Distinguishing axions from WIMPs as *Cold Dark Matter?*

1603.04249
1405.1139
1307.8024

Sacha Davidson (IN2P3/CNRS), M Elmer, T Schwetz

1. (squeletal) intro to the $axion$, $\mathcal{A}$ in the Alphabet of Bsm Curiosities
   - the strong CP problem (theoretically popular light (pseudo)-scalar)
   - astrophysical constraints/hints
2. the (QCD) axion in cosmology
   - $assume$ born after inflation
   - becomes (C)DM at QCDPT (despite $m_a \sim m_\nu$) : redshift as $1/R^3(t)$
     : grow small $\delta\rho$ on LSS scales like WIMPs
3. growing Large Scale Structure
   - variables and equations
   - initial conditions
   - (dynamics, often non-linear)
   - statics

Why the axion : the strong CP problem of QCD

Problem: in QCD, can put a renormalisable, CP-violating interaction for gluons:

$$-\underset{\vec{E}^2+\vec{B}^2}{\frac{1}{4}G^A_{\mu\nu}G^{\mu\nu A}} - \underset{\vec{E}\cdot\vec{B}}{\boldsymbol{\theta}\frac{g_s^2}{32\pi^2}G^A_{\mu\nu}\widetilde{G}^{\mu\nu A}} + \sum_i \overline{q}_i(\not{D}-m_i)q_i \qquad A:1..8, \quad \widetilde{G}^{\mu\nu}=\varepsilon^{\alpha\beta\mu\nu}G_{\alpha\beta}$$

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But not observe electric dipole moment of neutron:

$$\Rightarrow \theta \lesssim 10^{-10}$$

Pich deRafael
Pospelov, Ritz

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⇒ How to get rid of θ?

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$\Rightarrow$ How to get rid of θ?

Try to use the axial anomaly? (obscure quantum field theory, but true, predicts $\pi_0 \to \gamma\gamma$)

Peccei-Quinn
Weinberg, Wilcek

1. Chiral phase rotns are a symmetry of classical theory of massless quarks.

Make rotn by θ, get $\quad \delta\mathcal{L} \propto \theta\partial_\mu J^\mu_5 = 0$

2. The axial *anomaly* is that 'tis not a symmetry of the quantum theory !

(?due to mass scale introduced for renormalisation?):

$$\delta\mathcal{L} \propto \theta\partial_\mu J^\mu_5 = \theta\frac{g_s^2 N}{8\pi^2}G\widetilde{G} \;\; \left(+\theta\sum_f m_f\overline{q}_f\gamma_5 q_f\right)$$

From the axial anomaly to axion models

Peccei Quinn
Kim , ShifmanVainshteinZakharov
DineFischlerSrednicki,Zhitnitsky
Srednicki NPB85

1. the axial anomaly says can remove θ by a chiral phase rotn on massless quarks

$$q_L \to e^{-i\theta/4} q_L \quad , \quad q_R \to e^{i\theta/4} q_R \quad \Rightarrow \quad \theta \frac{g_s^2}{32\pi^2} G\widetilde{G} \to 0 \times \frac{g_s^2}{32\pi^2} G\widetilde{G}$$

2. but SM quarks are not massless :(

$$m\overline{q_L} q_R \to e^{i\theta/2} m\overline{q_L} q_R$$

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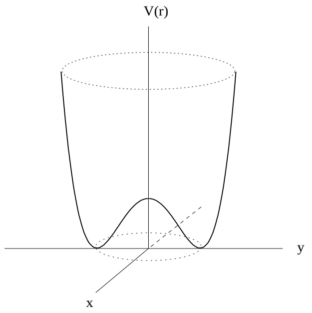
3. add ... quarks with a mass invariant under chiral rotns!
$\Rightarrow$ introduce new quarks Ψ, and new complex scalar $\Phi = |\Phi| e^{ia/f}$, with $\Phi \to e^{-i\theta/2}\Phi$, whose vev ($\sim 10^{11}$ GeV) gives mass to new quarks
$$\mathcal{L} = \mathcal{L}_{SM} + \partial_\mu \Phi^\dagger \partial^\mu \Phi + i\overline{\Psi} \not{D} \Psi + \{\lambda \Phi \overline{\Psi} \Psi + h.c.\} + V(\Phi)$$

4. θ is gone, $|\Phi|$ and new quarks are heavy...remains at low energy a, the axion.

Remains the axion at low energy

- summary: traded CPV parameter θ for a dynamical field a (with potential min at 0), who is phase of $\Phi \sim f e^{ia/f}$, $f \sim 10^{11}$ GeV.

$\Rightarrow$ only new particle at low-energy is the (pseudo-) goldstone a



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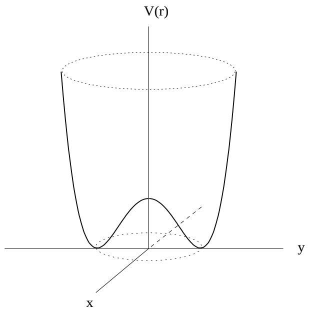

- chiral symmetry broken below $\sim \Lambda_{QCD} \Rightarrow$ tilt mexican hat:

$$V(a) \approx f_\pi^2 m_\pi^2 [1 - \cos(a/f)] \simeq \frac{1}{2} m^2 a^2 - \frac{1}{4!}\frac{m^2}{f^2} a^4 + \frac{1}{6!}\frac{m^2}{f^4} a^6 \ + \ldots$$

$$m_a \sim \frac{m_\pi f_\pi}{f} \simeq 6 \times 10^{-5} \frac{10^{11}\mathrm{GeV}}{f} \mathrm{eV} \qquad\qquad \lambda = \frac{m^2}{4! f^2} \simeq 10^{-49} \left(\frac{m}{.0001\mathrm{eV}} \right)^4$$

(but λ not small compared to grav:$\frac{1}{f^2} \gg \frac{1}{m_{pl}^2}$, and attractive...keep in *field* equations?)

