A Global Bayesian Analysis of Neutrino Mass Data

Alexander Merle
MPP Munich
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MAX-PLANCK-GESELLSCHAFT

Based on:

Caldwell, AM, Schulz, Totzauer:
1705.01945 [hep-ph]

IMPRS Young Scientist Workshop, Ringberg Castle, 19-07-2017

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Ph.D. in 1 week!

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Note to start:

This project touches nearly all groups at MPP!!!

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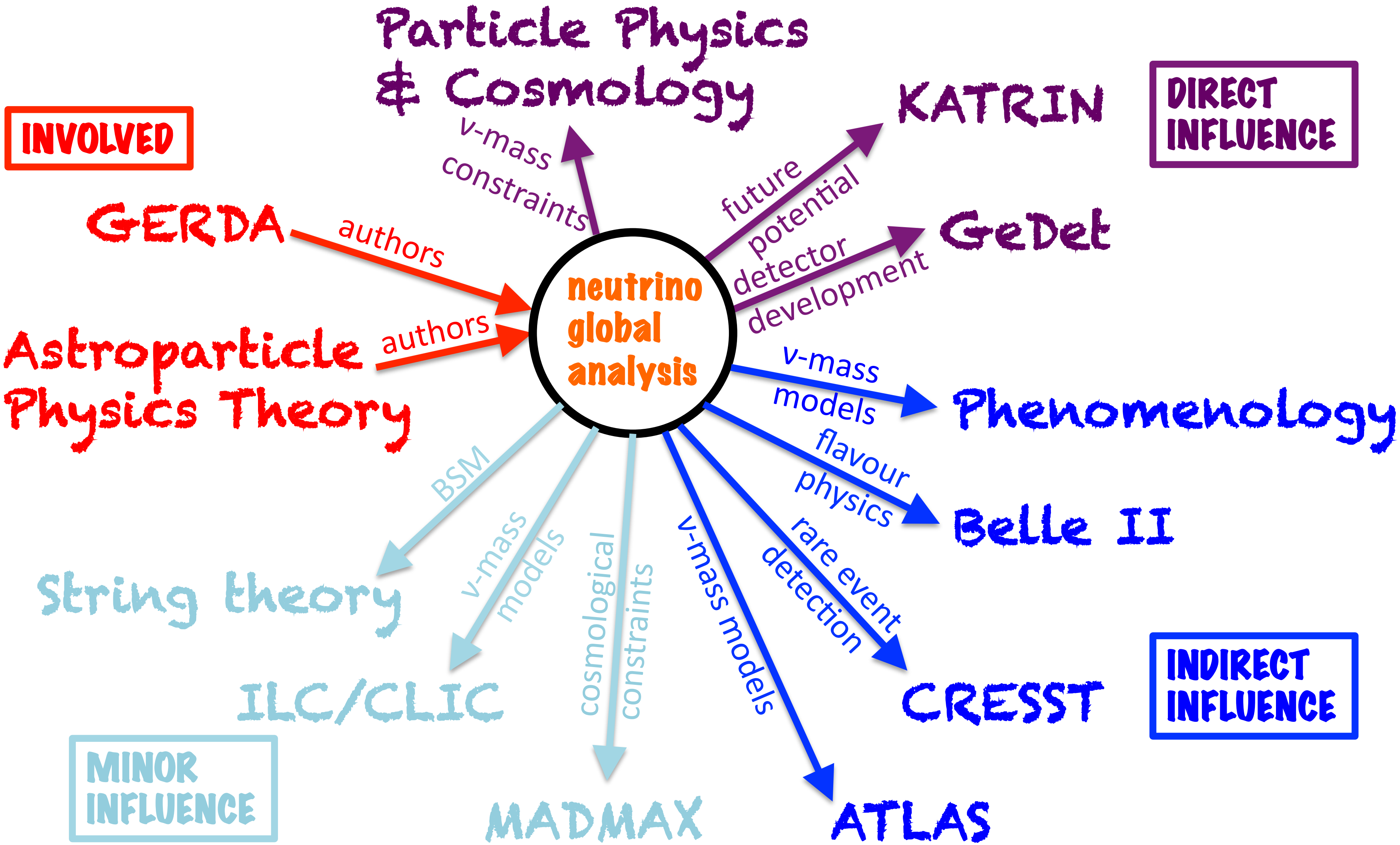

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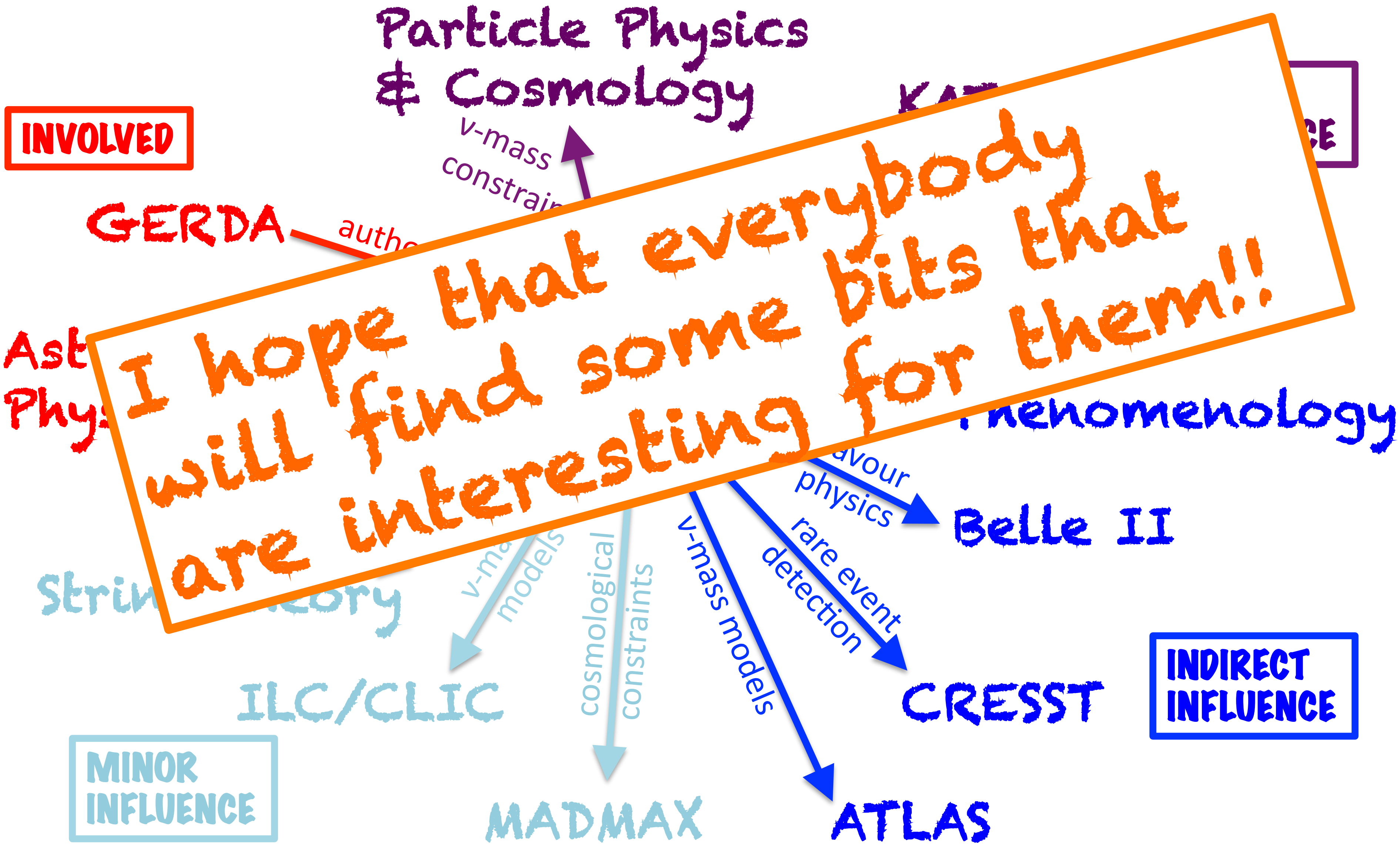

