pNRQCD determination of E1 radiative transitions

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Outline

1. Motivation & introduction
2. Potential non-relativistic QCD (pNRQCD)
3. Decay widths of electric dipole transitions
4. Numerical results
5. Summary & outlook

Motivation & introduction

- Most of the observable matter is bound state matter (atoms, molecules, hadrons) and ...
- ... a lot of it is non-relativistic (atoms, molecules, heavy quarkonium like $c\bar{c}$ or $b\bar{b}$).

Relevant scales in non-relativistic systems:

$E \sim \frac{\vec{p}^{\,2}}{2m} \sim V \sim mv^2$ and $p \sim \frac{1}{r} \sim mv$

If $v \ll 1$, then $m \gg mv \gg mv^2$ (hierarchy of scales $\Rightarrow$ EFT)

Hierarchy of scales for heavy quarkonia:

- m (hard scale) QCD ($\rightarrow$ NRQCD)
- $p \sim mv$ (soft scale) NRQCD ($\rightarrow$ pNRQCD)
- $E \sim mv^2$ (ultra soft scale) pNRQCD

Motivation & introduction

- EFTs are the tools of choice for systematic computations. In QCD:
  - we have a variety of EFTs depending on physical processes (χPT, HQET, SCET, NRQCD, pNRQCD, etc.).
  - we want to focus on a heavy quark-antiquark bound state (heavy quarkonium) especially bottomonium ($b\bar{b}$, where $m \equiv \overline{m}_b(\overline{m}_b) \sim 4.18$ GeV and $v^2 \sim 0.1 \ll 1$).
- Non-relativistic QCD (NRQCD)[1]:
  - arises from QCD by integrating out $m \Rightarrow \frac{1}{m}$ expansion in the Lagrangian.
  - hierarchy of scales: $m \gg p \sim mv, \Lambda_{\mathrm{QCD}}$.
  - d.o.f.: heavy quark and antiquark, light quarks, soft gluons.
- Potential NRQCD (pNRQCD)[2]:
  - arises from NRQCD by integrating out $p \sim mv \sim \frac{1}{r} \Rightarrow$ multipole expansion in the Lagrangian.
  - hierarchy of scales: $m \gg p \sim mv \gg E \sim mv^2$.
  - d.o.f.: quark-antiquark color singlet S and color octet O states, light quarks, ultra-soft gluons.

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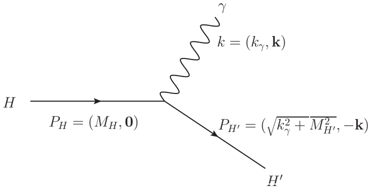
[1] Caswell and Lepage. *Phys. Lett.* B167 (1986); Bodwin, Braaten, and Lepage. *Phys. Rev.* D51 (1995). arXiv: 9407339 [hep-ph].

[2] Pineda and Soto. *Nucl. Phys. Proc. Suppl.* 64 (1998). arXiv: 9707481 [hep-ph]; Brambilla et al. *Nucl. Phys.* B566 (2000). arXiv: 9907240 [hep-ph].

Motivation & introduction



$$\Leftrightarrow \quad n\,^{2s+1}\ell_J \to n'\,^{2s'+1}\ell'_{J'} + \gamma$$

Why to study radiative decays?

- They are one of the possible ways to access heavy quarkonium states.
- In particular, they are one of the simplest transitions to be measured for quarkonia below open-flavor threshold.
- They provide a probe of the internal structure of hadrons.

$$\Gamma_{E1}^{(0)} = \frac{4}{9}\alpha_{e/m} e_Q^2 k_\gamma^3 \left[\int_0^\infty \mathrm{d}r\, r^2 R_{n'0}(r)\; r \; R_{n1}(r)\right]^2 \qquad \Delta\ell = 1, \Delta s = 0$$

$$\Gamma_{M1}^{(0)} = \frac{4}{3}\alpha_{e/m} e_Q^2 \frac{k_\gamma^3}{m^2} \qquad \Delta\ell = 0, \Delta s = 1$$

Motivation & introduction

Experimental branching ratios - PDG[3]

Mode	Fraction (Γ_i/Γ)
$\chi_{b0}(1P) \to \Upsilon(1S) + \gamma$	$(1.76 \pm 0.35)\%$
$\chi_{b1}(1P) \to \Upsilon(1S) + \gamma$	$(33.9 \pm 2.2)\%$
$\chi_{b2}(1P) \to \Upsilon(1S) + \gamma$	$(19.1 \pm 1.2)\%$
$h_b(1P) \to \eta_b(1S) + \gamma$	$(52^{+6}_{-5})\%$

The decay width in pNRQCD up to $\mathcal{O}(v^2)$ corrections

$$\Gamma_{n\,{}^3P_J \to n'\,{}^3S_1} = \Gamma^{(0)}_{E1}\Bigg[1 + R^{S=1}(J) - \frac{k_\gamma}{6m} - \frac{k_\gamma^2}{60}\frac{I_5^{(0)}(n1\to n'0)}{I_3^{(0)}(n1\to n'0)}$$

$$+\left(\frac{J(J+1)}{2} - 2\right)\left(-\left(1+\kappa_Q^{em}\right)\frac{k_\gamma}{2m} + \frac{1}{m^2}\left(1+2\kappa_Q^{em}\right)\frac{I_2^{(1)}(n1\to n'0) + 2I_1^{(0)}(n1\to n'0)}{I_3^{(0)}(n1\to n'0)}\right)\Bigg]$$

$$I_N^{(k)}(n\ell \to n'\ell') = \int_0^\infty \mathrm{d}r\, r^2 r^{N-2} R_{n'\ell'}\left(\frac{\mathrm{d}^k}{\mathrm{d}r^k} R_{n\ell}\right)$$

