# Using Spin Chains for Three-Point Functions in $\mathcal{N} = 4$ SYM

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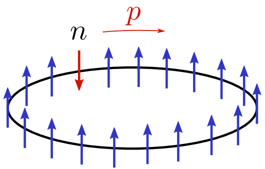


Christoph Dlapa

October 18, 2018

# Outline

# Motivation for working on $\mathcal{N} = 4$ supersymmetric Yang-Mills Theory

- $\mathcal{N} = 4$ SYM is dual to type IIB string theory on $AdS_5 \times S^5$
  - Strong/weak duality
  - $\Rightarrow$ Useful for computations
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- Solvable CFTs typically two-dimensional

- Integrability in planar limit $\rightarrow$ in principle exactly solvable

# Motivation for working on Three-Point Functions

- CFT has two basic ingredients:
  - Two-Point Functions (Spectrum of scaling dimensions $\Delta_A$)

$$\langle \mathcal{O}_A(x_1)\mathcal{O}_B(x_2)\rangle = \frac{\delta_{AB}}{|x_{12}|^{2\Delta_A}},$$

with $x_{12}^\mu = x_1^\mu - x_2^\mu$.

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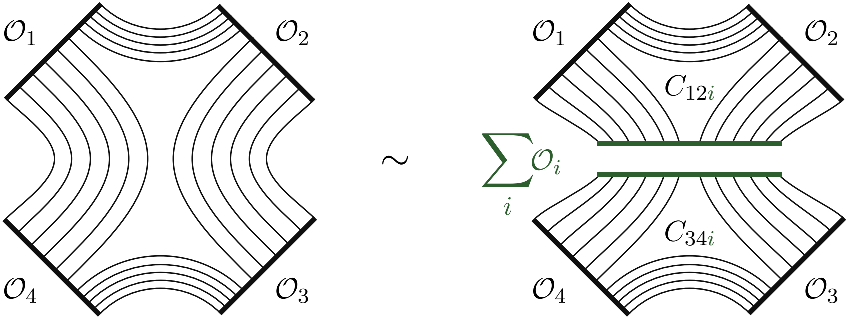
# Operator Product Expansion




General correlation functions can be computed through the Operator Product Expansion (OPE)

# Correlation Functions in $\mathcal{N} = 4$ SYM

- Restrict to scalar fields $\phi_m$, $m = 1, \dots, 6$
- R-symmetry: $SO(6)$ rotation of scalars
- Change of basis: $Z, X, Y, \bar{Z}, \bar{X}, \bar{Y}$

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- R-symmetry: $SO(6)$ rotation of scalars
- Change of basis: $Z, X, Y, \bar{Z}, \bar{X}, \bar{Y}$
- Gauge-invariant Operators are composed of multiple fields inside traces

$$\mathcal{O} = \mathrm{Tr}\left(Z\bar{Z}XYZ\ldots\right)$$

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- Minahan & Zarembo (2002): Mapping of operators to eigenstates of the Heisenberg spin chain Hamiltonian

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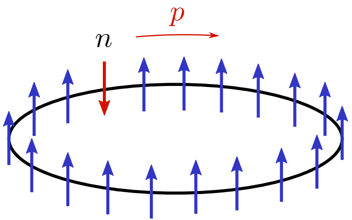

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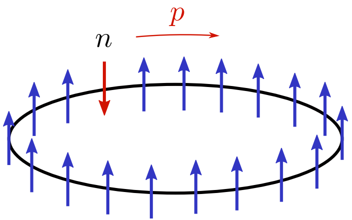
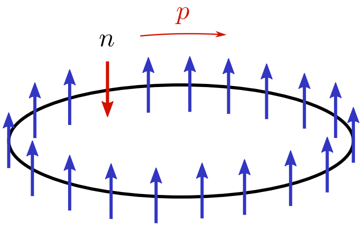
- Operators determined by excitations



- How to construct arbitrary operators in the spin chain framework?