Contents:

1. **Our basic idea**
2. **Aspects to know about:**
   - Leptonic mixing
   - Neutrinoless double beta decay
   - Neutrinos in Cosmology
   - Bayesian Statistics
3. **Our final results**
4. **Conclusions**

1. Our basic idea

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We obtain information on neutrinos from various sources:

- oscillation experiments -> Δm^2's, θ_{ij}
- 0νββ -> $|m_{ee}|=f(m_k, \theta_{ij}, \Phi_L)$
- cosmology -> $\Sigma=m_1+m_2+m_3$
- single beta decay -> $m_\beta^2=g(m_k, \theta_{ij})$

1. Our basic idea

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- o
- o
-
-

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$|m_{ee}| < 0.\ 2\text{-}0.\ 4$ eV [EXO-200: Nature **510** (2014) 229]
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$\Sigma < 0.194$ eV [PLANCK: Astron. Astrophys. **594** (2016) A13]
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- single beta decay -> $m_\beta^2=g(m_k, \theta_{ij})$

... in order to make sense of all that information (i.e., obtain information on m_k and θ_{ij}), obviously the best way is a global analysis

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- o…
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- …
- …

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For oscillations, this has already been recognised long ago!

… in order … information (i.e., obtai… m_k and θ_{ij}), obviously the best way is a … l analysis

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$\Sigma < 0.1$...

θ...

θ...

Δm...

$|\Delta m$...

$|\Delta m$...

While nowadays, we don't know the absolute neutrino mass scale yet, a global analysis will be the tool of choice for the future, also for neutrino mass data!!!

2. Aspects to know about

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Leptonic mixing

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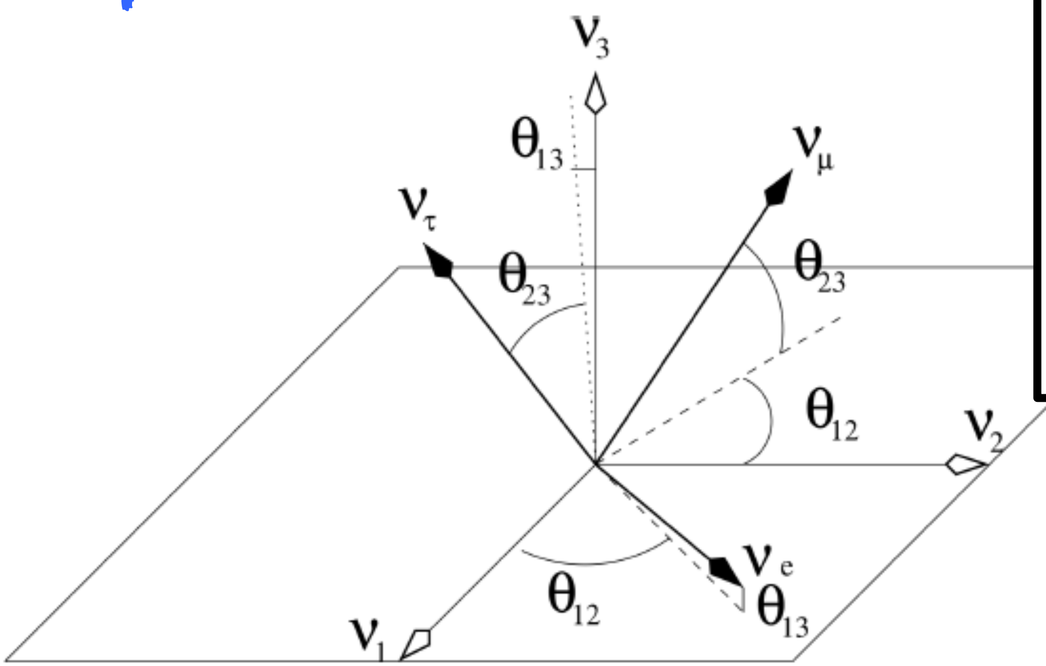

[King, Luhn: Rep. Prog. Phys. **76** (2013) 056201]

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[nu-fit.org, v3.0]

In a nutshell:

- neutrino mass basis ≠ flavour basis
- thus: a ν_e does NOT have an "electron neutrino mass", but it is a superposition

2. Aspects to know about

Leptonic mixing

- our main source of information:
  oscillation experiments
  - solar neutrinos
  - atmospheric neutrinos
  - reactor neutrinos
  - accelerator neutrinos
  - geoneutrinos

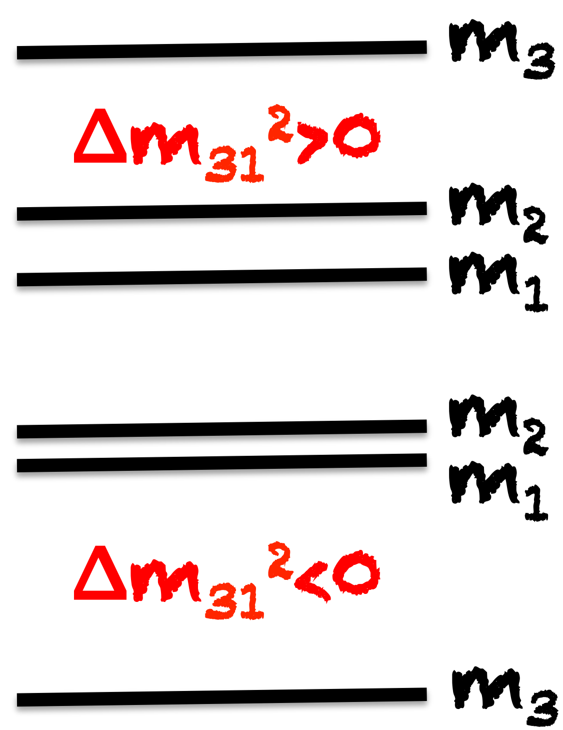
- some additional information:
  neutrino astrophysics
  - supernova neutrinos
  - high-energy cosmic neutrinos
  - diffuse backgrounds

2. Aspects to know about

Leptonic mixing

BUT: This does not tell us everything!!

