Quantum Black Holes and Global Symmetries

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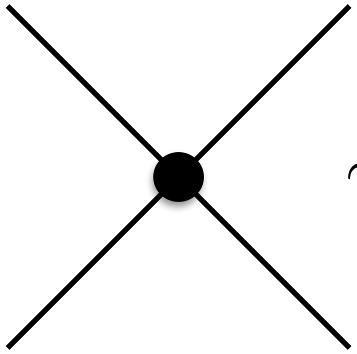
Outline

1) Quantum fields in curved spacetime
2) The Unruh effect
3) Quantum black holes and Hawking radiation
4) Problems with global symmetries

Quantum Gravity?

- Collider physics described by (perturbative) quantum field theory
- Prototype: scalar field theory

$$S = \int d^4x \left(\underbrace{\frac{1}{2}\eta_{\mu\nu}\partial^\mu\phi\partial^\nu\phi - \frac{1}{2}m^2\phi^2}_{\mathcal{L}_{\text{free}}} \underbrace{- \lambda\phi^4}_{\mathcal{L}_{\text{int}}} \right)$$



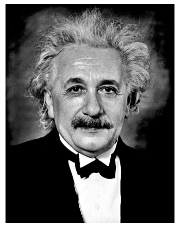
$\sim \lambda$

- Dictated by unification of quantum mechanics

$$i\hbar\partial_t|\psi\rangle = \hat{H}|\psi\rangle$$

- With special relativity

$$\eta_{\mu\nu} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$



$$d\tau^2 = \eta_{\mu\nu}x^\mu x^\nu = (ct)^2 - \vec{x}^2$$

Quantum Gravity?

- General relativity described by highly non-linear, complicated Einstein field equations for the metric $g_{\mu\nu}$ (spacetime geometry)

$$\eta_{\mu\nu} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \to g_{\mu\nu} = \begin{pmatrix} g_{00} & g_{01} & g_{02} & g_{03} \\ g_{10} & g_{11} & g_{12} & g_{13} \\ g_{20} & g_{21} & g_{22} & g_{23} \\ g_{30} & g_{31} & g_{32} & g_{33} \end{pmatrix}$$

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu} \qquad R \sim d(g^{-1}dg) + (g^{-1}dg)^2$$

- Naive treatment as perturbative QFT runs into big problems (non-renormalizability)

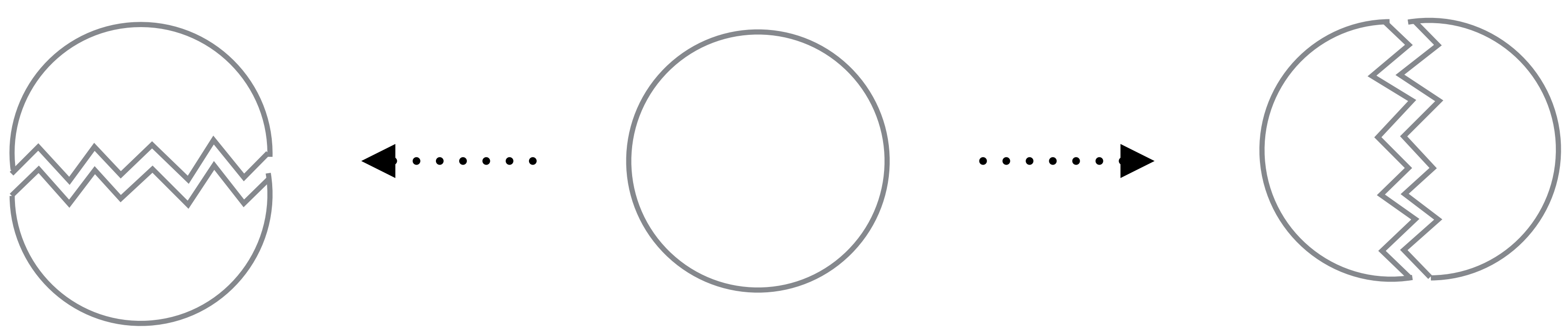
Gravity has to be quantized - we just don't know for sure how!