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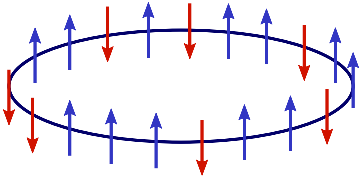
$$Z, X, Y$$



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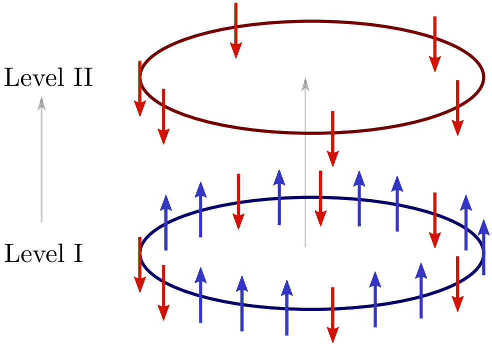

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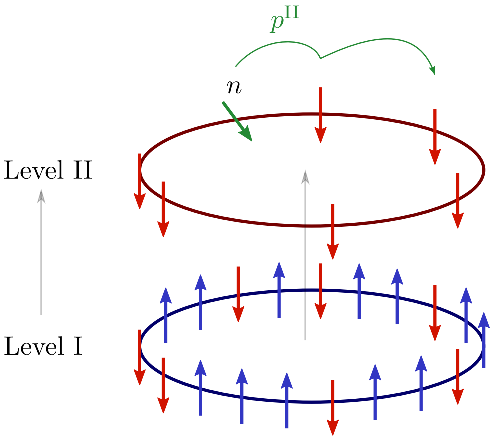

# Two-Point Functions in the Planar Limit

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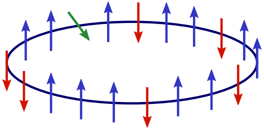

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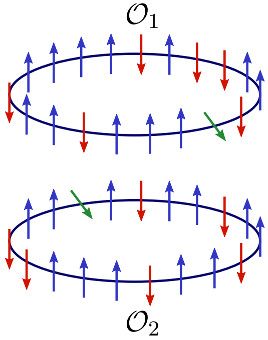

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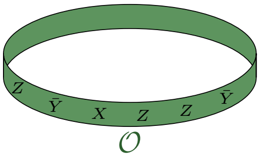


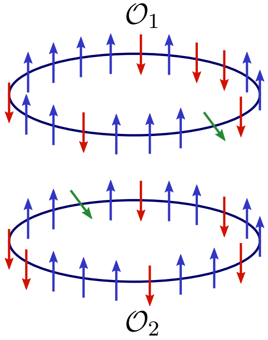

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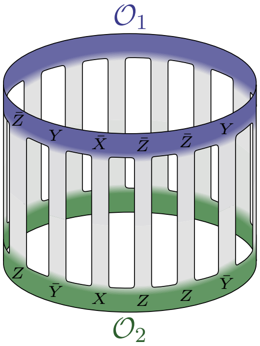


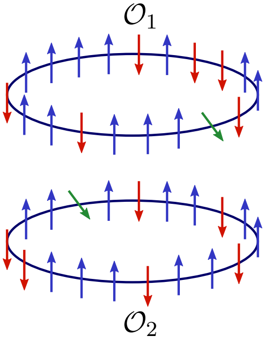


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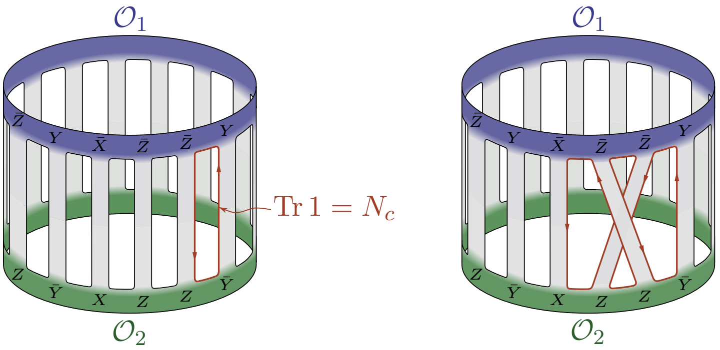


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- The planar or 't Hooft limit $N_c \to \infty$ only includes planar graphs of the $SU(N_c)$ gauge theory.