[3] Olive et al. *Chin. Phys.* C38 (2014)

pNRQCD - the Lagrangian at weak coupling

- Hierarchy of scales: $m \gg p \sim mv \gg E \sim mv^2$.
- At weak coupling ($\Lambda_{QCD} \leq mv^2$) the d.o.f. are color singlet S and color octet O configurations of the heavy $q\bar{q}$-pair. The effective Lagrangian is organized as an expansion in $\frac{1}{m}$ (NRQCD) and $r \sim \frac{1}{p}$ (pNRQCD):

$$\mathcal{L}_{\text{pNRQCD}} = \int \mathrm{d}^3r \, \text{tr}\Big\{ \text{S}^\dagger \Big(\mathrm{i}\partial_0 - \frac{\vec{p}^{\,2}}{m} + \cdots - V_s \Big) \text{S} + \text{O}^\dagger \Big(\mathrm{i}D_0 - \frac{\vec{p}^{\,2}}{m} + \cdots - V_o \Big) \text{O} \Big\}$$

$$-\frac{1}{4} F_{\mu\nu,a} F^{\mu\nu,a} + \sum_{i=1}^{n_f} \bar{q}_i \mathrm{i} \not{D} q_i + \Delta\mathcal{L} + \Delta\mathcal{L}_\gamma$$

$$\Delta\mathcal{L} = \int \mathrm{d}^3r \;\; V_A \, \text{tr}\{ \text{O}^\dagger \vec{r} \cdot g\vec{E} \text{S} + h.c. \} + \frac{V_B}{2} \text{tr}\{ \text{O}^\dagger \vec{r} \cdot g\vec{E} \text{O} + c.c. \} + \ldots$$

$$\Delta\mathcal{L}_\gamma = e e_Q \int \mathrm{d}^3r \, V^{r\cdot E} \, \text{tr}\{ \text{S}^\dagger \, \vec{r} \cdot \vec{E}^{\,e/m} \, \text{S} \} + V_o^{r\cdot E} \, \text{tr}\{ \text{O}^\dagger \vec{r} \cdot \vec{E}^{\,e/m} \text{O} \} + \ldots$$

- Power counting: $p = -\mathrm{i}\nabla_r \,, \frac{1}{r} \sim mv \,; \quad \partial_0 \,, P = -\mathrm{i}\nabla_R \,, \frac{1}{R} \,, A_\mu \sim mv^2 \,;$

$$E \,, B \sim (mv^2)^2 \,; \quad E^{e/m} \,, B^{e/m} \sim k_\gamma^2 \,; \quad k_\gamma \sim mv^2$$

- At strong coupling ($\Lambda_{QCD} \geq mv^2$) one only has color singlets.

pNRQCD - the e.o.m. and its solution

The singlet static potential is given by $V_s = -C_F \frac{\alpha_s(\nu)}{r} \left[1 + \sum_{k=1}^{\infty} \left(\frac{\alpha_s(\nu)}{4\pi}\right)^k a_k(\nu, r)\right]$

$$a_1(\nu, r) = a_1 + 2\beta_0 \ln(\nu e^{\gamma_E} r) \quad \text{(NLO)}$$

$$a_2(\nu, r) = a_2 + \frac{\pi^2}{3}\beta_0^2 + (4a_1\beta_0 + 2\beta_1)\ln(\nu e^{\gamma_E} r) + 4\beta_0^2 \ln^2(\nu e^{\gamma_E} r) \quad \text{(NNLO)}$$

The Schrödinger equation

$$\left(\frac{-1}{2m_\mathrm{r}}\nabla^2 + V_s^{(0)}(r)\right)\psi_{n\ell m}^{(0)}(\vec{r}) = E_n^{(0)}\psi_{n\ell m}^{(0)}(\vec{r})$$

with Coulombic bound state (pNRQCD at weak coupling) solutions:

$$\psi_{n\ell m}^{(0)}(\vec{r}) = R_{nl}(r) Y_{\ell m}(\Omega_r) = N_{n\ell}\, e^{-\frac{\rho_n}{2}} \rho_n^{\ell} L_{n-\ell-1}^{2\ell+1}(\rho_n) Y_{\ell m}(\Omega_r)$$

$$E_n^{(0)} = -\frac{m_\mathrm{r} C_F^2 \alpha_s^2(\nu)}{2n^2}, \quad \rho_n = \frac{2r}{na}, \quad a = \frac{1}{m_\mathrm{r} C_F \alpha_s(\nu)}$$

pNRQCD - corrections to the wave function

We can organize the NNLO relativistic corrections to the wave functions[4] as:

$$\delta H = -\frac{\nabla^4}{4m^3} + \frac{V^{(1)}}{m} + \frac{V_{SI}^{(2)}}{m^2} + \frac{V_{SD}^{(2)}}{m^2}$$

$$V_{SI}^{(2)} = V_r^{(2)} + \frac{1}{2}\{V_{p^2}^{(2)}, -\nabla^2\} + V_{L^2}^{(2)}\vec{L}^2$$

$$V_{SD}^{(2)} = V_{LS}^{(2)}\vec{L}\cdot\vec{S} + V_{S^2}^{(2)}\vec{S}^2 + V_{S_{12}}^{(2)}S_{12}$$