- oscillations depend on $\Delta m_{ij}^2 = m_i^2 - m_j^2$
  -> no information on absolute mass
- neutrino mass ordering unknown:
  -> normal ($m_1 < m_2 < m_3$)
  -> inverted ($m_3 < m_1 < m_2$)



2. Aspects to know about

Neutrinoless double beta decay

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Neutrinoless double beta decay

- IF neutrinos are Majorana fermions (i.e., identical to their antiparticles)...
  neutrinoless double beta decay can occur (but does not have to!):

$$(Z,A) \rightarrow (Z+2,A) + e^- + e^-$$

- this process violates lepton number by two units
- IF mediated by light neutrinos:

$$A \sim |m_{ee}| = |m_1 c_{12}^2 c_{13}^2 + m_2 s_{12}^2 c_{13}^2 e^{i\alpha} + m_3 s_{13}^2 e^{i\beta}|$$

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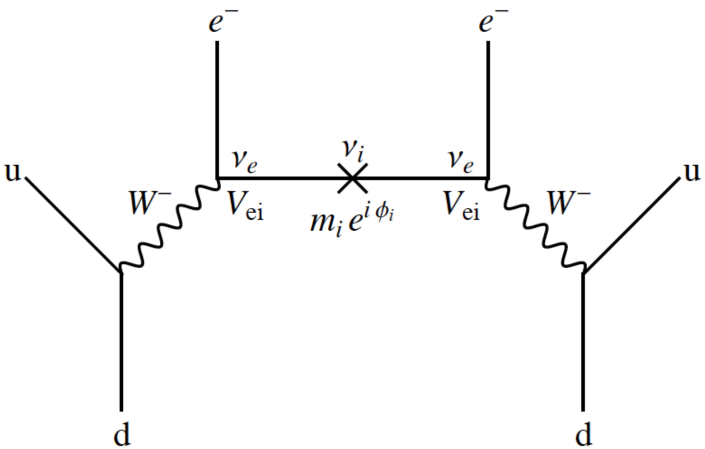
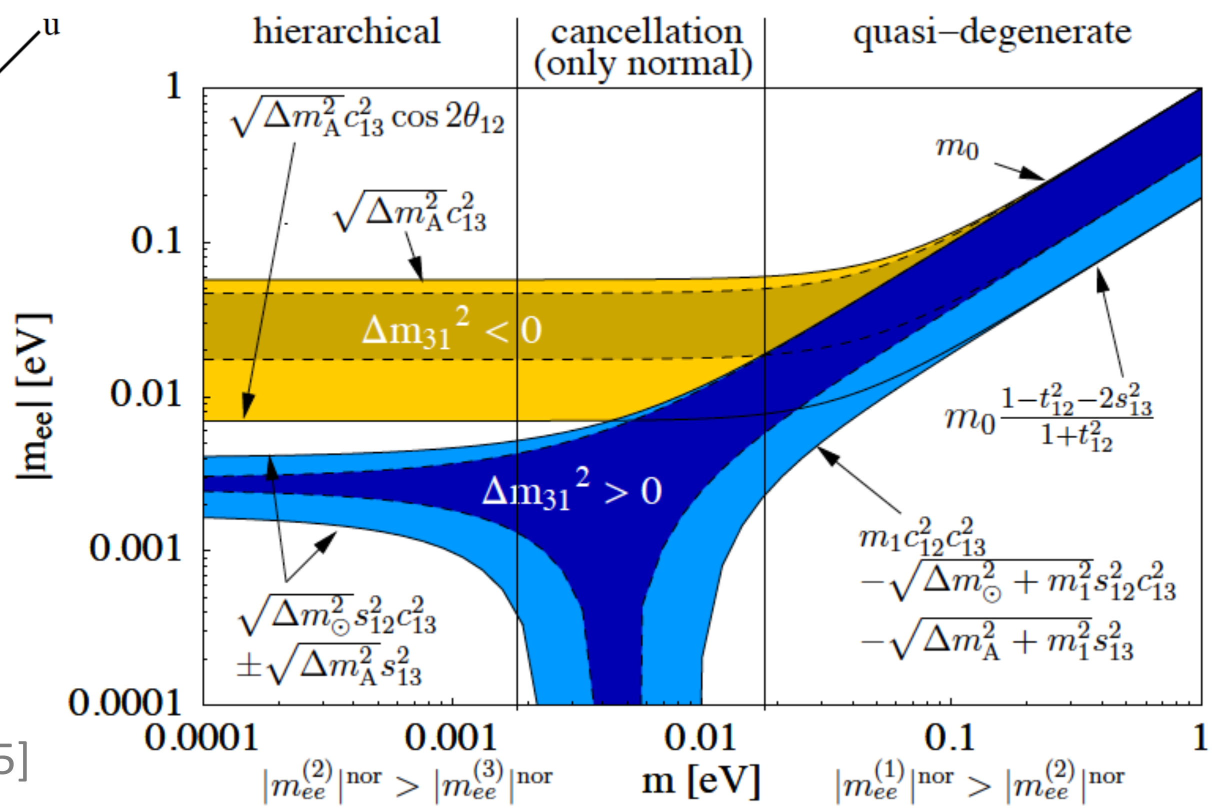
Neutrinoless double beta decay

- thus, neutrinoless double beta decay may give information on the neutrino mass:





[Lindner, **AM**, Rodejohann: Phys. Rev. **D73** (2006) 053005]

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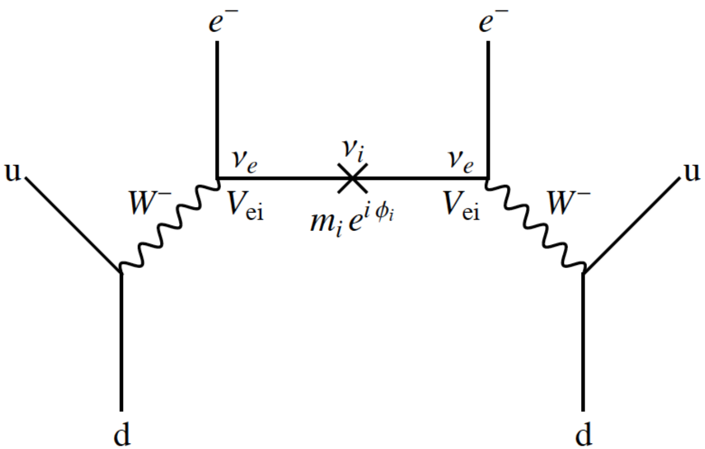