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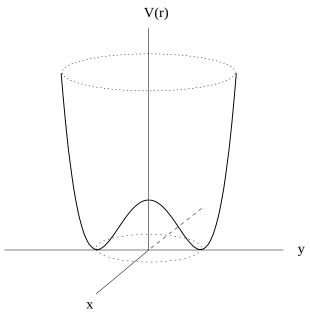

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- couplings to SM $\propto \frac{1}{f} \propto m_a$ (!! one-parameter NP model, almost)

Srednicki NPB85

upper bound on $\frac{1}{f}$ to avoid rapid stellar energy loss:

$$\boldsymbol{m_a \lesssim 10^{-2}}\ \mathrm{eV} \qquad\qquad (f_{PQ} \gtrsim 10^9\ \mathrm{GeV})$$

Raffelt...

The QCD axion in cosmology:

CDM despite $m_a \sim m_\nu$

- born at “Peccei Quinn” Phase Transition : $\Phi \to f e^{ia/f}$
- get a mass at QCD PhaseTransition
- CDM $\equiv$ redshift as $1/R^3$
  reproduce linear power spectrum for Large Scale Structure
  $\Rightarrow$ axion is ColdDM

Non-thermal axion production: *Cold* Dark Matter!

1. In the beginning, there was inflation

avoids CMB bounds on isocurvature fluctuations :

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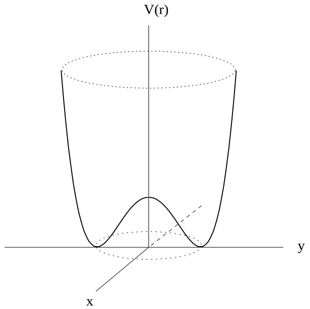
2. Then the axion is born

$$\Phi \to f e^{ia/f} \quad (f \sim 10^{12}\ \text{GeV})$$

$*$ a massless, random $-\pi f \leq a_0 \leq \pi f$ in each horizon

$$\langle a_0^2 \rangle_{U\ today} \sim \pi^2 f^2/3$$

$*$ ...one string/horizon



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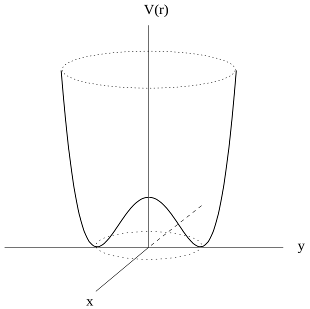


3. Laaaater: QCD Phase Transition ($T \sim 200$ MeV): ...m_π (tilt mexican hat)

$$m_a(t) : 0 \to f_\pi m_\pi / f \quad \Rightarrow \quad V(a) = f_{\rm PQ}^2 m_a^2 [1 - \cos(a/f_{\rm PQ})]$$

   $*$ ... at $H < m_a$, "misaligned" axion field starts oscillating around the minimum

   $*$ energy density $m_a^2 \langle a_0 \rangle^2 / R^3(t)$ density today higher for smaller mass ⇒ correct Ω for $m_a \gtrsim 10^{-5}$eV

   $*$ strings go away (radiate cold axion particles, $\vec{p} \sim H \lesssim 10^{-6} m_a$)

Hiramatsu etal 1012.5502
Klaer+Moore, 2017
(?) Redondo etal

axion after inflation ⇒ **oscillating axion field + cold particles** redshift like CDM

Linear Fluctuation Evolution

axion CDM inherits adiabatic density fluctuations on Large Scale Structure scales from radiation bath at QCDPT. Then, inside the horizon

$$\ddot{\delta} + 2H\dot{\delta} - 4\pi G_N \overline{\rho}_a \delta + c_s^2 \frac{k^2}{R^2(t)}\delta = \mathrm{non-lin-grav} \qquad \left(\delta \equiv \frac{\delta\rho_{mat}(\vec{k},t)}{\overline{\rho}_{mat}(t)} \ , \ \delta_{rad} = 0 \right)$$

For $\delta \sim 10^{-5}$ fluctuations on LSS scales:

- ⋆ black = eqn for WIMPs
- ⋆ pressure ($c_s^2 \sim \delta P/\delta\rho$) irrelevant because $k \to 0$
- ⋆ non-lin on LSS scales negligeable because $\delta \ll$. And $\delta \sim 1$ on small scales negligeable for large LSS scales because separation of scales

  (virial on small scales ⇒ cancellations among non-lin terms)

Peebles, LSS sec 28

axion DM : redshifts like WIMPs
grows small density fluctuations like WIMPs

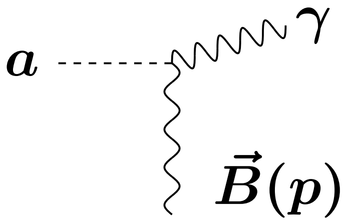
Is DM in our Universe made of Axions or WIMPs?

distinguish in Direct Detection?

in non-linear structure formation?

