Introduction to AdS/CFT

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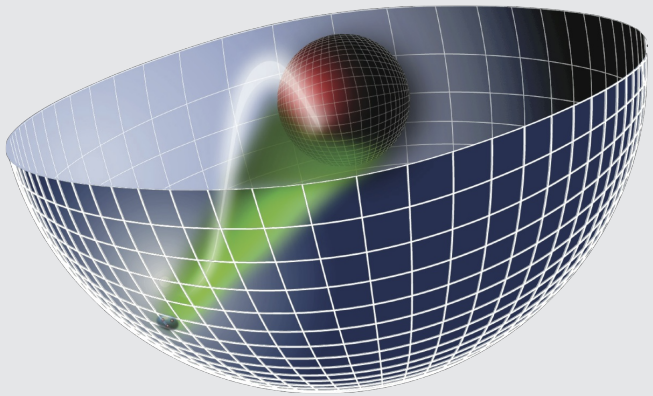


(Figure by Stan Brodsky)

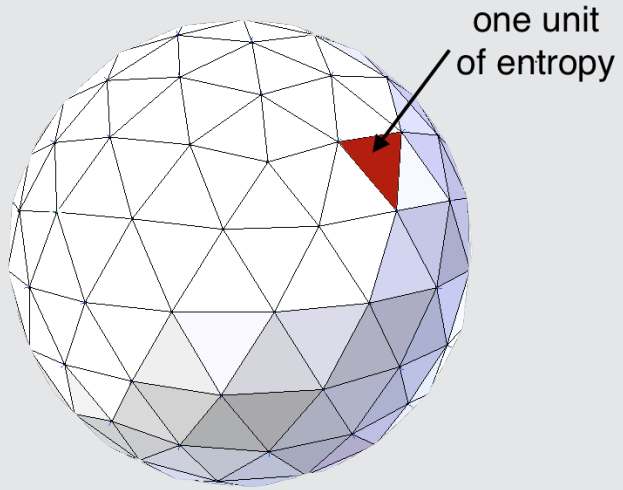
Intuitive motivation

What is meant by *holography*?

Black hole entropy

A theory of **gravity** contains black holes with entropy

$$S_{BH} = \frac{1}{4G_N} \, Area_{Horizon}.$$



Bekenstein bound

More generally, the entropy in a spacetime region A is bounded by

$$S(A) \leq \frac{1}{4G_N} \, Area(\partial A),$$

$$\Rightarrow \text{ bound on \# } DoFs \text{ in } A \propto S(A).$$

(Bekenstein 1972, 1981)

Intuitive motivation

What is meant by *holography*?

In contrast, in a **QFT** the entropy scales as

$$S(A) \propto Volume(A).$$

Holographic principle

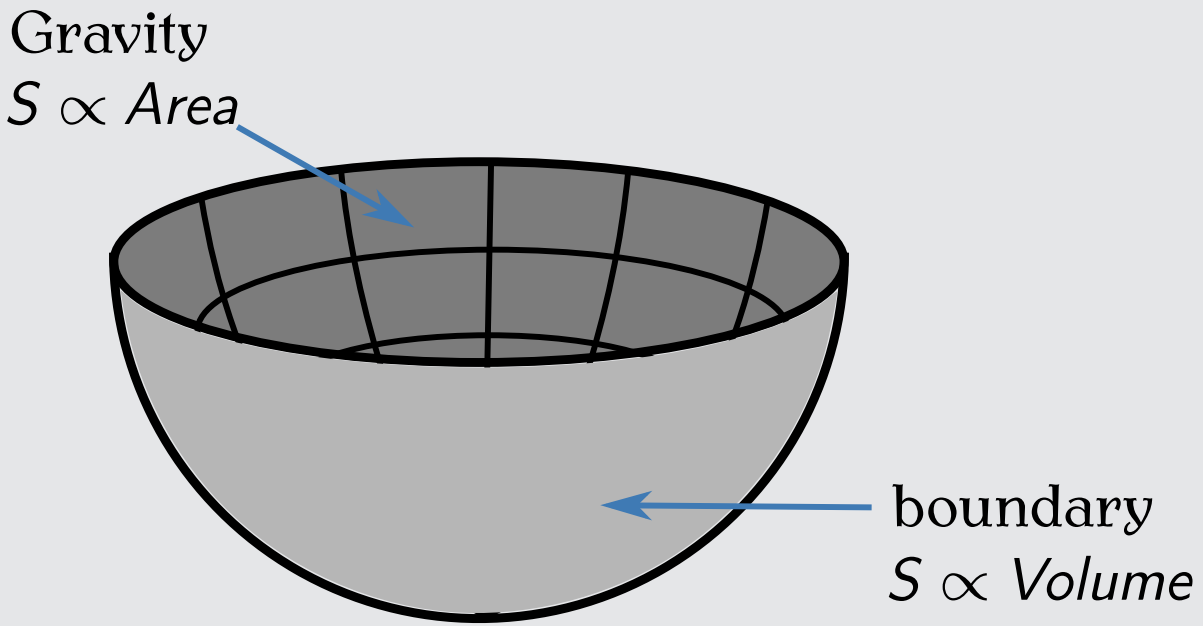
For theory of quantum gravity, the degrees of freedom in a region A can be described by a 'conventional' theory living on its boundary ∂A.

