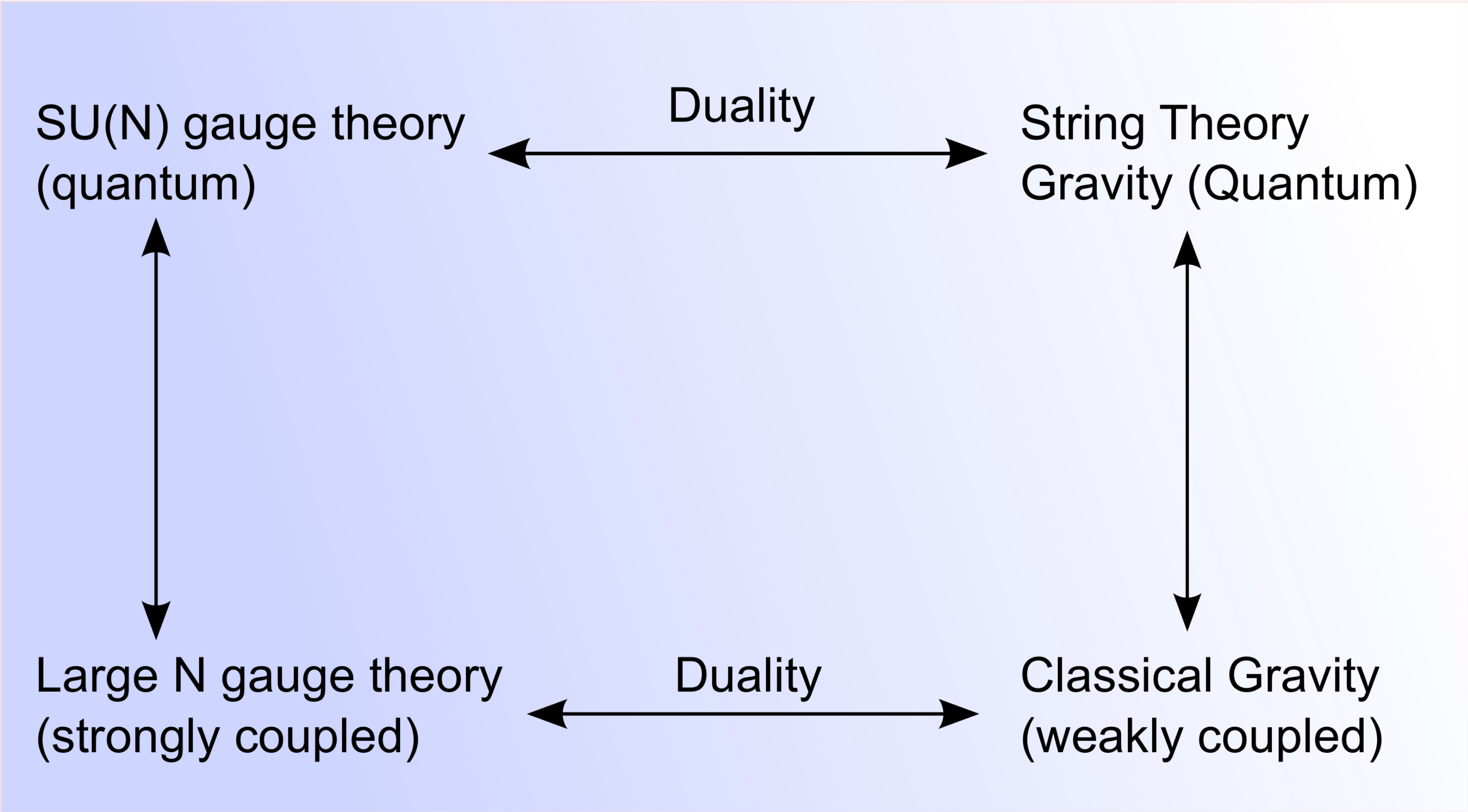
  $\Leftrightarrow$ Supergravity theory on $AdS_5 \times S^5$

- Arises from String theory in a particular low-energy limit:

  ’t Hooft coupling $\lambda = g^2 N$ large and fixed, $N \to \infty$

- 4d field theory lives at the boundary of 5d Anti-de Sitter space
  ($\mathcal{N} = 4$ Super Yang-Mills)

