Models of the signal response from SiPMs and PMTs

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cta cherenkov telescope array
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PMT response model

Photo-conversion and electron collection
Conversion of photons in electrons and subsequent collection by the dynode system is a random binary process: Poisson distribution

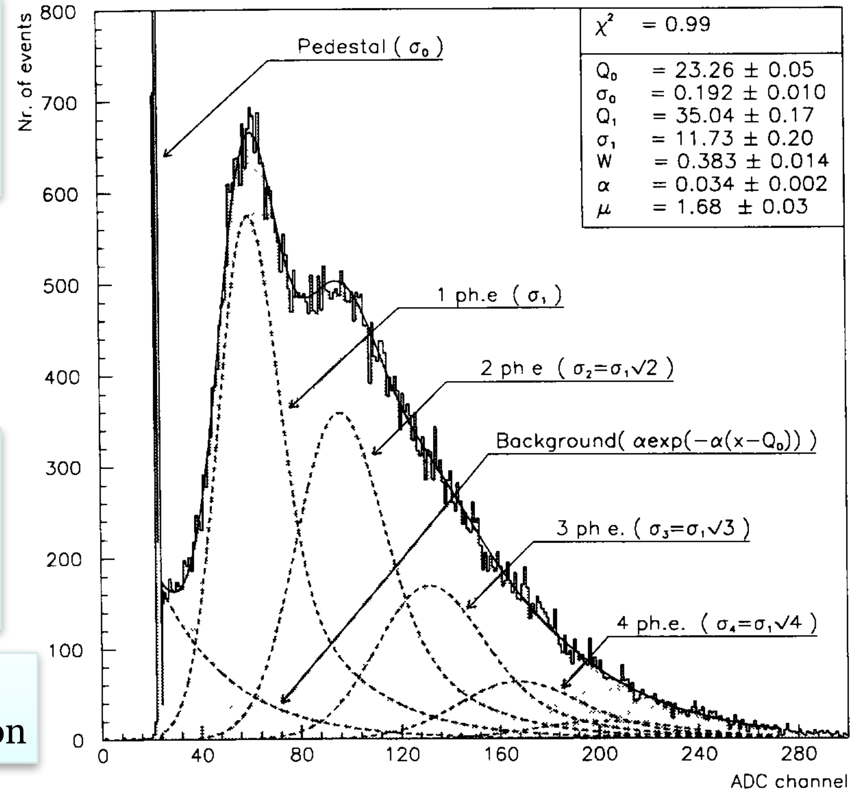
$$S_{\text{ideal}}(x) = P(n;\mu) \otimes G_n(x) = \sum_{n=0}^{\infty} \frac{\mu^n e^{-\mu}}{n!} \frac{1}{\sigma_1\sqrt{2\pi n}} \exp\left(-\frac{(x-nQ_1)^2}{2n\sigma_1^2}\right)$$

Amplification
The response of a multiplicative dynode system to a single photon electron can be approximated by a Gaussian distribution

Low charge processes
No photoelectron are emitted (Pedestal) - Gaussian distribution

$$B(x) = \frac{(1-w)}{\sigma_0\sqrt{2\pi}} \exp\left(-\frac{x^2}{2\sigma_0^2}\right) + w\theta(x)\alpha \exp(-\alpha x)$$

Discrete Processes
thermo-emission from photocathode or dynodes
Exponential function



PMT response function (charge distribution)
Convolution of these 2 functions
But …

Bellamy et. al. (1994) Absolute calibration and monitoring of a spectrometric channel using a photomultiplier

