Application of Double Field Theory to Flux Compactifications

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Outline

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   - Flux Compactifications
   - Double Field Theory (DFT)
2. Research part
   - Overview
   - Dimensional Reduction
   - Results and Discussion
   - Conclusion and Outlook

The Problem of Moduli Stabilization

Motivation from the perspective of string phenomenology

- String phenomenology:

$$\begin{array}{ccc} \text{String Theory} & & \text{Observable Physics} \\ D = 10 & \xrightarrow{\text{Dimensional Reduction}} & D = 4 \end{array}$$

- Traditional string compactifications: Vacuum degeneracy gives rise to large number of massless scalar fields (moduli) in effective four-dimensional theory

$\rightarrow$ obvious contradiction to experiments

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Flux Compactifications

Going beyond differential geometry

- Idea: Allow for non-vanishing background fields on internal manifold, e.g. for type II NS-NS two-form B

$$H_{\mathrm{flux}} = \langle \mathrm{d}B_{\mathrm{int}} \rangle$$

Non-zero flux of the fields gives rise to scalar potential depending on moduli

$\rightarrow$ moduli become massive

- Problem: Full moduli stabilization requires "non-geometric" fluxes

$$\underset{\text{NS-NS flux}}{H_{ijk}} \overset{T_i}{\longleftrightarrow} \underset{\text{geometric}}{{F^i}_{jk}} \overset{T_j}{\longleftrightarrow} \underset{\text{non-geometric}}{{Q_k}^{ij}} \overset{T_k}{\longleftrightarrow} \underset{\text{non-geometric}}{R^{ijk}}$$

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Double Field Theory (DFT)

Enhancing point particles with stringy features

- Double number of coordinates by adding "winding coordinates" $\tilde{x}_i$ conjugate to winding number $\tilde{p}_i$ (cf. $x^i \leftrightarrow p^i$)

$$\begin{aligned} X^I &= (\tilde{x}_i, x^i) \\ P^I &= (\tilde{p}_i, p^i) \end{aligned} \quad , \quad i = 1, \dots D; \quad I = 1, \dots 2D$$

- Enhance point particles with "stringy" features and enable them to transform under T-duality

$\rightarrow$ $D = 10$ SUGRAs arise as solutions to "strong constraint"

$\rightarrow$ T-duality transformations as rotations of ten-dimensional "physical section" through doubled spacetime

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Overview

Previous works and objectives

- Starting point: "flux formulation" of DFT
- Compactification of purely internal contribution on CY_3 gives rise to scalar potential of $\mathcal{N} = 2$ gauged SUGRA [Blumenhagen, Font, Plauschinn '15]
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Dimensional Reduction (1)

Type II DFT on a Calabi-Yau three-fold

- Starting point: Bosonic part of type II DFT action

$$
\begin{aligned}
S_{\text{NS-NS}} &= \tfrac{1}{2}\int \mathrm{d}^{20}X e^{-2d}\Big[\\
& \hat{\mathcal{F}}_{\hat{M}\hat{N}\hat{P}}\hat{\mathcal{F}}_{\hat{M}'\hat{N}'\hat{P}'}\Big(\tfrac{1}{4}\mathcal{H}^{\hat{M}\hat{M}'}\eta^{\hat{N}\hat{N}'}\eta^{\hat{P}\hat{P}'}-\tfrac{1}{12}\mathcal{H}^{\hat{M}\hat{M}'}\mathcal{H}^{\hat{N}\hat{N}'}\mathcal{H}^{\hat{P}\hat{P}'} \\
& -\tfrac{1}{6}\eta^{\hat{M}\hat{M}'}\eta^{\hat{N}\hat{N}'}\eta^{\hat{P}\hat{P}'}\Big)+\hat{\mathcal{F}}_{\hat{M}}\hat{\mathcal{F}}_{\hat{M}'}\Big(\eta^{\hat{M}\hat{M}'}-\mathcal{H}^{\hat{M}\hat{M}'}\Big)\Big]
\end{aligned}
$$

$$
S_{\text{R-R}} = \tfrac{1}{2}\int d^{20}X\Big(-\tfrac{1}{2}\textstyle\sum_n\big|\hat{\mathcal{G}}_n\big|^2\Big), \qquad \hat{\mathcal{G}}_n = (-1)^{\left\lfloor \frac{n}{2}\right\rfloor}\star\hat{\mathcal{G}}_n,
$$

Dimensional Reduction (2)