AdS/CFT correspondence: String theory origin



Gauge/gravity duality

Generalizations of AdS/CFT that break conformal and supersymmetry

Realize confinement

Generalizations of AdS/CFT that break conformal and supersymmetry

Realize confinement

Dual gravity theories more involved

Examples: Hard wall, soft wall (dilaton), consistent 10d gravity solutions

A particular strength of gauge/gravity duality is to provide universal predictions:

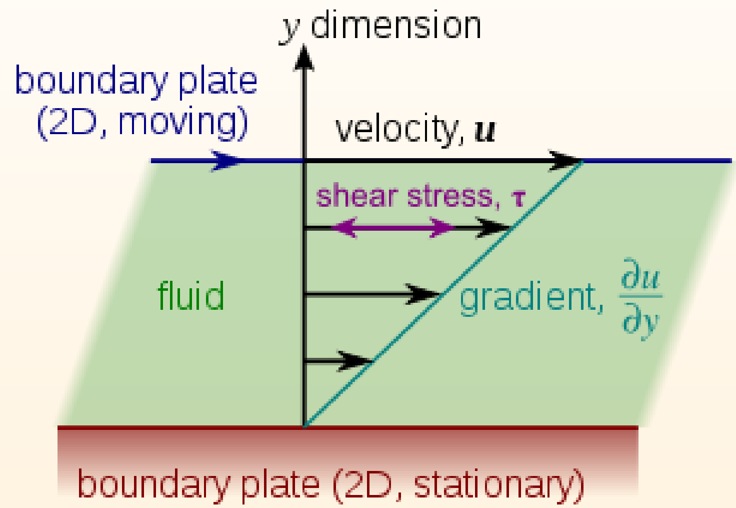
Examples:

- Shear viscosity
- Pomeron: Unified description of soft (Regge) and hard (BFKL) pomeron
- Froissart bound

Example: Shear viscosity over entropy density

Shear viscosity over entropy density

$$\frac{\eta}{s} = \frac{1}{4\pi} \frac{\hbar}{k_B}$$



Kovtun, Son, Starinets PRL 2004

- Universal lower bound (does not depend on details of theory)
- Bound satisfied by the most strongly coupled systems ($g \to \infty$)
- Experimentally observed for quark-gluon plasma at RHIC, ALICE

Example: Shear viscosity over entropy density

- Hydrodynamics: Long wavelength, low-frequency fluctuations in fluids
- Expand physical quantities in derivatives of the relativistic fluid velocity: u^μ, ∇u, $\nabla\nabla u \dots$
- Consider energy-momentum tensor $T_{\mu\nu}$

  Contains information about energy density, energy and momentum flux

- Hydrodynamic expansion to first order in derivatives:

$$T_{\mu\nu}(x) = T^{(0)}_{\mu\nu}(x) + T^{(1)}_{\mu\nu}(x) + \dots$$

$$T^{(0)}_{\mu\nu}(x) = (\epsilon + P)u_\mu u_\nu - P g_{\mu\nu}\,, \;\; T^{(1)}_{\mu\nu} = \eta\left(\partial_\mu u_\nu + \partial_\nu u_\mu - \tfrac{2}{3} g_{\mu\nu}\partial_\lambda u^\lambda\right) + \zeta g_{\mu\nu}\partial_\lambda u^\lambda$$

η shear viscosity, ζ bulk viscosity

Holographic calculation of shear viscosity

- Energy-momentum tensor $T_{\mu\nu}$ dual to graviton $g^{\mu\nu}$
- Calculate correlation function $\langle T_{xy}(x_1)T_{xy}(x_2)\rangle$ from propagation through AdS black hole space
- Shear viscosity is obtained from Kubo formula:

$$\eta = -\lim \frac{1}{\omega}\mathrm{Im}G^R_{xy,xy}(\omega)$$

- Shear viscosity $\eta = \pi N^2 T^3/8$,   Entropy density $s = \pi^2 N^2 T^3/2$

$$\frac{\eta}{s} = \frac{1}{4\pi}\frac{\hbar}{k_B}$$

Pomeron in AdS/CFT

Brower, Polchinski, Strassler, Tan JHEP 0712 (2007) 005

Pomeron:

Coherent color-singlet excitation in high-energy hadronic scattering

At large s, small t, large N

it contributes the leading singularity in the angular momentum plane

Pomeron in AdS/CFT: (large N)

Calculation of field theory amplitude from string amplitude in ten-dimensional $AdS_5 \times S^5$ space with cut-off

Four-dimensional scattering given by coherent sum over scattering in the six transverse dimensions