→ For this talk we will remain ignorant!

Necessity of Quantization

$$\underbrace{R_{\mu\nu} - \tfrac{1}{2}Rg_{\mu\nu}}_{\text{gravity}} = \underbrace{\tfrac{8\pi G}{c^4}T_{\mu\nu}}_{\text{matter}}$$

- Why not just quantize RHS? $R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = \frac{8\pi G}{c^4}\langle \hat{T}_{\mu\nu}\rangle$
- Superposition states: gravitational field is "average" over possible outcomes. Upon measurement we have collapse and discontinuous change of the gravitational field.
- Violates locality, Lorentz invariance, also $\nabla^\mu \langle \hat{T}_{\mu\nu}\rangle \neq 0$, which is inconsistent with the proposed equation.



Quantum Fields in Curved Spacetime!

- Study quantum fields in classical gravity background (e.g. black hole)

$$\int d^4x \left(\frac{1}{2}\eta_{\mu\nu}\partial^\mu\phi\partial^\nu\phi - \frac{1}{2}m^2\phi^2 - \lambda\phi^4\right)$$

$$\int d^4x\sqrt{\det(g)}\left(\frac{1}{2}g_{\mu\nu}\partial^\mu\phi\partial^\nu\phi - \frac{1}{2}m^2\phi^2 - \lambda\phi^4\right)$$

- Surprisingly, leads to non-trivial, robust insights about quantum gravity
- Works as long as curvature is not too strong (black hole singularity)

The Vacuum State

- QFT vacuum: state of lowest energy $\hat{H}|\Omega\rangle = E_{min}|\Omega\rangle$
- Equivalently killed by annihilation operators for every particle $\hat{a}_I|\Omega\rangle = 0$
- Lorentz symmetry: inertial observers agree on vacuum $[H, \Lambda_{\mu\nu}] = 0$
- In fact only true for inertial observers
- In general relativity: no privileged class of observers!

The definition of “vacuum” or “particle” in GR is inherently ambiguous

- Mathematically: creation/annihilation operators of two observers related by Bogolyubov transformation

$$\hat{b}_I = \sum_J \left(\alpha_{IJ}\hat{a}_J + \beta_{IJ}\hat{a}_J^\dagger\right) \quad \longrightarrow \quad \hat{b}_I|\Omega_A\rangle = \sum_J \beta_{IJ}\hat{a}_J^\dagger|\Omega_A\rangle \neq 0$$

The Unruh Effect

- Equivalence Principle: gravity is locally equivalent to accelerated frame of reference

- For a qualitative picture, we thus consider constantly accelerated observer (constant proper acceleration)

- Calculate expectation value of number density operator of the accelerated observer A in Minkowski vacuum