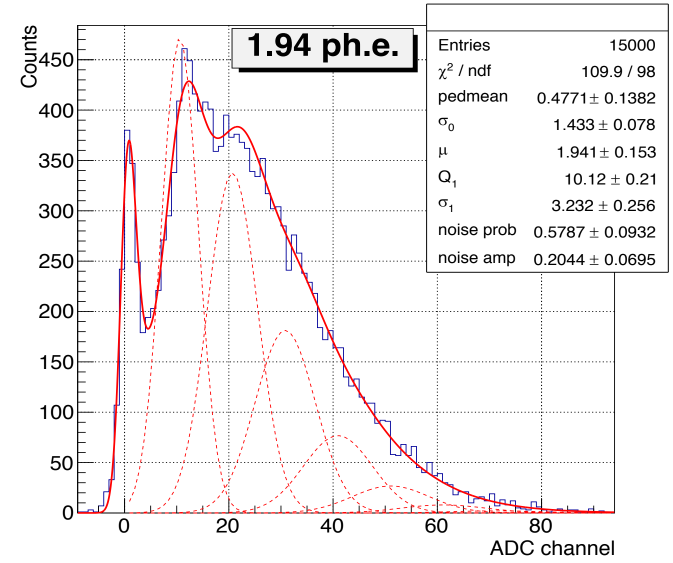
Tests of R12992-100 PMTs

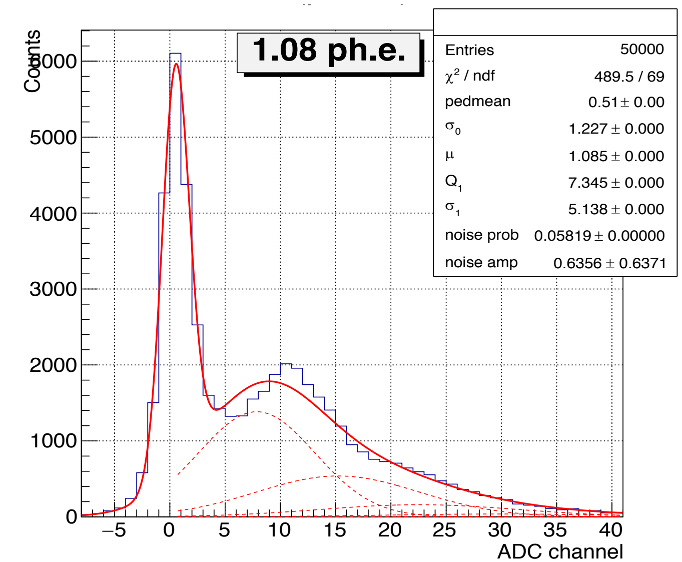
Controversial fit results:

1. When using custom amplifier resulting in high electronics noise (wide pedestal):
   - Stable good quality fit
   - No exp noise
2. When using FlashCam electronics resulting in low electronics noise and good signal-pedestal separation
   - Unstable fit with poor quality sometime
   - Probability of exp noise jumps from measurement to measurement for the same PMT
   - Pedestal taking in outside of the signal region reveals no exp noise

Reason:

Exp noise in fit just mimics another effect – non Gaussian shape of the single photo electron (s.p.e.) distribution



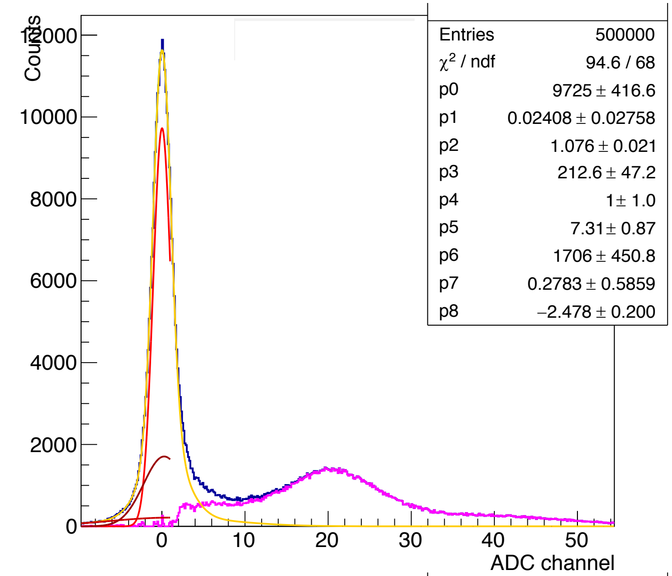


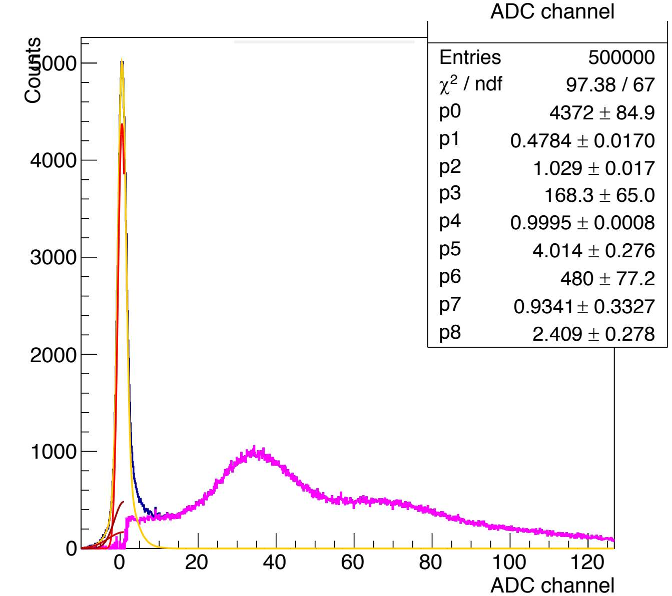
PMT response model

- No exp noise
- Long tail on the left side of s.p.e. distribution
- Pedestal described by more than one gauss

To be implemented in the fit model:

- S.p.e. distribution as a combination of constant and gauss functions
- Multiple p.e. as simple gauss functions using sigma and mean derived from s.p.e distribution