- variables and equations
- initial conditions
- (dynamics, often non-linear)
- statics

Direct detection (of axions)



1. $a \to \gamma$ conversion in $\vec{B}$ field. (with gradient, to transfer correct $\vec{p}$...a diff $\vec{B}$ for each m_a)

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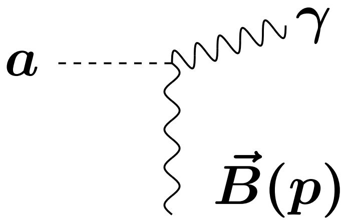
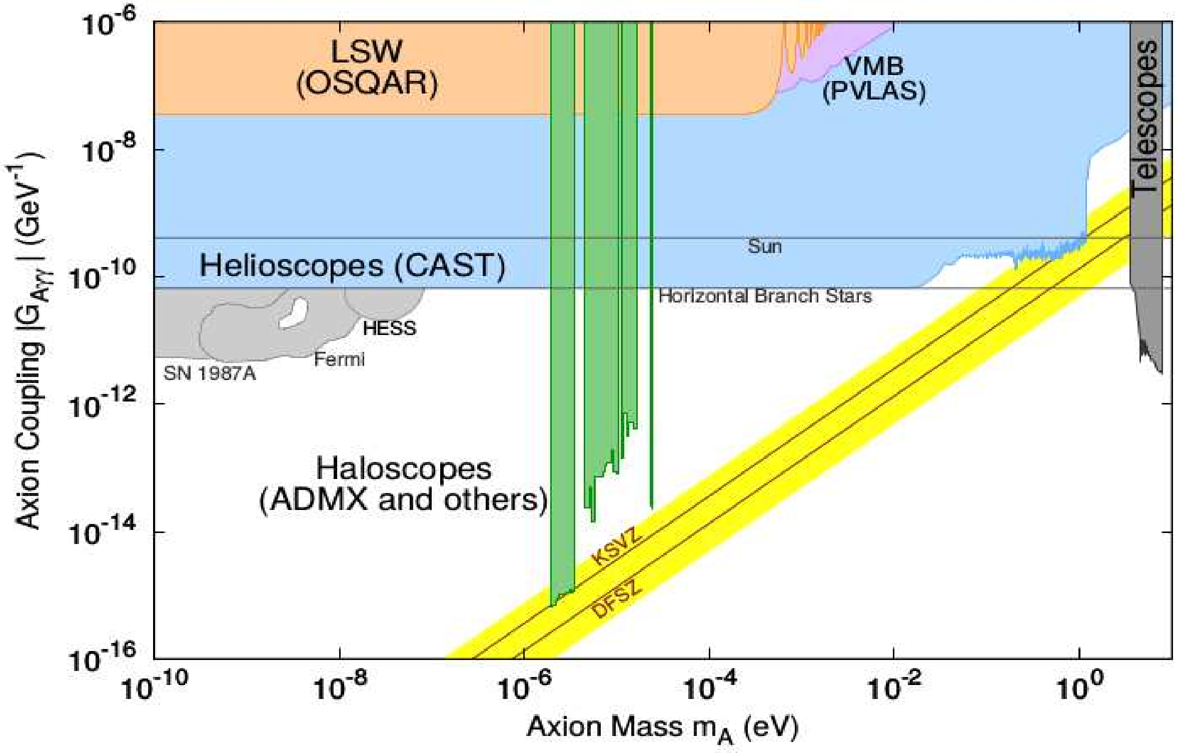


1. $a \to \gamma$ conversion in $\vec{B}$ field. (with gradient, to transfer correct $\vec{p}$...a diff $\vec{B}$ for each m_a)
   - (a) CernAxionSolarTel: LHC magnet, points at sun, convert solar a to γs (also Sumico)
   - (b) ADMX: dark matter axions ($E_\gamma \sim m_a \sim$ microwave)
2. WIMP direct detection expts look for axions too!

Edelweiss,...



ADMX,Coree

Variables + Eqns for axion CDM

Two sources/populations of CDM axions: from misalignment and strings

$$\left.\begin{array}{l}\left.\begin{array}{l}\text{classical field?}\\ \text{Bose Einstein Condensate?}\end{array}\right\}\text{?same?}\\ \text{made of particles}\end{array}\right\}\text{?different? ?? ???}$$

no matter! I only need to know: *how do they evolve?*

⇒ **consult the path integral**/(delphic oracle)

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- variables = expectation values of n-pt functions ($\phi \equiv$ axion)

$$\langle\phi\rangle \leftrightarrow \text{ classical field} = \text{misalignment axions } \phi_{cl}$$

$$\langle\phi(x_1)\phi(x_2)\rangle \leftrightarrow \text{ (propagator)} + \text{distribution of particles } f(x,p)$$

?put the string axions here?

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- get Eqns of motion for expectation values in Closed Time Path formulation

Einsteins Eqns with $T^{\mu\nu}(\phi_{cl}, f)$ + quantum corrections(λ, G_N)

(in 2 Particle Irreducible formulation, get EoM simultaneously for 1 , 2-pt fns)

$\Rightarrow$**simple @ leading order**(Saddle Pt of Path Int.): Einsteins Eqns with $T^{\mu\nu}(\phi_{cl}, f)$.
Quantum corrections as perturbative expansion in G_N, λ (both tiny)

...stress-energy tensors

non-rel axion particles are dust, like WIMPs (so not consider further):

$$T_{\mu\nu} = \begin{bmatrix} \rho & \rho\vec{v} \\ \rho\vec{v} & \rho v_i v_j \end{bmatrix}$$

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Classical field in non-relativistic limit

$$T_{\mu\nu} = \begin{bmatrix} \rho & \rho\vec{v} \\ \rho\vec{v} & \rho v_i v_j + \Delta T_{ij} \end{bmatrix} \qquad \Delta T^i_j \sim \partial_i a \partial_j a \ , \ \lambda a^4$$

Sikivie

classical field has different pressure + self-interactions at $\mathcal{O}(\lambda)$

⇒ Might extra pressures allow to distinguish axions from WIMPs in structure formation?