Handling the DFT Action

- Step 1: Separate kinetic terms and scalar potential
- Step 2: Reformulate action in terms of twisted differential

$$\mathcal{D} = d \;\; - H\wedge \;\; - F\circ \;\; - Q\bullet \;\; - R\llcorner \;\; \underbrace{-Y\wedge \;\; - Z\blacktriangledown}_{\text{gen. dilaton}}$$

with, e.g.,

$$\begin{aligned} Q\bullet : \quad \Omega^p(CY_3) &\longrightarrow \Omega^{p-1}(CY_3) \\ \omega_p &\mapsto \frac{1}{2!} Q_i^{jk}\, dx^i \wedge \iota_j \wedge \iota_k \wedge \omega_p \end{aligned}$$

- Step 3: Expand in terms of cohomology basis, integrate equations of motion and duality constraints over internal space

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Effective Four-Dimensional Action

Partially dualized $\mathcal{N} = 2$ gauged SUGRA

$$
\begin{aligned}
S_{\mathrm{IIA}} &= \int \tfrac{1}{2} R^{(4)} \star \mathbf{1}^{(4)} - \mathrm{d}\phi \wedge \star \mathrm{d}\phi - \frac{e^{-4\phi}}{4} \mathrm{d}B \wedge \star \mathrm{d}B - g_{\mathrm{ij}} \mathrm{d}t^{\mathrm{i}} \wedge \star \mathrm{d}t^{\mathrm{j}} - g_{\mathrm{a}\bar{\mathrm{b}}} \mathrm{d}z^{\mathrm{a}} \wedge \star \mathrm{d}\bar{z}^{\bar{\mathrm{b}}} \\
&+ \tfrac{1}{2} \mathrm{Im}\mathcal{N}_{\mathrm{IJ}} \mathrm{F}_2^{\mathrm{J}} \wedge \star \mathrm{F}_2^{\mathrm{J}} + \tfrac{1}{2} \mathrm{Re}\mathcal{N}_{\mathrm{IJ}} \mathrm{F}_2^{\mathrm{J}} \wedge \mathrm{F}_2^{\mathrm{J}} + \tfrac{1}{2} \widetilde{\Delta}_{\mathbb{AB}} \mathrm{d}\mathrm{C}_0^{\mathbb{A}} \wedge \star \mathrm{d}\mathrm{C}_0^{\mathbb{B}} \\
&+ \tfrac{1}{2} (\Delta^{-1})^{\mathrm{IJ}} \left(\mathrm{d}(\widetilde{\mathcal{O}}_{\mathrm{I}\,\mathbb{A}} \widetilde{\mathrm{C}}_2^{\mathbb{A}}) + \widetilde{\mathcal{O}}_{\mathrm{I}\,\mathbb{A}} \mathrm{C}_0^{\mathbb{A}} \mathrm{d}B \right) \wedge \star \left(\mathrm{d}(\widetilde{\mathcal{O}}_{\mathrm{J}\,\mathbb{B}} \widetilde{\mathrm{C}}_2^{\mathbb{B}}) + \widetilde{\mathcal{O}}_{\mathrm{J}\,\mathbb{B}} \mathrm{C}_0^{\mathbb{B}} \mathrm{d}B \right) \\
&+ \left(\mathrm{d}(\widetilde{\mathcal{O}}_{\mathrm{I}\,\mathbb{A}} \widetilde{\mathrm{C}}_2^{\mathbb{A}}) + \widetilde{\mathcal{O}}_{\mathrm{I}\,\mathbb{A}} \mathrm{C}_0^{\mathbb{A}} \mathrm{d}B \right) \wedge \left(e^{2\phi} (\Delta^{-1})^{\mathrm{IJ}} (\mathcal{O}^T)_{\mathrm{J}}{}^{\mathbb{B}} \mathbb{M}_{\mathbb{BC}} \mathrm{d}\mathrm{C}_0^{\mathbb{C}} \right) - \tfrac{1}{2} \mathrm{d}B \wedge \mathrm{C}_0^{\mathbb{A}} S_{\mathbb{AB}} \mathrm{d}\mathrm{C}_0^{\mathbb{B}} \\
&- \left(\widetilde{\mathcal{O}}_{\mathrm{I}\,\mathbb{A}} \widetilde{\mathrm{C}}_2^{\mathbb{A}} - \mathrm{G}_{\mathrm{I}\,\mathrm{flux}} B \right) \wedge \left(\mathrm{d}\mathrm{C}_1^{\mathrm{I}} + \tfrac{1}{2} \widetilde{\mathcal{O}}^{\mathrm{I}}{}_{\mathbb{B}} \widetilde{\mathrm{C}}_2^{\mathbb{B}} - \tfrac{1}{2} \mathrm{G}^{\mathrm{I}}_{\mathrm{flux}} B \right) + V_{\mathrm{scalar}} \star \mathbf{1}^{(4)}
\end{aligned}
$$