- *holography:*
  encoding the DoFs in a volume on its boundary
- quite general, no explicit construction of dual theory

(Hooft, 1993; Susskind 1994)

Intuitive motivation

What is meant by *holography*?



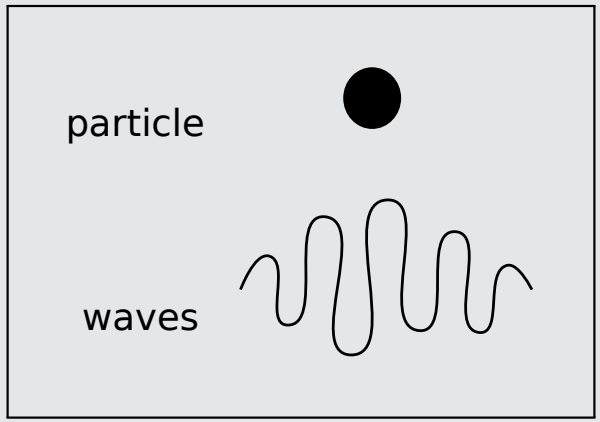
AdS/CFT duality

AdS/CFT theory in Anti-de Sitter $\longleftrightarrow$ conformal field theory

- realisation of the holographic principle
- origin in string theory

Duality **two theories** describe **same physics**

- same dynamics
- one-to-one map between DoFs
- vastly different theories
  - different spacetime dimension
  - different coupling regimes
  - …
- example:
  particle-wave duality



(Maldacena, 1997)

AdS/CFT duality

Anti-de Sitter space (AdS)

- solution to the vacuum Einstein equations with $\Lambda < 0$ space with constant negative curvature $R < 0$
- $d+1$ dimensional AdS space embedded in $d+2$ dimensions as

