Breakdown of Perturbation Theory in Multi-Photon-Annihilation Processes

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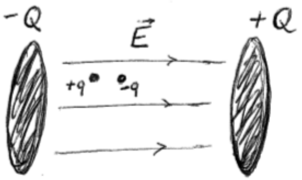

Figure : Pair-creation in electric field **E** of capacitor plates

e^+e^--creation in electric fields

Long-standing prediction

Rate for e.g. **E** $= const$:

$\Gamma_{pair}(E) \sim e^{-\frac{m^2}{gE}}$

Relevance in physics

- **Prototype** for particle production in bosonic backgrounds
- Accessible in **lab**

Common theoretical treatment

Electric field as (classical) **background** $\mathbf{E}_B$

→ Idea:

Quantum-resolution of $\mathbf{E}_B$

e.g. in laser: $n\gamma \rightarrow e^+e^-$

Coherent State

Overview Calculations

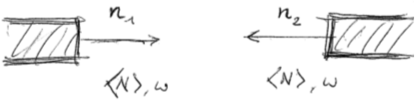

Figure : Experimental setup: 2 colliding **laser** beams, monochromatic

$\langle N \rangle$: mean occupation

ω: photon energy

n: number of photons annihilated

$n_1 + n_2 = n$

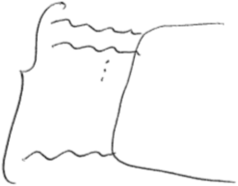

Figure : Leading order perturbation theory terms

Quantities computed:

$\Gamma_{tree}(n_1, n_2 \rightarrow e^+e^-)$

Treatment as **scattering**

Photons on-shell

Collinearity-effects negligible

Loss of Perturbative Unitarity

Generic to all processes $n_1, n_2 \to e^+ e^-$:

$$\Gamma_{tree} \sim n! \left(1 + \mathcal{O}\left(\frac{1}{n}\right)\right)$$

- For n sufficiently large, perturbation theory breaks down for **all** parameter-values α, m, ω, V.
- Onset of unlimited growth:

$$n_* \sim \frac{1}{\alpha} V m^2 \omega$$

α: coupling
V: volume
m: electron mass

- 2nd case of lost tree-level unitarity in **weakly** coupled regime of SM besides known case of Multi-Higgs/-W production[1] $q\bar{q} \to nh + mV$.

[1] **Goldberg (1990)**, **Khoze et al. (2016)**.

Relating to Experiment: Coherent State

Consider initial **coherent** superposition

$$|z\rangle \equiv \exp\left(\sum_s \int_k z(\mathbf{k}, s)\hat{a}^\dagger(\mathbf{k}, s) + h.c.\right)|0\rangle$$

with spectrum

$$z(\mathbf{k}, s) = \sqrt{\langle N\rangle}\left(\delta^3\left(\mathbf{k} - \omega\mathbf{e}_z\right) + \delta^3\left(\mathbf{k} + \omega\mathbf{e}_z\right)\right)\delta_{s+}.$$

$\Rightarrow$ Associated mean electric field:

$$\langle\hat{\mathbf{E}}\rangle = \sqrt{\frac{\langle N\rangle\omega}{V}}\cos(\omega z)\begin{pmatrix}\cos(\omega t)\\ \sin(\omega t)\\ 0\end{pmatrix}$$

Rate from Coherent State

$n_1 + n_2 = n$

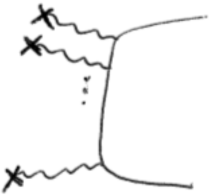

Figure : Coherent state acts like a background field

e^+e^--creation rate from coherent state $\Gamma_{coh} = f(\omega, \langle E \rangle)$ is **weighted** sum of **individual rates**:

$$\Gamma_{coh} \equiv \sum_n^{\infty} \left(\sum_{n_1=1}^{n-1} \frac{\langle N \rangle^n}{n_1! n_2!} \Gamma_{tree}\left(n_1, n_2 \to e^+e^-\right) \right) \equiv \sum_n^{\infty} a_n \alpha^n$$

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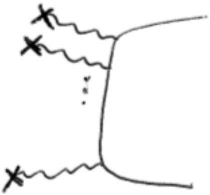