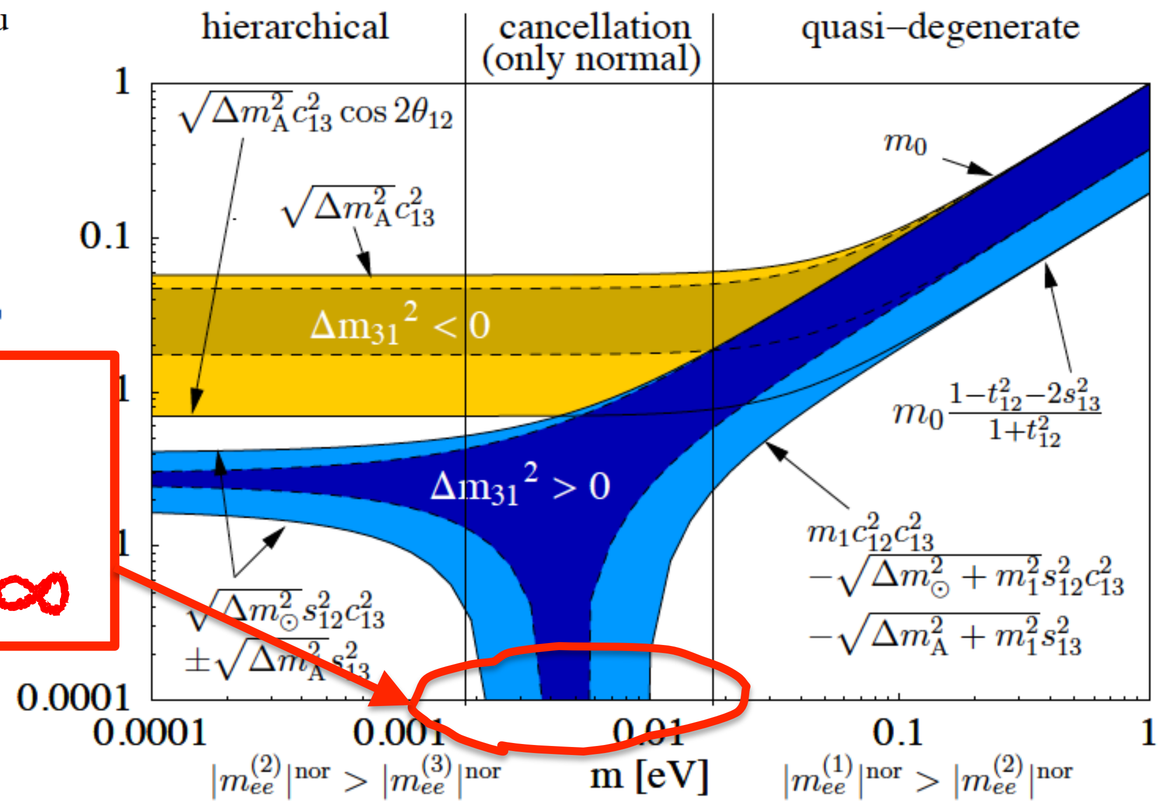

:-(You don't wanna be here: $|m_{ee}|=0 \rightarrow T_{1/2}=\infty$

[Lindner, **AM**, Rodejohann: Phys. Rev. **D73** (2006) 053005]

2. Aspects to know about

Cosmology

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Cosmology

- neutrinos do in fact act as Dark Matter (HOT Dark Matter, if thermalised – i.e., with relativistic velocities)

-> IF thermalised, everything depends on the amount of the neutrinos (i.e., their "abundance" Ω_ν)

-> for late times: $\Omega_\nu \sim \Sigma = m_1 + m_2 + m_3$

2. Aspects to know about

Cosmology

- relativistic neutrinos impact cosmology:

  - large free streaming
    -> suppression of small structures
  - modification of CMB anisotropies
    - redshift of matter-radiation equality
      -> position & amplitude of peaks
    - modified matter density (late times)
      -> overall position of spectrum & slope for low l)

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Strongest impact: Lyman-α forest
... even Σ < 0.12 eV can be reached
[Palanque-Delabrouille *et al.*: JCAP **1511** (2015) 011]

2. Aspects to know about

Bayesian statistics

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Bayesian statistics

In modern statistics, there are two main directions:

- frequentist statistics

$$\text{probability} = \lim_{N \to \infty} \frac{\#\ \text{successful trials}}{\#\ \text{total trials}}$$

-> looks objective

- Bayesian statistics

probability = degree of belief

-> looks subjective

2. Aspects to know about

Bayesian statistics

While subjectivity may appear strange, probability is NOT objective:

- urn with R red balls and W white balls

$$P(\text{red on 1st trial}) = \frac{R}{R+W}$$

P(red on 2nd trial after first draw's colour is unknown)

$$= P(2{=}R|1{=}R) \times P(1{=}R) + P(2{=}R|1{=}W) \times P(1{=}W)$$

$$= \frac{R-1}{R+W-1} \times \frac{R}{R+W} + \frac{R}{R+W-1} \times \frac{W}{R+W} = \frac{R}{R+W}$$

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Bayesian statistics

While subjectivity may appear strange, probability is NOT objective:

- urn with R red balls and W white balls

$$P(\text{red on 1st trial}) = \frac{R}{R+W}$$

Same result, although the system has clearly changed!!

P(red on 2nd trial after fir[st]
colour is unknown)

$=P(2=R|1=R)\times P(1=R) + P(2=R|1=W)\times P(1=W$[)]

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2. Aspects to know about

Bayesian statistics

Basic formula: Bayes' theorem

Likelihood

Prior

$$P(H|D) = \frac{P(D|H) \times P(H)}{P(D)}$$

Posterior

"Evidence"
(=normalisation)

2. Aspects to know about

Bayesian statistics

Implications of Bayes' theorem:

- if we know the likelihood, we can construct a posterior probability density for any given prior probability
- for "good" (i.e., constraining) data, the posterior is nearly independent of the choice of prior (except for pathologic cases); for "bad" data, it is not....
- in practice, the evaluation of the evidence can be very tricky
  -> Bayesian Analysis Toolkit (BAT)

3. Our final results

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We have performed a Bayesian analysis of neutrino mass data!!

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INPUTS:

- oscillation data -> nu-fit.org: v3.0
- 0νββ -> GERDA, KamLAND-Zen, EXO
- cosmology -> Planck + lensing (+BAO)
- NME computations (nuclear part of 0νββ)

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We have performed a Bayesian analysis of neutrino mass data!!