$$
\begin{aligned}
V_{\mathrm{scalar}} &= \frac{e^{-2\phi}}{2} V^{\mathbb{I}} (\mathcal{O}^T)_{\mathbb{I}}{}^{\mathbb{A}} \mathbb{M}_{\mathbb{AB}} \mathcal{O}^{\mathbb{B}}{}_{\mathbb{J}} V^{\mathbb{J}} + \frac{e^{-2\phi}}{2} W^{\mathbb{A}} (\widetilde{\mathcal{O}}^T)_{\mathbb{A}}{}^{\mathbb{I}} \mathbb{N}_{\mathbb{IJ}} \widetilde{\mathcal{O}}^{\mathbb{J}}{}_{\mathbb{B}} \overline{W}^{\mathbb{B}} \\
&- \frac{e^{-2\phi}}{4\mathcal{K}} W^{\mathbb{A}} S_{\mathbb{AC}} \mathcal{O}^{\mathbb{C}}{}_{\mathbb{I}} \left(V^{\mathbb{I}} \overline{V}^{\mathbb{J}} + \overline{V}^{\mathbb{I}} V^{\mathbb{J}} \right) (\mathcal{O}^T)_{\mathbb{J}}{}^{\mathbb{D}} S_{\mathbb{DB}} \overline{W}^{\mathbb{B}} \\
&+ \frac{e^{4\phi}}{2} \left(\mathrm{G}^{\mathbb{I}}_{\mathrm{flux}} + \widetilde{\mathcal{O}}^{\mathbb{I}}{}_{\mathbb{A}} \mathrm{C}_0^{\mathbb{A}} \right) \mathbb{N}_{\mathbb{IJ}} \left(\mathrm{G}^{\mathbb{J}}_{\mathrm{flux}} + \widetilde{\mathcal{O}}^{\mathbb{J}}{}_{\mathbb{B}} \mathrm{C}_0^{\mathbb{B}} \right).
\end{aligned}
$$

Conclusion (1)

- Consistency check: Type II DFT compactified on Calabi-Yau three-fold gives rise to full four-dimensional action of $\mathcal{N} = 2$ gauged SUGRA

- DFT allows for description of non-constant background fluxes, yielding the same effective four-dimensional action

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Conclusion (2)

- Trace-terms and generalized dilaton fluxes

$$\mathcal{F}_i = F^m{}_{mi} + 2Y_i, \qquad \mathcal{F}^i = Q_m{}^{mi} + 2Z^i$$

give rise to additional terms in flux matrices, e.g.

$$\mathcal{O}^{\mathbb{A}}{}_{\mathbb{I}} = \begin{pmatrix} -\left(\tilde{f}^{\mathrm{A}}{}_{\mathrm{i}} + \tilde{y}^{\mathrm{A}}{}_{\mathrm{i}}\right) & \left(\tilde{q}^{\mathrm{Ai}} + \tilde{z}^{\mathrm{Ai}}\right) \\ \left(f_{\mathrm{Ai}} + y_{\mathrm{Ai}}\right) & -\left(q_{\mathrm{A}}{}^{\mathrm{i}} + z_{\mathrm{A}}{}^{\mathrm{i}}\right) \end{pmatrix}$$

$\rightarrow$ Ten-dimensional origin of non-unimodular gaugings in $\mathcal{N} = 4$ gauged SUGRA?

Conclusion (3)

- $\mathrm{IIA} \leftrightarrow \mathrm{IIB}$ Mirror Symmetry restored:

$$
\begin{aligned}
t^{\mathrm{i}} &\leftrightarrow z^{\mathrm{a}}, & g_{\mathrm{ij}} &\leftrightarrow g_{\mathrm{a}\bar{\mathrm{b}}}, \\
\mathcal{M}_{\mathrm{AB}} &\leftrightarrow \mathcal{N}_{\mathrm{IJ}}, & h^{1,1} &\leftrightarrow h^{1,2}, \\
V^{\mathbb{I}} &\leftrightarrow W^{\mathbb{A}}, & S_{\mathbb{I}\mathbb{J}} &\leftrightarrow S_{\mathbb{A}\mathbb{B}} \\
\mathrm{C}_n^{\mathbb{I}} &\leftrightarrow \mathrm{C}_n^{\mathbb{A}}, & \mathrm{G}_{\mathrm{flux}}^{\mathbb{I}} &\leftrightarrow \mathrm{G}_{\mathrm{flux}}^{\mathbb{A}}, \\
\mathcal{O}^{\mathbb{A}}{}_{\mathbb{I}} &\leftrightarrow \widetilde{\mathcal{O}}^{\mathbb{I}}{}_{\mathbb{A}}.
\end{aligned}
$$

Outlook

- DFT provides a useful framework to handle non-geometric flux compactifications
- Possibilities for further research: orientifold compactifications, heterotic DFT, Exceptional Field Theory, ...

End of Talk

Questions?