Introduction to the Standard Model

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Outline

1. Elements of Quantum Field Theory
2. Gauge Theories
3. QCD
4. Higgs mechanism
5. Electroweak interaction and Standard Model
6. Phenomenology of W and Z bosons, precision tests
7. Higgs bosons

Notations and Conventions

$$\mu, \nu, .. = 0, 1, 2, 3; \qquad k, l, .. = 1, 2, 3$$

$$x = (x^\mu) = (x^0, \vec{x}), \quad x^0 = t \qquad (\hbar = c = 1)$$

$$p = (p^\mu) = (p^0, \vec{p}), \quad p^0 = E = \sqrt{\vec{p}^2 + m^2}$$

$$a_\mu = g_{\mu\nu}\, a^\nu, \qquad (g_{\mu\nu}) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

$$a^2 = a_\mu a^\mu, \quad a \cdot b = a_\mu b^\mu = a^0 b^0 - \vec{a} \cdot \vec{b}$$

$$\partial_\mu = \frac{\partial}{\partial x^\mu} = g_{\mu\nu}\, \partial^\nu, \quad \partial^\nu = \frac{\partial}{\partial x_\nu} \qquad [\, \partial^0 = \partial_0, \;\; \partial^k = -\partial_k \,]$$

$$\Box = \partial_\mu \partial^\mu = \frac{\partial^2}{\partial t^2} - \Delta$$

1. Elements of Quantum Field Theory

Fields in the Standard Model

- spin 0 particles: scalar fields $\phi(x)$
- spin 1 particles: vector fields $A_\mu(x), \quad \mu = 0, \ldots 3$
- spin 1/2 fermions: spinor fields $\psi(x) = \begin{pmatrix} \psi_1 \\ \psi_2 \\ \psi_3 \\ \psi_4 \end{pmatrix}$

Lagrangian: $\mathcal{L}(\phi, \partial_\mu \phi)$ Lorentz invariant

Action: $S = \int \mathrm{d}^4x \, \mathcal{L}(\phi(x), \cdots)$ Lorentz invariant

Hamilton's principle: $\delta S = 0 \Rightarrow e.o.m.$ Lorentz covariant

free fields: $\mathcal{L}$ is quadratic in the fields $\Rightarrow$ e.o.m. are linear diff. eqs.

equations of motions from $\delta S = S[\phi + \delta\phi] - S[\phi] \; = \; 0$

$\Rightarrow$ Euler-Lagrange equations

- mechanics of particles: $L(q_1, \dots q_n, \dot{q}_1, \dots \dot{q}_n)$

  e.o.m. $\frac{\mathrm{d}}{\mathrm{d}t} \frac{\partial L}{\partial \dot{q}_i} - \frac{\partial L}{\partial q_i} \; = \; 0$

- field theory: $q_i \to \phi(x), \quad \dot{q}_i \to \partial_\mu \phi(x)$

  e.o.m. $\partial_\mu \frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi)} - \frac{\partial \mathcal{L}}{\partial \phi} \; = \; 0$ field eq.

example: scalar field $\phi(x), \quad \mathcal{L} = \frac{1}{2} (\partial_\mu \phi)^2 - \frac{m^2}{2} \phi^2$

field equation $\Box \phi + m^2 \phi = 0$

solution $\phi(x) = \frac{1}{(2\pi)^{3/2}} \int \frac{\mathrm{d}^3 k}{2k^0} \left[a(k)\, e^{-ikx} \; + \; a(k)^\dagger \, e^{ikx} \right]$

Scalar field [spin 0 , mass m]

neutral: $\phi = \phi^{+}$, $\mathcal{L} = \frac{1}{2}(\partial_\mu \phi)(\partial^\mu \phi) - \frac{m^2}{2}\phi^2$

charged: $\phi \neq \phi^{+}$, $\mathcal{L} = (\partial_\mu \phi)^{+}(\partial^\mu \phi) - m^2 \phi^{+}\phi$

$\delta S = 0 \xrightarrow{\text{e.o.m.}}$ $\boxed{(\Box + m^2)\phi = 0}$ Klein-Gordon-Eq.

Propagator (= Green's function) : $D(x-y)$

consider point-like source at y:

$(\Box + m^2) D(x-y) = \delta^4(x-y) \xrightarrow[\text{Fourier-Transf.}]{}$ $(-k^2+m^2)D(k) = 1$

$$\boxed{iD(k) = \frac{i}{k^2 - m^2 + i\varepsilon}}$$

$x^0 > y^0$:	particle $y \rightarrow x$
$y^0 > x^0$:	anti-particle $x \rightarrow y$

causality

arrow = flow of particle charge

Vector field [Spin 1, mass $m \neq 0$]   "massive photon"

$A_\mu(x)$ $(\mu = 0, \dots 3)$, $\quad F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ field strength tensor

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{1}{2} m^2 A_\mu A^\mu$$

$$\delta S = 0 \xrightarrow{\text{e.o.m}} \underbrace{\left[(\Box + m^2) g^{\mu\nu} - \partial^\mu \partial^\nu \right]}_{=: K^{\mu\nu}} A_\nu = 0$$

solutions: $\sim \varepsilon_\mu e^{\pm ikx}$, with $\varepsilon \cdot k = 0$, $\varepsilon^2 = -1$

3 independent polarization vectors $\varepsilon_\mu^{(\lambda)}(k)$ $(\lambda = 1, 2, 3)$

polarization sum:
$$\boxed{\sum_{\lambda=1}^{3} \varepsilon_\mu^{(\lambda)} \varepsilon_\nu^{(\lambda)} = -g_{\mu\nu} + \frac{k_\mu k_\nu}{m^2}}$$