INPUTS:

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- $0\nu\beta\beta$ -> GERDA, KamLAND-Zen, EXO
- cosmology -> Planck + lensing (+BAO)
- NME computations (nuclear part of $0\nu\beta\beta$)

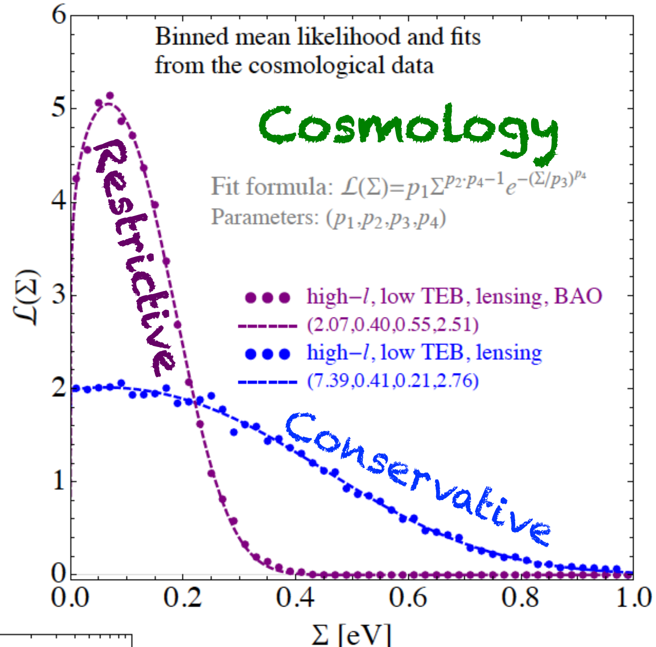
ASSUMPTIONS:

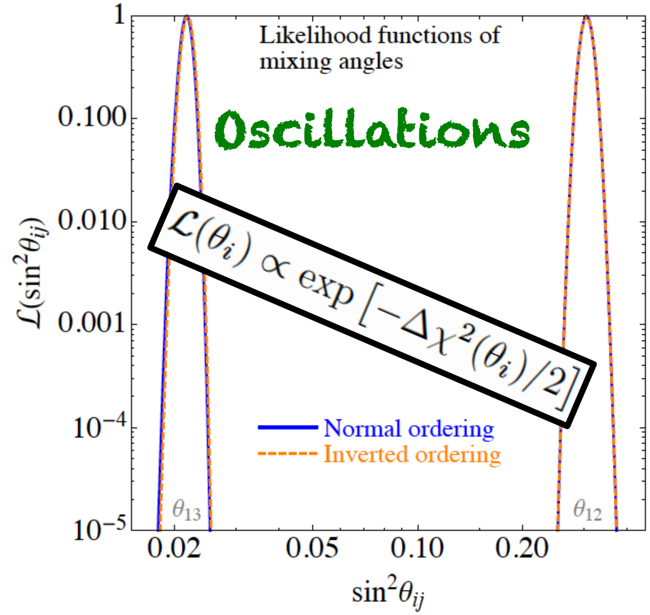
- $0\nu\beta\beta$ transmitted by light neutrinos
  -> mostly the dominant contribution
- cosmology only depends on Σ
  -> minimal, but may still be wrong

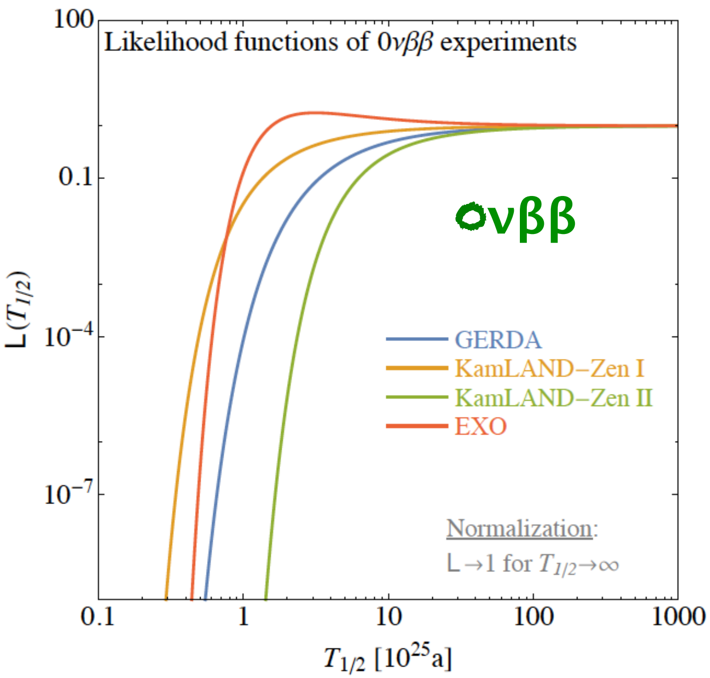
3. Our final results

Priors

NMEs: discrete prior for all values







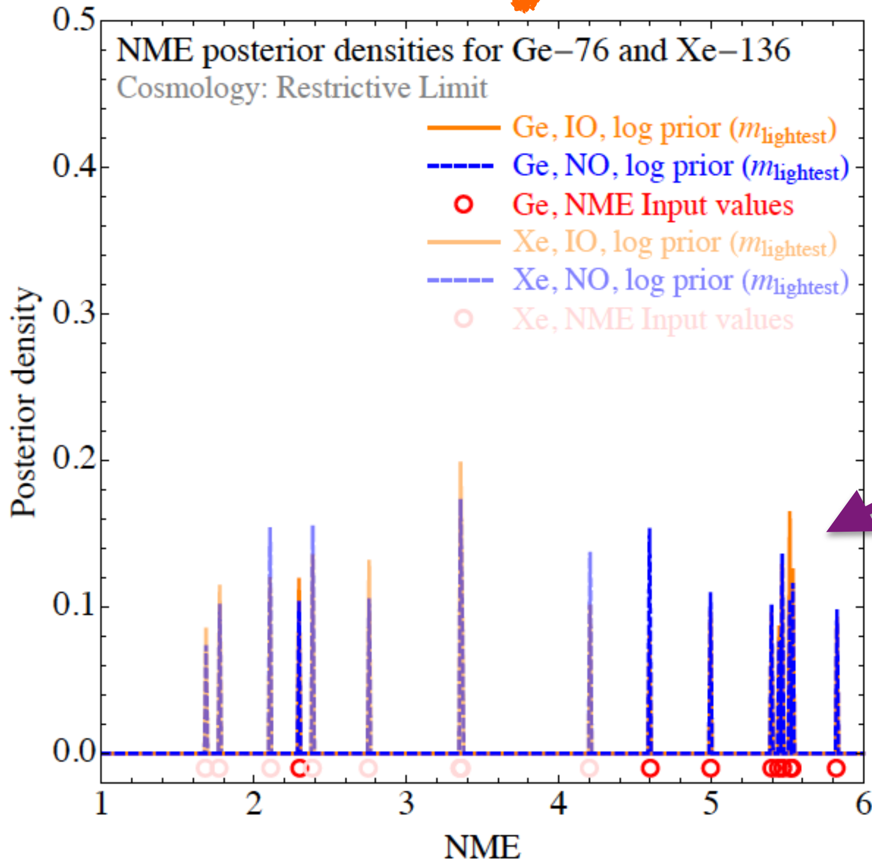
Masses: P(m)=const. vs. P(m)~1/m

NOTE: absolute scale not constrained -> priors **MUST** yield different results

3. Our final results

Posteriors: Round 1 – NMEs

Known fact:
NMEs are hardly constrained by the data
-> one basically gets back the prior:



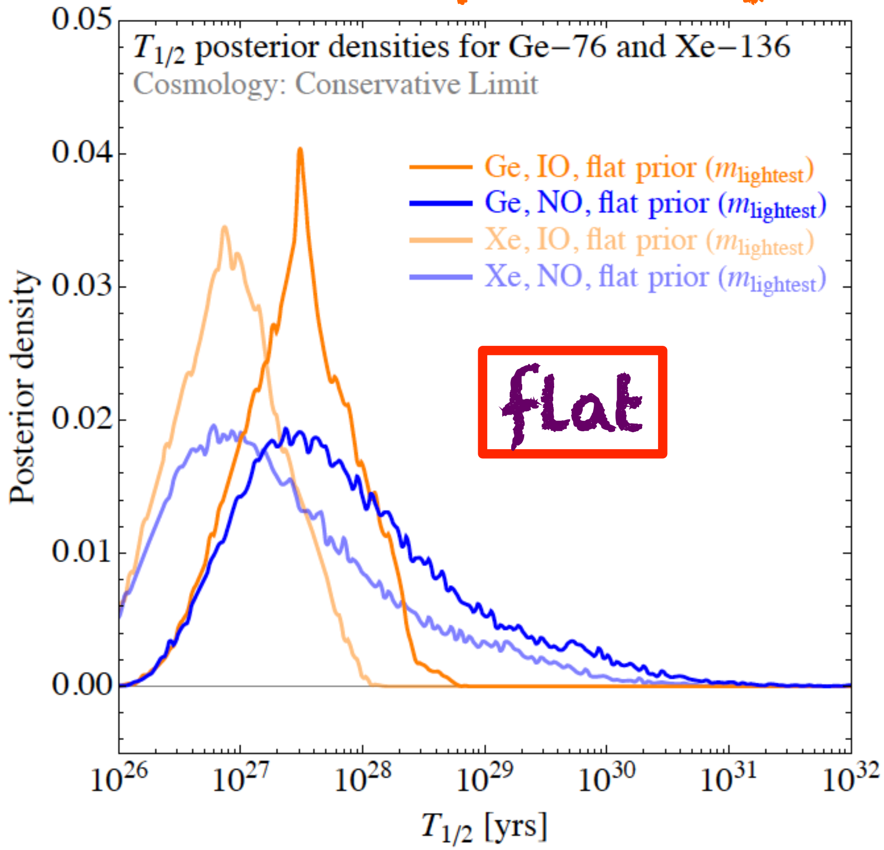
the posteriors simply tracks the prior

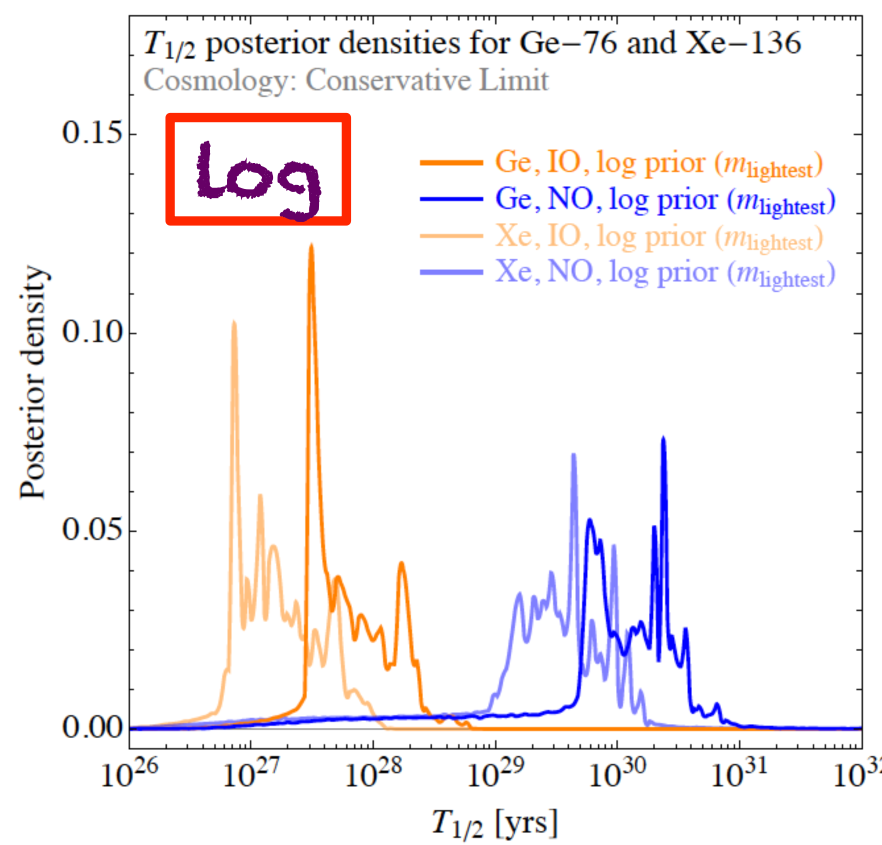
3. Our final results

Posteriors: Round 2 - half-lives

The half-lives strongly depend on $|m_{ee}|$, and thus on the neutrino mass scale

-> flat/log priors yield visible differences:





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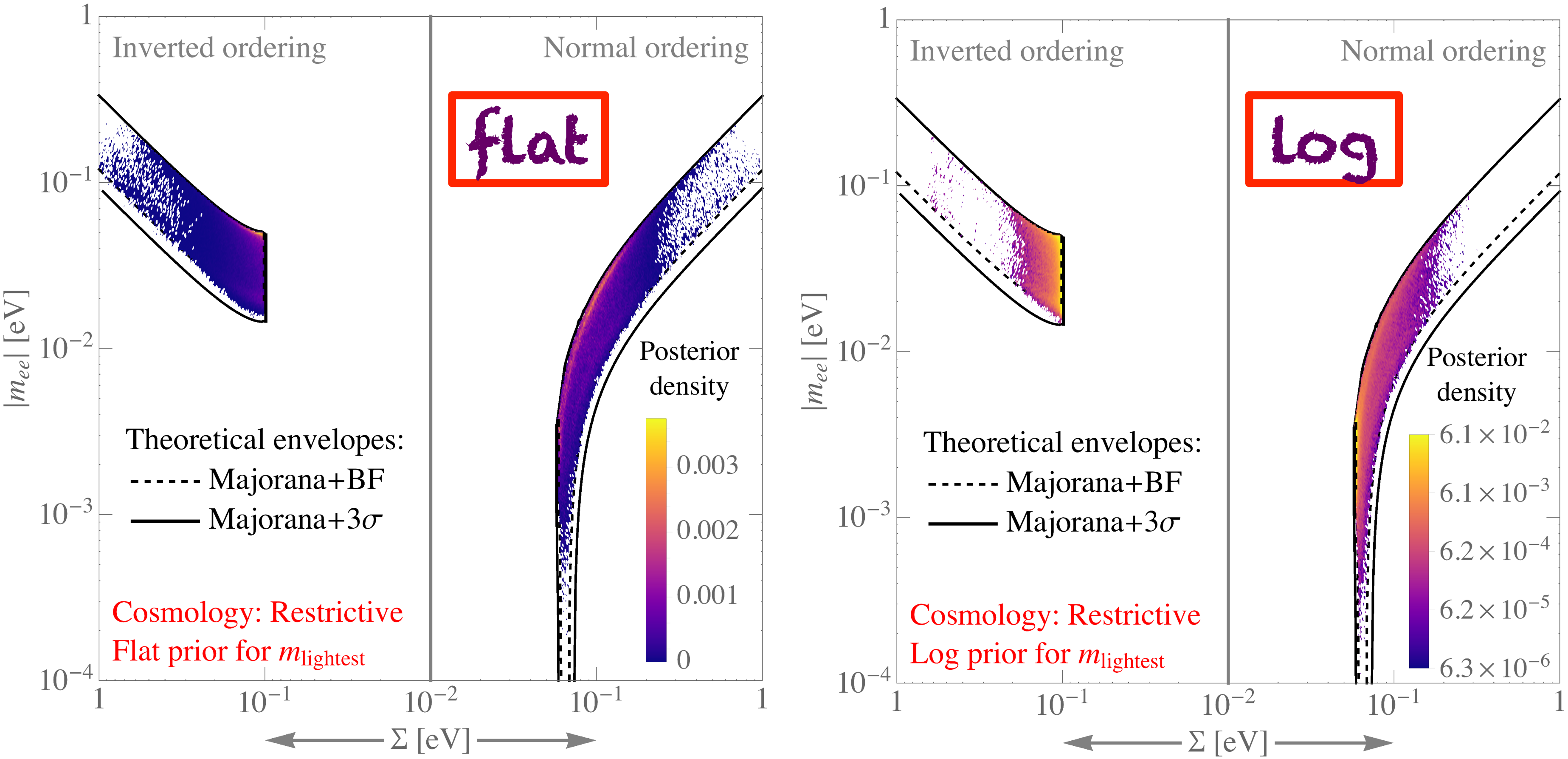
Posteriors: Round 3 - $|m_{ee}|$ vs. Σ

Fortunately:

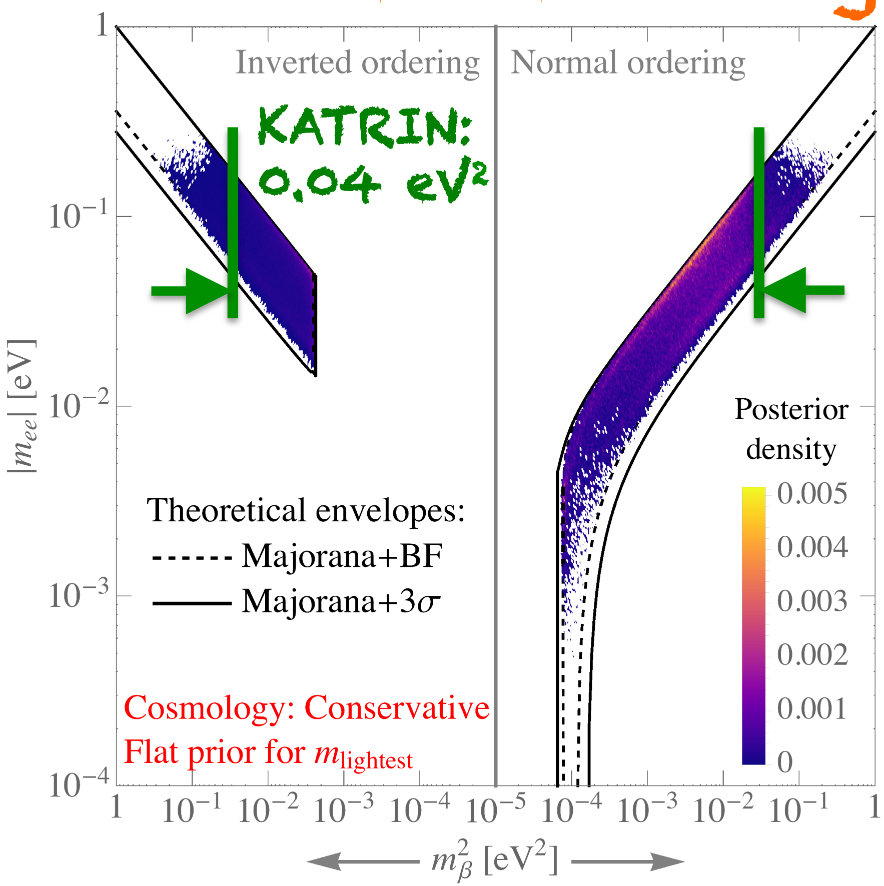
It is unlikely that we're in the throat!

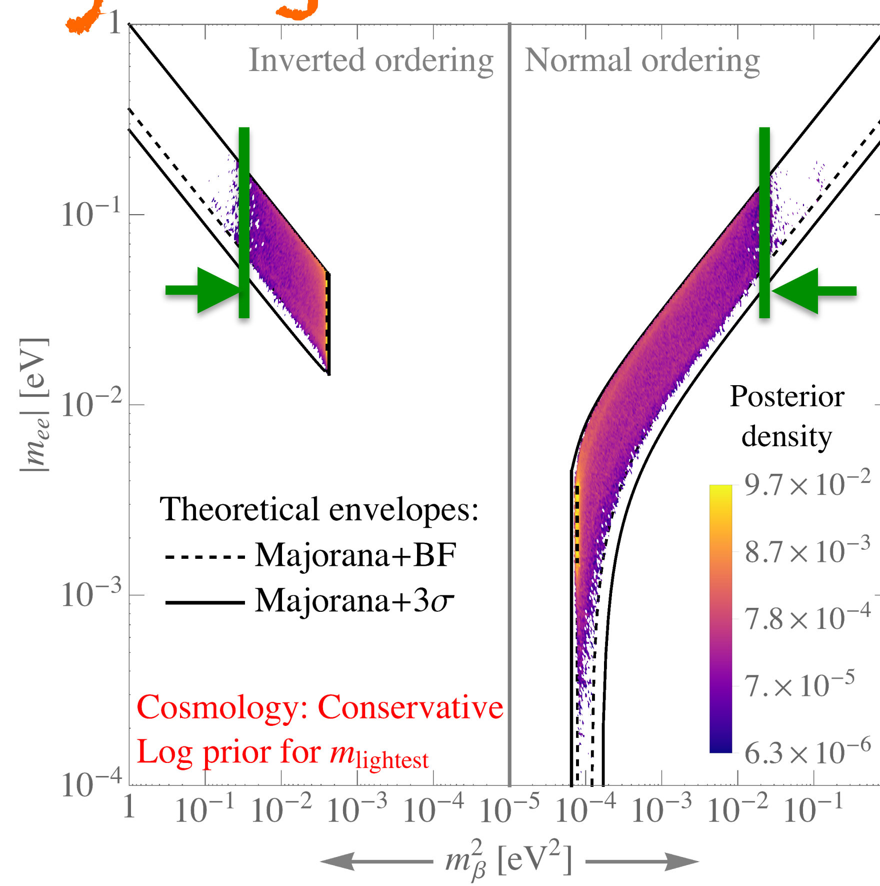


3. Our final results

Posteriors: Round 4 - $|m_{ee}|$ vs. m_β^2

Unfortunately: even for conservative cosmo bounds, single beta experiments will have a hard time measuring anything





3. Our final results

Bayes factor: posterior "odds"

Would you bet on normal ordering?!?