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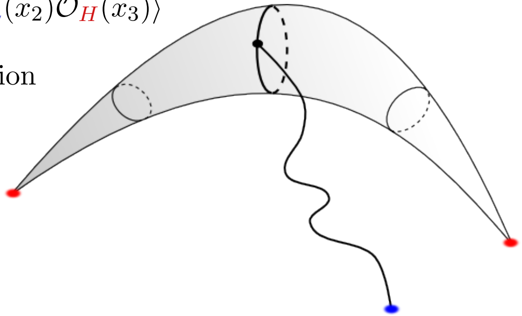

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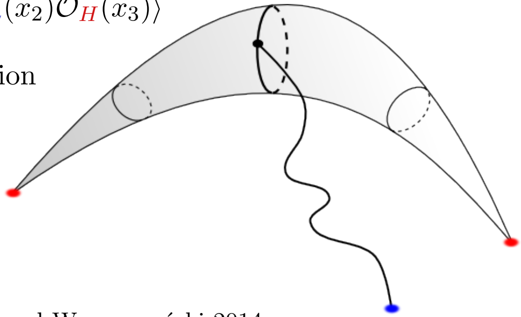
  - Related to diagonal form factors[3]

$$\langle p_1, \dots, p_N | \, \hat{\mathcal{O}}_L \, | p_1, \dots, p_N \rangle$$



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# "Easy" Three-Point Function: The Lagrangian density

- Path-integral expression for the two-point function

$$\left\langle \tilde{\mathcal{O}}_A(x_1)\tilde{\mathcal{O}}_B(x_2)\right\rangle = \int [\mathrm{D}W] e^{i\int \mathrm{d}^D x_0\, \mathcal{L}[W]}\tilde{\mathcal{O}}(x_1)\tilde{\mathcal{O}}(x_2)$$

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- Compare to[5]

$$2\pi^2 C_{A\mathcal{L}A} = -g^2 \frac{\partial \Delta_A}{\partial g^2}$$

[4] Eden, Korchemsky, and Sokatchev 2011, [5] Costa et al. 2010

# The Hexagon Approach[6]

- Currently most advanced method for three-point functions
  - Non-perturbative, but a conjecture
  - Corrections: $\sim \mathrm{Pol}(1/L)$ but not $\sim e^{-L}$

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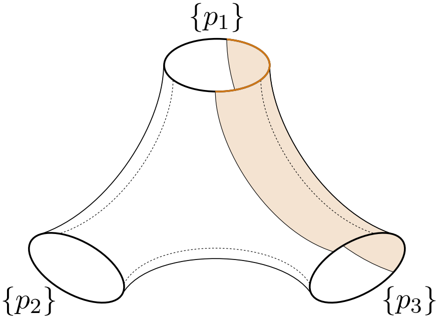


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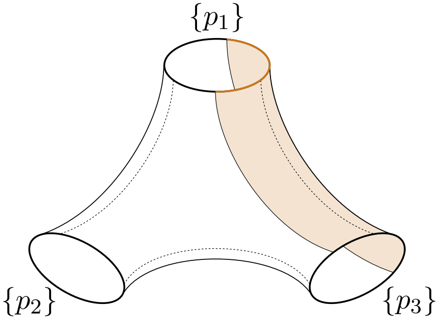


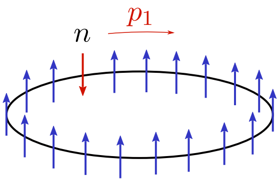


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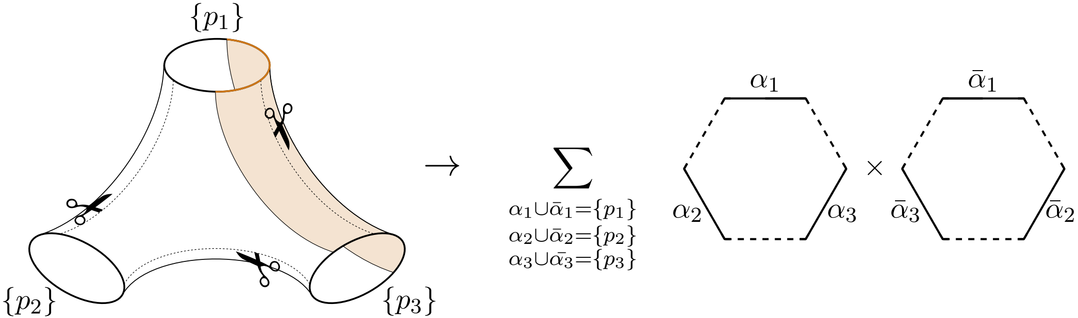


