A numerical approach for the evaluation of helicity scattering amplitudes

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November 13, 2017

Outline of the talk

1. Motivation
2. Review of basic concepts
3. Introduction to the method
4. Conclusions

Motivation

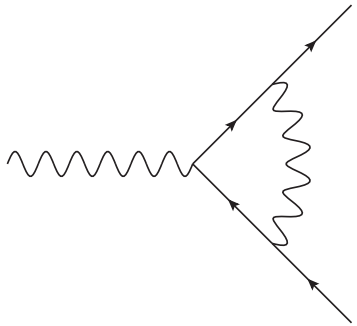
The second run of the LHC comes with an increase in accuracy. In order to keep up with experimental results we need to work on the ways to increase precision of theoretical predictions. Currently available methods are sufficient to obtain QCD amplitudes up Next-to-Leading Order. In some cases NNLO amplitudes have also been obtained, but there is still a long way before we find general approach. Therefore, we need to look out for new possibilities that would bring us closer to this goal.

UV divergence

UV divergences happen when we are dealing with loop integrals. The origin of these divergences is related to the fact that we integrate over infinite momentum. The solution to this problem is the well-known renormalisation. It results in redefining parameters of the theory and introducing counterterms. While it is a perfectly working solution in analytical calculations, it is not enough if we want to calculate the same thing numerically.

IR divergences

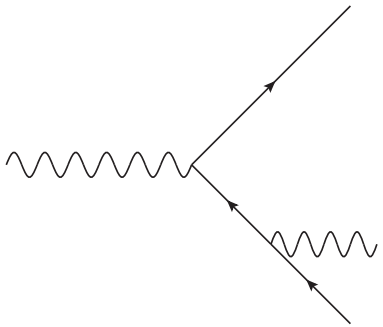
IR divergences occur when final states exchange massless particle, for example



The are two reasons for this type of divergence. The first one is the situation when exchanged particle is soft. The second one is when the exchanged particle goes collinear to one of the final states but that is only true when the final states are massless.

IR divergences

The solution turns out to be an inclusion of diagrams that have an extra final state such as



It is related to the fact that this final state is indistinguishable from the original one in the limit when emitted particle is soft or collinear. Also this solution is not enough for numerical integration.

Subtraction terms

In order to solve the problem of UV and IR singularities in numerical calculations, we need to introduce so-called subtraction terms. The reason why counterterms are not sufficient is that they are defined in integrated form, so for numerical integration we need to find an alternative expressions that approximate them but are in the form of integrand. The same is true for real emission diagrams which are integrated over different phase space than loop integral. Hence, for each type of divergence we need to use separate subtraction term.

Challenges of numerical method

1. Finding subtraction terms that would deal with UV and IR divergences.
2. Finding transformations that would allow for stable Monte-Carlo integration.

Current method

However, there already is a general numerical method that meets these two criteria up to NLO.

MZ-TH/10-38

Numerical NLO QCD calculations

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Abstract

We present an algorithm for the numerical calculation of one-loop QCD amplitudes. The algorithm consists of subtraction terms, approximating the soft, collinear and ultraviolet divergences of one-loop amplitudes and a method to deform the integration contour for the loop integration into the complex space. The algorithm is formulated at the amplitude level and does not rely on Feynman graphs. Therefore all required ingredients can be calculated efficiently using recurrence relations. The algorithm applies to massless partons as well as to massive partons.

Current method

As it was pointed out in this paper finding an algorithm that works for NLO case is quite challenging.

> *It is highly non-trivial to find a general algorithm which avoids these singularities and which leads to stable Monte Carlo results.*

However, while the contour deformation proposed in this method works for NLO, it fails for higher order corrections. Hence it is desirable to look out for alternative approaches that could be used to obtain more precise results.

Goal of the project

The main goal of the project was to find a new approach to integrate scattering amplitudes after adding subtraction terms.

Instead of contour deformation which was used in the method by Weinzierl et al., we wanted to transform initial expression in order to use the following identity

$$\lim_{\epsilon \to 0^+} \frac{1}{x + i\epsilon} = P\left(\frac{1}{x}\right) - i\pi\delta(x). \tag{1}$$

Sketch of the project

We develop the method on the simplest 3-leg process. In general the one-loop integral for such a process has a form

$$\int \frac{d^4k}{(2\pi)^4 i} \frac{P(k, p_1, p_2)}{[k^2 + i\epsilon][(k - p_1)^2 + i\epsilon][(k - p_2)^2 + i\epsilon]}. \tag{2}$$

