Heavy quark potentials in effective string theory

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Long-distance regime in QCD

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Confinement in QCD [Wilson, PRD (1974)]

- Definition and characteristics
  - No single color charged object isolated at low-energy (i.e., $E \leq \Lambda_{QCD}$)
  - Detect only hadrons in the form of jets
  - Perturbative approach breaks down at low-energy ($\Lambda_{\mathrm{QCD}} \sim 200 MeV$)
- Any (analytical) solution?
  - Not in the form of perturbative method
  - Phenomenologically treat hadrons as DOFs (e.g., χPT, CQM, etc)
  - Non-perturbative approach: Lattice Gauge Theory (Lattice QCD)
- Another approach:
  QCD flux tube model for heavy meson [Nambu, Phys. Lett. B (1979)]

QCD flux tube model [Bali, Schilling, Schlichter, PRD (1995)]

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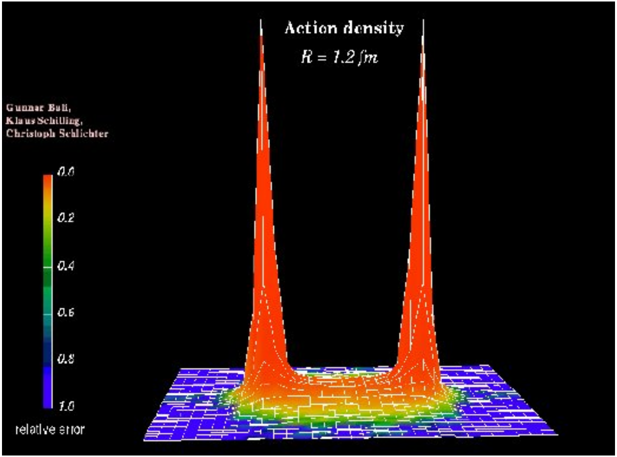

R: distance between $Q\overline{Q}$ rescaled by lattice spacing

Outline

Brief guide towards analytic calculation of long-distance heavy quark potentials in EFT framework

1. Non-relativistic EFTs of QCD and Heavy quark potentials
2. Effective string theory (EST)
3. Heavy quark potentials in EST at LO
4. Heavy quark potentials in EST at NLO
5. Summary and outlook

Non-relativistic EFTs of QCD

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1. NRQCD: Non-relativistic description of QCD for the heavy quark-antiquark pair [Caswell, Lepage, Phys. Lett. B (1986), Bodwin, Braaten, Lepage PRD (1995)]
   - Hierarchy of scales: $M \gg Mv, \Lambda_{QCD}$
   - Integrate out M from QCD
   - Heavy DOFs: Pauli spinors for heavy quarks/antiquarks
2. Potential NRQCD: effective theory for heavy quarkonium

   [Brambilla, Pineda, Soto, Vairo, Nucl. Phys. B (2000)]
   - Hierarchy of scales: $M \gg Mv \gg Mv^2$
   - Integrate out momentum transfer $Mv \sim 1/r$ from NRQCD
   - Heavy DOFs: Color singlet $S(t, \mathbf{r}, \mathbf{R})$ and octet $O^a(t, \mathbf{r}, \mathbf{R})$
   - Appearance of potentials

$$\mathcal{L}_{\rm pNRQCD} \ni \int d^3r \Big\{ \mathrm{Tr}[S^\dagger(i\partial_0 - V_s(r) + \dots)S + O^\dagger(iD_0 - V_o(r) + \dots)O]$$
$$+ g V_A(r) \mathrm{Tr}[O^\dagger \mathbf{r}\cdot\mathbf{E} S + S^\dagger \mathbf{r}\cdot\mathbf{E} O] + g\frac{V_B(r)}{2} \mathrm{Tr}[O^\dagger \mathbf{r}\cdot\mathbf{E} O + O^\dagger O \mathbf{r}\cdot\mathbf{E}] \Big\}$$

Heavy quark potentials

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Relativistic corrections to the static potentials in pNRQCD