where:

$$V^{(1)} = -\frac{C_F C_A \alpha_s^2(\nu)}{2r^2}\,, \quad V_r^{(2)} = \pi C_F \alpha_s(\nu)\delta^{(3)}(\vec{r})\,,$$

$$V_{p^2}^{(2)} = -\frac{C_F\alpha_s(\nu)}{r}\,, \quad V_{L^2}^{(2)} = \frac{C_F\alpha_s(\nu)}{2r^3}\,, \quad V_{LS}^{(2)} = \frac{3C_F\alpha_s(\nu)}{2r^3}\,,$$

$$V_{S^2}^{(2)} = \frac{4\pi C_F\alpha_s(\nu)}{3}\delta^{(3)}(\vec{r})\,, \quad V_{S_{12}}^{(2)} = \frac{C_F\alpha_s(\nu)}{4r^3}$$

with:

$$\vec{p} = -\mathrm{i}\vec{\nabla}\,, \quad \vec{L} = \vec{r}\times\vec{p}\,, \quad \vec{S} = \vec{S}_1 + \vec{S}_2\,, \quad \vec{S} = \frac{\vec{\sigma}}{2}\,,$$

$$\vec{L}\cdot\vec{S} = \frac{1}{2}(\vec{J}^2 - \vec{L}^2 - \vec{S}^2)\,, \quad S_{12} = 3(\hat{r}\cdot\vec{\sigma}_1)(\hat{r}\cdot\vec{\sigma}_2) - \vec{\sigma}_1\cdot\vec{\sigma}_2$$

[4] Brambilla, Pietrulewicz, and Vairo. *Phys. Rev.* D85 (2012). arXiv: 1203.3020.

pNRQCD - Perturbation theory

Corrections to the wave functions

$$|n\ell\rangle^{(1)} = \sum_{n'\neq n} \frac{\langle n'\ell'|V|n\ell\rangle}{E_n^{(0)} - E_{n'}^{(0)}}|n'\ell'\rangle = \sum_{n'\neq n} \frac{|n'\ell'\rangle\langle n'\ell'|}{E_n^{(0)} - E_{n'}^{(0)}} V|n\ell\rangle\,, \qquad |n\ell\rangle^{(2)} = \ldots$$

$$\sum_{n'\neq n} \frac{|n'\ell'\rangle\langle n'\ell'|}{E_n^{(0)}-E_{n'}^{(0)}} = \sum_{n'} \frac{|n'\ell'\rangle\langle n'\ell'|}{E_n^{(0)}-E_{n'}^{(0)}} - \sum_{n'=n} \frac{|n'\ell'\rangle\langle n'\ell'|}{E_n^{(0)}-E_{n'}^{(0)}} = \lim_{E\to E_n^{(0)}} \left(\frac{\mathbb{1}}{E-H} - \frac{\mathcal{M}(n)}{E-E_n^{(0)}}\right) \equiv \frac{1}{(E_n-H)'}$$

The Coulomb green function

$$G'(\vec{r}_1,\vec{r}_2) \equiv (-1)\times \lim_{E\to E_n} \left(G(\vec{r}_1,\vec{r}_2,E) - \frac{|\psi_{n\ell}|^2}{E-E_n}\right)$$

- $G(\vec{r}_1,\vec{r}_2,E) = \sum_{\ell=0}^{\infty} \frac{2\ell+1}{4\pi} P_\ell(\hat{r}_1\cdot\hat{r}_2) G_\ell(r_1,r_2)$

  $G_\ell(r_1,r_2) = \sum_{\nu=\ell+1}^{\infty} G_{\nu\ell}(r_1,r_2)$

  $G_{\nu\ell}(r_1,r_2) = m_{\mathrm{r}} a^2 \left(\frac{\nu^4}{\lambda}\right) \frac{R_{\nu\ell}(\rho_{\lambda,1})R_{\nu\ell}(\rho_{\lambda,2})}{\nu-\lambda}$
- $E = -\frac{m_r C_F^2\alpha_s^2}{2\lambda^2} = E_n(1-\epsilon)\big|_{\epsilon\to 0} = -\frac{m_r C_F^2\alpha_s^2}{2n^2}(1-\epsilon)\Big|_{\epsilon\to 0} \quad\Rightarrow\quad \lambda = \frac{n}{\sqrt{1-\epsilon}}\Big|_{\epsilon\to 0}$

pNRQCD - Perturbation theory - an Example

$$\langle n'\ell'|\mathcal{O}|n\ell\rangle^{(1)} = \langle n'\ell'|\mathcal{O}\frac{1}{(E_n - H)'}V|n\ell\rangle$$

$$= \int \mathrm{d}^3 r_1\, \mathrm{d}^3 r_2\, \psi^*_{n'\ell'}(\vec{r}_2)\, \mathcal{O}(\vec{r}_2)\, G'(\vec{r}_2, \vec{r}_1)\, V(\vec{r}_1)\, \psi_{n\ell}(\vec{r}_1)$$

$$= \lim_{\epsilon\to 0, \mathcal{O}(\epsilon^0)} -N_{n'\ell'} N_{n\ell} \frac{4m_\mathrm{r}}{a\lambda} \sum_{\ell''=0}^{\infty} \sum_{s=0}^{\infty} \frac{s!}{(s+\ell''+1-\lambda)(s+2\ell''+1)!}$$