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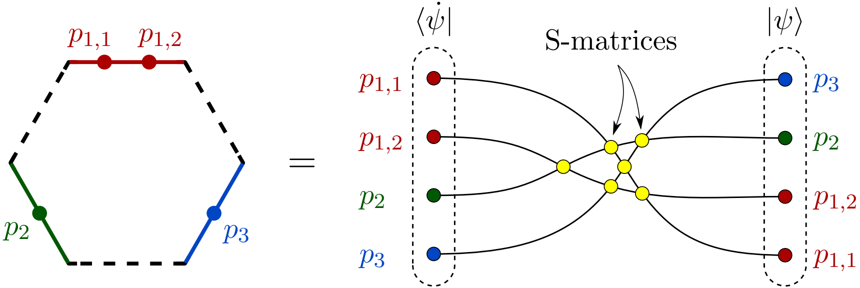
- Hexagon form factors can be restricted by symmetry to be: $H = \langle \dot{\psi} | \, \mathcal{S} \, | \psi \rangle$



# Example: Three non-BPS Operators

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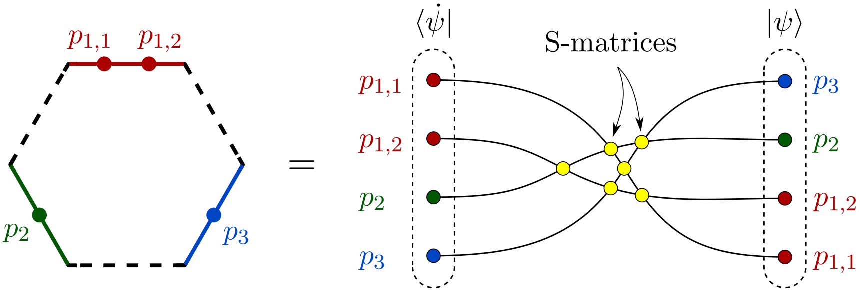


- Quantum Integrability:
  - Infinitely many integrals of motion
  - → Conservation of individual momenta
  - → Factorized scattering

# Result for general Operators

- Asymptotic three-point function for one excited operator:

$$\langle \mathcal{O}_1^{\bullet}(\mathbf{p})\mathcal{O}_2^{\circ}\mathcal{O}_3^{\circ}\rangle = \frac{C_{123}^{\bullet\circ\circ}}{|x_{12}|^{\Delta_{AB}}|x_{13}|^{\Delta_{AC}}|x_{23}|^{\Delta_{BC}}},$$

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with

$$\left(\frac{C_{123}^{\bullet\circ\circ}}{C_{123}^{\circ\circ\circ}}\right)^2 = \frac{\langle \mathbf{p}^{\mathrm{II},\dots} | \mathbf{p}^{\mathrm{II},\dots} \rangle^2 \prod_{k=1}^{K} \mu(p_k)}{\langle \mathbf{p} | \mathbf{p} \rangle \prod_{i<j} S(p_i, p_j)} \mathcal{A}^2,$$

where the sum over partitions onto the two hexagons is given by

$$\mathcal{A} = \prod_{i<j} h(p_i, p_j) \sum_{\alpha \cup \bar{\alpha} = \mathbf{p}} (-1)^{|\bar{\alpha}|} \prod_{j \in \bar{\alpha}} f(p_j) e^{i p_j \ell_{21}} \prod_{i \in \alpha, j \in \bar{\alpha}} \frac{1}{h(p_i, p_j)}.$$