- Heuristically: $V = V^{(0)} + V^{(1/M)} + V^{(1/M^2)} + \dots$
- Static singlet potential up to $1/M^2$

$$V(r) = V^{(0)}(r) + \frac{2}{M} V^{(1,0)}(r) + \frac{1}{M^2}\left\{ \left[2\frac{V^{(2,0)}_{\mathbf{L}^2}(r)}{r^2} + \frac{V^{(1,1)}_{\mathbf{L}^2}(r)}{r^2} \right] \mathbf{L}^2 \right.$$

$$+ \left[V^{(2,0)}_{LS}(r) + V^{(1,1)}_{L_2 S_1}(r) \right] \mathbf{L}\cdot\mathbf{S} + V^{(1,1)}_{S^2}(r)\left(\frac{\mathbf{S}^2}{2} - \frac{3}{4} \right) + V^{(1,1)}_{\mathbf{S}_{12}}(r)\mathbf{S}_{12}(\hat{\mathbf{r}})$$

$$\left. + \left[2V^{(2,0)}_{p^2}(r) + V^{(1,1)}_{p^2}(r) \right] \mathbf{p}^2 + 2V^{(2,0)}_{r}(r) + V^{(1,1)}_{r}(r) \right\}$$

- Notations
  - (a, b): a order of $1/M_1$ and b order of $1/M_2$ $(1/M_1 = 1/M_2)$
  - $S_i = \frac{\sigma^i}{2}$ $(i \in \{1, 2\})$: spin of the heavy quark/antiquark
  - Vs: Wilson coefficients to be matched to NRQCD

Wilson loop and heavy quark potential (1/2)

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NRQCD and pNRQCD matching [Brambilla, Pineda, Soto, Vairo, PRD (2000)]

- Static potential:

$$V^{(0)}(r) = \lim_{T\to\infty} \frac{i}{T} \ln \langle W_\Box \rangle$$

  - Matching $Q\overline{Q}$ correlator to singlet propagator
  - Wilson loop: $W_\Box \equiv P \exp\left\{-ig \oint_{r\times T} dz^\mu A_\mu(z)\right\}$
  - $\langle \ldots \rangle$: average value over Yang-Mills action
- 1st order relativistic correction:

$$V^{(1,0)}(r) = -\frac{1}{2} \lim_{T\to\infty} \int_0^T dt t \langle\langle g\mathbf{E}_1(t) \cdot g\mathbf{E}_1(0)\rangle\rangle_c$$

  - $\langle\langle \ldots \rangle\rangle \equiv \langle \ldots W_\Box \rangle / \langle W_\Box \rangle$
  - $\langle\langle O_1(t_1) O_2(t_2)\rangle\rangle_c \equiv \langle\langle O_1(t_1) O_2(t_2)\rangle\rangle - \langle\langle O_1(t_1)\rangle\rangle\langle\langle O_2(t_2)\rangle\rangle$, for $t_1 \geq t_2$
  - $\mathbf{E}_{1,2}(t) \equiv \mathbf{E}(t, \pm\mathbf{r}/2)$; heavy quark/antiquark located at $(0, 0, \pm r/2)$

Wilson loop and heavy quark potential (2/2)

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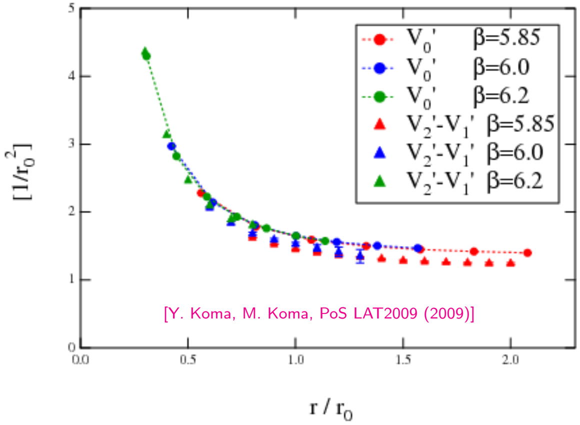
- 2nd order corrections [Pineda, Vairo, PRD (2001)]
  1. Spin-independent part: $V_{\mathbf{p}^2}^{(2,0)}(r)$, $V_{\mathbf{p}^2}^{(1,1)}(r)$, $V_{\mathbf{L}^2}^{(2,0)}(r)$, $V_{\mathbf{L}^2}^{(1,1)}(r)$, $V_r^{(2,0)}(r)$, $V_r^{(1,1)}(r)$