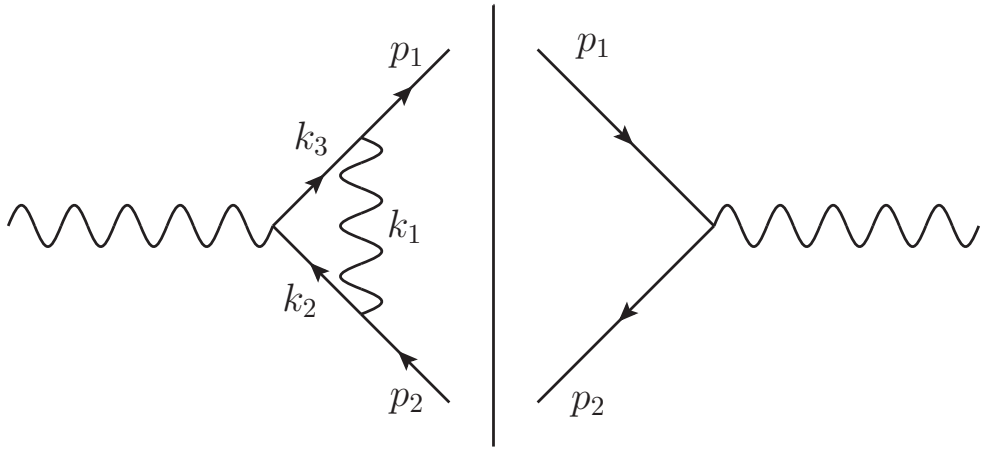
After few transformations we obtain the following form

$$\int dx_1 dx_2 dx_3 \frac{f(x_1, x_2, x_3)}{[x_1 + i\epsilon][x_2 + i\epsilon][x_3 + i\epsilon]}, \tag{3}$$

which allows to use the identity

$$\lim_{\epsilon \to 0^+} \frac{1}{x + i\epsilon} = P\left(\frac{1}{x}\right) - i\pi\delta(x).$$

Starting point



$$B = C\,Re\int \frac{d^4k}{(2\pi)^4 i}\left\{-\frac{1}{[k_2^2+i\epsilon][k_3^2+i\epsilon]}\left(3+\frac{2(p_2-p_1)\cdot(k_2+k_3)}{2s}\right)-\right.$$
$$\left.+\frac{2p_1\cdot k_3}{s[k_1^2+i\epsilon][k_3^2+i\epsilon]}+\frac{2p_2\cdot k_2}{s[k_1^2+i\epsilon][k_2^2+i\epsilon]}+\frac{2}{[\bar{k}^2-\mu_{UV}^2+i\epsilon]^3}\left(2\bar{k}^2-\frac{(2p_1\cdot\bar{k})(2p_2\cdot\bar{k})}{s}\right)\right\}, \tag{4}$$

Transformations

Firstly, we use Sudakov basis

$$k^\mu = \alpha p_1^\mu + \beta p_2^\mu + \gamma_1 b_1^\mu + \gamma_2 b_2^\mu \tag{5}$$

$$d^4k \to \frac{s^2}{2} d\alpha d\beta d\gamma_1 d\gamma_2 \tag{6}$$

Next steps include transformation to polar coordinates with integration over angular variable and less trivial reparametrisation of variables.

Result

After applying these transformations we obtain

$$B = C\,Re\int \frac{dw_1 dw_2 dw_3}{(2\pi)^3 i}\Theta(-w_1^2 + w_1(1+w_2+w_3) - w_2 w_3)$$
$$\left\{-\frac{1}{(w_3-i\epsilon)(w_2-i\epsilon)}(4-2w_1+w_2+w_3) + \frac{w_2-w_1}{(w_1-i\epsilon)(w_2-i\epsilon)} + \frac{w_3-w_1}{(w_1-i\epsilon)(w_3-i\epsilon)} + \right.$$
$$\left. + \frac{2}{(w_1+\mu^2-i\epsilon)^3}\left(-2w_1^2 - 2w_2 w_3 + 2w_1(2+w_2+w_3)\right)\right\}, \tag{7}$$

which is in (nearly) the form we would like.

Implementation of $P(\frac{1}{x})$

It turns out that we can effectively implement the principal value by remapping $(-\infty,\infty)^3 \to (0,\infty)^3$. It is simply achieved by

$$\begin{aligned} G = {} & g(x_1,x_2,x_3) + g(-x_1,x_2,x_3) + g(x_1,-x_2,x_3) + \\ & + g(x_1,x_2,-x_3) + g(-x_1,-x_2,x_3) + g(-x_1,x_2,-x_3) + \\ & + g(x_1,-x_2,-x_3) + g(-x_1,-x_2,-x_3), \end{aligned} \quad , \qquad (8)$$

where g is the integrand.

Open problems

The last remapping should allow us to successfully implement

$$\lim_{\epsilon \to 0^+} \frac{1}{x + i\epsilon} = P\left(\frac{1}{x}\right) - i\pi\delta(x).$$

to Monte-Carlo integration.

However, it turns out that it is still not enough to ensure stability of Monte-Carlo integration for both real and imaginary parts of the integrand.

Therefore ensuring stability for real and imaginary part remains still an open problem.

Conclusions

1. Current state of particle physics encourages to look out for new methods that would allow automation of NNLO calculations.
2. In our attempt we have managed to derive a potential approach that can be an alternative to contour deformation.
3. However, despite promising beginning there are still some problems that need to be addressed before the method will be reliable.

Thank you!