- propagator = Green's function $D_{\mu\nu}(x-y)$

  propagation of field from point-like source at y

$$K^{\mu\rho} D_{\rho\nu}(x-y) = g^{\mu}{}_{\nu}\ \delta^4(x-y)$$

Fourier transformation: $D_{\mu\nu}(x-y) = \int \frac{d^4k}{(2\pi)^4}\, D_{\mu\nu}(k)\, e^{ik(x-y)}$

$$K^{\mu\nu} = (\Box + m^2)\, g^{\mu\nu} - \partial^\mu \partial^\nu \longrightarrow (-k^2+m^2)\, g^{\mu\nu} + k^\mu k^\nu$$

$\rightarrow$ algebraic equation: $\left[(-k^2+m^2)\, g^{\mu\rho} + k^\mu k^\rho\right] D_{\rho\nu}(k) = g^{\mu}{}_{\nu}$

solution [with $+i\varepsilon$ convention $\rightarrow$ causality]

$$\boxed{i D_{\rho\nu}(k) = \frac{i}{k^2 - m^2 + i\varepsilon}\left(-g_{\rho\nu} + \frac{k_\rho k_\nu}{m^2}\right)}$$

Vector field for mass $m=0$ (photon)

$A_\mu(x)$ 4-potential. $\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu}$, $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$

e.o.m. $\underbrace{\left(\Box g^{\mu\nu} - \partial^\mu \partial^\nu\right)}_{= K^{\mu\nu}} A_\nu = 0$ (Maxwell's eqs.)

- 2 physical solutions $\sim \varepsilon_\mu^{(\lambda)} e^{\pm ikx}$ with $\varepsilon \cdot k = 0$, $\vec{\varepsilon} \cdot \vec{k} = 0$
  transverse polarization
- unphysical solution: $\varepsilon_\mu \sim k_\mu$ longitudinal polarization

$$A_\mu(x) = k_\mu e^{\pm ikx} = \partial_\mu\left(\mp i\, e^{\pm ikx}\right) \equiv \partial_\mu \chi$$

$$A_\mu \to A_\mu + \partial_\mu \chi \quad \text{gauge transformation}$$

$K^{\mu\nu} k_\nu = 0$: k_ν eigenvector with eigenvalue $= 0$, $\det(K^{\mu\nu}) = 0$

$(K^{\mu\nu})^{-1}$ does not exist $\rightarrow$ propagator $K^{\mu\nu} D_{\nu\rho} = g^\mu_{\ \rho}$ (?)

- reason: gauge invariance of $\mathcal{L}$
- way out: break gauge invariance by $\mathcal{L} \to \mathcal{L} + \mathcal{L}_{fix}$

$\mathcal{L}_{fix} = -\frac{1}{2\xi}\left(\partial_\mu A^\mu\right)^2$ "gauge fixing term", ξ real parameter (free)

$K^{\mu\nu} \longrightarrow K'^{\mu\nu}$

$\Rightarrow$ propagator:

$$\boxed{iD_{\mu\nu}(k) = \frac{i}{q^2 + i\varepsilon}\left[-g_{\mu\nu} + (1-\xi)\frac{k_\mu k_\nu}{k^2}\right]}$$

$\xi = 1$: "Feynman gauge" $\sim \frac{-ig_{\mu\nu}}{q^2 + i\varepsilon}$

<u>Remark:</u> $\mathcal{L}_{fix}$ has no physical impact;

$k_\mu \leftrightarrow \partial_\mu$

photon couples to conserved current

$\partial_\mu j^\mu = 0$

Dirac field [spin $\frac{1}{2}$, mass m]

- spinor: $\psi(x) = \begin{pmatrix} \psi_1 \\ \vdots \\ \psi_4 \end{pmatrix}$, adjoint spinor: $\bar{\psi} = \psi^\dagger \gamma^0 = (\psi_1^*, \psi_2^*, -\psi_3^*, -\psi_4^*)$

- Dirac matrices: γ^μ $(\mu = 0,1,2,3)$, $\gamma^0 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$, $\gamma^k = \begin{pmatrix} 0 & \sigma_k \\ -\sigma_k & 0 \end{pmatrix}$

  notation: $\not{a} = a_\mu \gamma^\mu = \gamma^\mu a_\mu$ $\qquad$ σ_k : Pauli matrices

- Lagrangian: $\mathcal{L} = \bar{\psi} (i\gamma^\mu \partial_\mu - m) \psi$

- Dirac eq. $\delta S = 0 \overset{\text{e.o.m.}}{\Longrightarrow}$ $\boxed{(i\gamma^\mu \partial_\mu - m)\psi = 0}$

- two types of solutions:

  ① $u(p)\, e^{-ipx}$: $(\not{p} - m)\, u(p) = 0$ $\qquad$ ② $v(p)\, e^{ipx}$: $(\not{p} + m)\, v(p) = 0$

particle

anti-particle

<u>propagator</u> : point-like source at $y \longrightarrow S(x-y)$

$$(i\gamma^\mu \partial_\mu - m)\, S(x-y) = \mathbb{1}\ \delta^4(x-y) \xrightarrow[\text{Transf.}]{\text{Fourier}} (\not{k} - m)\, S(k) = \mathbb{1}$$

solution (with $+i\varepsilon$ convention):

$$\boxed{iS(k) = \frac{i}{\not{k} - m + i\varepsilon} = \frac{i(\not{k}+m)}{k^2 - m^2 + i\varepsilon}}$$

$$S(x-y) = \int \frac{d^4k}{(2\pi)^4}\, S(k)\, e^{ik(x-y)}$$

causality behaviour

arrow = direction of charge flow of the <u>particle</u> (momentum space)