$$\begin{aligned} V_{\mathbf{p}^2}^{(2,0)}(r) &= \frac{i}{2}\hat{\mathbf{r}}^{\mathbf{i}}\hat{\mathbf{r}}^{\mathbf{j}} \int_0^\infty dt t^2 \langle\langle g\mathbf{E}_1^i(t) g\mathbf{E}_1^j(0)\rangle\rangle_c \\ V_{\mathbf{p}^2}^{(1,1)}(r) &= i\hat{\mathbf{r}}^{\mathbf{i}}\hat{\mathbf{r}}^{\mathbf{j}} \int_0^\infty dt t^2 \langle\langle g\mathbf{E}_1^i(t) g\mathbf{E}_2^j(0)\rangle\rangle_c \\ &\cdots \end{aligned} \tag{1}$$

  2. Spin-dependent part: $V_{LS}^{(2,0)}(r)$, $V_{L_2S_1}^{(1,1)}$, $V_{S^2}^{(1,1)}$, $V_{\mathbf{S}_{12}}^{(1,1)}$,

$$\begin{aligned} V_{LS}^{(2,0)}(r) &= -\frac{c_F^{(1)}}{r^2} i\mathbf{r}\cdot \int_0^\infty dt t \langle\langle g\mathbf{B}_1(t) \times g\mathbf{E}_1(0)\rangle\rangle + \frac{c_s^{(1)}}{2r^2}\mathbf{r}\cdot(\nabla_r V^{(0)}) \\ V_{L_2S_1}^{(1,1)}(r) &= -\frac{c_F^{(1)}}{r^2} i\mathbf{r}\cdot \int_0^\infty dt t \langle\langle g\mathbf{B}_1(t) \times g\mathbf{E}_2(0)\rangle\rangle \\ &\cdots \end{aligned} \tag{2}$$

Lattice fit for spin-orbit potential



Confirmation of the Gromes relation: $\frac{1}{2r}\frac{dV^{(0)}}{dr} = V^{(1,1)}_{L_1 S_2} - V^{(2,0)}_{LS}$

Limitation

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Lattice only?

- Fits nicely to Poincaré invariance [D. Gromes, Z. Phys. C (1984)]
- But 3- and 4-gauge field insertions unknown from lattice
- Need other ways to calculate V_r (contains 3 and 4 gauge fields insertions)
  - Effective String Theory: an analytic description at long-distance
  - Dynamics of strings instead of gluodynamics between heavy quark-antiquark
  - Effective description below confinement scale (non-perturbative)

Effective string theory: hypothesis

Wilson loop-string partition function equivalence conjecture

[Lüscher, Nucl. Phys. B180, 317 (1981)]

$$\lim_{T\to\infty} \langle W_\Box \rangle = Z \int \mathcal{D}\xi^1 \mathcal{D}\xi^2 e^{iS_{\text{String}}[\xi^1,\xi^2]}$$

- ξ^1, ξ^2: string coordinates
- Z: normalization constant
- S_{String}: string action

Effective string theory: construction

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1. Hierarchy: $r\Lambda_{QCD} \gg 1$, r distance between heavy quark-antiquark
2. Boundary conditions : $\xi^l(t,z) = 0$; $z = \pm r/2$
   - Transversal string vibrations: $\xi^l(t,z)$, for $l = x, y$ (or 1, 2)
3. Power counting: $\partial_a \sim 1/r$, $\xi^l \sim 1/\Lambda_{QCD}$ (i.e., $\partial_a \xi^l \sim (r\Lambda_{QCD})^{-1}$)
4. String action in 4D with static gauge (starting from Nambu-Goto action):

$$
\begin{aligned}
S &= -\sigma \int dtdz \sqrt{-\det(\eta_{ab} + \partial_a \xi^l \partial_b \xi^l)} \\
&= -\sigma \int dtdz \left(1 - \frac{1}{2}\partial_a \xi^l \partial^a \xi^l + \dots \right)
\end{aligned}
$$

- σ: string tension ($\sim \Lambda_{QCD}^2$), determined by lattice
- Truncation up to $(r\Lambda_{QCD})^{-2}$
- $a = z, t$
- Static potential: $V^{(0)} \approx \sigma r$ (leading order)