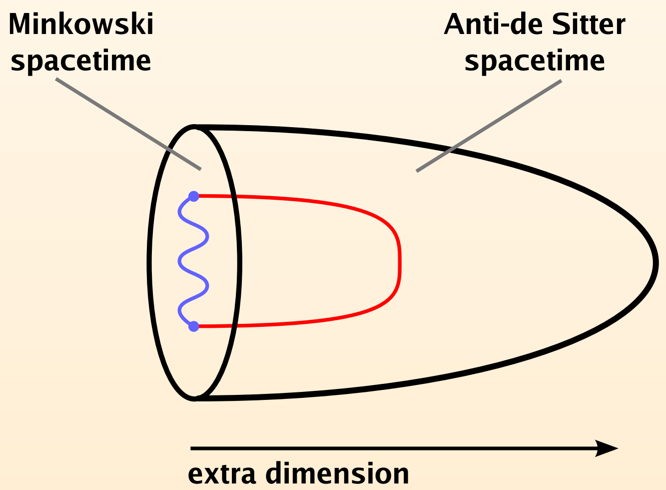
Hard scattering

Holographic encoding of gauge theory physics:

Low energy states at small r, high energy states at large r (near boundary)

Warped space:

$$ds^2 = \frac{r^2}{L^2}\eta_{\mu\nu}dx^\mu dx^\nu + \frac{L^2}{r^2}dr^2 + L^2 ds_X^2$$



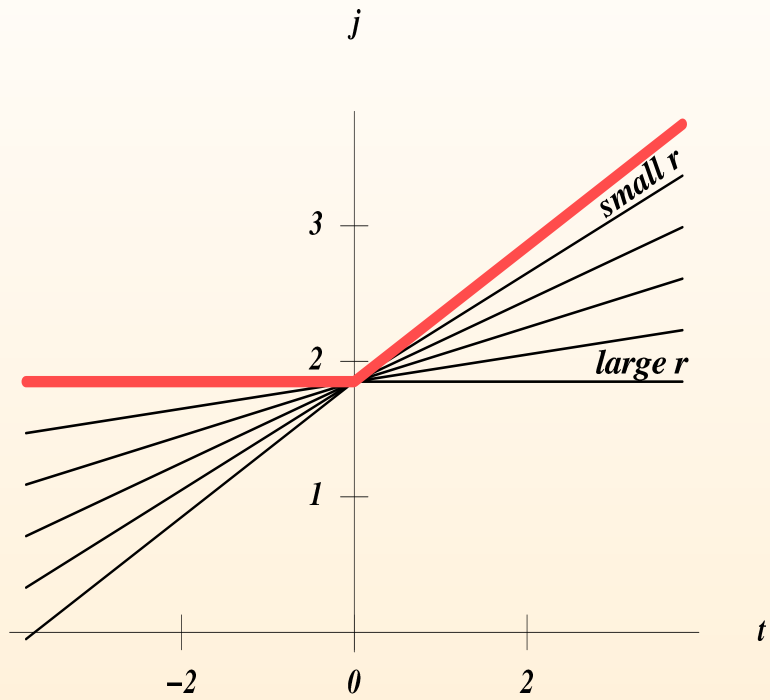
$$p_\mu = \frac{r}{L}\tilde{p}_\mu$$

$$\mathcal{A}(s,t) \propto s^{\alpha(t,r)}$$

p_μ conserved momentum, corresponding to invariance under translation of x^μ

$\tilde{p}_\mu$ momentum in local inertial coordinates for momenta localized at r

Pomeron in gauge/gravity duality



At large s, highest trajectory will dominate:

t positive: r small: soft (Regge) pomeron, properties determined by confining dynamics: glueball