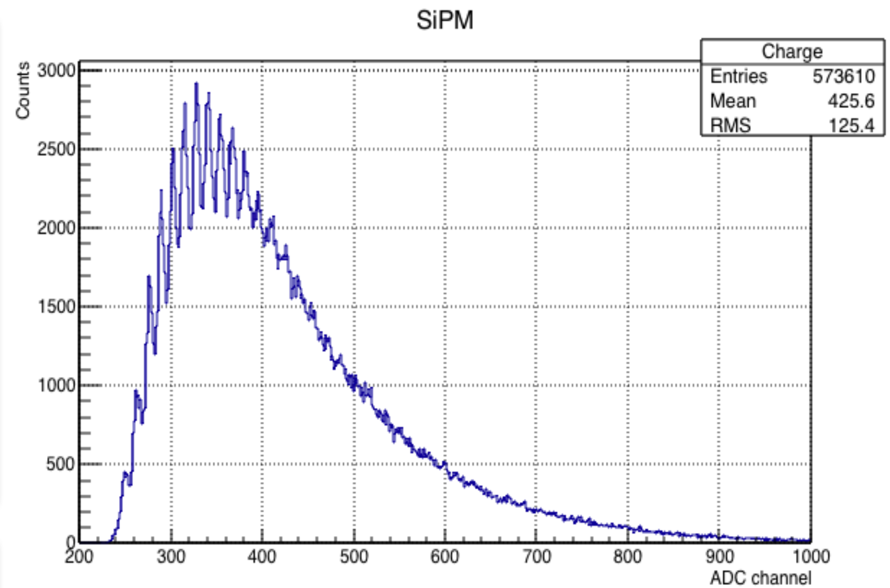
SiPM response model

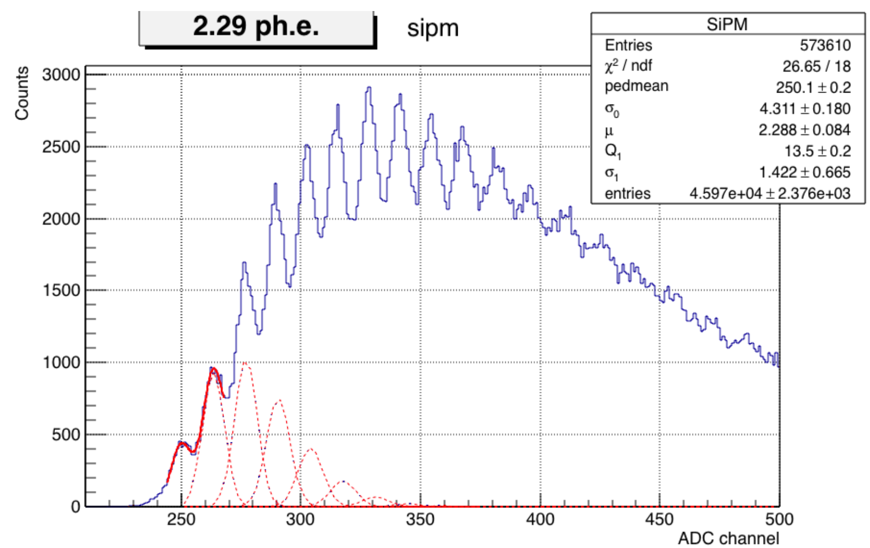
- The same basic model: Poisson+Gauss
- Cross-talk as probabilities *P0, P1, …, Pm* that 1 p.e. produces *0, 1, …, m-1* additional p.e.
- Number of p.e. from Poisson changed with cross-talk probabilities using multinomial distribution

SiPM measured with CTA TARGET ASIC

Expected μ – mean #p.h. easily estimated from k=0 of the Poisson $N_k = N\mu^k e^{-\mu}/k!$
$\mu = -\log(N_0/N) = 4.77$ ph

Fit without cross-talk in the range 0-1 p.e. leads to much smaller μ





Multinomial distribution

$$f(x_1,\ldots,x_k;n,p_1,\ldots,p_k) = \Pr(X_1 = x_1 \text{ and } \ldots \text{ and } X_k = x_k)$$
$$= \begin{cases} \dfrac{n!}{x_1!\cdots x_k!} p_1^{x_1} \times \cdots \times p_k^{x_k}, & \text{when } \sum_{i=1}^{k} x_i = n \\ 0 & \text{otherwise,} \end{cases}$$

n – number of p.e. in Poisson

$x_1, \ldots, x_k$ => $0, \ldots, k\text{-}1$ – number of cross-talk electrons

$p_1, \ldots, p_k$ $(\Sigma p_i=1)$ – probabilities of cross-talks

Example:

Poisson probabilities P_0, P_1, P_2

probabilities of cross-talks $p_0=0.5, p_1=0.2, p_2=0.2, p_3=0.1$

#p.e.	0	1	2	3	4	5	6	7	8
P_0	P_0								
$P_1 \times$		0.5	0.2	0.2	0.1				
$P_2 \times$			0.25	0.2	0.24	0.18	0.08	0.04	0.01

Satisfying of the condition $\Sigma p_i=1$

Redefinition of probabilities of cross-talk in fit as following (each in range [0-1]):

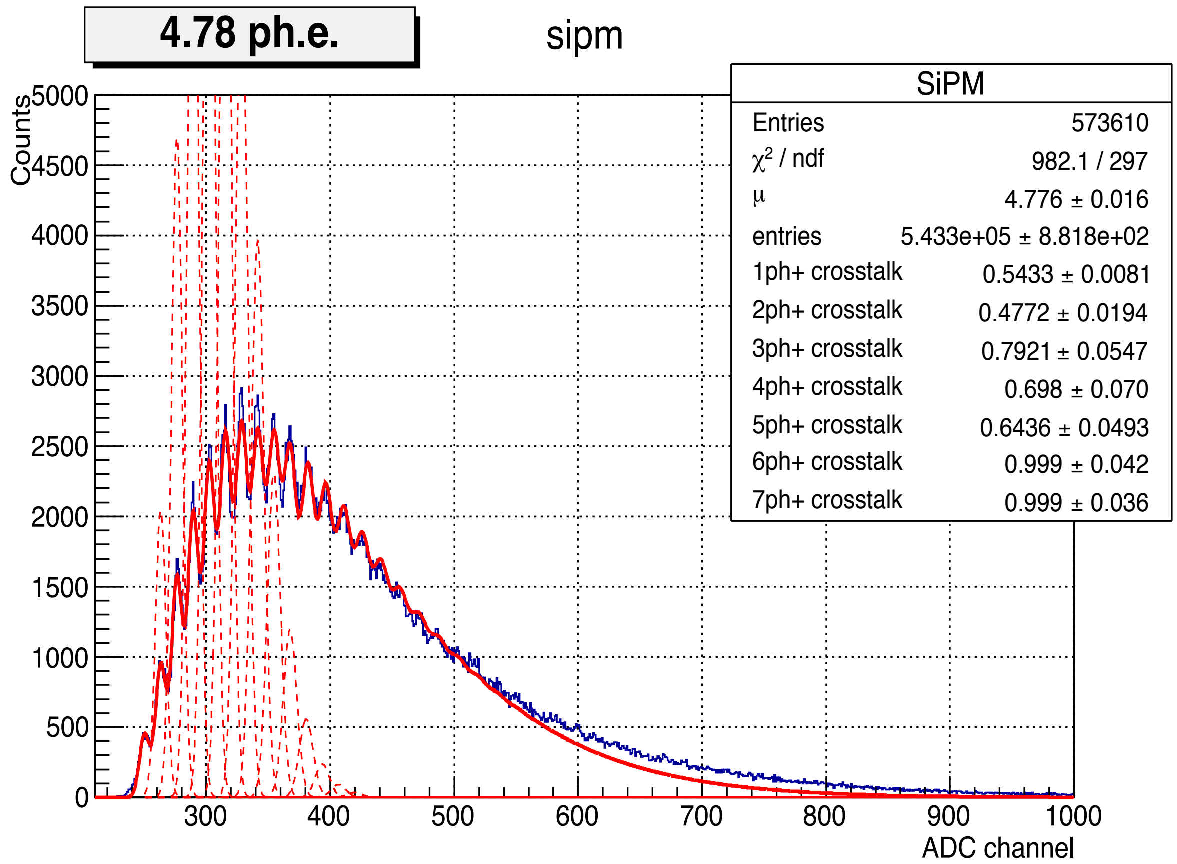
- $P(\geq 1 \text{ p.e.})=Pr(1+)$ => $p_0=1-Pr(1+)$
- $P(\geq 2 \text{ p.e.})=Pr(2+)\times Pr(1+)$=> $p_1=Pr(1+)\times(1-Pr(2+))$
- $P(\geq 3 \text{ p.e.})=Pr(3+)\times Pr(2+)\times Pr(1+)$=> $p_2=Pr(1+)\times Pr(2+)\times(1-Pr(3+))$
- …

SiPM response model

Some parameters fixed to speed up the fit

1ph – 28.4%
2ph – 5.4%
3ph – 6.2%
4ph – 5.1%
5ph – 0%
6ph – 0%
7ph – 9.2%

Known problem of non-linearity at large amplitudes may cause longer tail



SiPM response model summary

Cross-talk modelled with multinomial distribution

Advantage:
Probabilities of cross-talk multiplicities per one initial photo electron

Disadvantage:
Fit duration grows rapidly with mean p.e. number and with cross-talk multiplicity.
Therefore, more effective and useful for small mean p.e.