$S(x-y)$ describes

- particle propagation from $y \to x$ if $y^0 < x^0$
- anti-particle propagation from $x \to y$ if $x^0 < y^0$

Interaction

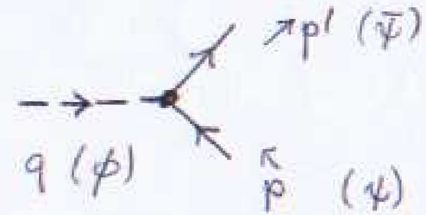
higher powers of fields in $\mathcal{L}$ (> 2)

Example 1: Yukawa interaction

spinor $\psi(x)$, mass m + scalar $\phi(x)$, mass M

$$\mathcal{L} = \underbrace{\bar{\psi}(i\gamma^\mu \partial_\mu - m)\psi + \tfrac{1}{2}(\partial_\mu \phi)(\partial^\mu \phi) - \frac{\;}{2}\phi^2}_{\text{free part, } \mathcal{L}_0} + \underbrace{g\,\bar{\psi}\psi\phi}_{\mathcal{L}_{int},\ g:\ \text{coupling constant}}$$

graphical symbol for interaction: vertex



momentum conservation:

$$q = p' - p$$

Example 2: QED Quantum Electrodynamics

spinor $\psi(x)$, mass m + vector $A_\mu(x)$, mass $= 0$

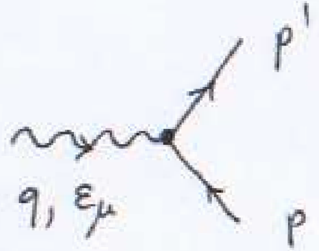
$$\mathcal{L} = \underbrace{\bar{\psi}(i\gamma^\mu \partial_\mu - m)\psi - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \mathcal{L}_{fix}}_{\text{free part, } \mathcal{L}_0} + \underbrace{e\, \bar{\psi}\gamma^\mu \psi A_\mu}$$

$\mathcal{L}_{int} = j^\mu A_\mu$

$j^\mu = \bar{\psi}\gamma^\mu\psi$ current

e : coupling constant (charge)

vertex :



$q = p' - p$ momentum conservation

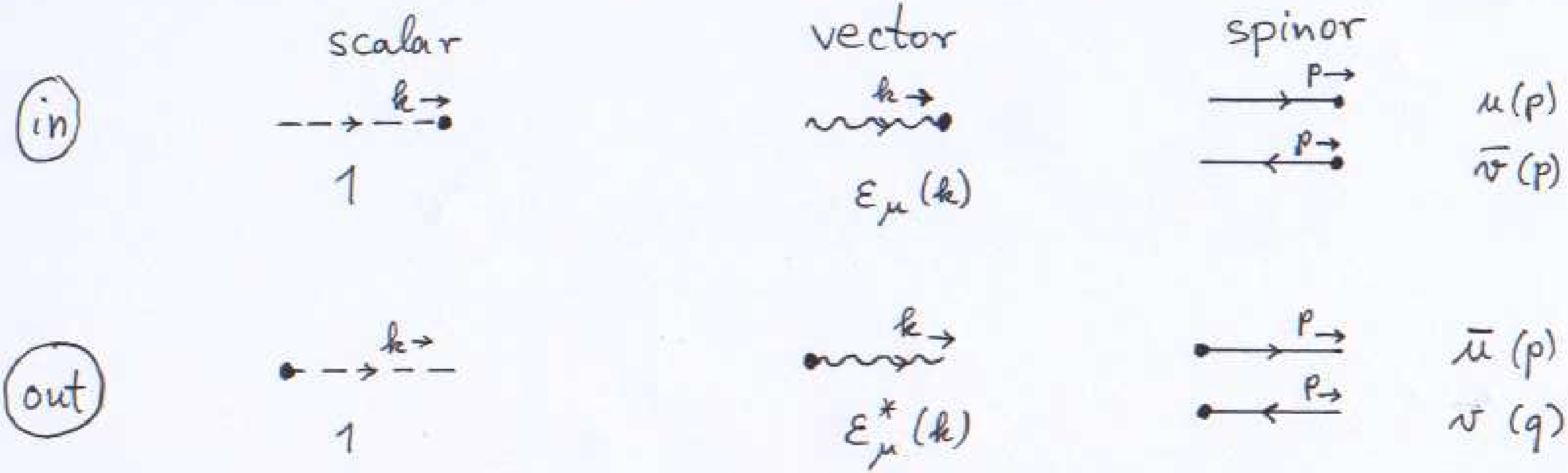
Feynman graphs

perturbation theory for scattering processes
= expansion in coupling constant(s)

QED: $\alpha = \frac{e^2}{4\pi} = \frac{1}{137}$ small $(\alpha > \alpha^2 > \alpha^3 \ldots)$

- Feynman rules: (i) wave functions (ii) propagators (iii) vertices

(i) solution of field equations (e.o.m.) in momentum space



(ii) propagators from inhomogeneous wave eqs, point-like source

scalar

k

$$\frac{i}{k^2 - m^2 + i\varepsilon}$$

vector

k

$$\frac{i}{k^2 - m^2 + i\varepsilon}\left(-g_{\mu\nu} + \frac{k_\mu k_\nu}{m^2}\right)$$

$$\frac{i}{k^2 + i\varepsilon} \cdot (-g_{\mu\nu})$$

for $m = 0$

spinor

k

$$\frac{i(\not{k} + m)}{k^2 - m^2 + i\varepsilon}$$

(iii) vertices from $\mathcal{L}_{int}$ ⟶ momentum space

Yukawa

$i g \mathbb{1}$

QED

$i e \gamma^\mu$

... (others)