- prior: oscillation data slightly favour NO
- posterior odds (after our analysis):

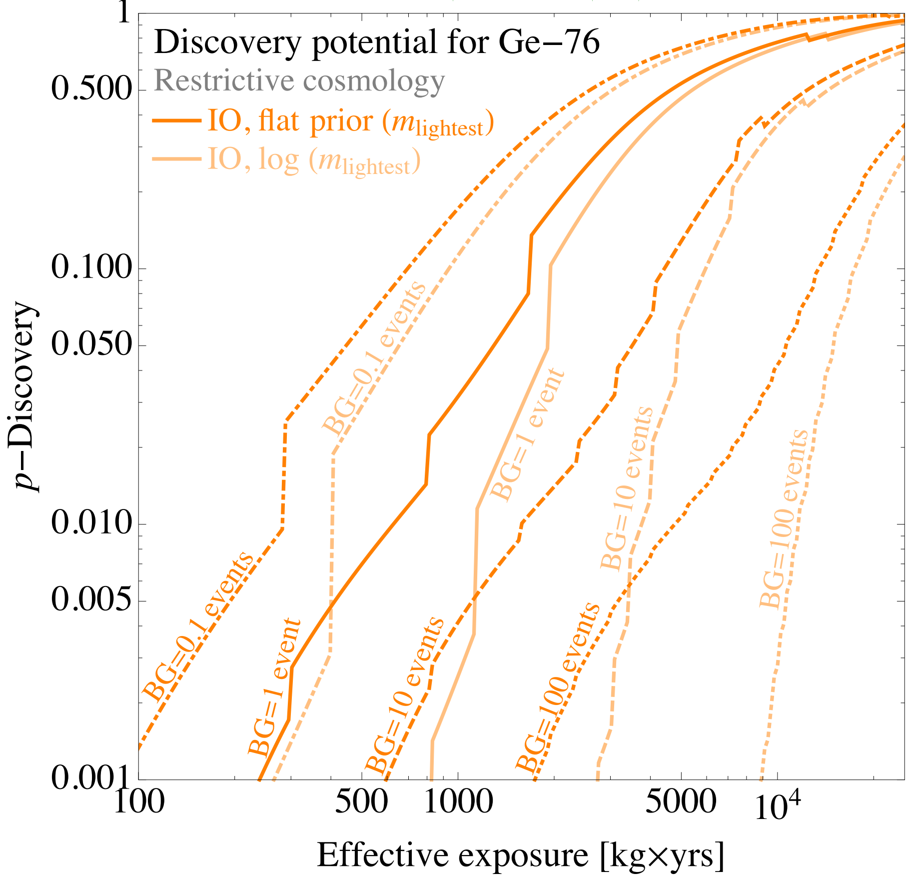
Cosmology	Prior on m_{lightest}	Bayes factor NO/IO
restrictive	flat	2.3
conservative	flat	1.6
restrictive	log	1.9
conservative	log	1.7

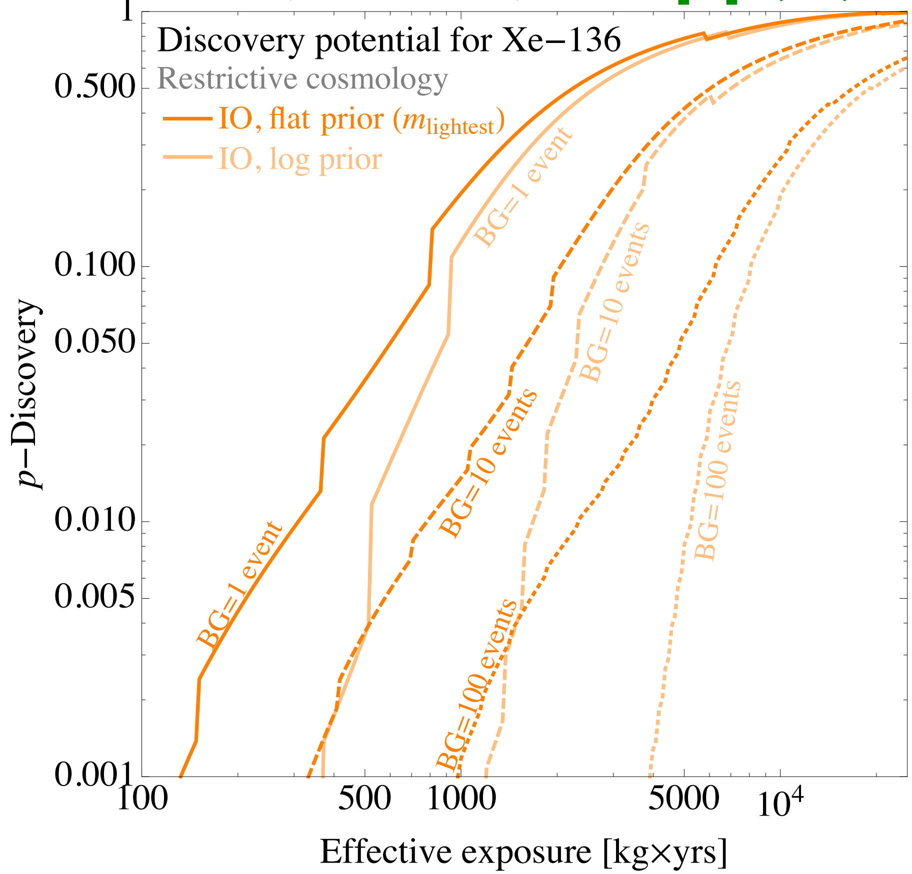
-> preference only mildly enhanced (to be expected: ordering not yet measured)

3. Our final results

Discovery potential

What do we need to do to measure 0νββ?!?





-> we better be background-"free", so that GERDA can beat the Xe-experiments! ;-)

4. Conclusions

- **Neutrinos...** ***will hopefully teach us a lot about new physics in the coming years!***
- **Neutrino mass data...** ***will be available in the future, and we need to know how to handle it!***
- **A global analysis...** ***is the tool of choice!***
- **Bayesian inference...** ***is the best approach to combine the different data sets from rather distinct sectors!***

- **Interdisciplinary projects and cross-collaborations... are always the greatest fun!!**


THANK YOU!!!