Equations of motion

- (in linear evolution, on LSS scales, same for WIMPs and axions ↔ defn of CDM)

Ratra, Hwang+Noh

- non-linear dynamics, inside horizon, Newtonian V_N satisfies Poisson (black=eqns for dust):

$$T^{\mu}_{\ \nu;\mu} = 0 \quad \Leftrightarrow \quad \begin{cases} \partial_t \rho + \nabla \cdot (\rho \vec{v}) = 0 \\ \\ \partial_t \vec{v} + (\vec{v} \cdot \nabla)\vec{v} = -\nabla V_N \pm \ \text{extra pressures from field} \end{cases}$$

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⇒ to see if extra pressures affect structure formation, solve with extra pressures and compare to N-body (= dust)?

Broadhurt etal

- But first need to know — does gravity/self-interactions move axions between the field and particle bath? ⇔ does it condense cold axion particles/evaporate the field?

not at lowest order= classical:

$$\langle n, a | \hat{T}_{\mu\nu} | n, a \rangle = T_{\mu\nu}(a) + T_{\mu\nu}(part)$$

⇒ at $\mathcal{O}(G_N^2, \lambda^2)$? Not according to the calculations I understand)

Equations of Motion

- *fluid* dynamics (with non-rel axion $= \phi = \sqrt{\frac{\rho}{m}} e^{-iS}$ and $v^j = -\partial_j S/m$),

$$\begin{aligned} \partial_t \rho = &\quad -\nabla \cdot \rho \vec{v} && \text{continuity} \\ \rho \partial_t \vec{v} + \rho \vec{v} \cdot \nabla \vec{v} = &\quad \rho \nabla \left(\frac{\nabla^2 \sqrt{\rho}}{2m^2 \sqrt{\rho}} + |g| \frac{\rho}{m^2} - V_N \right) && \text{Euler} \quad , \end{aligned}$$

$*$ eqns for dust

$*$ extra terms for axion field, $|g| \sim 1/f^2 \sim \lambda/m^2$ $V(a) \supset -\lambda a^4$)
self-interaction pressure *inwards*: $\frac{\partial}{\partial r} r^{-n} < 0$

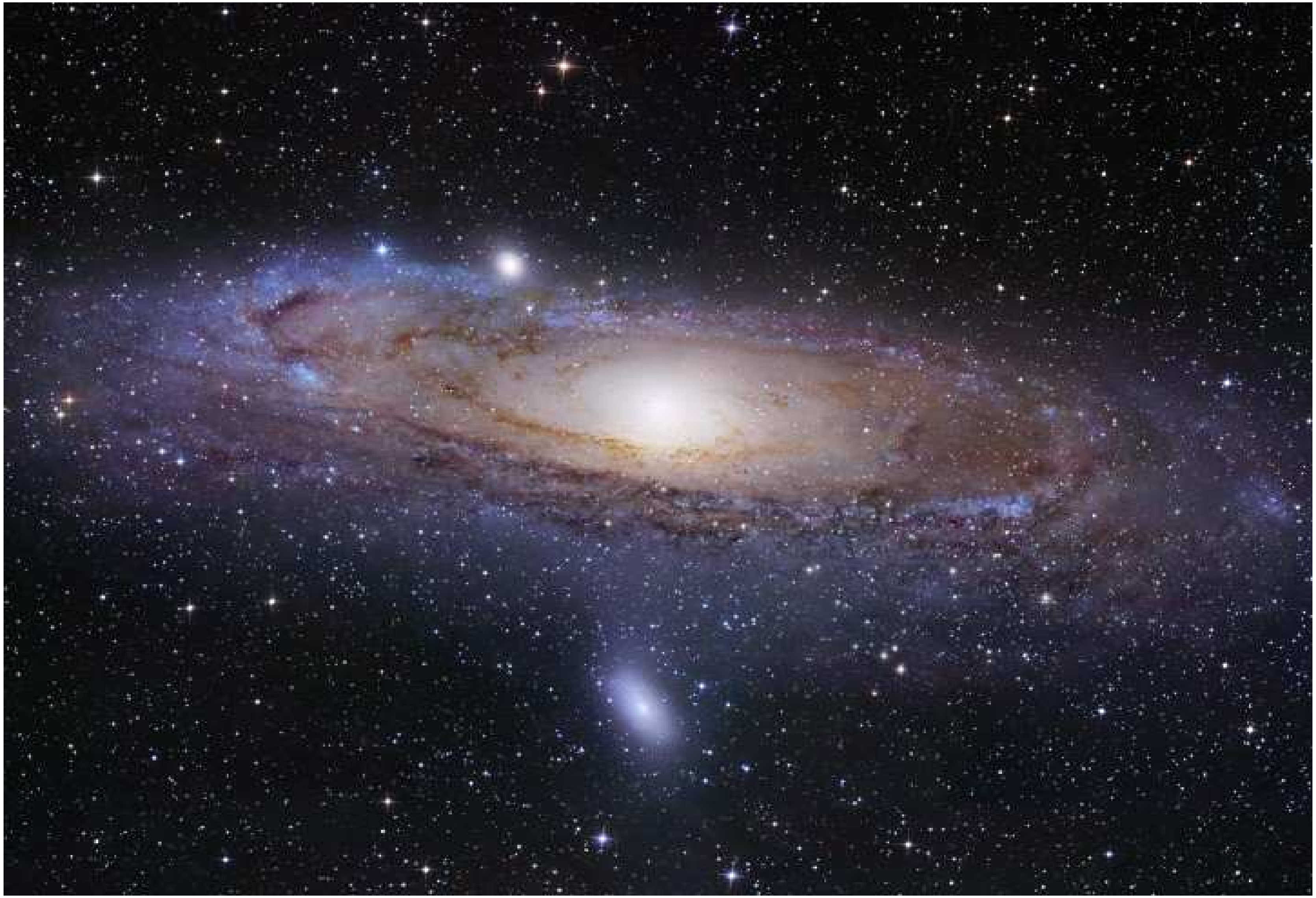
$*$ fluid parameters single-valued (no shell-crossing ⇒ shocks, etc.) ... *different* from $f(x, p)$

"Bose Stars" in GR, Broadhurt etal (numerics)

Simple first step: are stable/stationary solutions different for axion-field vs dust?