Embedding

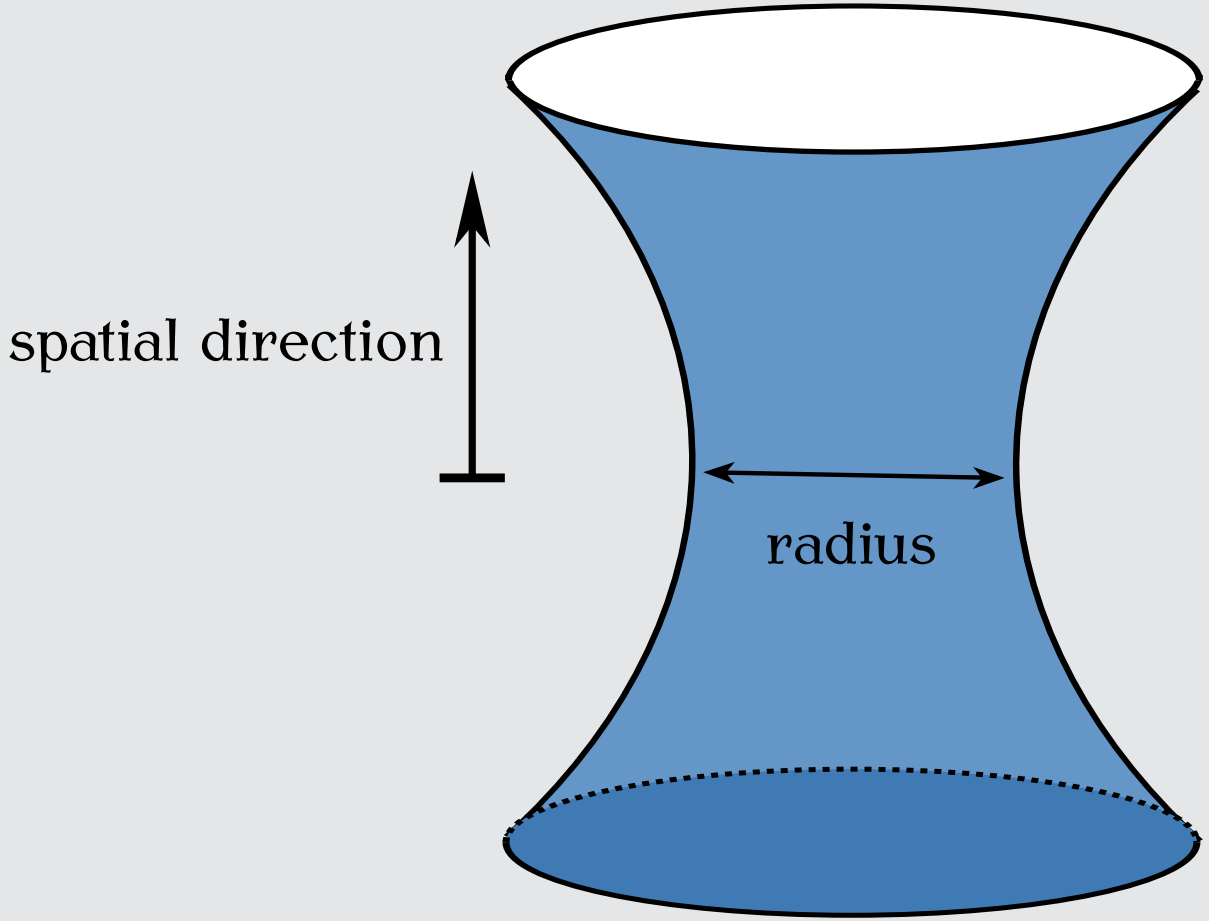
$$-(x^0)^2 + \sum_{i=1}^{d} (x^i)^2 - (x^{d+1})^2 = const. < 0$$

- symmetry group: orthogonal group of $\mathbb{R}^{d,2}$

$$SO(d,2)$$

AdS/CFT duality

Anti-de Sitter space (AdS)



