Kinetic Field Theory and Cosmic Fluctuation-Dissipation Relations

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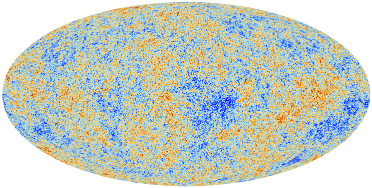
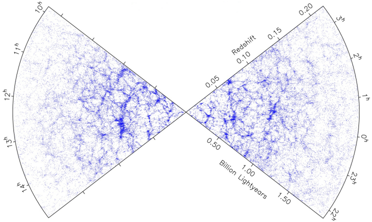
Cosmic Structure Formation



$\Rightarrow$



- System: Initially correlated, gravitationally interacting Dark Matter
- Conventional analytic approach: Hydrodynamic description
- Numerical approach: N-body Simulations
- New approach: Kinetic Field Theory

System

- $6N$-dimensional Phase-space: $\mathbf{x}$
- Gaussian initial conditions: $P(\mathbf{x}^{(\mathrm{i})}) \propto \exp(-\frac{1}{2}\vec{p}_j^{(\mathrm{i})} C^{-1}_{p_j p_k} \vec{p}_k^{(\mathrm{i})})$
- Hamiltonian equations of motion: $\mathbf{E}(\mathbf{x}) = 0$
- Transition probability:

$$T(\mathbf{x}(t), \mathbf{x}^{(\mathrm{i})}) = \int_{\mathbf{x}^{(\mathrm{i})}}^{\mathbf{x}(t)} \mathcal{D}\mathbf{x}\, \delta_D(\mathbf{E}(\mathbf{x})) = \int_{\mathbf{x}^{(\mathrm{i})}}^{\mathbf{x}(t)} \mathcal{D}\mathbf{x} \int \mathcal{D}\boldsymbol{\chi} \mathrm{e}^{\mathrm{i}\boldsymbol{\chi}\cdot\mathbf{E}(\mathbf{x})}$$

$$P(\mathbf{x}(t)) = \int \mathrm{d}\mathbf{x}^{(\mathrm{i})} P(\mathbf{x}^{(\mathrm{i})}) \int_{\mathbf{x}^{(\mathrm{i})}}^{\mathbf{x}(t)} \mathcal{D}\mathbf{x} \int \mathcal{D}\boldsymbol{\chi} \mathrm{e}^{\mathrm{i}\boldsymbol{\chi}\cdot\mathbf{E}(\mathbf{x})}$$

Generating Functional

- Generating Functional:

$$Z[\mathbf{J},\mathbf{K}] = \int \mathrm{d}\mathbf{x}^{(\mathrm{i})} P(\mathbf{x}^{(\mathrm{i})}) \int_{\mathbf{x}^{(\mathrm{i})}} \mathcal{D}\mathbf{x} \int \mathcal{D}\boldsymbol{\chi} \mathrm{e}^{\mathrm{i}\boldsymbol{\chi}\cdot\left(\mathbf{E}(\mathbf{x})+\mathbf{K}\right)+\mathrm{i}\mathbf{x}\cdot\mathbf{J}}$$

$$\Rightarrow \langle \mathbf{x}(t)\rangle \rightarrow \frac{\delta}{\mathrm{i}\delta \mathbf{J}(t)}, \qquad \langle \boldsymbol{\chi}(t)\rangle \rightarrow \frac{\delta}{\mathrm{i}\delta \mathbf{K}(t)}$$

- Field operators:

$$\rho_j(t,\vec{k}) = \exp\left(-\mathrm{i}\vec{k}\cdot\vec{q}_j(t)\right) \quad \rightarrow \quad \hat{\rho}_j(t,\vec{k}) = \exp\left(-\mathrm{i}\vec{k}\cdot\frac{\delta}{\mathrm{i}\delta \vec{J}_{q_j}(t)}\right)$$

$$\hat{B}_j(t,\vec{k}) = \left(\mathrm{i}\vec{k}\cdot\frac{\delta}{\mathrm{i}\delta \vec{K}_{p_j}(t)}\right)\hat{\rho}_j$$