$$\times \int_0^\infty \mathrm{d}r_1\, r_1^2 V(r_1) \rho_{\lambda,1}^{\ell''} \rho_{n,1}^{\ell} \mathrm{e}^{-\frac{1}{2}\rho_{\lambda,1}} \mathrm{e}^{-\frac{1}{2}\rho_{n,1}} L_s^{2\ell''+1}(\rho_{\lambda,1}) L_{n-\ell-1}^{2\ell+1}(\rho_{n,1})$$

$$\times \int_0^\infty \mathrm{d}r_2\, r_2^2 \mathcal{O}(r_2) \rho_{\lambda,2}^{\ell''} \rho_{n',2}^{\ell'} \mathrm{e}^{-\frac{1}{2}\rho_{\lambda,2}} \mathrm{e}^{-\frac{1}{2}\rho_{n',2}} L_s^{2\ell''+1}(\rho_{\lambda,2}) L_{n'-\ell'-1}^{2\ell'+1}(\rho_{n',2})$$

$$\times \sum_{m''=-\ell''}^{\ell''} \int \mathrm{d}\Omega_1\, Y_{\ell''}^{m''*}(\Omega_1) V(\Omega_1) Y_\ell^m(\Omega_1) \int \mathrm{d}\Omega_2\, Y_{\ell'}^{m'*}(\Omega_2) \mathcal{O}(\Omega_2) Y_{\ell''}^{m''}(\Omega_2)$$

- We calculate with $\lambda = n$ the finite contribution, which means performing the sum in s without the pole ($s \neq n - \ell'' - 1$)
- We compute the divergent term of the sum using $\lambda = \frac{n}{\sqrt{1-\epsilon}}$, expand in ϵ and finally pick up the finite term only ($\mathcal{O}(\epsilon^0)$ when $\epsilon \to 0$).

The tensor potential: s-d-wave mixing

- $V_T(\vec{r}) = \frac{1}{m^2}\frac{C_F\alpha_s}{4r^3} S_{12}(\hat{r}) \equiv V_{S_{12}} S_{12}(\hat{r})$ mixes states with $\Delta\ell = 2$, since $S_{12}(\hat{r}) = 3(\hat{r}\cdot\vec{\sigma}_1)(\hat{r}\cdot\vec{\sigma}_2) - \vec{\sigma}_1\cdot\vec{\sigma}_2 = 3\sqrt{5}\left\{\{\hat{r}\otimes\hat{r}\}_2 \otimes \{\vec{\sigma}_1\otimes\vec{\sigma}_2\}_2\right\}_0$
- diagonal part is included in the spectrum[5], perturbative treatment of off-diagonal matrix elements is derived in[6]

$$\langle n'\ell's'J'm_{J'}|S_{12}|n\ell sJm_J\rangle = 3\delta_{J'J}\delta_{m_{J'}m_J}(-1)^{J+\ell+s'}\begin{Bmatrix}\ell' & \ell & 2\\ s & s' & J\end{Bmatrix}\langle\ell'||\{\hat{r}\otimes\hat{r}\}_2||\ell\rangle\langle s'||\{\vec{\sigma}_1\otimes\vec{\sigma}_2\}_2||s\rangle$$

using the Wigner-Eckart theorem we find

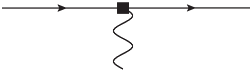
$$\langle n'\ell's'J'm_{J'}|S_{12}|n\ell sJm_J\rangle = \delta_{J'J}\delta_{m_{J'}m_J}\delta_{s's}\delta_{s1}(-1)^{J+s'}2\sqrt{30}\sqrt{(2\ell+1)(2\ell'+1)}\begin{Bmatrix}\ell' & \ell & 2\\ s & s' & J\end{Bmatrix}\begin{pmatrix}\ell' & 2 & \ell\\ 0 & 0 & 0\end{pmatrix},$$

where $\begin{pmatrix}\ell' & 2 & \ell\\ 0 & 0 & 0\end{pmatrix} \neq 0 \quad \Leftrightarrow \quad \Delta\ell = 2$

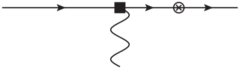
[5] Peset, Pineda, and Stahlhofen. *JHEP* 05 (2016). arXiv: 1511.08210.

[6] Kiyo, Mishima, and Sumino. *Phys. Lett.* B761 (2016). arXiv: 1607.05510.

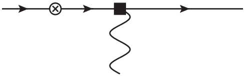
Decay widths of electric dipole transitions



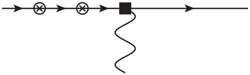
$$\propto |\,^{(0)}\langle n', \ell', s', J'; \gamma | \mathcal{O}_{E1} | n, \ell, s, J; 0 \rangle^{(0)}|^2 \to \Gamma_{E1}^{(0)}$$



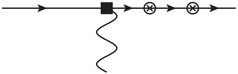
$$\propto |\,^{(1)}\langle n', \ell', s', J'; \gamma | \mathcal{O}_{E1} | n, \ell, s, J; 0 \rangle^{(0)}|^2 \to \Gamma_{E1}^{(1,2)}$$