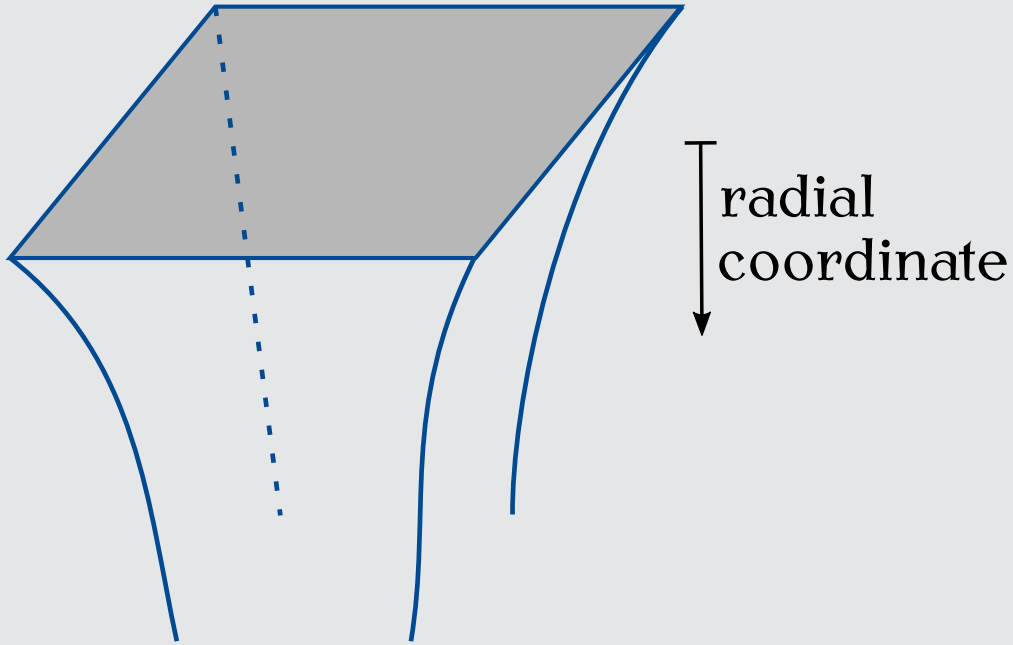
AdS/CFT duality

Anti-de Sitter space (AdS)

metric

$$ds^2 \propto \frac{1}{z^2}\left(dz^2 - dt^2 + d\vec{x}^2\right)$$

- radial coordinate z
- metric singular at $z = 0 \Rightarrow$ boundary ∂AdS



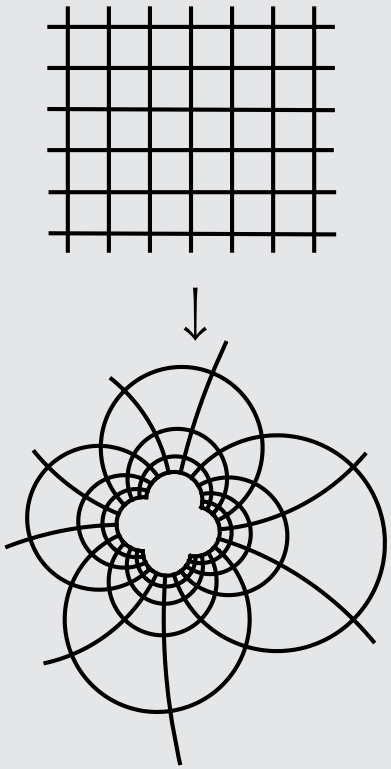
AdS/CFT duality

Conformal field theory

- conformal symmetry
  - extension of Poincare symmetry
  - coordinate transformation such that

$$g_{\mu\nu}(x) \to e^{2\sigma(x)} \cdot g_{\mu\nu}(x)$$

  - preserves angles and causality
- no length-scale, i.e. only dimensionless coupling constants
- vanishing β-functions



AdS/CFT duality

Comparison of Symmetries

- $SO(d,2)$ symmetry of AdS $\leftrightarrow$ conformal symmetry

How can we understand this?

- coordinate transformation

  $\downarrow$

  conformal transformation on boundary

$$ds^2_{AdS} \propto \frac{1}{z^2}\left(dz^2 + \hat{g}_{\mu\nu}dx^\mu dx^\nu\right)$$

- AdS metric fixes boundary metric only up to conformal factor:

$$ds^2_{\partial AdS} = \lim_{z\to 0} \omega(z,t,\vec{x})\; ds^2_{AdS}|_{z=const.}$$

$\rightarrow$ 'defining function' $\omega(z,t,\vec{x})$,
second order zero at the boundary

Boundary metric

AdS/CFT duality

Example: $\mathcal{N} = 4$ SYM theory, $d = 4$

- supersymmetric Yang-Mills theory
- adjoint DoFs with gauge group $SU(N)$

field content

one	gauge field	
four	Weyl fermions	$\longrightarrow$ in adjoint rep.
six	(real) scalars	

- dimensionless coupling-constant g_{YM}
- $\mathcal{N} = 4$: number of supersymmetries