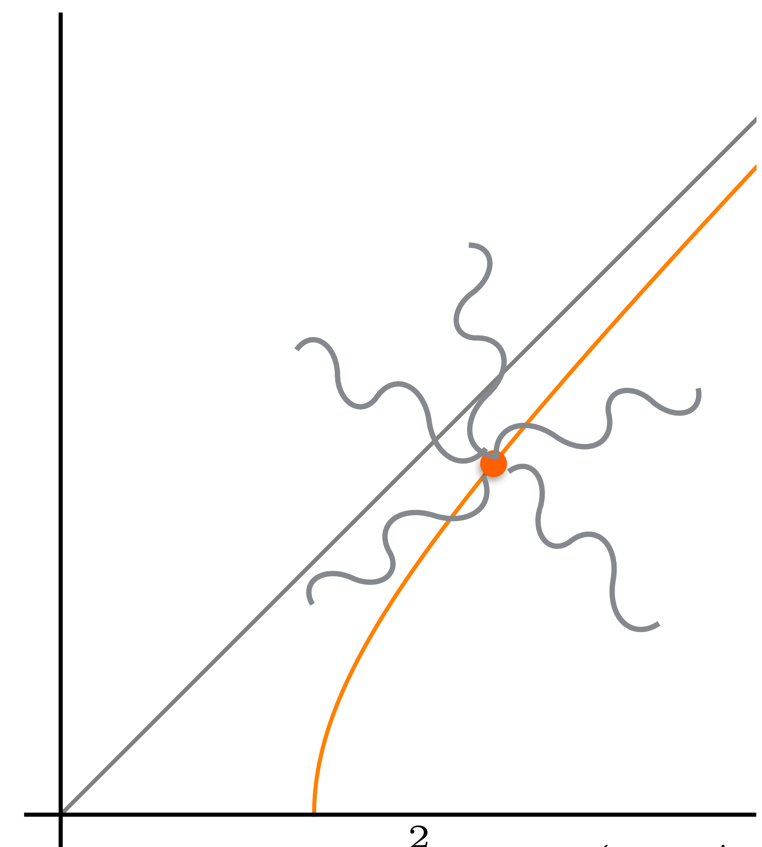
$$\langle \Omega_M | \hat{n}_A(E) | \Omega_M \rangle = \frac{1}{\exp\left(\frac{2\pi c E}{\hbar \alpha}\right) - 1}$$

- This is precisely Planck's law! A sees radiation!

→ Unruh Effect

$$T = \frac{\hbar k_B}{c} \frac{\alpha}{2\pi}$$

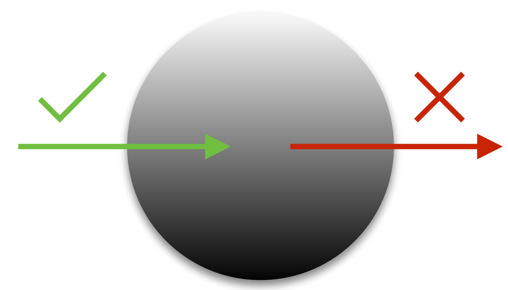
$$ct = \frac{c^2}{\alpha} \sinh(\alpha\tau/c)$$



$$x = \frac{c^2}{\alpha} \cosh(\alpha\tau/c)$$

Quantum Black Holes

- Classical black hole: nothing can escape from within the horizon



- Hawking showed: vacuum of collapse that of free falling observer
- Observer at infinity has relative acceleration and sees Hawking radiation
- Black holes lose mass and evaporate after all!
- Black hole thermodynamics:

$$T = \frac{\hbar c^3}{8\pi G k_B} \frac{1}{M} \qquad dM = T dS \qquad S = k_B \frac{4\pi G}{\hbar c^3} M^2 = \frac{A}{4\ell_P^2}$$

Some Ballpark Figures

- Both Unruh and Hawking radiation are hard to measure
- Measuring Hawking radiation requires getting close to small black holes - only option: micro black holes at accelerators
- For a 1K black hole we are looking at

$$M \simeq 10^{-8} M_{\odot}$$

- Unruh radiation is in principle easy to measure but the amount of acceleration is huge, again for 1K we need

$$\alpha \simeq 10^{20} \frac{m}{s^2}$$

Global Symmetries and Black Holes

- Imagine a world with gravity, matter and a continuous global symmetry (no gauge symmetry! no associated force!)
- Noether's theorem guarantees associated charge, e.g. baryon number