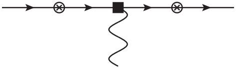
$$\propto |\,^{(0)}\langle n', \ell', s', J'; \gamma | \mathcal{O}_{E1} | n, \ell, s, J; 0 \rangle^{(1)}|^2 \to \Gamma_{E1}^{(1,2)}$$



$$\propto |\,^{(0)}\langle n', \ell', s', J'; \gamma | \mathcal{O}_{E1} | n, \ell, s, J; 0 \rangle^{(2)}|^2 \to \Gamma_{E1}^{(2)}$$



$$\propto |\,^{(2)}\langle n', \ell', s', J'; \gamma | \mathcal{O}_{E1} | n, \ell, s, J; 0 \rangle^{(0)}|^2 \to \Gamma_{E1}^{(2)}$$



$$\propto |\,^{(1)}\langle n', \ell', s', J'; \gamma | \mathcal{O}_{E1} | n, \ell, s, J; 0 \rangle^{(1)}|^2 \to \Gamma_{E1}^{(2)}$$

Decay widths of electric dipole transitions

$$\Gamma = \frac{k_\gamma}{(2\pi)} \overline{|\mathcal{M}_{fi}|^2} = \frac{k_\gamma}{(2\pi)} \frac{1}{N_\lambda} \sum_{\lambda,\lambda',\sigma} |\mathcal{M}_{fi}|^2$$

$$\mathcal{O}_{E1} = ee_Q(\vec{r} \cdot \vec{E}^{\,e/m}) = ee_Q(\hat{e}_r \cdot \vec{E}^{\,e/m}) \sum_{\mu=-1}^{1} \sqrt{\frac{4\pi}{3}} r Y_1^{\mu *}(\Omega_r)$$

$$\Phi^{(0)}_{n^3P_0}(\mathbf{r}) = \sqrt{\frac{1}{4\pi}}\, R_{n1}(r)\, \frac{\vec{\sigma}\cdot\vec{r}}{\sqrt{2}}, \qquad \Phi^{(0)}_{n^3P_1(\lambda)}(\mathbf{r}) = \sqrt{\frac{3}{8\pi}}\, R_{n1}(r)\, \frac{\vec{\sigma}\cdot(\vec{r}\times\vec{e}_{n^3P_1}(\lambda))}{\sqrt{2}},$$

$$\Phi^{(0)}_{n^3P_2(\lambda)}(\mathbf{r}) = \sqrt{\frac{3}{4\pi}}\, R_{n1}(r)\, \frac{\vec{\sigma}^i\, h^{ij}_{n^3P_2}(\lambda)\, \hat{r}^j}{\sqrt{2}}, \qquad \Phi^{(0)}_{n^3S_1(\lambda)}(\mathbf{r}) = \frac{1}{\sqrt{4\pi}}\, R_{n0}(r)\, \frac{\vec{\sigma}\cdot\vec{e}_{n^3S_1}(\lambda)}{\sqrt{2}},$$

where the polarization vectors and tensors are normalized as:

$$\vec{e}^{\,*}_{n^3S_1}(\lambda)\cdot\vec{e}_{n^3S_1}(\lambda') = \vec{e}^{\,*}_{n^1P_1}(\lambda)\cdot\vec{e}_{n^1P_1}(\lambda') = \vec{e}^{\,*}_{n^3P_1}(\lambda)\cdot\vec{e}_{n^3P_1}(\lambda') = \delta_{\lambda\lambda'}$$

$$h^{ij\,*}_{n^3P_2}(\lambda)\, h^{ji}_{n^3P_2}(\lambda') = \delta_{\lambda\lambda'}$$

Numerical results

$$n_f = 3, \quad e_Q = -\frac{1}{3}, \quad \alpha_{e/m} = \frac{1}{137.0359991}$$

state	$\Upsilon(1S)$	$\chi_{b_0}(1P)$	$\chi_{b_1}(1P)$	$\chi_{b_2}(1P)$	$\eta_b(1S)$	$h_b(1P)$
exp. masses[7] [GeV]	9.46030	9.85944	9.89278	9.91221	9.398	9.899

$$k_\gamma = \frac{m_i^2 - m_f^2}{2m_i} = \begin{cases} 391.1\,\text{MeV}\,, & \chi_{b0}(1P) \to \Upsilon(1S) + \gamma \\ 423.0\,\text{MeV}\,, & \chi_{b1}(1P) \to \Upsilon(1S) + \gamma \\ 441.6\,\text{MeV}\,, & \chi_{b2}(1P) \to \Upsilon(1S) + \gamma \\ 488.3\,\text{MeV}\,, & h_b(1P) \to \eta_b(1S) + \gamma \end{cases}$$

$$M^{\text{exp.}}(\Upsilon(1S)) = 2m_b + E_n^{(0)}(m_b, \alpha_s(\nu)) \quad (\Upsilon\text{-scheme})$$

$$a(\nu) = \frac{1}{C_F m_r \alpha_s(\nu)}, \quad \kappa_Q^{em} \equiv C_F^{e/m} - 1 = C_F \frac{\alpha_s(m_b)}{2\pi} \sim v^2 \text{ (beyond NNLO)}$$

$\nu\,[GeV]$	1.0	1.5	2.0	2.5	3.0
$\alpha_s^{4\text{ loops},\ n_f=3}(\nu)$[8]	0.479778	0.345836	0.295478	0.265205	0.245092
$a(\nu)\,[GeV^{-1}]$	0.627151	0.885286	1.0524	1.17705	1.27659
$m_b\,[GeV]$	4.98515	4.86138	4.82374	4.80525	4.79415

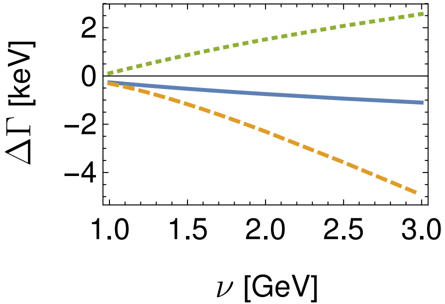
[7] Olive et al. *Chin. Phys.* C38 (2014)