Rindler-Daller+Shapiro, Chavanis, ...

Can I find a (static) solution of QCD axion + gravity that could be the halo of Andromeda?



Diversion: initial spectrum of axion density fluctuations

(QCDPT $=$ complicated...start a bit after)

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1a: misalignment axions spatially random on co-moving QCDPT-horizon scale

$\equiv$ **miniclusters**: $\frac{\delta\rho}{\rho} \sim \mathcal{O}(1)$ on scale $1/H_{QCD}$
fall off like random walk on larger scales (white noise)

Hogan,Rees
Tkachev+Kolb

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$$M_{mini} \sim V_{osc} m_a n_{osc} E$$

where $m(T_{osc}) = 3H(T_{osc})$, $E \sim 2-8$: Turner86
Lyth92,BaeEtal08

$$M_{mini} \sim \frac{\pi^2 m f_{PQ}^2}{H^2(T_{osc})} \sim \left\{ \begin{array}{r} 3 \times 10^{-13} M_\odot \\ 10^{-10} M_\odot \end{array} \right.$$

1b: axion field(+string-decay-prods) inherit adiabatic $\delta\rho/\rho$ on LSS scales from bath

2.phase space distribution of NR axions from strings

??fluctuation spectrum?? $\frac{\delta\rho_a}{\rho_a} \sim 1$ on scale H^{-1}_{QCDPT}??

Stable solution that could occur after collapse

1 recall *fluid* eqns (with $\phi = \sqrt{\frac{\rho}{m}}e^{-iS}$ and $v^j = -\partial_j S/m$),

$$\begin{aligned} \partial_t \rho = & \quad -\nabla \cdot \rho\vec{v} && \text{continuity} \\ \rho\partial_t\vec{v} + \rho\vec{v}\cdot\nabla\vec{v} = & \quad \rho\nabla\left(\frac{\nabla^2\sqrt{\rho}}{2m^2\sqrt{\rho}} + |g|\frac{\rho}{m^2} - V_N\right) && \text{Euler} \quad , \end{aligned}$$

- $*$ eqns for dust extra terms for axion field, $|g| \sim 1/f^2 \sim \lambda/m^2$
- $*$ self-interaction pressure *inwards*: $\frac{\partial}{\partial r} r^{-n} < 0$
- $*$ fluid parameters single-valued ($\Rightarrow$ shocks, etc.) ... different from $f(x,p)$

"Bose Stars" in GR, Rindler-Daller+Shapiro, Chavanis, ... Broadhurt etal (numerics)

2 Set LHS of Euler $\simeq 0$ (stable soln —but LHS $\neq 0$ for dust halo!)
By dim analysis:

$$\left(\frac{1}{2m^2R^2} - |g|\frac{M}{m^2R^3} - G_N\frac{M}{R}\right) \simeq 0 \quad \Rightarrow \quad R \sim \frac{m_{pl}^2}{4m^2M}\left(1 \pm \sqrt{1 \mp \frac{m^2M^2}{2f^2m_{pl}^2}}\right)$$

$M_\odot \simeq 10^{57}$ GeV $\sim 2 * 10^{30}$ kg
$kpc \simeq 3 * 10^{21}$ cm

Stable solutions

BarrancoBernal
Rindler-DallerShapiro
Chavanis+
DavidsonSchwetz
...

approx stationary soln to $Euler$, with self-int sign from $\mathcal{L}_a \supset \pm\frac{\lambda}{4!}a^4$

$$R \sim \frac{m_{pl}^2}{4m^2}\frac{1}{M}\left(1 \pm \sqrt{1 \pm \lambda\frac{48M^2}{m_{pl}^2}}\right)$$

the QCD axion, $m \sim 10^{-4}$ eV, $\lambda \sim -\frac{m^2}{f^2} \sim -10^{-45}$,

$$\Rightarrow \quad R \sim \frac{m_{pl}^2}{4m^2 M} \quad , \quad M \lesssim \frac{m_{pl} f}{m} .$$

$$R \sim 100 \text{ km} \quad , \quad M_{max} \sim 10^{-(14\to 13)} M_\odot \left(\frac{10^{-4}\text{eV}}{m}\right)^2 \simeq \left\{ \begin{array}{l} \text{asteroid!} \\ \lesssim \text{minicluster} \end{array} \right.$$

(numerical ansätz for the radial fn, allowing breathing mode Chavanis)

heavier, smaller solutions, if account for $1 - \cos(a)$ potential

BraatenMohapatraZhang

If allow for rotation, can we get a bigger object?

arXiv:1603.04249
with Thomas Schwetz
Rindler-DallerShapiro,
ALP halos, $m \ll$, repulsive SI.