[Lüscher, K. Symanzik, P. Weisz, Nucl. Phys. B173, 365 (1980)]

Gauge fields insertion and string mapping

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- Correlator in Euclidean time ($t \to -it$):

$$G(t,t';z,z') = \frac{\delta^{lm}}{4\pi\sigma} \ln \left(\frac{\cosh[(t-t')\pi/r] + \cos[(z+z')\pi/r]}{\cosh[(t-t')\pi/r] - \cos[(z-z')\pi/r]} \right) \quad (3)$$

- Mapping from gauge fields insertion into Wilson loop to EST:

[Kogut, Parisi, PRL (1981), Perez-Nadal, Soto, PRD (2009), Brambilla, Groher, Martinez, Vairo PRD (2014)]

1. Symmetries: rotations w.r.t. z-axis, reflection w.r.t. xz plane, CP, T
2. Match the mass dimensions
3. Truncate up to $(r\Lambda_{QCD})^{-2}$

$$\begin{aligned}
\langle\langle \dots \mathbf{E}^l_{1,2}(t) \dots \rangle\rangle &= \langle \dots \Lambda^2 \partial_z \xi^l(t, \pm r/2) \dots \rangle \\
\langle\langle \dots \mathbf{E}^3_{1,2}(t) \dots \rangle\rangle &= \langle \dots \Lambda''^2 \dots \rangle \\
\langle\langle \dots \mathbf{B}^l_{1,2}(t) \dots \rangle\rangle &= \langle \dots \pm \Lambda' \epsilon^{lm} \partial_t \partial_z \xi^m(t, \pm r/2) \dots \rangle \\
\langle\langle \dots \mathbf{B}^3_{1,2}(t) \dots \rangle\rangle &= \langle \dots \pm \Lambda''' \epsilon^{lm} \partial_t \partial_z \xi^l(t, \pm r/2) \partial_z \xi^m(t, \pm r/2) \dots \rangle
\end{aligned} \quad (4)$$

Long-distance potentials in EST

Potentials at leading order

[Perez-Nadal, Soto, PRD (2009), Brambilla, Groher, Martinez, Vairo PRD (2014)]

- Wilson loop expectation values by using Eq. (3) and (4):

$$\langle\langle \mathbf{E}_1^i(-it)\mathbf{E}_1^j(0)\rangle\rangle_c = \tilde{\delta}^{ij}\frac{\pi\Lambda^4}{4\sigma r^2}\sinh^{-2}\left(\frac{\pi t}{2r}\right), \quad \langle\langle \mathbf{E}_1^3(-it)\mathbf{E}_1^3(0)\rangle\rangle_c = 0,$$

$$\langle\langle \mathbf{E}_1^i(-it)\mathbf{E}_2^j(0)\rangle\rangle_c = -\tilde{\delta}^{ij}\frac{\pi\Lambda^4}{4\sigma r^2}\cosh^{-2}\left(\frac{\pi t}{2r}\right), \quad \langle\langle \mathbf{E}_1^3(-it)\mathbf{E}_2^3(0)\rangle\rangle_c = 0,$$

$$\ldots$$

- Relativistic correction to static potential

$$V^{(1,0)}(r) = \frac{g^2\Lambda^4}{2\pi\sigma}\ln(\sigma r^2) + \mu_1, \quad V^{(2,0)}_{\mathbf{p}^2}(r) = V^{(1,1)}_{\mathbf{p}^2}(r) = 0, \quad V^{(2,0)}_{\mathbf{L}^2}(r) = -V^{(1,1)}_{\mathbf{L}^2}(r) = -\frac{g^2\Lambda^4 r}{6\sigma},$$

$$\ldots$$

Poincaré invariance in QCD

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Emergence of parameters: $\mu_1, \mu_2, \mu_3, \mu_4, \mu_5, \ldots$

- μ_i: renormalization constants
- Exploit symmetries of the fundamental theory $\to$ reduce parameters
- Poincaré invariance
  - Theory independent of the reference frame interaction observed
  - Spatial rotations, boosts, as well as spacetime translations
- Poincaré invariance in QCD constrains parameters

[Brambilla, Gromes, Vairo, Phys. Lett. B (2003), Berwein, Brambilla, SH, Vairo, TUM-EFT 74/15]:

  - Gromes relation: $\frac{1}{2r}\frac{dV^{(0)}}{dr} + V_{LS}^{(2,0)} - V_{L_2 S_1}^{(1,1)} = 0 \longrightarrow \mu_2 = \frac{\sigma}{2}$
  - Momentum dependent potentials: $\frac{r}{2}\frac{dV^{(0)}}{dr} + 2V_{\mathbf{L}^2}^{(2,0)} - V_{\mathbf{L}^2}^{(1,1)} = 0 \longrightarrow g\Lambda^2 = \sigma$
  - $-\nabla_1 V^{(0)} = \langle\langle g\mathbf{E}_1 \rangle\rangle$ [Brambilla, Pineda, Soto, Vairo, PRD (2000)]: $g\Lambda''^2 = -\sigma$

Full result at LO mapping

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- In CM, Hamiltonian of the system given, $H = \mathbf{p}^2/M + V$
  [Brambilla, Groher, Martinez, Vairo PRD (2014)]

$$V(r) = V^{(0)}(r) + \frac{2}{M}V^{(1,0)}(r) + \frac{1}{M^2}\left\{\left[2\frac{V^{(2,0)}_{\mathbf{L}^2}(r)}{r^2} + \frac{V_{\mathbf{L}^2}(r)}{r^2}\right]\mathbf{L}^2 + \left[V^{(2,0)}_{LS}(r) + V^{(1,1)}_{L_2S_1}(r)\right]\mathbf{L}\cdot\mathbf{S}\right.$$
$$\left. + V^{(1,1)}_{S^2}(r)\left(\frac{\mathbf{S}^2}{2} - \frac{3}{4}\right) + V^{(1,1)}_{\mathbf{S}_{12}}(r)\mathbf{S}_{12}(\hat{\mathbf{r}}) + 2V^{(2,0)}_r(r) + V^{(1,1)}_r(r)\right\}$$
$$= \sigma r + \frac{1}{M}\frac{\sigma}{\pi}\ln(\sigma r^2) + \frac{1}{M^2}\left(-\frac{\sigma}{6r}\mathbf{L}^2 - \frac{\sigma}{2r}\mathbf{L}\cdot\mathbf{S} - \frac{9\zeta_3\sigma^2 r}{2\pi^3}\right) + \mathcal{O}(r^{-2})$$

- Significance
  - Potential depends only on string tension and heavy quark mass
  - These parameters determined by lattice simulations
- Next-to-leading order
  - Non-trivial impact of the sub-leading terms to the potentials
  - NLO in the gauge field insertions: $(r\Lambda_{QCD})^{-2}$ suppressed
  - Subleading contributions to the action might also be needed

NLO calculation

Possible contributions at NLO:

1. Non-gaussian action
   - Add $(\partial_a\xi^l\partial^a\xi^l)^2$, $(\partial_a\xi^l\partial_b\xi^l)(\partial^a\xi^m\partial^b\xi^m)$ [Lüscher, Weisz, JHEP (2004)]
   - Solution via perturbative expansion
   - But the Green's function is at $\mathcal{O}(\sigma^{-3}r^{-6})$, counted as NNLO
2. String Mapping
   - Gaussian action
   - Symmetries from $Q\overline{Q}$ bound state (CP, T, rotation, reflection):

$$\langle\langle \dots \mathbf{E}^l_{1,2}(t)\dots\rangle\rangle = \langle\dots\Lambda^2\partial_z\xi^l(t,\pm r/2) + \overline{\Lambda}^2\partial_z\xi^l(t,\pm r/2)(\partial_a\xi^m(t,\pm r/2))^2\dots\rangle$$

$$\langle\langle \dots \mathbf{B}^l_{1,2}(t)\dots\rangle\rangle = \langle\dots\pm\Lambda'^2\epsilon^{lm}\partial_t\partial_z\xi^m(t,\pm r/2) \pm \overline{\Lambda'}^2\epsilon^{lm}\partial_t\partial_z\xi^m(t,\pm r/2)(\partial_a\xi^n(t,\pm r/2))^2\dots\rangle$$