- scattering amplitudes $\equiv$ S-matrix elements

  $a_1 + a_2 \longrightarrow b_1 + b_2 + \cdots + b_n$, $\qquad a \longrightarrow b_1 + \cdots + b_n$

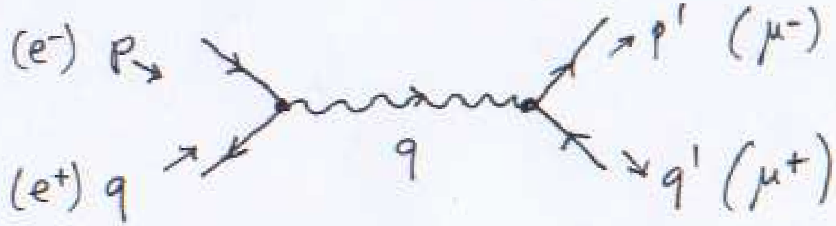
  matrix element $\quad \langle b_1 \cdots b_n | S | a_1 a_2 \rangle$, $\qquad S = \lim\limits_{\substack{t_0 \to -\infty \\ t \to \infty}} \mathcal{U}(t, t_0)$

  $$\equiv \langle f | S | i \rangle \equiv S_{fi}$$

- Feynman graphs $\Rightarrow S_{fi}$ in given order of perturbation theory

<u>lowest order</u>: connect in- and outgoing particles by minimum number of vertices and propagators

<u>example (i)</u>: $e^+ e^- \longrightarrow \mu^+ \mu^-$ in QED



$$\left[q = p + q = p' + q' \right]$$

$$\bar{v}(q)\, ie\gamma^\mu u(p) \left(\frac{-i g_{\mu\nu}}{q^2 + i\varepsilon} \right) \bar{u}(p')\, ie\gamma^\nu v(q') \quad \sim \quad e^2 = \mathcal{O}(\alpha)$$

Feynman graphs

perturbation theory for scattering processes
= expansion in coupling constant(s)

QED: $\alpha = \frac{e^2}{4\pi} = \frac{1}{137}$ small $(\alpha > \alpha^2 > \alpha^3 \ldots)$

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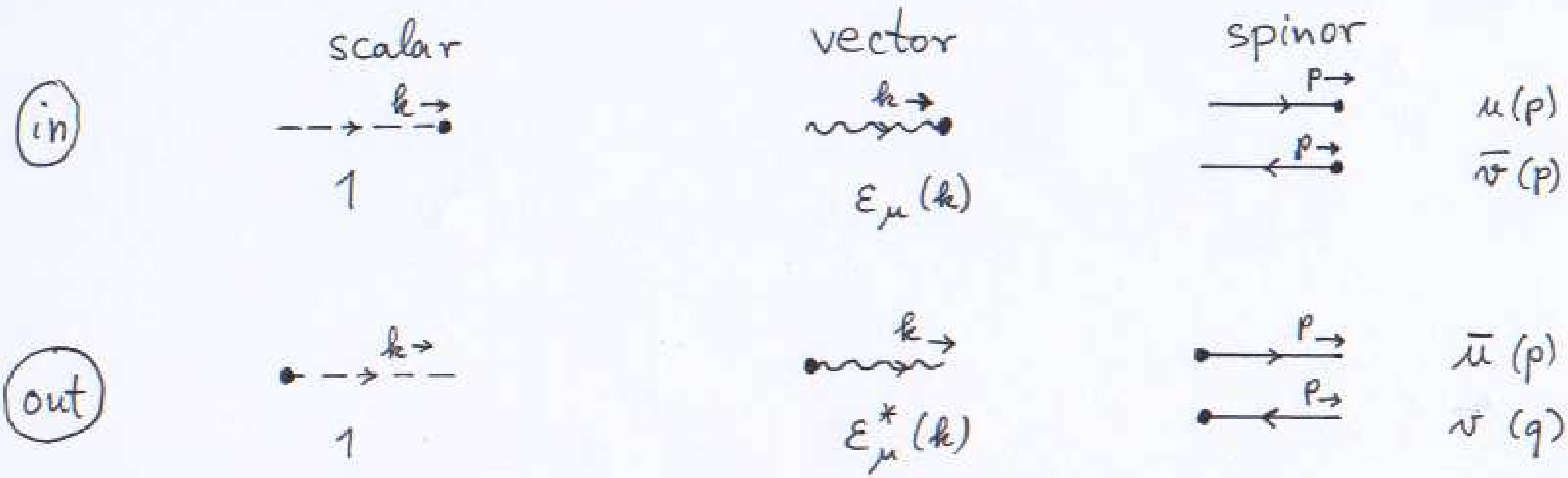
(i) solution of field equations (e.o.m.) in momentum space



(ii) propagators from inhomogeneous wave eqs, point-like source

scalar

k

$$\frac{i}{k^2 - m^2 + i\varepsilon}$$

vector

k

$$\frac{i}{k^2 - m^2 + i\varepsilon}\left(-g_{\mu\nu} + \frac{k_\mu k_\nu}{m^2}\right)$$

$$\frac{i}{k^2 + i\varepsilon} \cdot (-g_{\mu\nu})$$

for $m = 0$

spinor

k

$$\frac{i(\not{k} + m)}{k^2 - m^2 + i\varepsilon}$$

(iii) vertices from $\mathcal{L}_{int} \longrightarrow$ momentum space

Yukawa

$i g \mathbb{1}$

QED

$i e \gamma^\mu$

... (others)

