Hamiltonian, Quasilocal Energy and Boundary Terms in Kinetic Gravity Braiding

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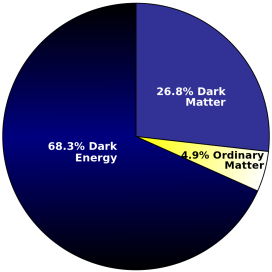
What is it?

What is dark energy?

Dark energy

- Expansion of the universe is accelerating



- Cosmological constant?
- Dynamical model? (Quintessence)
- k-essence? (kinetic quintessence)
  (Armendariz-Picon, Mukhanov, Steinhardt, 2001)

Kinetic Gravity Braiding

- "Imperfect Dark Energy from Kinetic Gravity Braiding" (Deffayet, Pujolàs, Sawicki, Vikman, 2010)

$$S = \int_{\mathcal{M}} \mathrm{d}^4x\sqrt{-g}\left[\frac{1}{2}R + K(\phi, X) + G(\phi, X)\Box\phi\right]$$

(with $X = -\frac{1}{2}g^{\mu\nu}\nabla_\mu\phi\nabla_\nu\phi$)

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  - Crossing of boundary $w = -1$ (cosmological constant) without classical instabilities (phantom energy)
  - Violation of null energy condition by a physical plausible theory

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  - Crossing of boundary $w = -1$ (cosmological constant) without classical instabilities (phantom energy)
  - Violation of null energy condition by a physical plausible theory
- Usually, $\ddot{\phi}$ in the action leads to $\dddot{\phi}$ in equation of motion (eom)
  $\rightarrow$ Hamiltonian not bounded from below
- In "kinetic gravity braiding theories" (KGB) : mixing of 2nd derivatives on the level of eom's

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Total energy?

Boundary Terms

- 2nd derivative in the action is problematic
- Variation of the normal derivative of ϕ?
- Similar case in the General Relativity (GR) action
- For total energy of system: useful to perform calculations with boundaries and later send them to infinity

Objective

Use appropriate boundary terms and derive the contribution to the total energy coming from the scalar sector of a system with Kinetic Gravity Braiding

Total energy in GR and the foliation of spacetime

- Total energy is an ambigious concept in GR
- Action is invariant under arbitrary coordinate transformations
  $$S_{\mathsf{G}} = \frac{1}{2}\int\limits_{\mathcal{M}} \mathrm{d}^4x\sqrt{-g}R + \oint\limits_{\partial\mathcal{M}} \mathrm{d}^3y\sqrt{|h|}\mathcal{K}$$
  (with trace of extrinsic curvature $\mathcal{K}$)
- No **global** Lorentz frame $\rightarrow$ no **global** time

Total energy in GR and the foliation of spacetime

- Foliation of spacetime

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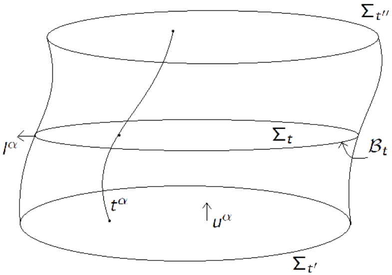

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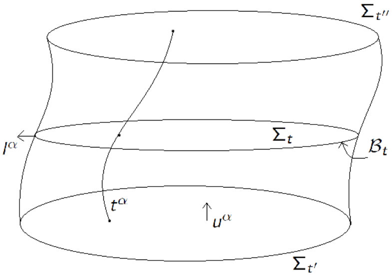
- Foliation of spacetime

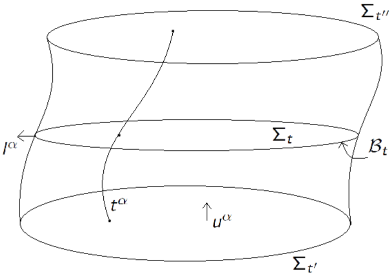


$$t^\alpha = N u^\alpha + N^a e^\alpha_a \ , \qquad a \in \{1,2,3\}$$

(with basis e^α_a and unit normal u^α on Σ_t)

Total energy in GR and the foliation of spacetime

- Foliation of spacetime



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- $g^{\mu\nu}$ is split into induced metric h_{ab} intrinsic to the hypersurface and into 4 foliation-dependent functions time lapse N and time shift N^a