t negative: r large: hard (BFKL) pomeron, two-gluon perturbative small object

Recent refinements

Ballon-Bayona, R. Quevedo, Costa 1704.08280

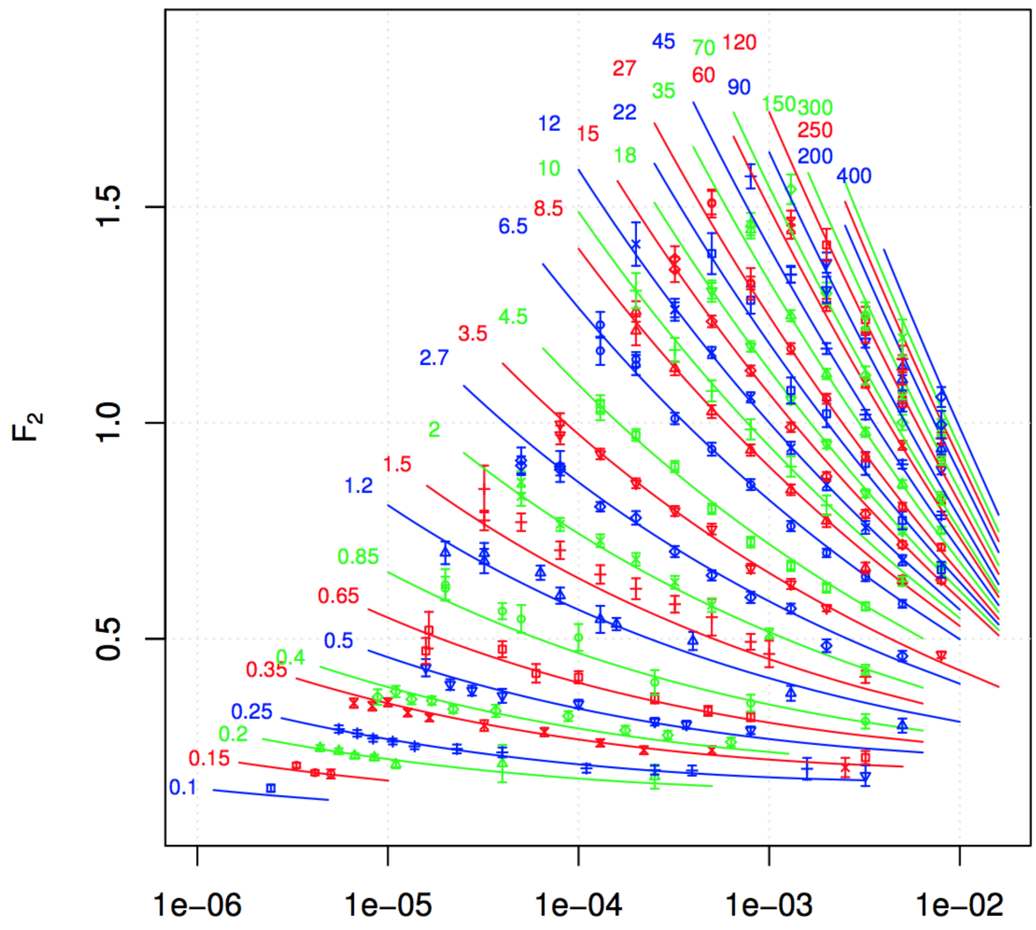
Exchange of higher-spin fields in the graviton Regge trajectory
dual to glueball states of twist two

First four pomeron trajectories are considered; fit to HERA data

$x < 0.01$

$0.1 < Q^2 < 400$ in GeV^2

$\chi^2_{\text{d. o. f.}} = 1.7$



Example: Froissart bound

High-energy behaviour of total cross-sections in two-particle scattering

Heisenberg 1952:

A target hadron is surrounded by a pion field with energy density $\propto e^{-m_\pi r}$

Inelastic processes will occur when the collision is close enough to locally yield enough energy to create a pion pair

$\Rightarrow$

$$\sigma \propto \frac{1}{m_\pi^2} \ln^2 \frac{E}{m_\pi}$$

Example: Froissart bound

Froissart 1961:

At high energies, the total cross-section for two-particle scattering (protons) has an upper bound

$$\sigma \propto \ln^2 \frac{s}{s_0}$$

s centre-of-mass energy, s_0 energy scale

General argument based on unitarity of S matrix and
analyticity properties of the scattering amplitude

Example: Froissart bound

Froissart 1961:

At high energies, the total cross-section for two-particle scattering (protons) has an upper bound

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General argument based on unitarity of S matrix and analyticity properties of the scattering amplitude

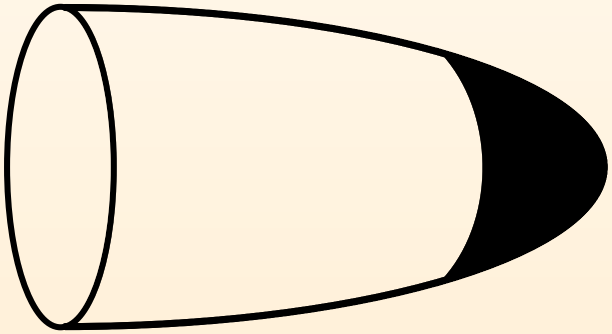
QCD considerations link the Froissart bound at high energies to the dynamics of ultra-soft gluons (strongly coupled)