- scattering amplitudes $\equiv$ S-matrix elements

$a_1 + a_2 \longrightarrow b_1 + b_2 + \cdots + b_n$, $\qquad a \longrightarrow b_1 + \cdots + b_n$

matrix element $\langle b_1 \cdots b_n | S | a_1 a_2 \rangle$, $\qquad S = \lim\limits_{\substack{t_0 \to -\infty \\ t \to \infty}} \mathcal{U}(t, t_0)$

$$\equiv \langle f | S | i \rangle \equiv S_{fi}$$

- Feynman graphs $\Rightarrow S_{fi}$ in given order of perturbation theory

<u>lowest order</u>: connect in- and outgoing particles by minimum number of vertices and propagators

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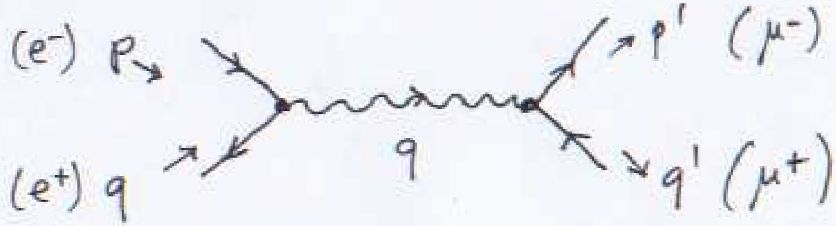
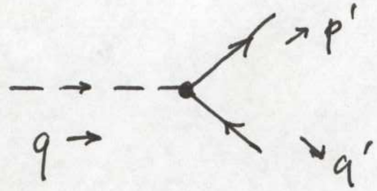


$[\, q = p + q = p' + q' \,]$

$$\bar{v}(q)\, ie\gamma^\mu u(p) \left(\frac{-i g_{\mu\nu}}{q^2 + i\varepsilon} \right) \bar{u}(p')\, ie\gamma^\nu v(q') \quad \sim \quad e^2 = \mathcal{O}(\alpha)$$

example (ii) : $\phi^0 \longrightarrow f\bar{f}$ (scalar ϕ^0 decays to $f\bar{f}$)



$q = p' + q'$, $q^2 = M^2$

$M =$ mass of ϕ^0

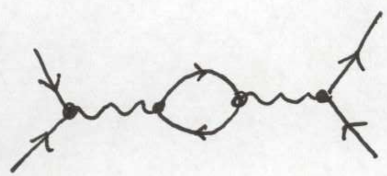
$\bar{u}(p')\, ig\, v(q') \sim g$

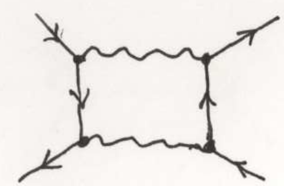
Higher Order

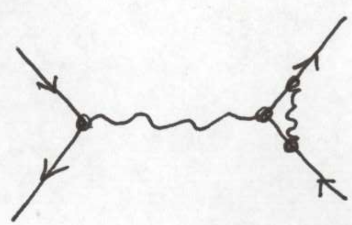
Feynman graphs with closed loops

example :

$e^+e^- \longrightarrow \mu^+\mu^-$







$\sim e^4$

2. Gauge theories

Constructing QED – main steps

- start with $\mathcal{L}_0 = \overline{\psi}\,(i\gamma^\mu \partial_\mu - m)\,\psi$ for free fermion field ψ

  symmetric under global gauge transformations

  $\psi' = e^{i\alpha}\psi, \ \alpha \text{ real}$

- perform minimal substitution $\partial_\mu \ \rightarrow \ D_\mu = \partial_\mu - ieA_\mu$

  $\Rightarrow$ invariance under local gauge transformations

  $$\psi' = e^{i\alpha(x)}\,\psi, \qquad A'_\mu = A_\mu + \frac{1}{e}\,\partial_\mu \alpha(x)$$

  - involves additional vector field A_μ
  - induces interaction between A_μ and ψ

  $$e\ (\overline{\psi}\gamma^\mu\psi)\ A_\mu \ \equiv \ e\ j^\mu A_\mu$$

- make A_μ a dynamical field by adding

  $$\mathcal{L}_A = -\frac{1}{4}\,F_{\mu\nu}\,F^{\mu\nu}, \qquad F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

Non-Abelian gauge theories

Generalization: “phase” transformations that do not commute

$$\psi \to \psi' = U\psi \quad \textit{with} \quad U_1 U_2 \neq U_2 U_1$$

requires matrices, *i.e.* ψ is a multiplet

$$\psi = \begin{pmatrix} \psi_1 \\ \psi_2 \\ \vdots \\ \psi_n \end{pmatrix}, \qquad U = n \times n \text{ -}\textit{matrix}$$

each $\psi_k = \psi_k(x)$ is a Dirac spinor

(i) global symmetry

starting point: $\mathcal{L}_0 = \overline{\psi}\,(i\gamma^\mu\,\partial_\mu - m)\,\psi$

where $\overline{\psi} = (\overline{\psi}_1, \cdots, \overline{\psi}_n)$

consider unitary matrices: $U^\dagger = U^{-1}$

$$\psi' = U\psi \quad \Rightarrow \quad \overline{\psi'} = \overline{\psi}\,U^\dagger = \overline{\psi}\,U^{-1}$$

$$\Rightarrow \quad \overline{\psi'}\psi' = \overline{\psi}\psi, \quad \overline{\psi'}\gamma^\mu\partial_\mu\psi' = \overline{\psi}\gamma^\mu\partial_\mu\psi$$

if U does not depend on x

$\Rightarrow$ $\mathcal{L}_0$ is invariant under $\psi \to U\psi$

U: global gauge transformation

similar for **scalar fields:**

$$\phi \to \phi' = U\phi, \qquad \phi = \begin{pmatrix} \phi_1 \\ \phi_2 \\ \vdots \\ \phi_n \end{pmatrix}$$

each $\psi_k = \phi_k(x)$ is a scalar field, $\phi^\dagger = (\phi_1^\dagger, \cdots, \phi_n^\dagger)$

terms $\phi^\dagger\phi, \quad (\partial_\mu\phi)^\dagger(\partial^\mu\phi)$ are invariant