Total energy in GR and the foliation of spacetime

- ADM mass

Total energy in GR and the foliation of spacetime

- ADM mass
  - If spacetime is asymptotically flat, we can choose boundary $\mathcal{B}_t$ at infinity to coincide with a 2-sphere of fixed Minkowski-time
  - And we set $N = 1$ and $N^a = 0$

Total energy in GR and the foliation of spacetime

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$$M_{\mathrm{ADM}} = -\lim_{\mathcal{B}_t \to \infty} \oint_{\mathcal{B}_t} \mathrm{d}^2\theta \sqrt{\sigma} \left(k - k_0\right)$$

- k (k_0) is the trace of the extrinsic curvature of $\mathcal{B}_t$ embedded in Σ_t (spatial slice of Minkowski spacetime)
- M_{ADM} contains entire information about the interior (gravitational energy of spacetime and matter energy)

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Hamiltonian in Kinetic Gravity Braiding

$$S = S_{\mathrm{G}} + \int_{\mathcal{M}} \mathrm{d}^4x \sqrt{-g}\, [K(X) + G(X)\Box\phi] - \oint_{\partial\mathcal{M}} \mathrm{d}^3y \sqrt{|h|} F$$

- Boundary term has been recently derived for a generic class of scalar-tensor-theories (Padilla, Sivanesan, 2012)

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$$E_{\mathrm{ADM}} \equiv - \lim_{\mathcal{B}_t\to\infty}\oint_{\mathcal{B}_t} \mathrm{d}^2\theta\sqrt{\sigma}\,(k - k_0 + G\phi_l - F^{\mathcal{B}} + F^{\mathcal{B}}_U\phi_u^2 - F^{\Sigma}_Y\phi_u\phi_l)$$

  (with $\phi_u \equiv \nabla_\mu\phi u^\mu$, $\phi_l \equiv \nabla_\mu\phi l^\mu$ and scalar functions $F^{\mathcal{B}}$, $F^{\mathcal{B}}_U$, F^{Σ}_Y coming from the boundary)
- Divergent for a generic scalar field configuration
- Check, if it is possible to find a configuration for which the expression for ADM energy is both applicable and finite!

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Quasilocal Energy (Brown, York, 1993)

Finite ADM energy

- Ansatz: static, spherically symmetric metric and scalar field

Finite ADM energy

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$$\rightarrow \left.\begin{matrix} K \sim \frac{2}{3}(-X)^{3/2} \\ G \sim 2\mu\sqrt{-X} \end{matrix}\right\} \qquad \text{for} \qquad r \to \infty$$

$$\partial_r \phi \sim -\frac{6\mu}{r} - \frac{6\mu m}{r^2} \qquad \text{for} \qquad r \to \infty$$

(with $\mu < 0$, $[\mu] = [\text{mass}]^{1/3}$)

- m is just the name of integration constant

Finite ADM energy

$$
\begin{aligned}
ds^2 \quad \approx \quad & -\left[1 - \frac{2m}{r} - \frac{\mu^3 m \cdot const}{r^2}\right] dt^2 \\
& + \left[1 + \frac{2m}{r} + \frac{4m^2 + \mu^3 m \cdot const}{r^2}\right] dr^2 \\
& + r^2 d\Omega^2 \qquad \text{for} \qquad r \to \infty
\end{aligned}
$$

Finite ADM energy

$$
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& + r^2 \mathrm{d}\Omega^2 \qquad \text{for} \qquad r \to \infty
\end{aligned}
$$

$$
\begin{aligned}
E_{\mathrm{ADM}} \quad & = \quad m + 72\sqrt{2}\pi|\mu|^3 \\
& > \quad m
\end{aligned}
$$

In theories with KGB it is possible for scalar to have contribution to total energy without contributing to leading order in metric

Thank You

Questions?

Quasilocal Energy

- Quasilocal Energy (Brown, York, 1993)

$$\lim_{\mathcal{B}_t\to\infty} E_{\mathrm{QL}} = - \lim_{\mathcal{B}_t\to\infty} \oint_{\mathcal{B}_t} \mathrm{d}^2\theta\sqrt{\sigma}\,\Big(k - k_0 + G\phi_l - F^{\mathcal{B}} + F^{\mathcal{B}}_u\phi_u^2 - \xcancel{F^{\Sigma}_Y\phi_u\phi_l}\Big)$$

- Differs from E_{ADM}, because time lapse function does not appear linearly in the boundary action for KGB