- Include rotation via Virial thm:

$$E_{grav} + 2E_{cin} + 3E_{si} = 0$$

$$E_{grav} = \int dV \frac{\rho}{2} V_N \,, \qquad E_{si} = g \int dV \frac{\rho^2}{2m^2} \,, \qquad E_{cin} = \frac{1}{2} \int dV \left[\frac{(\nabla\rho)^2}{4\rho m^2} + \rho|\vec{v}|^2 \right] .$$

- Implement rotn in field Eqns, because simple to impose continuity of phase

$$\phi(r, \theta, \varphi) \simeq \mathrm{top-hat} \times \sin^l \theta e^{il\varphi}$$

- See that $E_{grav}, E_{SI} \sim \sqrt{l+1}$ (drop flattens to disk for large l)
  $E_{cin} \sim l^2$ (angular momentum + gradient in θ)

$$M \lesssim \frac{m_{pl} f}{m} \frac{1 + 4l(l+1)}{\sqrt{l+1}} \lesssim \frac{1 + 4l(l+1)}{\sqrt{l+1}} \times 10^{-13} \, M_\odot \, .$$

- Asteroids have masses, radii $\sim$ non-rotating axion drops, rotation periods ~ 6 hours. Equatorial rotation frequency of drop at r_c, $\omega \simeq l/(r_c^2 m) \simeq 6l$/day, $\Rightarrow$ (?) low l are realistic.

$\Rightarrow M_{max}$ grows by $\sim$ order of mag (confirmed numerics)

Dynamics !

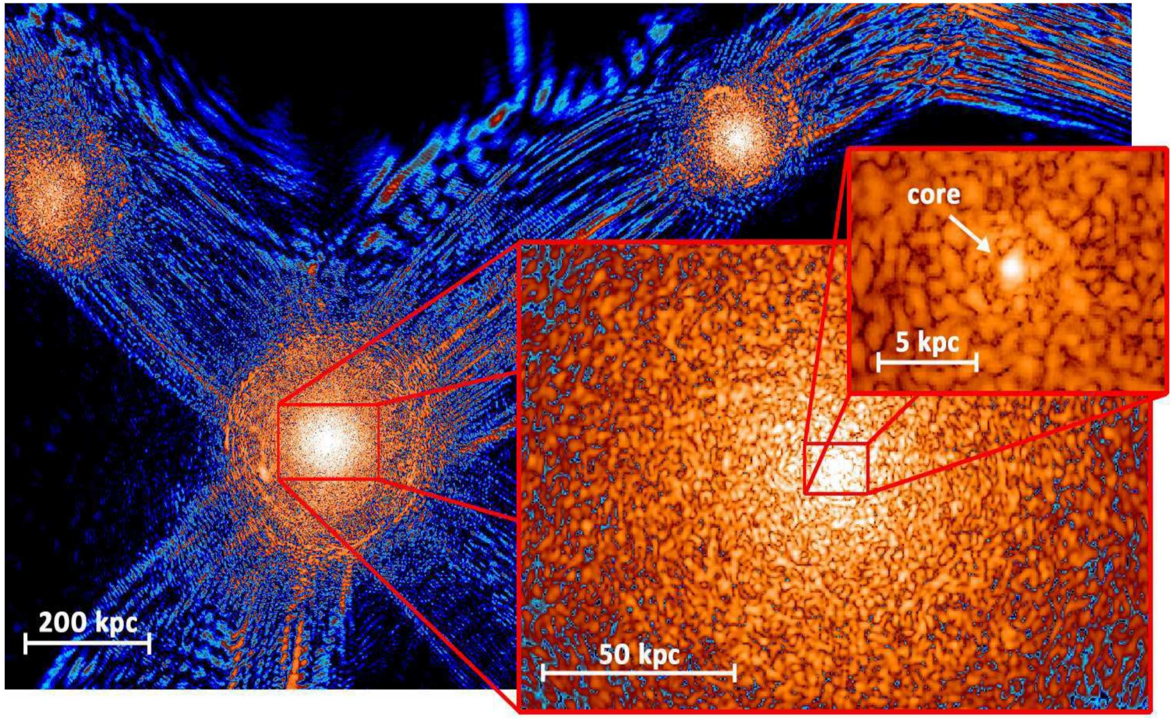


Figure 2: A slice of density field of ψDM simulation on various scales at $z = 0.1$. This scaled sequence (each of thickness 60 pc) shows how quantum interference patterns can be clearly seen everywhere from the large-scale filaments, tangential fringes near the virial boundaries, to the granular structure inside the haloes. Distinct solitonic cores with radius $\sim 0.3 - 1.6$ kpc are found within each collapsed halo. The density shown here spans over nine orders of magnitude, from 10^{-1} to 10^{8} (normalized to the cosmic mean density). The color map scales logarithmically, with cyan corresponding to density $\lesssim 10$.

SchiveChiuehBroadhurst, Nature , $m \sim 10^{-22}$ eV

Constraints on DM of the size of asteroids?

Jacobs Starkman Lynn
Zurek etal
FairbairnMarshQuevillon

window where Primordial Black Holes can contribute $\Omega_{BH} \sim .1$:

$$\text{(femtolensing) } 10^{-13} M_\odot \lesssim M_{PBH} \lesssim 10^{-9} M_\odot \text{ (microlensing)}$$

(PBH $\lesssim 10^{-18} M_\odot$ evaporate)

Micro-lensing:halo object amplifies light from nearby stars (LMC)

Femtolensing: source = GRBs, lensing objects in intervening space, signal = oscillation in energy spectrum (interference between light that took two different paths round the lensing object)

BATSE: exclude $\Omega \sim 0.2$ for $10^{-16} \rightarrow 10^{-13} M_\odot$

(+ picolensing bounds = 1 σ sensitivity to $\Omega \sim 1$ of compact objects in the mass range $10^{-12.5} M_\odot \rightarrow 10^{-9} M_\odot$.)