$$\langle\langle \dots \mathbf{E}^3_{1,2}(t)\dots\rangle\rangle = \langle\dots\Lambda''^2 + \overline{\Lambda''}^2(\partial_a\xi^m(t,\pm r/2))^2\dots\rangle$$

$$\langle\langle \dots \mathbf{B}^3_{1,2}(t)\dots\rangle\rangle = \langle\dots\pm\Lambda'''\epsilon^{lm}\partial_t\partial_z\xi^l(t,\pm r/2)\partial_z\xi^m(t,\pm r/2)$$
$$\pm\overline{\Lambda'''}\epsilon^{lm}\partial_t\partial_z\xi^l(t,\pm r/2)\partial_z\xi^m(t,\pm r/2)(\partial_a\xi^n(t,\pm r/2))^2\dots\rangle$$

Divergence and regularization

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4-string field correlator

- Product of two Green's functions
- Three sources of divergence
  - $\partial_z \partial_{z'} G(t, t'; z, z')|_{t=t', z=z'=\pm r/2}$
  - $\partial_t \partial_z \partial_{z'} G(t, t'; z, z')|_{t=t', z=z'=\pm r/2}$
  - $\partial_t \partial_{t'} G(t, t'; z, z')|_{t=t', z=z'=\pm r/2}$
- Infinite sum over vibrational modes and integral over all momentum space in the Green's function, Eq. (3)

$$G(t, t'; z, z') = \frac{1}{\pi \sigma r} \sum_{n=0}^{\infty} \sin\left(\frac{n\pi}{r} z - \frac{n\pi}{2}\right) \sin\left(\frac{n\pi}{r} z' - \frac{n\pi}{2}\right) \int_{-\infty}^{\infty} dk \frac{e^{-ik(t-t')}}{k^2 + \left(\frac{n\pi}{r}\right)^2}$$

- Zeta function and dimensional regularization
  - $\partial_z \partial_{z'} G|_{t=t', z=z'=\pm r/2} \propto \sum_n n = -1/12$
  - $\partial_t \partial_z \partial_{z'} G|_{t=t', z=z'=\pm r/2} \propto \sum_n n^2 = 0$
  - $\partial_t \partial_{t'} G|_{t=t', z=z'=\pm r/2} \propto 0 \cdot \int dy [y^2/(y^2+1)] = 0 \cdot (-2\pi)$

NLO result

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- Poincaré invariance $\rightarrow$ reduce parameters
  - Additional equality: $-4V_{p^2}^{(2,0)} + 2V_{p^2}^{(1,1)} - V^{(0)} + r\frac{dV^{(0)}}{dr} = 0$
- Exact result with few free parameters:

$$V_{p^2}^{(2,0)}(r)|_{NLO} = \left(\frac{1}{12\pi} + \frac{\pi}{36}\right)\frac{1}{r} - \frac{\mu}{4}, \quad V_{L^2}^{(2,0)}(r)|_{NLO} = -\frac{\sigma r}{6} + \left(\frac{11}{36\pi} + \frac{2\pi}{27}\right)\frac{1}{r},$$
$$V_{p^2}^{(1,1)}(r)|_{NLO} = \left(\frac{1}{6\pi} - \frac{\pi}{36}\right)\frac{1}{r}, \quad V_{L^2}^{(1,1)}(r)|_{NLO} = \frac{\sigma r}{6} + \left(\frac{1}{9\pi} + \frac{5\pi}{216}\right)\frac{1}{r},$$
$$\cdots$$

- Full expression up to $\mathcal{O}(M^{-2}; r^{-1})$:

$$V(r) = \left\{\sigma + \left[-\frac{9\zeta_3}{2\pi^3} + \frac{a+b}{4\pi^3} + \left(\frac{\pi}{540} - \frac{1}{9\pi}\right)\left(\frac{1}{3} + \frac{1}{4\pi^2}\right)^2\right]\frac{\sigma^2}{M^2}\right\} r^1$$
$$+ \frac{\sigma}{\pi M}\ln\left(\sigma r^2\right) + \left(\mu + \frac{2\mu_1'}{M} - \frac{\mu}{2M^2}\mathbf{p}^2 + \frac{\mu_2}{M^2}\right) r^0$$
$$+ \left\{-\frac{\pi}{12} + \left[\frac{1}{3\pi} + \frac{\pi}{36}\right]\frac{\mathbf{p}^2}{M^2} + \left(-\frac{\mathbf{L}^2}{6} - \frac{\mathbf{L}\cdot\mathbf{S}}{2} + \left[\frac{50}{27\pi^2} - \frac{2}{9\pi^2} - \frac{1}{12\pi^6}\right]\right)\frac{\sigma}{M^2}\right\} r^{-1} + \mathcal{O}(r^{-2}) \quad (5)$$