Perturbation Theory

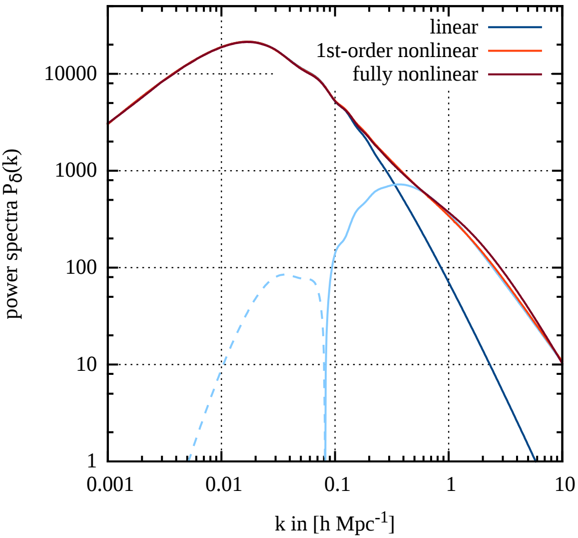
- Perturbative expansion:

$$Z[\mathbf{J},\mathbf{K}] = \mathrm{e}^{\mathrm{i}\hat{S}_I} Z_0[\mathbf{J},\mathbf{K}] = \Big(1 + \mathrm{i}\hat{S}_I + \dots\Big) Z_0[\mathbf{J},\mathbf{K}]$$

- Interaction Operator:

$$\hat{S}_I = -\int \mathrm{d}t \int \mathrm{d}^3\vec{k}\, \hat{B}(t,-\vec{k}) v(\vec{k}) \hat{\rho}(t,\vec{k})$$

Non-linear Power Spectrum



Fluctuation-Dissipation Relations

- Linear Response Theory:

$$\chi(t,t') = -\beta\Theta(t-t')\frac{\partial}{\partial t}\langle\mathcal{O}(t)\mathcal{O}(t')\rangle_0.$$

- Kinetic Field Theory:

$$\left\langle B_j(t')\rho_j(t)\right\rangle_0 = \mathrm{i}\frac{3}{\sigma_1^2}\Theta(t-t')\frac{\partial}{\partial t}\left\langle\rho_j(t')\rho_j(t)\right\rangle_0$$

Time-Reversal Symmetry

- Symmetry of the generating Functional:

$$\mathcal{T}: \begin{cases} \vec{q}_j(t) & \rightarrow \quad \vec{q}_j(-t), \\ \vec{p}_j(t) & \rightarrow \quad -\vec{p}_j(-t), \\ \vec{\chi}_{q_j}(t) & \rightarrow \quad -\vec{\chi}_{q_j}(-t), \\ \vec{\chi}_{p_j}(t) & \rightarrow \quad \vec{\chi}_{p_j}(-t) - \mathrm{i} C^{-1}_{p_j p_k} \vec{p}_k(-t) - \vec{c}_j[\mathbf{J}]. \end{cases}$$

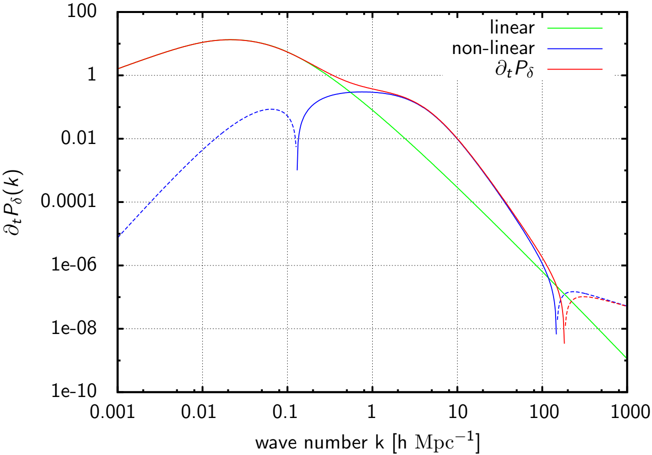
- General 'FDR-type' relations:

$$\langle B\rho \ldots \rho \rangle_0 \sim \partial_t \langle \rho \ldots \rho \rangle_0 + \text{correction terms}$$

$$\langle B \ldots B\rho \ldots \rho \rangle_0 \sim \partial_t \ldots \partial_t \langle \rho \ldots \rho \rangle_0 + \text{correction terms}$$

Cosmological Application

- Time evolution of the Power Spectrum



Summary

- FDR in initially correlated systems
- Time-reversal symmetry
- Time evolution of the Power Spectrum