Enhanced Entropy for Quantum Critical State of Attractive Bose Gas

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(Based on a collaborative work with Gia Dvali and Sebastian Zell.)

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Outline

1. Motivation: Light Modes
2. Prototype Systems: Cold Bose Gas
3. Emergence of Gapless Collective Mode
4. Summary

Bekenstein Entropy

- Bekenstein-Hawking entropy:

$$S_B \sim \frac{r_g^2}{L_{Pl}^2}$$

- Number of microstates

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Need light modes for quantum information storage

Prototype Systems

- Idea: light modes at quantum phase transition of attractive Bose Gas[1]
- Apply to 1-D Gas in a box
- Goal: confirm existence of light mode

[1] G. Dvali and C. Gomez, *Black Holes as Critical Point of Quantum Phase Transition*, arXiv:1207.4059.

1-D Attractive Bose Gas in a Box

- Weakly interacting cold Bose gas:

$$\hat{H} = \int_0^L dz \left[\partial_z \hat{\psi}^\dagger \partial_z \hat{\psi} - \alpha \hat{\psi}^\dagger \hat{\psi}^\dagger \hat{\psi} \hat{\psi} \right]$$

$$\hat{\psi}(z) = \sum_{k=1}^{\infty} \hat{a}_k \sin\left(\frac{\pi z k}{L}\right)$$

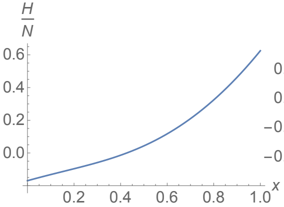
- Particle number N
- Control parameter: collective coupling αN

Bogoliubov Approximation

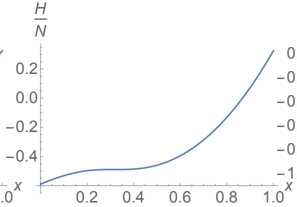
- Replace operators by expectation values
- Good approximation for large occupation numbers
- $k = 2$-mode occupation:

$$x = \frac{\langle a_2^\dagger a_2 \rangle}{N}$$

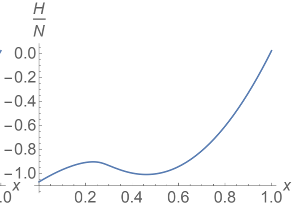
Change of Ground State



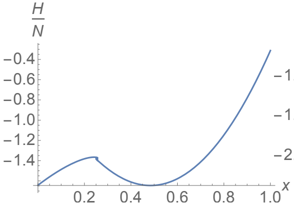
(a) $\alpha N = 1$



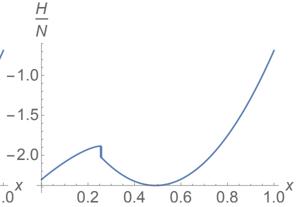
(b) $\alpha N = 1.8$



(c) $\alpha N = 2.6$



(d) $\alpha N = 3.5$



(e) $\alpha N = 4.5$

Mean Occupation in Full Quantum Ground State

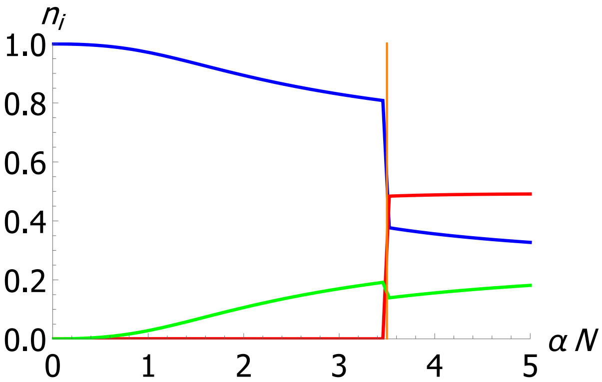


Figure : $N = 300$
$k = 1$-mode, $k = 2$-mode, $k = 3$-mode occupation

Appearance of Light Mode

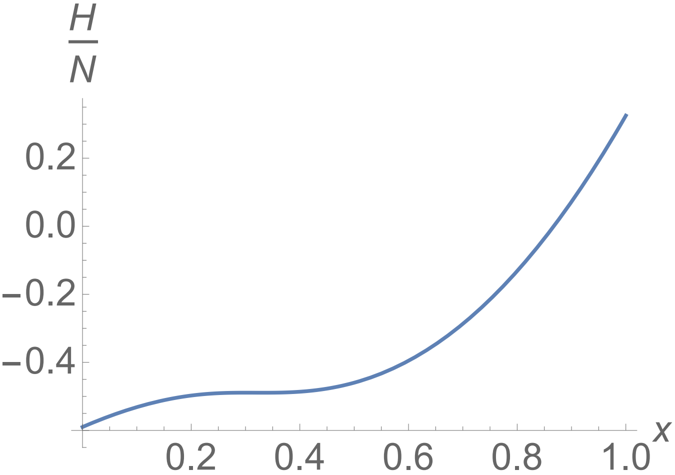
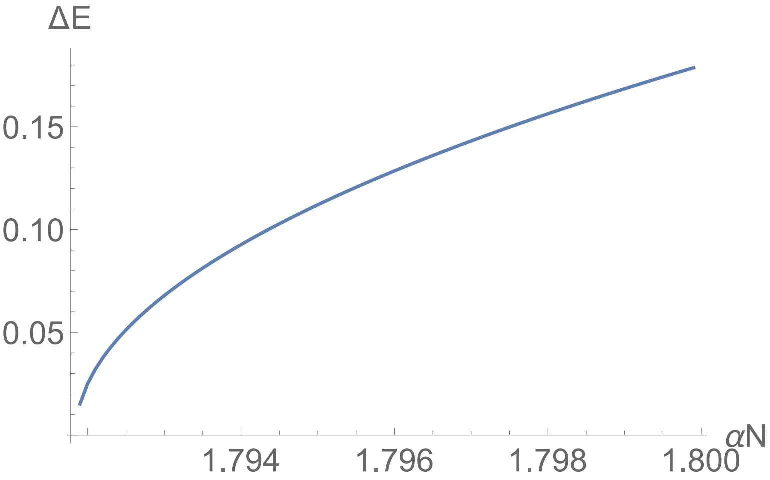
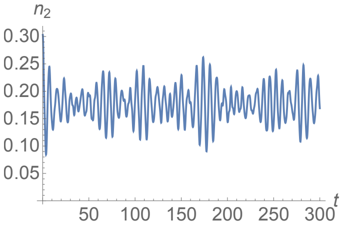


Figure : $\alpha N = 1.8$

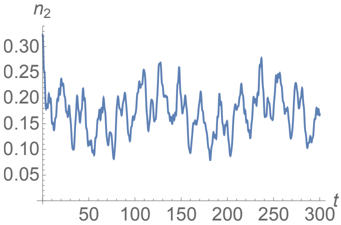
Spectrum of Fluctuations



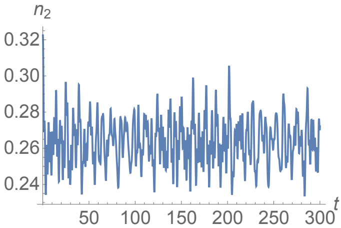
Full Quantum Time-Evolution of Light Mode



(a) $\alpha N = 1.6$



(b) $\alpha N = 1.99$



(c) $\alpha N = 2.15$

Summary

Attractive 3-level Bose gas in box

- First-order phase transition
- Appearance of modes with small mass gap
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Relationship to black holes

Similar mechanism?

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Quantum computing

Good quantum information storer

Thank You!