Comparison to Lattice QCD (1/5)

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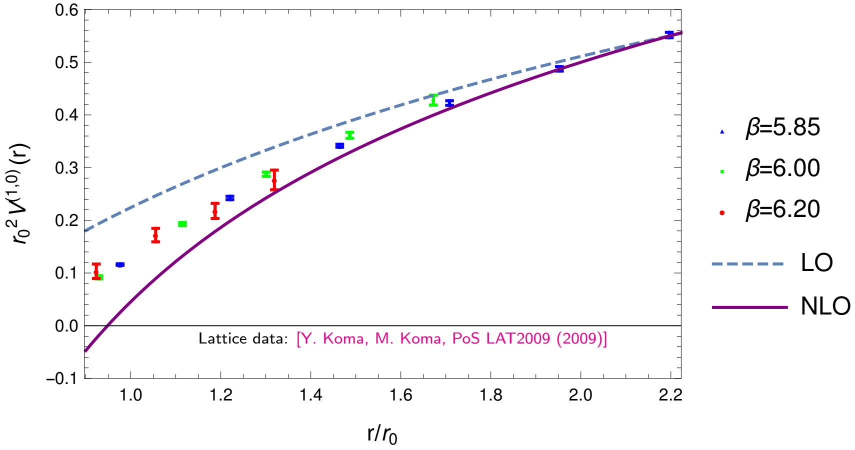
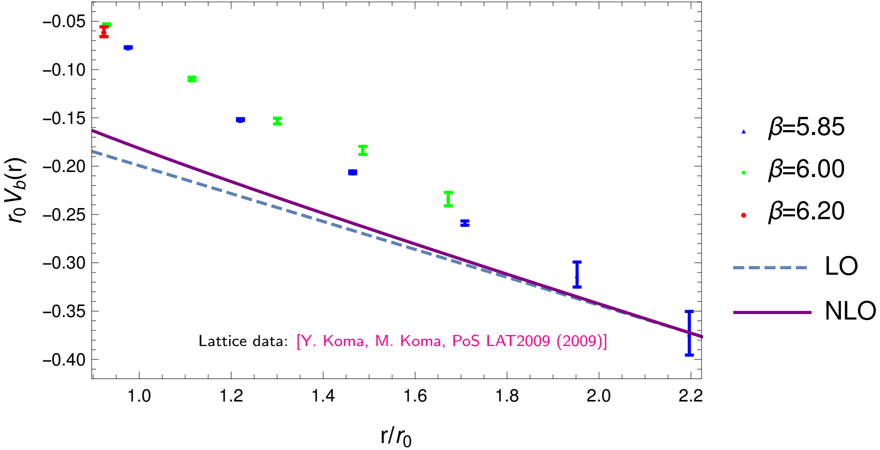


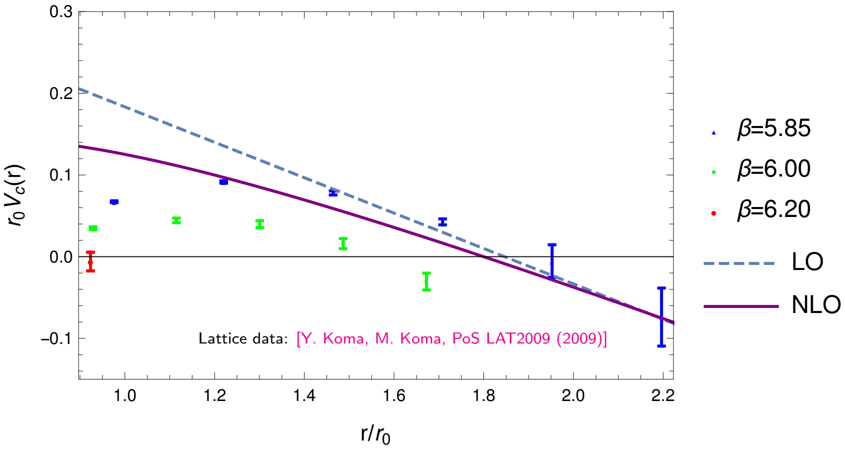
1st order relativistic correction via analytic prediction at NLO vs. Lattice data (prelim.)