- Translation to spin chain states not known in general

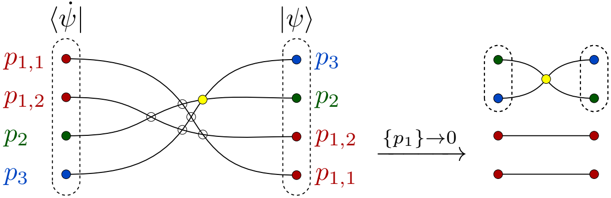
# Results in Subsectors

- Volume dependence of Heavy-Heavy-Light three-point functions[7]
- Three-point functions in $SU(1|1)$ sector (fermions)[8]

[7] Jiang 2017, [8] Caetano and Fleury 2016

# Results in Subsectors

- Volume dependence of Heavy-Heavy-Light three-point functions[7]
- Three-point functions in $SU(1|1)$ sector (fermions)[8]
- Descendants: Calculation in $SO(6)$ sector reproduces results from Wick-contractions




$$\lim_{p_1\to 0} S(p_1,p_2) = 1 + \mathcal{O}(p_1) \qquad \Rightarrow \qquad \mathrm{Tr}\left(Y\bar{Y} - X\bar{X}\right)$$

$$\Rightarrow \qquad C_{123}^{\bullet\bullet\circ} \sim \left(1 - e^{i\ell_{12}p_2}\right)\left(1 - e^{i\ell_{23}p_3}\right)$$

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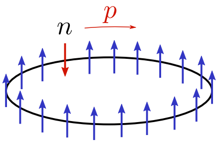
# Summary and Conclusions

- Integrability: Powerful tool to calculate correlation functions in $\mathcal{N} = 4$ SYM in the planar limit



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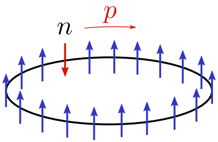
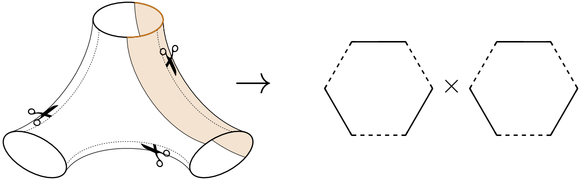



- Efficient computation of three-point functions through the Hexagon Approach



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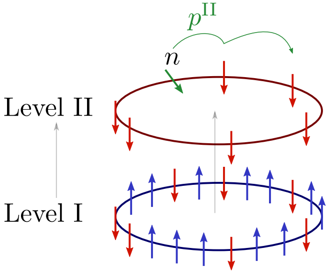
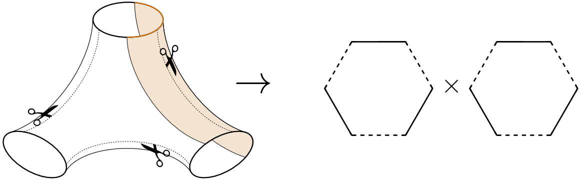
- Efficient computation of three-point functions through the Hexagon Approach
- Difficult for higher rank sectors



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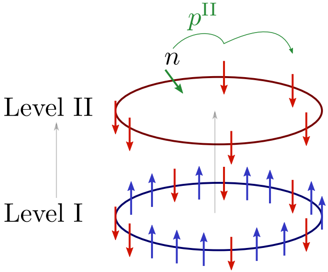
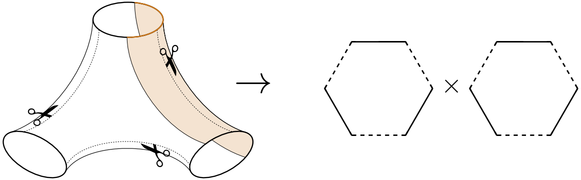



- Efficient computation of three-point functions through the Hexagon Approach

- Difficult for higher rank sectors
  - ⇒ Calculate three-point function known to all orders

$$2\pi^2 C_{A\mathcal{L}A} = -g^2 \frac{\partial \Delta_A}{\partial g^2}$$

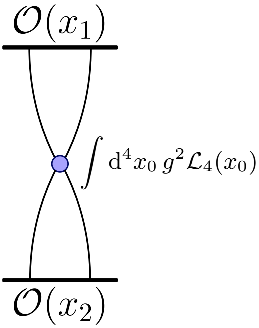


Backup Slides

# Anomalous Dimension Matrix

- Loop corrections lead to divergencies
- Example of quartic vertex:

$$\int \frac{d^4x_0}{|x_{01}|^4|x_{02}|^4} = \frac{2\pi^2 i}{|x_{12}|^4} \log|\Lambda x_{12}|^2$$