Example: Froissart bound in gauge/gravity duality

Giddings Phys.Rev. D67 (2003) 126001; Kang, Nastase Phys.Rev. D72 (2005) 106003

AdS metric with IR cutoff ('hard wall'), point mass m is placed on this IR wall

This creates perturbations of the AdS space which may lead to the formation of a black hole in AdS space



Geometrical cross section of this black hole ⇔
maximum possible scattering cross section in the field theory

$$\sigma \leq \sigma_{\rm BH} = \pi r_h^2 \propto \ln^2 \frac{E}{E_0}$$

Subleading corrections to Froissart bound from AdS black holes

Diez, Godbole, SInha, Phys.Lett. B746 (2015) 285

Subleading corrections $\propto -\ln(s/s_0)$ and $\propto \ln s/s_0 \ln \ln s/s_0$,
from higher curvature corrections

improve fits to cosmic ray and LHC data

Subleading corrections to Froissart bound from AdS black holes

Diez, Godbole, SInha, Phys.Lett. B746 (2015) 285

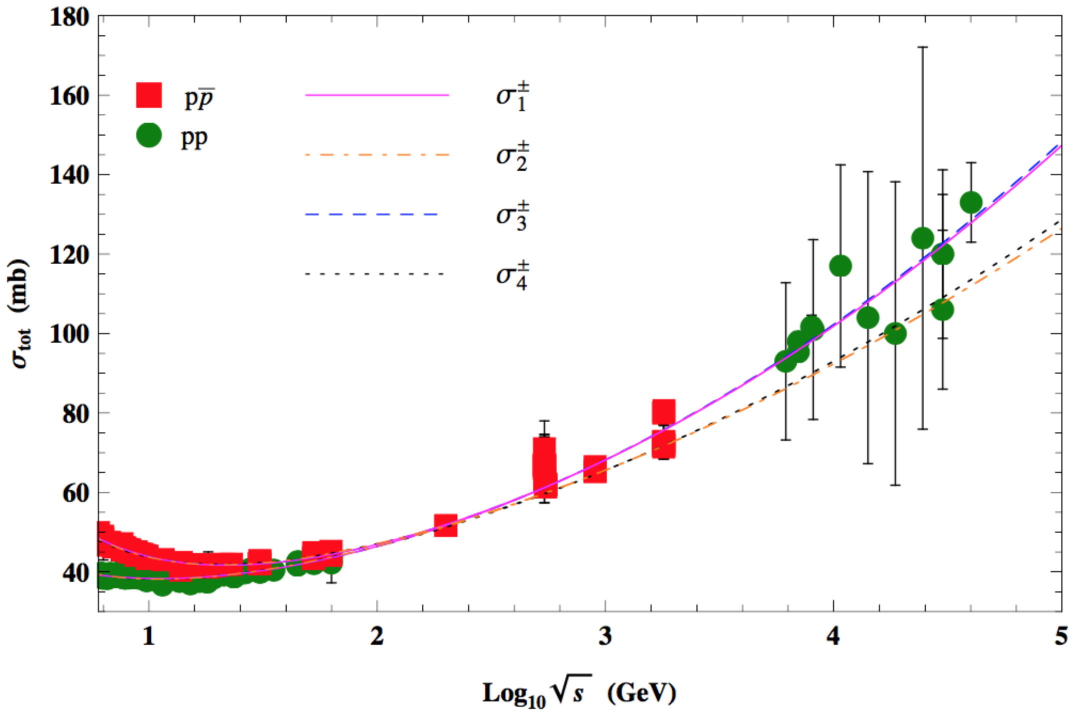
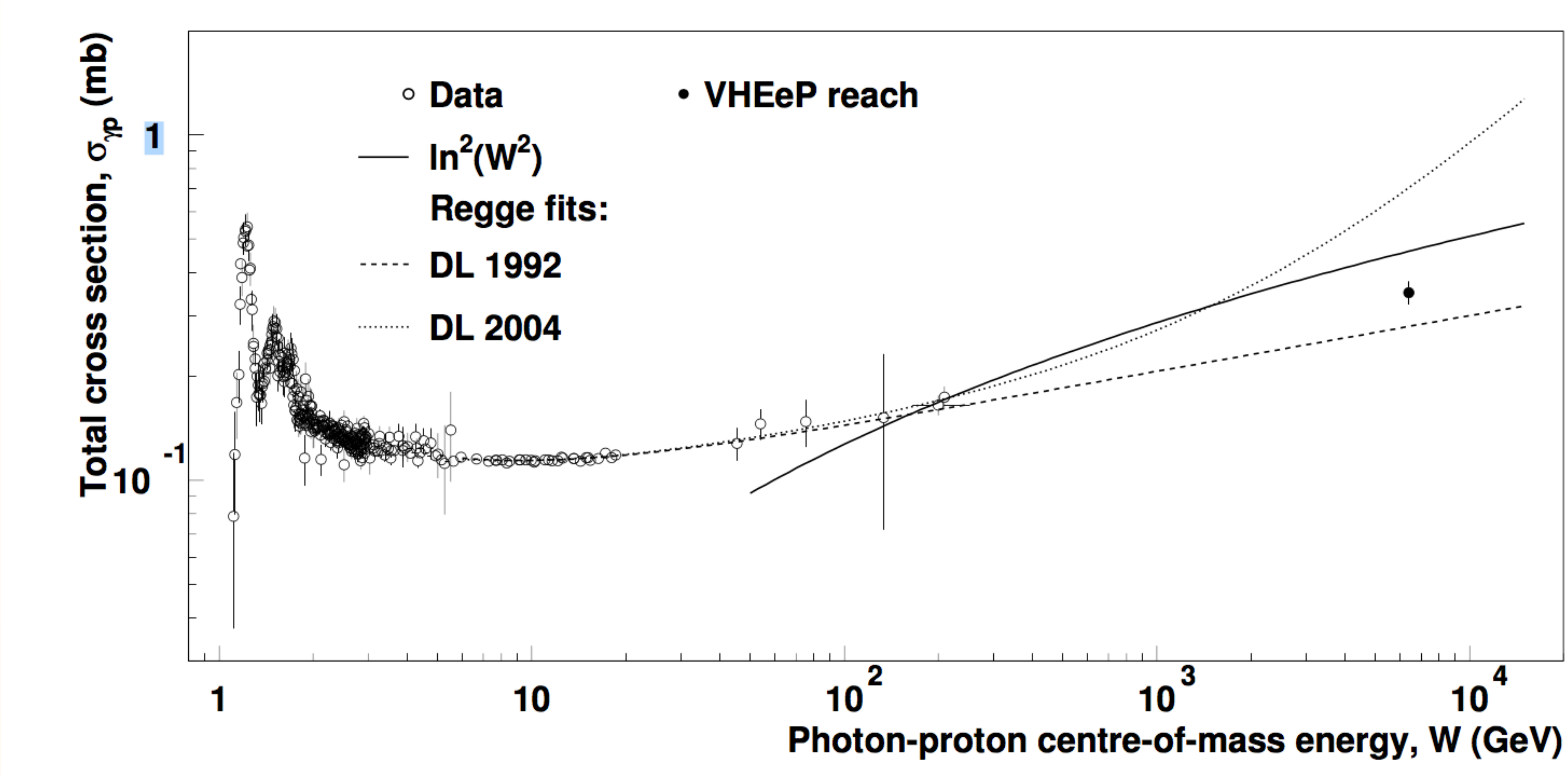


FIG. 1: (Colour online.) Fit results to experimental values of σ_{tot}^{pp} and $\sigma_{tot}^{p\bar{p}}$. The magenta solid, orange dot-dashed, blue dashed and black dotted curves are the (57)-(60) fits to the pp (green circles) and $p\bar{p}$ (red squares) data points, respectively. The data are from CDF, E710, E811, UA1, UA4, UA5 experiments [35–42]. The pp data points also include σ_{tot}^{pp} results from the LHC (at $\sqrt{s} = 7, 8$ TeV) [43–45] and cosmic-ray data [46].

Total γp cross section

Caldwell, Wing Eur.Phys.J. C76 (2016) 463



Coherence length dependence of cross section

First approach within gauge/gravity duality

Find argument based on black hole cross section

Conclusions and outlook

- Gauge/gravity duality provides a new approach to universality at strong coupling
- Examples:Deep inelastic scattering, pomeron, Froissart bound
- Techniques available to investigate saturation of $\sigma^{\gamma p}$ cross section at very high energies
- Much more to explore!