$\Rightarrow \quad \mathcal{L}_0 = (\partial_\mu\phi)^\dagger(\partial^\mu\phi) - m^2\,\phi^\dagger\phi$ is invariant relevant in physics:

the special unitary $n \times n$-matrices with det=1

group $SU(n)$

examples:

$$SU(2): \quad \psi = \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix} \quad e.g. \quad \psi = \begin{pmatrix} \psi_\nu \\ \psi_e \end{pmatrix}$$ *isospin*

$$SU(3): \quad \psi = \begin{pmatrix} \psi_1 \\ \psi_2 \\ \psi_3 \end{pmatrix} \quad e.g. \quad \psi = \begin{pmatrix} \psi_r \\ \psi_g \\ \psi_b \end{pmatrix}$$ *colour*

$\boxed{SU(n)}$ matrices U can be written as exponentials

$$U(\theta_1, \cdots, \theta_N) = e^{i\theta_a T_a} \qquad \textit{sum over } a = 1, \cdots N$$

$\theta_1, \cdots \theta_N$: real parameters

$T_1, \cdots T_N$: $n \times n$-matrices, generators, $T_a^\dagger = T_a$

infinitesimal θ : $U = \mathbf{1} + i\theta_a T_a \quad (+O(\theta^2))$

N-dimensional Lie Group

det=1 and unitarity $\Rightarrow$ $\boxed{N = n^2 - 1}$

$n = 2: \quad N = 3,$ $\qquad n = 3: \quad N = 8$

commutators $[T_a, T_b] \neq 0$ non-Abelian

$$\boxed{[T_a, T_b] = i\, f_{abc}\, T_c}$$

Lie Algebra

f_{abc} : real numbers, structure constants

$f_{abc} = -f_{bac} = \cdots$ antisymmetric

commutators $[T_a, T_b] \neq 0$ non-Abelian

$$[T_a, T_b] = i\, f_{abc}\, T_c$$

Lie Algebra

f_{abc} : real numbers, structure constants

$f_{abc} = -f_{bac} = \cdots$ antisymmetric

$SU(2)$ $f_{abc} = \epsilon_{abc}$ *(like angular momentum)*

$T_a = \frac{1}{2}\,\sigma_a,$ σ_a : *Pauli matrices (a=1,2,3)*

commutators $[T_a, T_b] \neq 0$ non-Abelian

$$\boxed{[T_a, T_b] \,=\, f_{abc}\, T_c}$$

Lie Algebra

$f_{abc}:$ real numbers, structure constants

$f_{abc} = -f_{bac} = \cdots$ antisymmetric

$\boxed{SU(2)}$ $f_{abc} = \epsilon_{abc}$ *(like angular momentum)*

$T_a = \frac{1}{2}\,\sigma_a,$ $\sigma_a:$ *Pauli matrices (a=1,2,3)*

$\boxed{SU(3)}$ $T_a = \frac{1}{2}\,\lambda_a,$ $\lambda_a:$ *Gell-Mann matrices (a=1,...8)*

$$\lambda_1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad \ldots$$

$$\lambda_1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad \lambda_2 = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad \lambda_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix},$$

$$\lambda_4 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}, \quad \lambda_5 = \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix},$$

$$\lambda_6 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \lambda_7 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix}, \lambda_8 = \frac{1}{\sqrt{3}}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}$$

(ii) local symmetry

now: $\theta_a = \theta_a(x)$ for $a = 1, \dots N$

covariant derivative $\partial_\mu \to D_\mu \,=\, \partial_\mu - ig\,\mathbf{W}_\mu$

vector field $\mathbf{W}_\mu$ is $n \times n$ matrix: $\mathbf{W}_\mu(x) = T_a\,W_\mu^a(x)$

induces interaction term $\mathcal{L}_0 \to \mathcal{L}_0 + \mathcal{L}_{\rm int}$

with $\mathcal{L}_{\rm int} = g\,\overline{\Psi}\gamma^\mu \mathbf{W}_\mu \Psi \;= g\,(\overline{\Psi}\gamma^\mu T_a \Psi)\,W_\mu^a \,\equiv\, j_a^\mu\,W_\mu^a$

For a multiplet of scalar fields Φ:

$$\mathcal{L}_0 = (\partial_\mu \Phi)^\dagger (\partial^\mu \Phi) \quad \to \quad \mathcal{L} = (D_\mu \Phi)^\dagger (D^\mu \Phi)$$