- Renormalise with

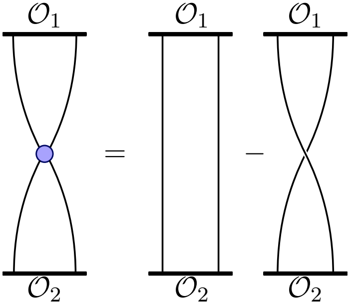
$$\tilde{\mathcal{O}}_A = Z_A{}^B\mathcal{O}_B, \quad Z_A{}^B = 1+\tfrac{1}{2}\underbrace{\Gamma_A{}^B}_{\gamma_A}\log|\Lambda/\mu|^2$$

$$\left\langle \tilde{\mathcal{O}}_A(x_1)\tilde{\mathcal{O}}_B(x_2)\right\rangle_{\text{one-loop}} = \frac{\delta_{AB}}{|x_{12}|^{2L}}\left(1-\gamma_A\log|\mu x_{12}|^2\right) \approx \frac{\delta_{AB}}{|x_{12}|^{2L}|\mu x_{12}|^{2\gamma_A}}$$

- Scaling Dimension: $\Delta = \Delta_0 + \gamma = L + \gamma$

# Diagonalising Γ

Minahan & Zarembo (2002):




- Heisenberg spin chain Hamiltonian ($SU(2)$ sector)

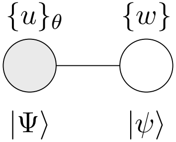
$$\Gamma = H \sim \sum_{l=1}^{L} \left( I_{l,l+1} - P_{l,l+1} \right), \qquad L+1 \equiv 1$$

# Diagonalising $\Gamma$ in higher rank sectors

- Change notation to $Z \cong \theta,\ X \cong \overset{u(p_1)}{\theta}$
- Roots (or rapidities) $u_k \equiv u(p_k)$ instead of momenta $p_k$
  - e.g. two-magnon state reads

$$|p_1 < p_2\rangle \equiv \sum_{n_1<n_2} e^{ip_1n_1+ip_2n_2}\,|\theta_1 \dots \overset{u_1}{\theta}_{n_1} \dots \overset{u_2}{\theta}_{n_2} \dots \theta_L\rangle$$

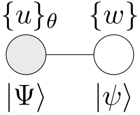
- Nested Ansatz: For $SU(3)$ put additional set of roots $\{w\}$ on top of roots $\{u\}$
  - Fields: $Z \cong \theta,\ X \cong \overset{u_1}{\theta},\ Y \cong \overset{\overset{w_1}{u_1}}{\theta}$
  - Results in sum over states like




$$|u_1 \dots \overset{w_1}{u}_{k_1} \dots \overset{w_2}{u}_{k_2} \dots u_M\rangle \to |\theta_1 \dots \overset{u_1}{\theta}_{n_1} \dots \overset{\overset{w_1}{u_{k_1}}}{\theta}_{n_{k_1}} \dots \overset{\overset{w_2}{u_{k_2}}}{\theta}_{n_{k_2}} \dots \overset{u_M}{\theta}_{n_M} \dots \theta_L\rangle$$

# Higher Rank Sectors

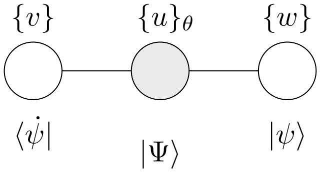
- Nested Ansatz: For $SU(3)$ put additional set of roots $\{w\}$ on top of roots $\{u\}$




  - Fields: $Z \cong \theta$, $X \cong \overset{u_1}{\theta}$, $Y \cong \overset{w_1}{\overset{u_1}{\theta}}$

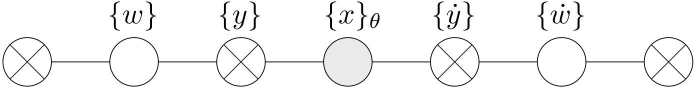
- All six scalar fields: $SO(6) \cong SU(4)$