$\updownarrow$

- string theory on

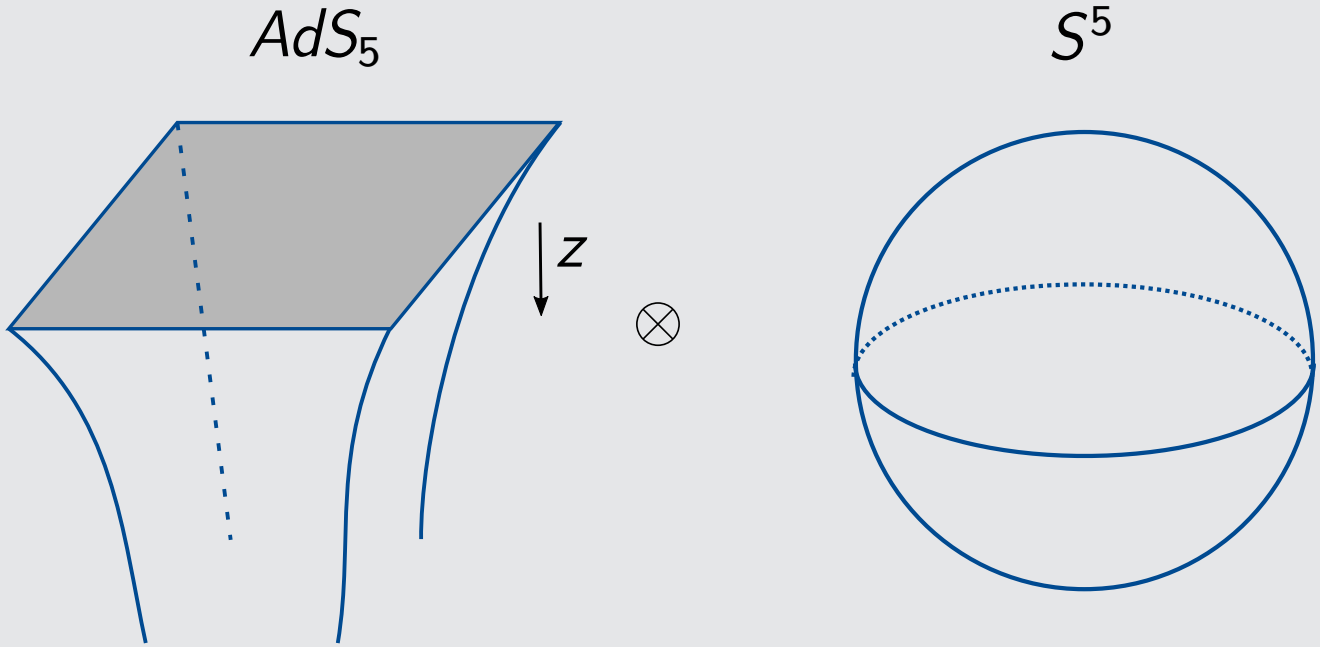
bulk geometry

$AdS_5 \times S^5$

- compactify $S^5 \rightarrow$ effective theory in five dimensions
- symmetry $SO(6) \longleftrightarrow$ additional $\mathcal{N} = 4$ SUSY

AdS/CFT duality

Example: $\mathcal{N} = 4$ SYM theory



AdS/CFT duality

Example: $\mathcal{N} = 4$ SYM theory

dictionary:

$$g_{YM}^2 \propto \text{string coupling}$$
$$\lambda = g_{YM}^2 N \propto \text{string length}^{-4}$$

strongest form	N, λ arbitrary	$\longleftrightarrow$	quantum string theory
strong form	$N \to \infty$, λ arbitrary	$\longleftrightarrow$	classical string theory, only tree level diagrams
weak form	$N \to \infty$, λ large	$\longleftrightarrow$	point-particle limit, classical field theory

AdS/CFT duality

How does this work?

Field-operator map

I promised you a one-to-one map:

CFT operator $\mathcal{O}$ $\quad\Longleftrightarrow\quad$ **AdS field** φ with mass m

$$x \to \lambda x$$

$$\mathcal{O}(x) \to \lambda^{-\Delta}\mathcal{O}(x)$$

$$m^2 \propto \Delta(\Delta - d)$$

near-boundary expansion:

$$\varphi(z, t, \vec{x}) \sim \underset{\uparrow}{\varphi_{(0)}}(t, \vec{x}) \cdot z^{d-\Delta} + \underset{\uparrow}{\langle \mathcal{O}(t, \vec{x}) \rangle} \cdot z^{\Delta} + \cdots$$

source of $\mathcal{O}$ expectation value

(Witten, 1998; Gubser, Klebanov, Polyakov 1998)

AdS/CFT duality

How does this work?