[8] Chetyrkin, Kühn, and Steinhauser. *Comput. Phys. Commun.* 133 (2000). arXiv: 0004189 [hep-ph]

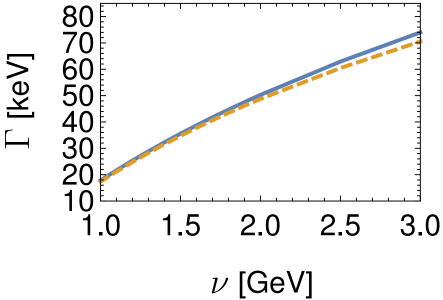
Relativistic corrections to the Lagrangian ($\chi_{b1} \to \Upsilon + \gamma$)

$$\Gamma_{n^3P_J \to n'^3S_1} = \Gamma^{(0)}_{E_1}\Bigg[1 + R^{S=1}(J) - \frac{k_\gamma}{6m} - \frac{k_\gamma^2}{60}\frac{I_5^{(0)}(n1 \to n'0)}{I_3^{(0)}(n1 \to n'0)}$$
$$+ \left(\frac{J(J+1)}{2} - 2\right)\left(-\left(1 + \cancel{\kappa^{em}_Q}\right)\frac{k_\gamma}{2m} + \frac{1}{m^2}\left(1 + 2\cancel{\kappa^{em}_Q}\right)\frac{I_2^{(1)}(n1 \to n'0) + 2I_1^{(0)}(n1 \to n'0)}{I_3^{(0)}(n1 \to n'0)}\right)\Bigg]$$

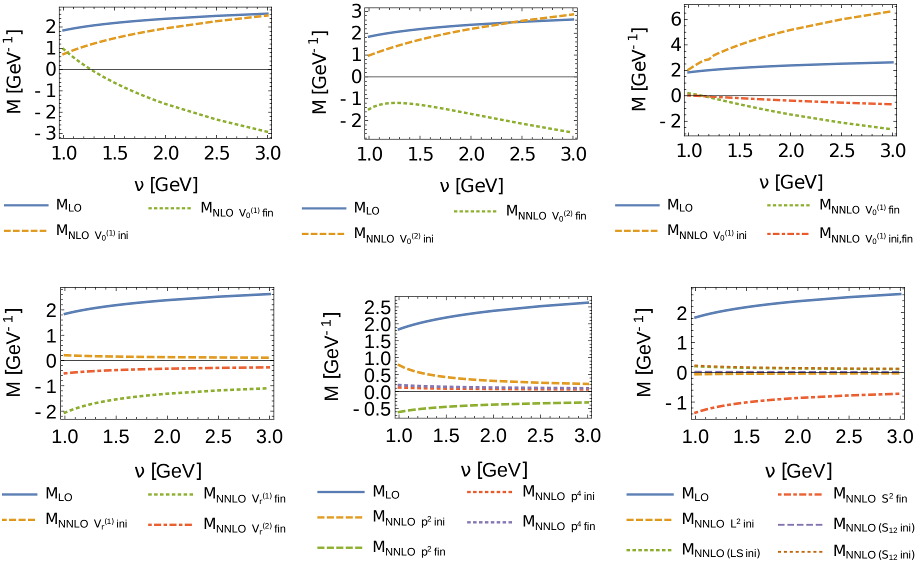
individual contributions:



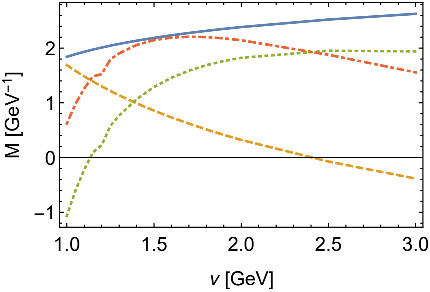
combined effect:

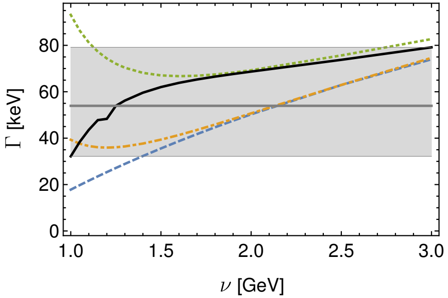


Relativistic wave function corrections ($\chi_{b1} \to \Upsilon + \gamma$)



Total matrix elements and decay widths ($\chi_{b1} \to \Upsilon + \gamma$)





- Sizable scale dependence induced by the running of $\alpha_s(\nu)$ and by the radiative corrections to the static potential.
- Final value taken at $\nu = 1.25$ GeV ($\frac{1}{a} = \frac{mC_F\alpha_s(\frac{1}{a})}{2}$).
- Uncertainty band is fully dominated by the scale-dependence.

Final results - comparison and predictions

Mode	LO	NLO	NNLO	CQM[9]
$\chi_{b0}(1P) \to \Upsilon(1S) + \gamma$	21	29	45^{+20}_{-18}	28.07
$\chi_{b1}(1P) \to \Upsilon(1S) + \gamma$	27	36	54^{+25}_{-22}	35.66
$\chi_{b2}(1P) \to \Upsilon(1S) + \gamma$	31	41	55^{+27}_{-24}	39.15
$h_b(1P) \to \eta_b(1S) + \gamma$	42	55	125^{+3}_{-27}	43.7

$\Rightarrow$ LO and NLO results agree with the constituent quark model.