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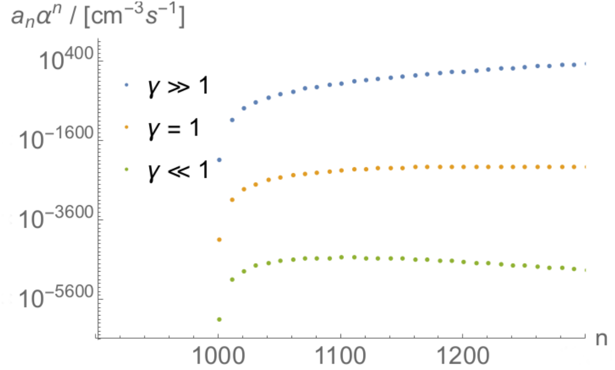
Transition with Parameters

n-**scaling** of series terms in $\Gamma_{coh} \equiv \sum_n^\infty a_n \alpha^n$:

$$a_n\alpha^n \sim r^n \left(1 + \mathcal{O}(1/n)\right), \qquad r \sim \frac{\alpha\langle N\rangle}{Vm^2\omega} \sim \left(\frac{g\langle E\rangle}{m\omega}\right)^2 \equiv \gamma^2$$

$\Rightarrow$ **Perturbativity** depends on values of parameters: $\gamma \div 1$.



Plot parameters:

- $2m/\omega = 1000$
- $\langle E\rangle$: 3 different values

Figure : Scaling of series terms $a_n\alpha^n$ with n

Comparison Background-Field Result

For **background**-field (equal to above mean field $\langle \hat{\mathbf{E}} \rangle$ of coherent state in anti-nodes)

$$\mathbf{E}_B = E \begin{pmatrix} \cos(\Omega t) \\ \sin(\Omega t) \\ 0 \end{pmatrix},$$

non-perturbative result[2] for rate $\Gamma_B(E, \Omega)$ is:

$$\Gamma_B(E,\Omega) \sim (gE)^2 \begin{cases} e^{-\frac{m^2}{gE}} & \text{for} \quad \gamma \gg 1 \\ \left(\frac{gE}{m\Omega}\right)^{2\frac{2m}{\Omega}} & \text{for} \quad \gamma \ll 1, \end{cases} \qquad \gamma \equiv \frac{gE}{m\Omega}$$

interpolates between multi-photon- and non-perturbative regime.
$\rightarrow$ **This transition qualitatively captured by Γ_{coh}.**

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[2] **H.Gies et al. (2014)**.

Quantum Corrections?

Figure : Terms entering background-field treatment

No Quantum corrections from **coherent** state

- Coherent superposition generates same weights as background
- $|i\rangle \neq |z\rangle$ **will** give Quantum corrections

Truncations different

- $n' \neq 0$: Stimulated-emission-type processes
- Our truncation: $n' = 0$

Summary and Outlook

Loss of perturbative unitarity

for elementary processes $n_1\gamma, n_2\gamma \to e^+e^-$
Constitutes 2nd occurrence in **weakly** coupled regime of SM
besides known case $q\overline{q} \to nh + mV$

Coherent state

- Coherent state generating same terms as background-field
- **Smaller truncation** able to **capture** qualitatively transition between multi-photon- and NP regime

Quantum Corrections

Calculation of $\Gamma_{tree}(n_1, n_2 \to e^+e^-)$ provides basis for finding Quantum corrections at **tree**-level:
Arising from **initial state's deviation** from coherent state

Backup: Quantitative Comparison

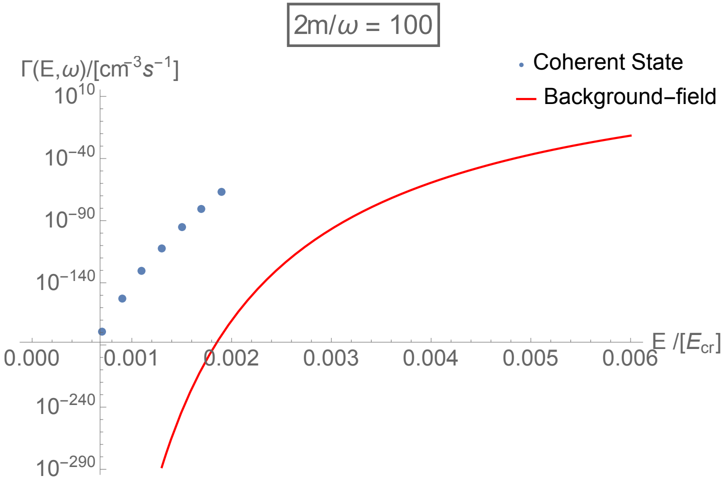

Figure : Scaling of e^+e^--creation rate Γ with field strength E (in units of $E_{cr} \equiv m^2/g$)