GKP-Witten relation

We identify the partition functions on both sides

$$\begin{aligned} Z_{CFT}[\varphi_{(0)}] &= Z_{AdS}[\varphi] \\ &= \exp^{iS_{AdS}[\varphi]}, \end{aligned}$$

where $\varphi_{(0)}$ is the source of the operator $\mathcal{O}$.

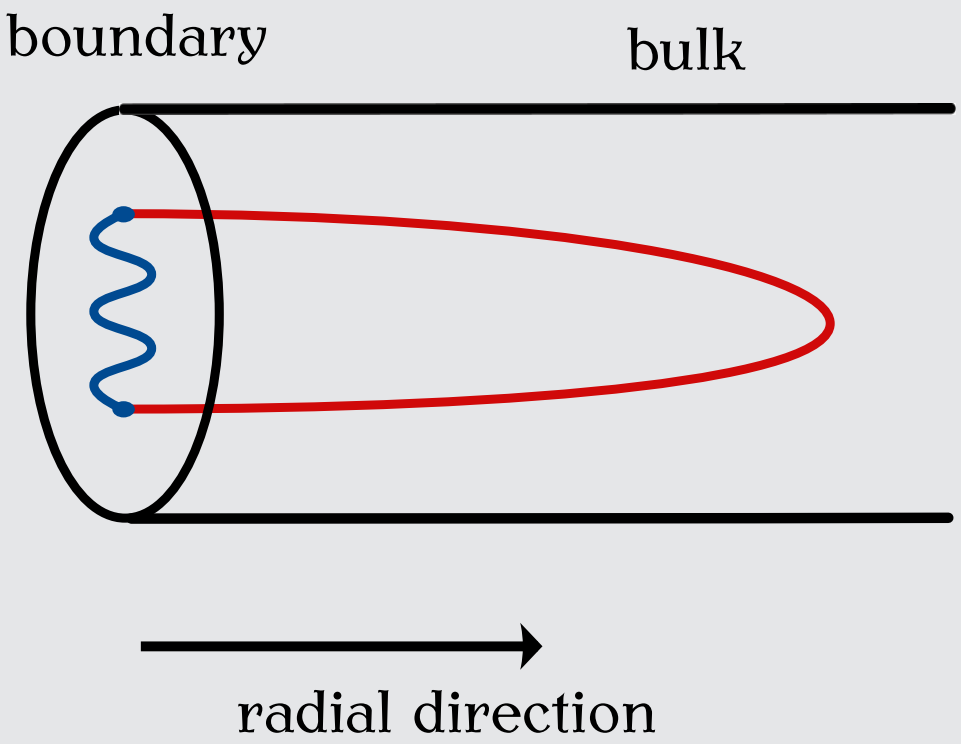
Correlation functions

$$\langle \mathcal{O}_1(x_1) \cdots \mathcal{O}_n(x_n) \rangle = \frac{1}{i^n} \frac{\delta^n Z_{AdS}}{\delta \varphi^1_{(0)}(x_1) \varphi^n_{(0)}(x_n)}$$

(Witten, 1998; Gubser, Klebanov, Polyakov 1998)

AdS/CFT duality

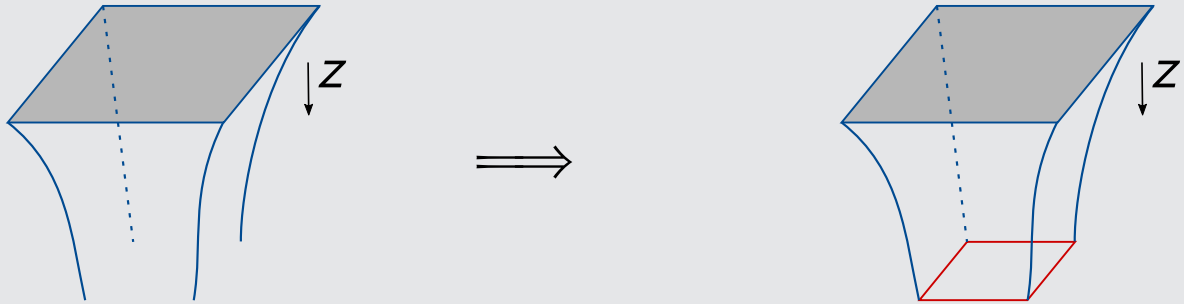
How does this work?