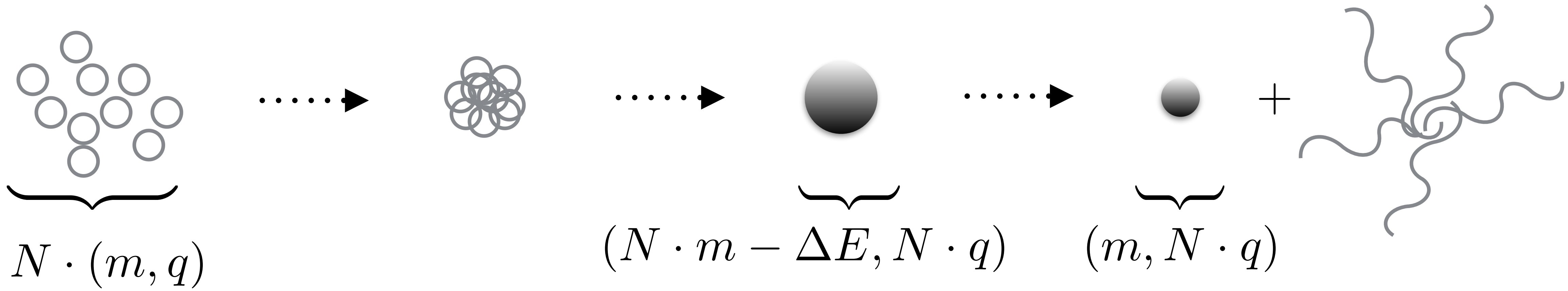
$$|q\rangle \rightarrow e^{\epsilon/3}|q\rangle \qquad |\bar{q}\rangle \rightarrow e^{-\epsilon/3}|\bar{q}\rangle \qquad \longrightarrow \qquad B = \frac{1}{3}\left(N_q - N_{\bar{q}}\right)$$

- To avoid subtleties, assume single particle species $|p\rangle$ interacting only through gravity and with charge p under such $U(1)$ symmetry

$$|p\rangle \rightarrow e^{\epsilon p}|p\rangle$$

- By collapsing N of these to a black hole with mass $N \cdot m$ and waiting until it evaporates to mass m we get black holes with arbitrary charge, all of the same mass/energy

No Global Symmetries

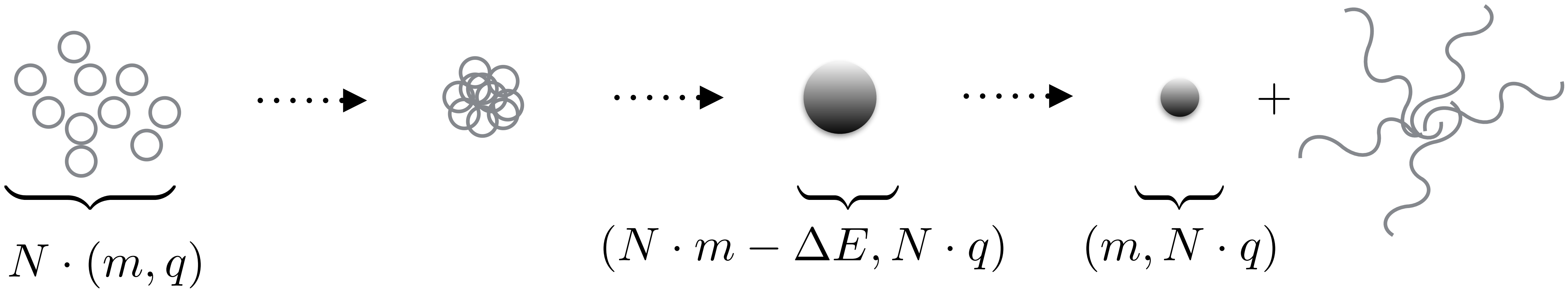


- Important: Hawking radiation contains same number of + and - charged particles, so black hole cannot lose charge
- If we let the black hole completely evaporate, charge is gone!
- We have created a process