Mode	Fraction $\mathcal{B} = \frac{\Gamma_i}{\Gamma}$ [PDG]	Total width Γ [keV]
$\chi_{b0}(1P) \to \Upsilon(1S) + \gamma$	$(1.76 \pm 0.35)\%$	$(2.6^{+1.3}_{-1.1}) \cdot 10^3$
$\chi_{b1}(1P) \to \Upsilon(1S) + \gamma$	$(33.9 \pm 2.2)\%$	159^{+75}_{-65}
$\chi_{b2}(1P) \to \Upsilon(1S) + \gamma$	$(19.1 \pm 1.2)\%$	290^{+142}_{-124}
$h_b(1P) \to \eta_b(1S) + \gamma$	$(52^{+6}_{-5})\%$	240^{+28}_{-57}

$\Rightarrow$ Large ($\sim 50\%$) uncertainties due to the scale dependence!

[9] Segovia et al. *Phys. Rev.* D93 (2016). arXiv: 1601.05093

The RGI approach

Back to the singlet static potential: $V_s = -C_F \frac{\alpha_s(\nu)}{r} \left[1 + \sum_{k=1}^{\infty} \left(\frac{\alpha_s(\nu)}{4\pi}\right)^k a_k(\nu, r)\right]$

The Schrödinger equation

$$\left(\frac{-1}{2m_{\mathrm{r}}}\nabla^2 + V_s(r)\right)\psi^{(0)}_{n\ell m}(\vec{r}) = E^{(0)}_n \psi^{(0)}_{n\ell m}(\vec{r})$$

- The logs in the $a_k(\nu, r)$ steam from soft gluons and should be resummed[10].
- This then incorporates the correct short distance behavior of the potential and is achieved by substituting $\nu \to \frac{1}{r}$ for $r < \nu_r^{-1}$ and also $\alpha_s(\nu) \to \alpha_{V_s}(\frac{1}{r}) \equiv \alpha_s(\frac{1}{r}) \left[1 + \sum_{k=1}^{\infty} \left(\frac{\alpha_s(\frac{1}{r})}{4\pi}\right)^k a_k(\frac{1}{r}, r)\right]$
- We now need to solve the Schrödinger equation numerically!
- Not treating the logs as perturbations may reduce the scale dependence and including RGI should improve the results further (success in M1 transitions[11]).

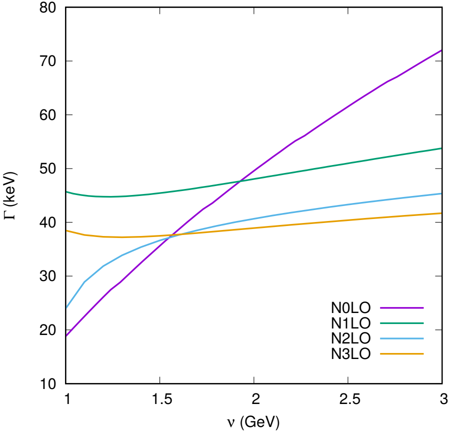
[10] Brambilla et al. *Rev. Mod. Phys.* 77 (2005). arXiv: 0410047 [hep-ph].

[11] Brambilla, Jia, and Vairo. *Phys. Rev.* D73 (2006). arXiv: 0512369 [hep-ph]; Pineda and Segovia. *Phys. Rev.* D87 (2013). arXiv: 1302.3528.

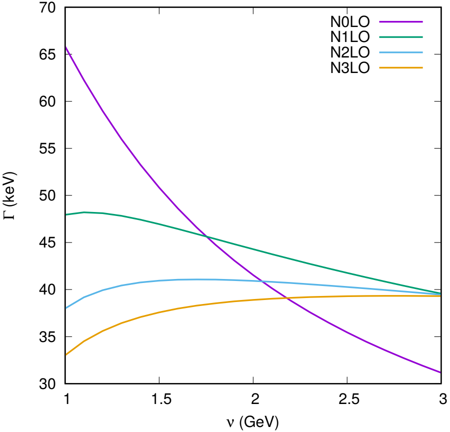
Preliminary results for the LO decay width

$$\chi_{b1} \to \Upsilon + \gamma :$$

$\nu_r = \infty$



$\nu_r = 1$ GeV



- Good and fast convergence.
- Heavily reduced scale dependence.
- Including RGI improves the convergence even more.

Summary & outlook

Summary

- We presented numerical results for the E1 transitions $2\,^3P_{J=0,1,2} \to 1\,^3S_1 + \gamma$ and $2\,^1P_1 \to 1\,^1S_0 + \gamma$ up to NNLO ($\mathcal{O}(v^2)$) in pNRQCD at weak coupling.
- The results are reasonable but a non-negligible dependence on the scale ν is observable via:
  - Direct ν dependence induced by the corrections to the static potential.
  - Indirect ν dependence via $\alpha_s(\nu)$ and $a(\alpha_s(\nu))$.
- Incorporating the full static potential in the leading order + RGI diminishes the scale dependence strongly.

Outlook

- Computing the relativistic corrections to the initial and final state wave functions is work in progress.
- Non-perturbative effects might be of the same size as the perturbative corrections and should be incorporated.