FERMI :GRBs at measured redshift, exclude $\Omega > 0.03$ in compact objects of mass between

$$5 \times 10^{-17} \rightarrow 5 \times 10^{-15} M_\odot$$

BarnackaGlicensteinModerski

(assumes GRB = point source. Is GRB projected onto lens plane smaller than Einstein radius?)

$\Rightarrow$ axion asteroids allowed as (at least part of) DM

? hierarchical clustering ? (need more coherence among analyses before excluding :))

Other constraints?

1. Do the drops evaporate due to self-interactions?

Tkachev,Riotto

2. Do axion drops drops shine like comets (could be bound on $\lesssim 10^{-14} M_\odot$)?

3. What is cross-section in CMB? geometric ? (Starkmann et al argue for "collisional damping" constraints if yes. Might depend on whether drops accumulate baryons?

4. One can ask what happens if a drop meets an ordinary star, a white dwarf, a neutron star, or a black hole?

disk stars
Dokuchaev Eroshenko Tkachev

5. The "explosion" of axion drops was recently proposed as a possible source for Fast Radio Bursts.

Tkachev

Summary and Speculations

The QCD axion is a BSM curiosity: one parameter (pseudo)scalar with the mass of a neutrino, beloved of theorists.

The axion is born, massless, around the time of inflation. At the QCDPT, the axion mass turns on, and the energy density (=cold axion particles (from strings) or the misalignment field) redshifts as $1/R(t)^3$. Also large-scale linear fluctuations grow as in WIMPs... so the axion is a CDM candidate!

In non-linear structure formation, cold axion particles should behave like WIMPs. But the axion *field* cannot support itself with velocity dispersion... there is a stable gravitationally bound configuration the size of an asteroid (not a galaxy).

How does a galaxy halo of axion field form? (numerical problem?)
IF axion born after inflation: CDM = cold particles from strings + field with "miniclusters" ($\mathcal{O}(1)$ density fluctuations, $M_{mini} \gtrsim M_{asteroid}$). Do miniclusters collapse to asteroids? ...do the bigger ones fragment? (or BH?)
IF axion born before inflation: all axions in the field, no miniclusters. ...what is small scale $\delta\rho/\rho$? How collapses to what?

(Do asteroids evaporate?)

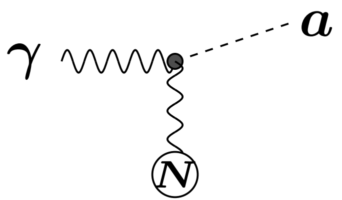
Backup

Astrophysical bounds

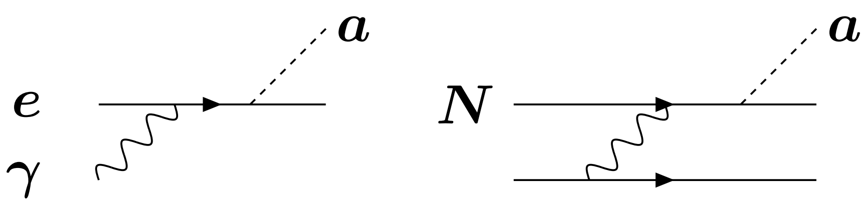
Raffelt...

axion light and (feebly) coupled to SM $\propto \dfrac{1}{f_{PQ}} \propto m_a$

$\Rightarrow$ produce in sun, He-burning stars(g_{ae}), supernovae(g_{aN})...



Primakoff



(axion couplings to e vs N vary across models by ~ 10)

upper bound on coupling to avoid rapid stellar energy loss:

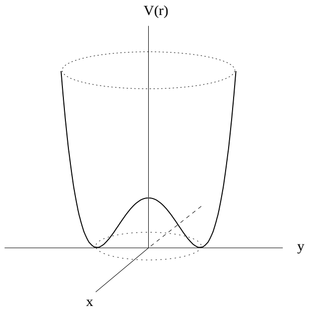
$$\boldsymbol{m_a \lesssim 10^{-2}}\ \mathrm{eV} \qquad\qquad (f_{PQ} \gtrsim 10^9\ \mathrm{GeV})$$

...or, are some/many astro objects observed to cool a wee bit faster than theory predicts?
??? hint for an Axion-Like-Particle just beyond current bounds on the coupling?

GiannottiIrastorzaRedondoRingwaldSaikawa

(This talk interested in lighter, more weakly coupled QCD-axion)

Which first: inflation or the birth of the axion?



1. *IF* first the axion is born....

$$\Phi \to f e^{ia/f} \quad (f \sim 10^{12}\ \text{GeV})$$

$|\Phi|$ and new quarks heavy, a massless

2. ...then inflation

a constant across U, develops classical fluctuations

$$\frac{\delta a}{a} \sim \frac{H_I}{2\pi f}$$

different from inflaton $\Leftrightarrow$ isocurvature density fluctuations

Planck: $\Rightarrow H_I \lesssim 10^7 \sqrt{f/10^{12}}$ GeV

? or non-canonical kin.terms for a? ...

WantzShellard
HanannHRW
FolkertsCristianoRedondo

lets NOT consider this possibility

What is a Bose Einstein condensate? (I don't know. Please tell me if you do!)