$$Q = Nq \quad \cdots\cdots\blacktriangleright \quad Q = 0$$

which explicitly violates charge conservation

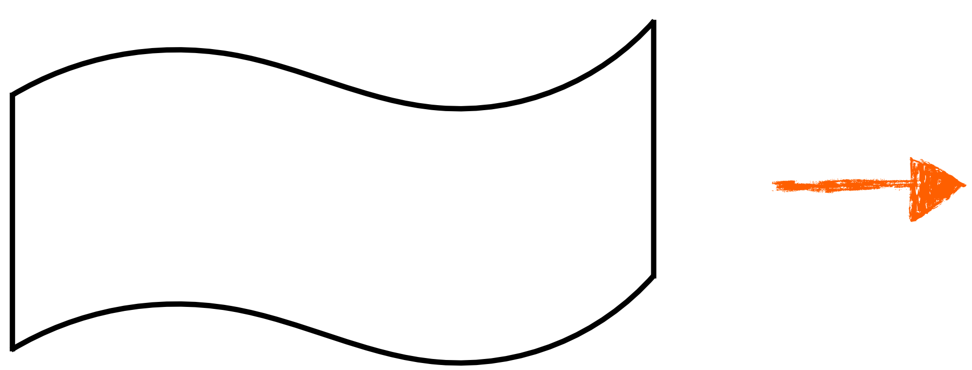
No Global Symmetries: Loophole



- Hawking calculation only valid until $M_{\rm BH} \simeq M_p$
- What if evaporation stops and remnant forms?
- No-Hair theorem: black holes with different Q but same M are indistinguishable from outside
- Since we can construct BH with arbitrary Q for a fixed M and thus energy, we see that black holes in the theory have infinite microcanonical entropy!
- Leads to various inconsistencies, violates entropy bounds!

Bonus: The Stringy Version

- We believe string theory is a consistent theory of quantum gravity
- Should rather forbid global symmetries then
- Explicit mechanism: perturbative string theory is described by two dimensional field theory on the string world sheet



$$S = \int dtdl \left(\eta_{MN} g^{ab} \partial_a X^M \partial_b X^N + \cdots \right)$$

$X^M(t,l)$ string position

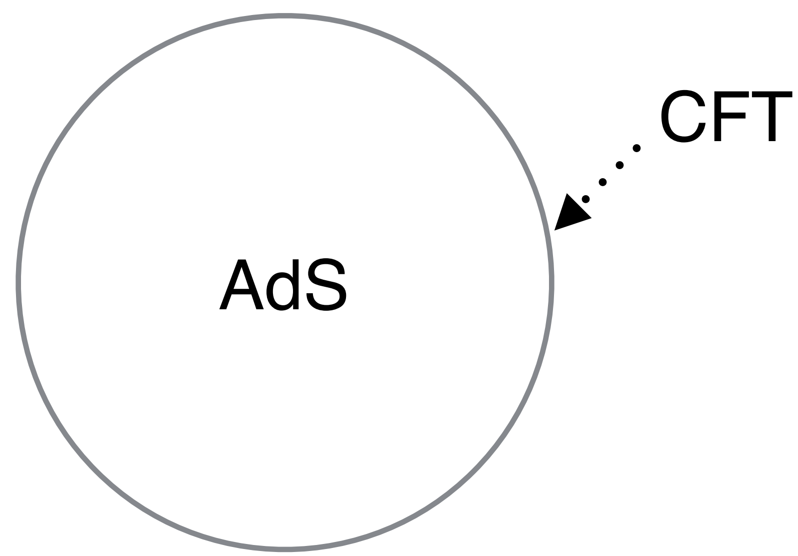
- Introducing a global symmetry on the world-sheet magically gives rise to a gauge symmetry with associated gauge bosons in the spacetime!

Bonus: The AdS/CFT version

- Reminder: AdS/CFT is an isomorphism of two very different theories

    1) Quantum gravity in Anti-de-Sitter (AdS) space

    2) Conformal field theory (non-gravitational) on the AdS boundary



global symmetry in CFT → gauge symmetry in AdS

Contradiction! ↯ ← global symmetry in AdS

Conclusion

- Even if ignorant about the details of quantum gravity, we can gain non-trivial insights by using usual QFT techniques in curved backgrounds
- Accelerated observers experience Unruh radiation
- A different manifestation of this is Hawking radiation of black holes
- Combining global symmetries with these expectations leads to paradoxes
- Hence global symmetries are not allowed in quantum gravity and thus nature!
- String theory seems to obey this!

Thank You