Comparison to Lattice QCD (2/5)



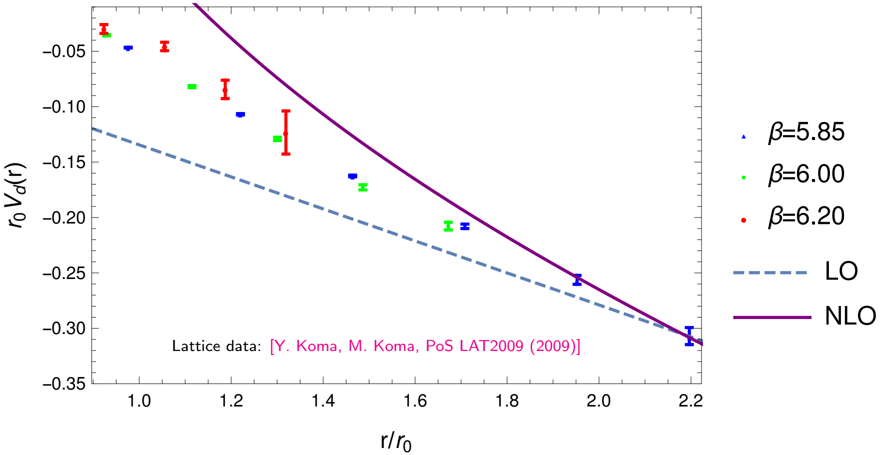
Momentum-dependent potential via analytic prediction at NLO vs. Lattice data (prelim.)

Comparison to Lattice QCD (3/5)



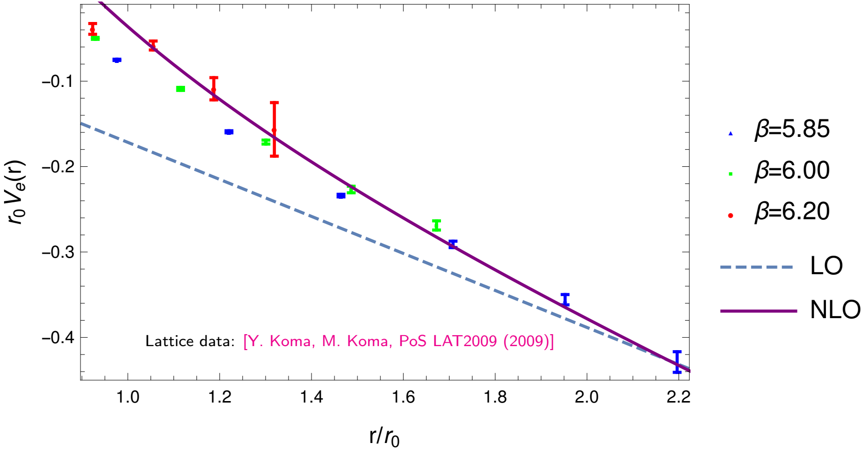
Momentum-dependent potential, $-V^{(1,1)}_{L^2}$, via analytic prediction at NLO vs. Lattice data (prelim.)

Comparison to Lattice QCD (4/5)



Momentum-dependent potential via analytic prediction at NLO vs. Lattice data (prelim.)

Comparison to Lattice QCD (5/5)



Momentum-dependent potential, $V_{L^2}^{(2,0)}$, via analytic prediction at NLO vs. Lattice data (prelim.)

Summary and outlook

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- Summary
  - Introduction to low-energy EFTs for QCD
  - Heavy quark potentials
  - Wilson loop and EST
  - Heavy quark potential via EST (at LO)
  - Correction via NLO mapping
  - Heavy quark potential up to NLO in EST
  - Comparison to Lattice data
- Outlook
  - Heavy quarkonium mass spectrum (in progress)
  - EST to heavy hybrids and baryonic states (in progress)

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