$\mathcal{L}$ is invariant under local gauge transformations

$$\Psi \to \Psi' = U\,\Psi\,,$$

$$\mathbf{W}_\mu \to \mathbf{W}'_\mu = U\,\mathbf{W}_\mu\,U^{-1} - \frac{i}{g}(\partial_\mu U)U^{-1}$$

for infinitesimal transformations:

$$W^a_\mu \to {W'}^a_\mu = W^a_\mu + \frac{1}{g}\,\partial_\mu \theta^a + f_{abc}\,W^b_\mu\,\theta^c$$

crucial property of covariant derivative

$$\boxed{D'_\mu U = U\,D_\mu}$$

(iii) dynamics of W_μ^a fields

need: additional term $\mathcal{L}_W \quad \Rightarrow \quad$ e.o.m., propagators

naive: $\quad \sum_a (\partial_\mu W_\nu^a - \partial_\nu W_\mu^a)^2 \quad$ *not gauge invariant*

instead:

$$\mathbf{F}_{\mu\nu} = D_\mu \mathbf{W}_\nu - D_\nu \mathbf{W}_\mu \equiv F_{\mu\nu}^a T_a$$

$$= \partial_\mu \mathbf{W}_\nu - \partial_\nu \mathbf{W}_\mu - ig\,[\mathbf{W}_\mu, \mathbf{W}_\nu]$$

$$= \frac{i}{g}\,[D_\mu, D_\nu]$$

gauge transformation: $\quad \mathbf{W}_\mu \to \mathbf{W}'_\mu, \quad D_\mu \to D'_\mu$

$$\Rightarrow \quad \mathbf{F}_{\mu\nu} \to \mathbf{F}'_{\mu\nu} = U\,\mathbf{F}_{\mu\nu}\,U^{-1}$$

$$\Rightarrow \mathrm{Tr}\,(\mathbf{F}'_{\mu\nu}\mathbf{F}'^{\,\mu\nu}) = \mathrm{Tr}\,(U\mathbf{F}_{\mu\nu}U^{-1}\,U\mathbf{F}^{\mu\nu}U^{-1}) = \mathrm{Tr}\,(\mathbf{F}_{\mu\nu}\mathbf{F}^{\mu\nu})$$

gauge invariant

Lagrangian:

$$\boxed{\mathcal{L}_W = -\frac{1}{2}\,\mathrm{Tr}\,(\mathbf{F}_{\mu\nu}\mathbf{F}^{\mu\nu}) = -\frac{1}{4}\sum_a F^a_{\mu\nu}F^{a,\mu\nu}}$$

components of $\mathbf{F}_{\mu\nu}$ *[using normalization* $\mathrm{Tr}(T_a T_b) = \frac{1}{2}\,\delta_{ab}$ *]*

$$\boxed{F^a_{\mu\nu} = \partial_\mu W^a_\nu - \partial_\nu W^a_\mu + g\, f_{abc}\, W^b_\mu W^c_\nu}$$

$F^a_{\mu\nu} =$ *Abelian* $+$ *non-Abelian*

$$\mathcal{L}_W = \underbrace{\text{quadratic}}_{\text{free part}} + \underbrace{\text{cubic} + \text{quartic}}_{\text{tri- and quadri-linear interactions}}$$

$$\mathcal{L}_W = -\frac{1}{4}(\partial_\mu W_\nu^a - \partial_\nu W_\mu^a)^2 \qquad \Rightarrow \text{ propagator}$$

$$= -\frac{1}{2}\, g f_{abc}\,(\partial_\mu W_\nu^a - \partial_\nu W_\mu^a)\, W^{b,\mu}\, W^{c,\nu}$$

$$= -\frac{1}{4}\, g^2 f_{abc}\, f_{ade}\, W_\mu^b\, W_\nu^c\, W^{d,\mu}\, W^{e,\nu}$$

new type of couplings:

- self-couplings of vector fields (gauge couplings)
- universal coupling constant g for fermions and vector fields

3. Quantum Chromodynamics (QCD)

each quark field $q = u, d, \ldots$ appears in 3 colours

$$\psi = \begin{pmatrix} q \\ q \\ q \end{pmatrix}, \qquad \overline{\psi} = (\overline{q}, \overline{q}, \overline{q})$$

$T_a = \frac{1}{2}\,\lambda_a \quad (a = 1, \ldots 8)$ *generators, group SU(3)*

$W^a_\mu \equiv G^a_\mu$ *8 gluon fields*

$G^a_{\mu\nu} = \partial_\mu G^a_\nu - \partial_\nu G^a_\mu + g_s\, f_{abc}\, G^b_\mu G^c_\nu$ *field strength*

$D_\mu = \partial_\mu - i g_s\, T_a\, G^a_\mu$ *covariant derivative*

g_s *coupling constant of strong interaction,* $\alpha_s = \frac{g_s^2}{4\pi}$

QCD Lagrangian

(for one flavor of quarks)

$$\mathcal{L}_{\mathrm{QCD}} = \mathcal{L}_{\mathrm{F}} + \mathcal{L}_{\mathrm{G}}, \qquad \mathcal{L}_{\mathrm{F}} = \overline{\psi}\,(i\gamma^{\mu} D_{\mu} - m)\,\psi\,,$$

$$\begin{aligned} \mathcal{L}_{\mathrm{QCD}} &= \overline{\psi}\,(i\gamma^{\mu}\partial_{\mu} - m)\,\psi \\ &+ g_s\,\overline{\psi}\,\gamma^{\mu}\frac{\lambda_a}{2}\,\psi\,G^a_{\mu} \\ &- \frac{1}{4}\,G^a_{\mu\nu}G^{a,\,\mu\nu} - \frac{1}{2\xi}\,(\partial_{\mu}G^{a,\,\mu})^2 \end{aligned}$$

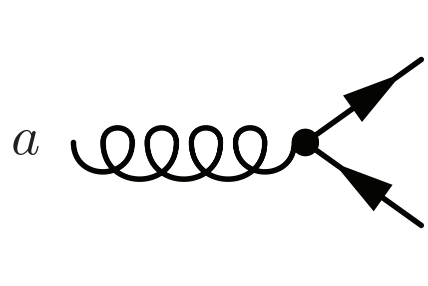
QCD Lagrangian

(for one flavor of quarks)

$$
\begin{aligned}
\mathcal{L}_{\mathrm{QCD}} &= \mathcal{L}_{\mathrm{F}} + \mathcal{L}_{\mathrm{G}} = \\
& \overline{\psi}\,(i\gamma^{\mu}\partial_{\mu} - m)\,\psi \\
&+ g_s\,\overline{\psi}\,\gamma^{\mu}\frac{\lambda_a}{2}\,\psi\,G^a_{\mu} \\
&- \frac{1}{4}\,G^a_{\mu\nu}G^{a,\,\mu\nu} - \frac{1}{2\xi}\,(\partial_{\mu}G^{a,\,\mu})^2
\end{aligned}
$$