Important characteristics of a BE condensate seem to be

1. a classical field,
2. carrying a conserved charge,
3. ? whose fourier modes are concentrated at a particular value — most of the "particles" who condense, should coherently do the same thing (but not necc the zero-momentum mode)

consistent with

- BE condensation in equilibrium stat mech, finite T FT, alkali gases.
- LO theory of BE condensates (Boguliubov → Pitaevskii) as a classical field

Using $T^{\mu\nu}{}_{;\nu}=0$ vs Eqns of motion of the field a

Eqns of motion for axion field cpled to gravity studied by Sikivie et al, Saikawa etal:

$$(\Box - m^2)a \sim G_N a^3 \qquad \Rightarrow \qquad i\frac{\partial n}{\partial t} \sim G_N \int a^4$$

Both obtained from $T^{\mu\nu}{}_{;\nu}=0$ and Poisson Eqn ($\to$ dynamics is equivalent?)

$$\begin{aligned}
T^{\mu\nu}{}_{;\nu} &= \nabla_\nu[\nabla^\mu a \nabla^\nu a] - \nabla_\nu[g^{\mu\nu}\left(\frac{1}{2}\nabla^\alpha a \nabla_\alpha a - V(a)\right)] \\
&= (\nabla_\nu\nabla^\mu a)\nabla^\nu a + \nabla^\mu a(\nabla_\nu\nabla^\nu a) - g^{\mu\nu}\nabla_\nu\nabla^\alpha a\nabla_\alpha a + g^{\mu\nu}V'(a)\nabla_\nu a \\
0 &= \nabla^\mu a[(\nabla_\nu\nabla^\nu a) + V'(a)]
\end{aligned}$$

1. eqns for $T_{\mu\nu}\sim a^2$ solvable during linear structure formation. Find $\delta \equiv \delta\rho(\vec{k},t)/\overline{\rho}(t)$ in dust or axion field has same behaviour on LSS scales ($c_s \simeq \partial P/\partial\rho \to 0$):

Ratra, Hwang+Noh

$$\ddot{\delta} + 2H\dot{\delta} - 4\pi G_N\overline{\rho}\delta + c_s^2\frac{k^2}{R^2(t)}\delta = 0$$

2. "better" handle on IR divs: ensures that long-wave-length gravitons see large objects (like MeV photons see the proton, and not quarks inside)

Particles vs fields

Develop field operator

$$\hat{a}(t,\vec{x}) = \frac{1}{[R(t)L]^{3/2}} \int \frac{d^3k}{(2\pi)^3} \Big\{ \hat{b}_{\vec{k}} \frac{\chi(t)}{\sqrt{2\omega}} e^{i\vec{k}\cdot\vec{x}} + \hat{b}^\dagger_{\vec{k}} \frac{\chi^*(t)}{\sqrt{2\omega}} e^{-i\vec{k}\cdot\vec{x}} \Big\}$$

then write the coherent state:

$$|a(\vec{x},t)\rangle \quad \propto \quad \exp\left\{ \int \frac{d^3p}{(2\pi)^3} a(\vec{p},t) b^\dagger_{\vec{p}} \right\} |0\rangle$$

which satisfies $\hat{b}_{\vec{q}} |a(\vec{x},t)\rangle = a(\vec{q},t) |a(\vec{x},t)\rangle$ (can check $\hat{b}_{\vec{q}} \{1 + \int \frac{d^3p}{(2\pi)^3} a(\vec{p},t) b^\dagger_{\vec{p}}\} |0\rangle = a(\vec{q},t)|0\rangle$)
where the classical field is

$$a(t,\vec{x}) = \frac{1}{[R(t)L]^{3/2}} \int \frac{d^3k}{(2\pi)^3} \Big\{ a(\vec{k},t) \frac{\chi(t)}{\sqrt{2\omega}} e^{i\vec{k}\cdot\vec{x}} + a^*(\vec{k},t) \frac{\chi^*(t)}{\sqrt{2\omega}} e^{-i\vec{k}\cdot\vec{x}} \Big\}$$

Brodsky+Heurer,Donoghue etal
Olive+Montonen...I+Z,C-T...

What is quantum?

Classical = saddle-point configurations of the path integral
$\Rightarrow$ attribute dimensions to fields/parameters $\ni$ [action]= E*t, and no $\hbar$ in selected classical limit (this is *not* unique)

Summary: particles or fields can be obtained in a "classical" (= no $\hbar$) limit. However, $\hbar$ is differently distributed in the Lagrangian in the two limits, so to get from one to another requires $\hbar$...

in particular, to define a number of quanta, in the field picture, requires $\hbar$.

ex 1: massive scalar electrodynamics

$$\mathcal{L} = (D_\mu \phi)^\dagger D^\mu \phi - \tilde{m}^2 \phi^\dagger \phi - \frac{1}{4} FF \qquad , \quad D_\mu = \partial_\mu - i\tilde{e} A_\mu$$

Classical field limit: $[\phi, A] = \sqrt{E/L}$, $[m] = 1/L$, $[\tilde{e}] = 1/\sqrt{EL}$.

No $\hbar$ in classical EoM. OK that $[m^2] = 1/L^2$ because gravity couples is the stress-energy tensor, function of the fields.

If in Maxwells Eqns, want $j^0 = i\tilde{e}(\dot{\phi}^\dagger \phi - \phi^\dagger \dot{\phi})$ to be eN/V, then need number of charge-carrying quanta $\Rightarrow e = \tilde{e}\hbar$.

De même, if classically m a particle mass, need $m = \tilde{m}\hbar$.

ex 2: the SHO Hamiltonian is (no $\hbar$)

$$H = \frac{1}{2m} P^2 + \frac{m\nu^2}{2} X^2$$

where ν is the oscillator frequency.

But to *quantise*, = introduce creation and annihilation ops, requires $\hbar$.
To write the total energy as $\omega(N + 1/2)$, requires $\hbar$ to convert frequency to energy $\omega = \hbar\nu$, and downstairs in the defn of N, because its the number of *quanta*.