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BACKUP SLIDES

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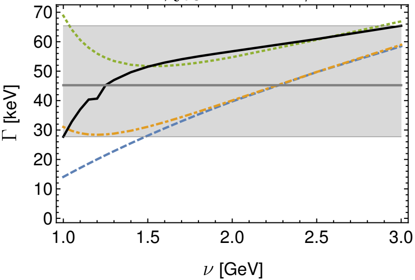
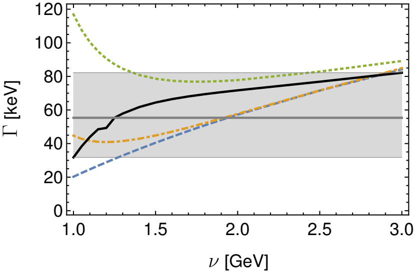
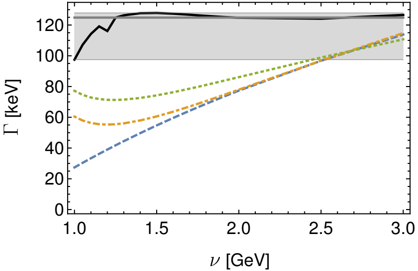
$\chi_{b0} \to \Upsilon + \gamma$:



$\chi_{b2} \to \Upsilon + \gamma$:



$h_b \to \eta_b + \gamma$:



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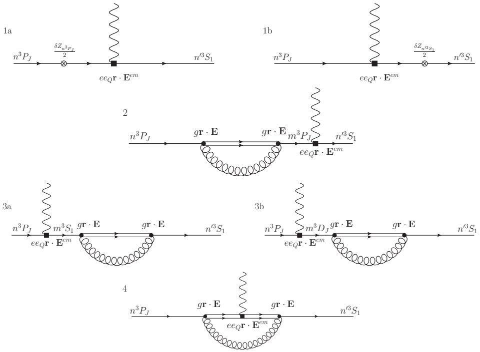
Non-perturbative color octet effects

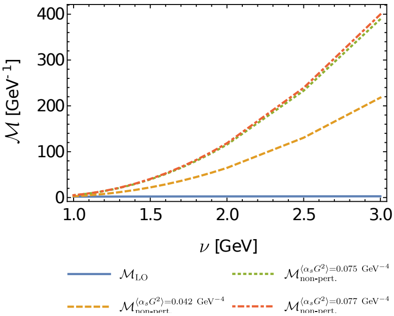


- A higher order Fock state consists of a quark and antiquark pair in a color octet configuration and at least one gluon: $\Delta\mathcal{L} \ni \int \mathrm{d}^3r\, V_A\, \mathrm{tr}\{\mathrm{O}^\dagger \vec{r}\cdot g\vec{E}\mathrm{S} + h.c.\}$
- $E \leq \Lambda_{\mathrm{QCD}}$: these contributions vanish (Λ_{QCD} integrated out).
- $E \gg \Lambda_{\mathrm{QCD}}$: contribution at NNLO; the chromo-electric correlator reduces to the two gluon condensate which factorizes.

$$\mathcal{M}^{\text{fig.},1a+1b}_{n\,^3P_J\to n'\,^3S_1+\gamma} = \frac{\mathcal{M}^{(0)}_{n\,^3P_J\to n'\,^3S_1+\gamma}}{12}\int_0^\infty \mathrm{d}t\, t\, \langle \mathrm{vac}|gE^{a\,i}(\vec{R},t)\phi(t,0)^{\mathrm{adj}}_{ab}gE^{b\,i}(\vec{R},0)|\mathrm{vac}\rangle$$
$$\times\left[{}^{(0)}\langle n\,^3P_J|r^j \mathrm{e}^{-\mathrm{i}(H^{(0)}_{o}-E^{(0)}_{n1})t} r^j|n\,^3P_J\rangle^{(0)} + {}^{(0)}\langle n'\,^3S_1|r^j \mathrm{e}^{-\mathrm{i}(H^{(0)}_{o}-E^{(0)}_{n'0})t} r^j|n'\,^3S_1\rangle^{(0)}\right]$$

Non-perturbative color octet effects

$$\mathcal{M}^{\text{fig.},1a+1b}_{n\,^3P_J\to n'\,^3S_1+\gamma} \simeq \mathcal{M}^{(0)}_{n\,^3P_J\to n'\,^3S_1+\gamma}\,\frac{1}{2}\left(-\frac{1}{6}\right)\langle \text{vac}|g^2E^{a\,i}\delta_{ab}E^{b\,i}|\text{vac}\rangle$$
$$\times\left[{}^{(0)}\langle n\,^3P_J|rG_o\mathbb{1}\,G_o r|n\,^3P_J\rangle^{(0)} + {}^{(0)}\langle n'\,^3S_1|rG_o\mathbb{1}\,G_o r|n'\,^3S_1\rangle^{(0)}\right]$$



- We know[12]:
  ${}^{(0)}\langle n|rG_o\mathbb{1}\,G_o r|n\rangle^{(0)} \sim \left(\frac{n}{\alpha_s}\right)^6$
  $\Rightarrow$ restriction to lowest lying states
  $\Rightarrow$ expect very strong scale dependence
- $\langle \text{vac}|g^2E^2|\text{vac}\rangle = -\pi\langle \text{vac}|\alpha_s G^2|\text{vac}\rangle$
  $\Rightarrow$ additional enhancement
- Non-perturbative contribution exceeds the LO by orders of magnitude. It is only smaller than the LO for $\nu \lesssim 1.1$ GeV.
  $\Rightarrow$ We did not incorporate this contribution in the analysis!

[12] Voloshin. *Nucl. Phys.* B154 (1979); Leutwyler. *Phys. Lett.* B98 (1981)