Neutrino propagation in dense media

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Introduction

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- medium influences behavior $\rightarrow$ MSW effect

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The plan:

- consider two neutrino generations
- electron neutrino source (monochromatic)
- analogy to spin precession in mag. field

Mathematical preliminaries

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  with $\hat{U}\left(\theta_0\right) = \begin{pmatrix} \cos\left(\theta_0\right) & \sin\left(\theta_0\right) \\ -\sin\left(\theta_0\right) & \cos\left(\theta_0\right) \end{pmatrix}$

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$$\boxed{\text{solve von Neumann equation } \mathrm{i}c\frac{\partial\hat{\rho}}{\partial x} = \left[\hat{H}_{\mathrm{F}},\, \hat{\rho}\right]}$$

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rotate to flavor space $\hat{H}_{\mathrm{F}} = \hat{U}\left(\theta_0\right) \hat{H}_{\mathrm{M}} \hat{U}^{\dagger}\left(\theta_0\right)$

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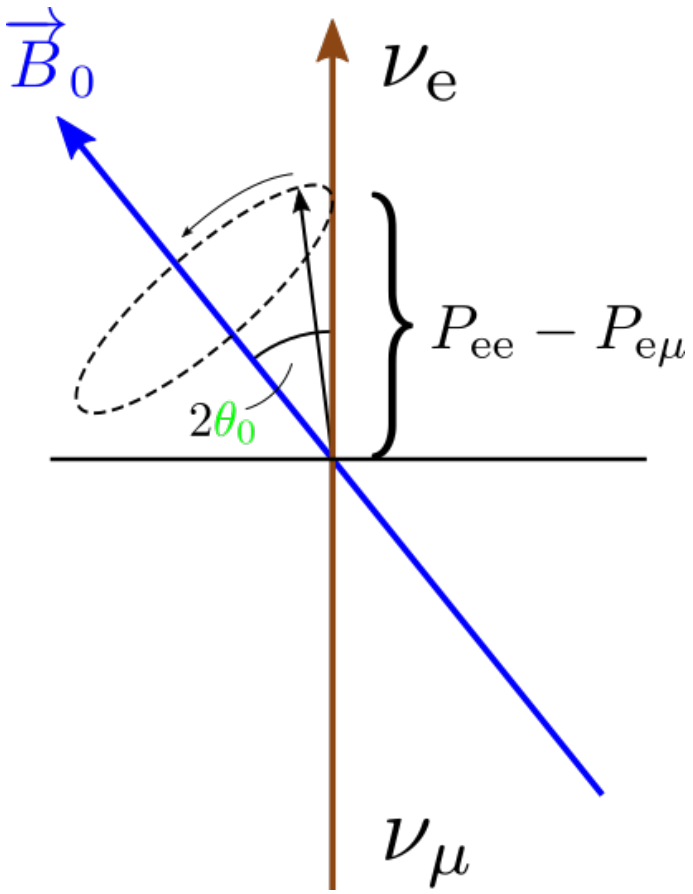
$$\boxed{\text{oscillation probability } P_{\mathrm{e}\mu} = 1 - P_{\mathrm{ee}} = \sin^2\left(2\theta_0\right)\sin^2\left(\frac{\Delta M^2}{2p}x\right)}$$

Spin analogy

convert 2×2 matrices to polarization vectors
$\hat{\rho} \equiv \vec{P} \cdot \hat{\vec{\sigma}}$ and $\hat{H}_{\mathrm{M}} \equiv \vec{B}_0 \cdot \hat{\vec{\sigma}}$

EoM $\hat{=}$ Bloch equation
$\dot{\vec{P}} = \vec{B} \times \vec{P}$



MSW effect

matter tilts rotation axis

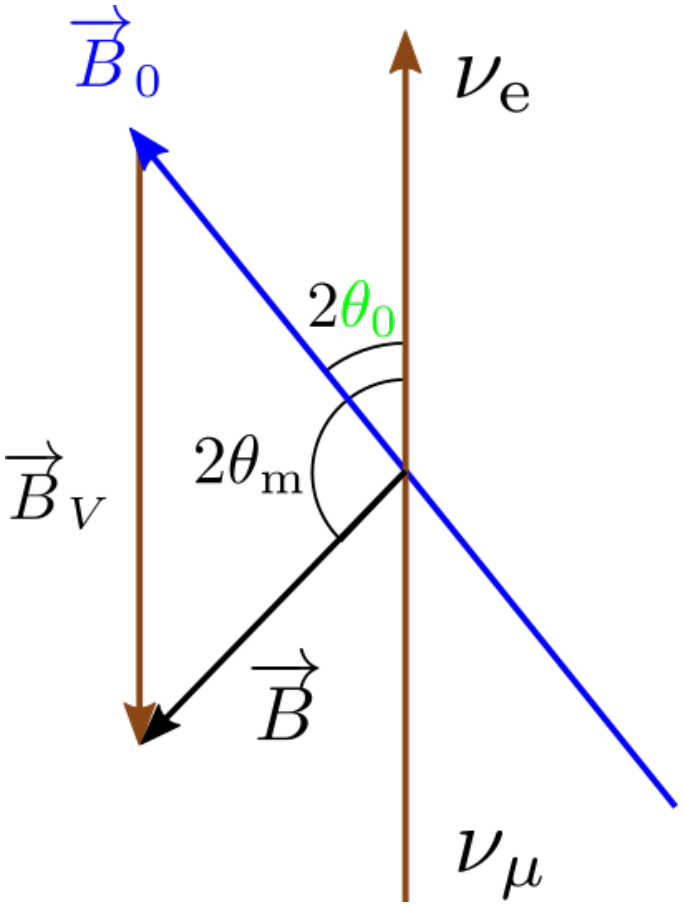
$\hat{H} = \hat{H}_{\mathrm{M}} + \hat{V}_{\mathrm{e}}$

with $\hat{V}_{\mathrm{e}} \sim \begin{pmatrix} n_{\mathrm{e}} & 0 \\ 0 & 0 \end{pmatrix}$

diagonalize $\hat{H}$ with effective mixing angle

$$\tan(2\theta_{\mathrm{m}}) = \frac{\sin(2\theta_0)}{\cos(2\theta_0) - V_{\mathrm{e}}/\Delta M^2}$$

$\rightarrow$ resonance possible



Outlook

stringent approach via Wigner function

$$\rho_{\alpha\beta}(x,\, p) = \int \mathrm{d}\Delta\, \mathrm{e}^{-\mathrm{i}p\Delta} \psi^{\dagger}_{\beta}\left(x - \frac{\Delta}{2}\right) \psi_{\alpha}\left(x + \frac{\Delta}{2}\right)$$

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von Neumann equation $\rightarrow$ Moyal equation

e.g. $\quad \mathrm{c}\,\frac{\partial \hat{\rho}}{\partial x} \rightarrow \left\{\frac{\partial \hat{H}}{\partial p}, \frac{\partial \hat{\rho}}{\partial x}\right\}$

Bibliography

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