Generalisations

Finite temperature

- finite temperature $\longleftrightarrow$ black hole



- thermodynamics
  - temperature
  - entropy
  - …

$\longleftrightarrow$

- black hole thermodynamics
  - Hawking temperature
  - Bekenstein entropy
  - …

Generalisations

- different dimensions:

  conformal field theory in d-dimensions

  $\updownarrow$

  theory in Anti-de Sitter space in $d + 1$ dimensions

- fundamental degrees of freedom

- string theory
  - highly symmetric theories
  - additional compact space

- alternative:

  tailor gravity background to desired properties

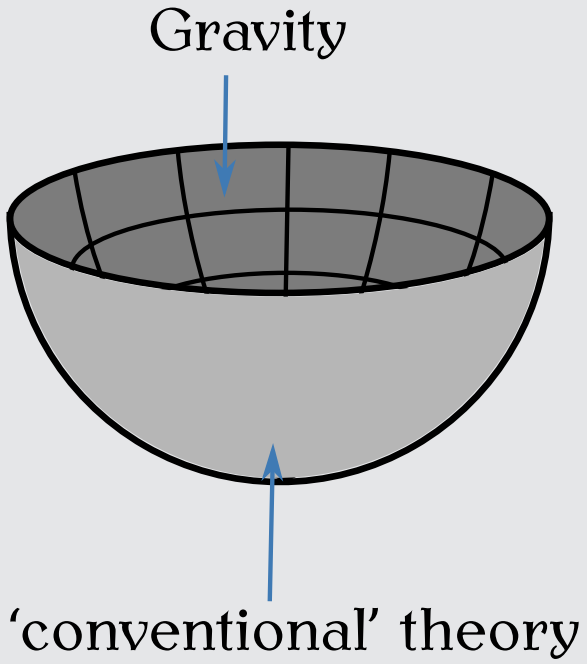
Summary

Holographic principle:

gravity theory in region A

$\updownarrow$

'conventional' theory on ∂A.



Summary

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gravity theory in region A

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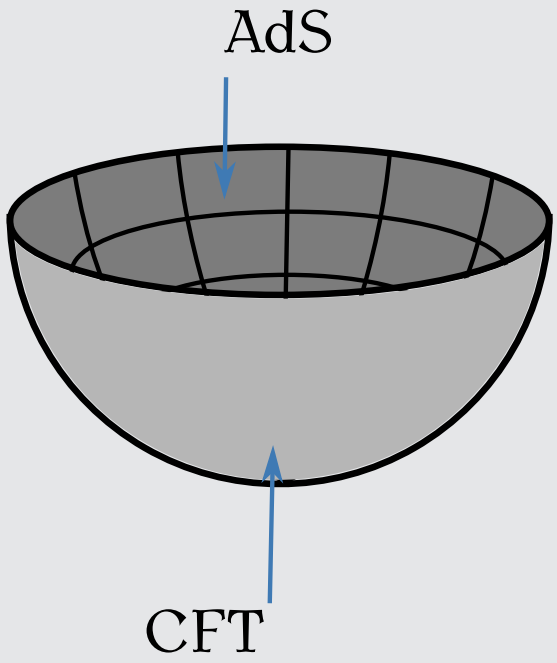

AdS/CFT:

CFT in d dimensions

$\updownarrow$

theory on AdS in $d + 1$ dimensions

- one-to-one map between DoFs, same dynamics
- symmetries match
- strong coupled field theories

$\rightarrow$ probes interesting new regime

Thank you for your attention