  - $Z \cong \theta, \quad X \cong \overset{u}{\theta}, \quad Y \cong \overset{w}{\overset{u}{\theta}}, \quad \bar{Y} \cong \overset{v}{\overset{u}{\theta}}, \quad \bar{X} \cong \overset{vw}{\overset{u}{\theta}}, \quad \bar{Z} \cong \overset{v\;\;w}{\overset{u_1 u_2}{\theta}}$



# Work in progress: The Lagrangian

- Lagrangian density is descendant of the full $PSU(2,2|4)$ symmetry




- Need to understand how to construct the Lagrangian in the spin chain framework

- Limits are highly ambiguous: Different orders give different results

# Goal

- Calculate $\langle \mathcal{O}_A(x_1)\mathcal{L}(x_2)\mathcal{O}_A(x_3)\rangle$
- Lagrangian density is a supersymmetry descendant $\rightarrow$ less understood

$$\mathcal{L} \sim Q^4 \,\mathrm{Tr}\,(\phi\phi)$$

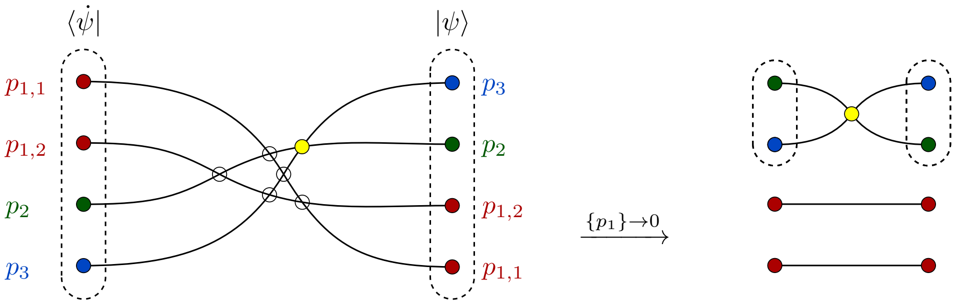
- Lagrangian insertion procedure[9] used to calculate correlators and amplitudes
- Currently most advanced method for Structure Constants: Hexagon Approach[5]
  - Non-perturbative, but a conjecture
  - Corrections: $\sim \mathrm{Pol}(1/L)$ but not $\sim e^{-L}$
- $C_{AA\mathcal{L}}$ known exactly[6]:

$$2\pi^2 C_{AA\mathcal{L}} = -g^2 \frac{\partial \gamma_A}{\partial g^2} \quad \text{with} \quad \gamma = \Delta - \Delta_0$$

[9] Eden, Korchemsky, and Sokatchev 2011, [5] Basso, Komatsu, and Vieira 2015, [6] Costa et al. 2010

# Limit to Descendant State

- Scattering becomes trivial: $\lim_{p_1 \to 0} S(p_1, p_2) = 1 + \mathcal{O}(p_1)$




- $SO(6)$ calculation reproduces expected result[7]

[7] Basso et al. 2017.

# Three-point functions and Structure Constants

- Coefficient appearing in the three-point function

$$\langle \mathcal{O}_A(x_1)\mathcal{O}_B(x_2)\mathcal{O}_C(x_3)\rangle = \frac{C_{ABC}}{|x_{12}|^{\Delta_{AB}}|x_{13}|^{\Delta_{AC}}|x_{23}|^{\Delta_{BC}}},$$

  where $\Delta_{AB} = \Delta_A + \Delta_B - \Delta_C$
- Special structure constant known to all orders[8]:

$$2\pi^2 C_{AA\mathcal{L}} = -g^2 \frac{\partial \gamma_A}{\partial g^2}$$

- Goal: Understand this relation in the integrability setup
- At one-loop

$$-2\pi^2 C^{(\text{one-loop})}_{AA\mathcal{L}} = -2\pi^2 \langle \mathcal{O}_A | \hat{\mathcal{L}} | \mathcal{O}_A \rangle = \langle \mathcal{O}_A | H | \mathcal{O}_A \rangle = \gamma_A^{(\text{tree})}$$

[8] Costa et al. 2010.

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