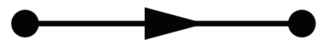
reality: *sum over quark flavors f, with* $\psi \to \psi_f,\ m \to m_f$

- Quark-Gluon-Vertex:



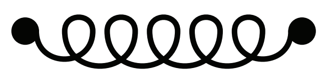
$$ig_S(T_a)_{kl}\gamma^{\mu}$$

- Quark-Propagator:



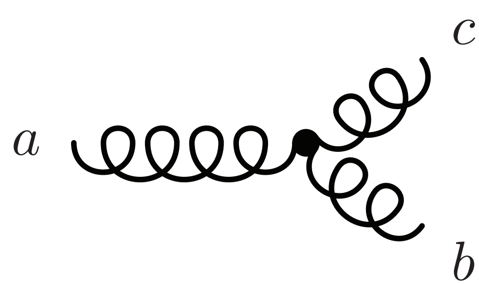
$$i\frac{\not{q}+m}{q^2-m^2+i\epsilon} \equiv \frac{i}{\not{q}-m+i\epsilon}$$

- Gluon-Propagator:

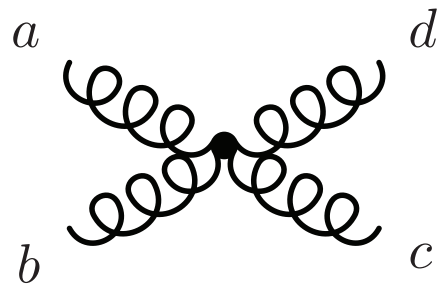


$$\frac{-ig_{\mu\nu}}{q^2+i\epsilon}$$

- Triple-Gluon-Vertex:



- Quartic-Gluon-Vertex:



4. Higgs mechanism

problem: weak interaction, gauge bosons are massive

mass terms $\sim M^2 W_\mu^a W^{a,\mu}$ spoil local gauge invariance

- bad high energy behaviour of amplitudes and cross sections, conflict with unitarity
  reason: longitudinal polarization $\epsilon^\mu \simeq \frac{k^\mu}{M} \sim k^\mu$
- bad divergence of higher orders with loop diagrams
  reason: propagators contain $-g_{\mu\nu} + \frac{k_\mu k_\nu}{M^2}$
  $\Rightarrow$ additional powers of momenta in loop integration
  $\Rightarrow$ spoil renormalizability

renormalizable theories: divergences can be removed by a finite number of counter terms

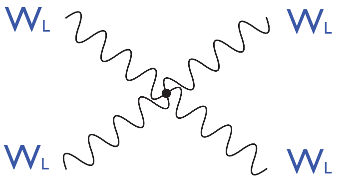
gauge invariant theories: counter terms for parameters (and fields)

W and Z are massive

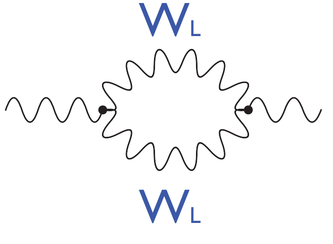
- W, Z have longitudinal polarization states

  polarization vectors of W (Z) $\epsilon_{\rm L} \sim k/M_{\rm W}$

  for large momentum k

- 

  bad high energy behaviour of WW scattering

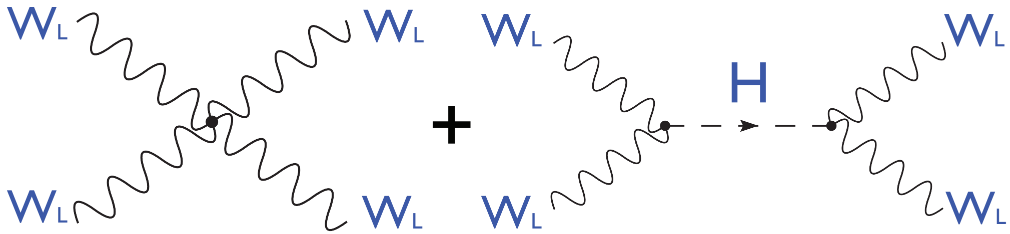
- 

  bad divergence of loop integrals

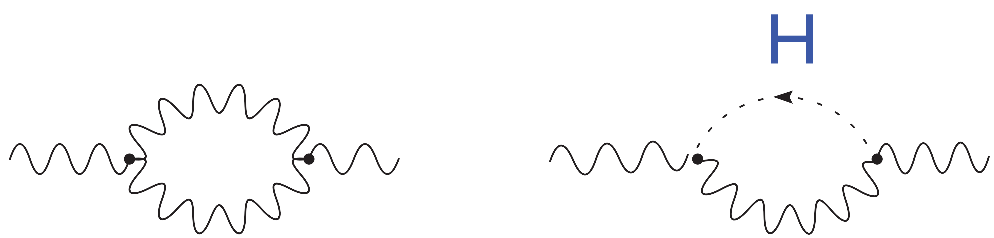
way out:

new scalar with appropriate couplings to W,Z

- restoration of unitarity



- restoration UV finiteness $\Rightarrow$ renormalizability

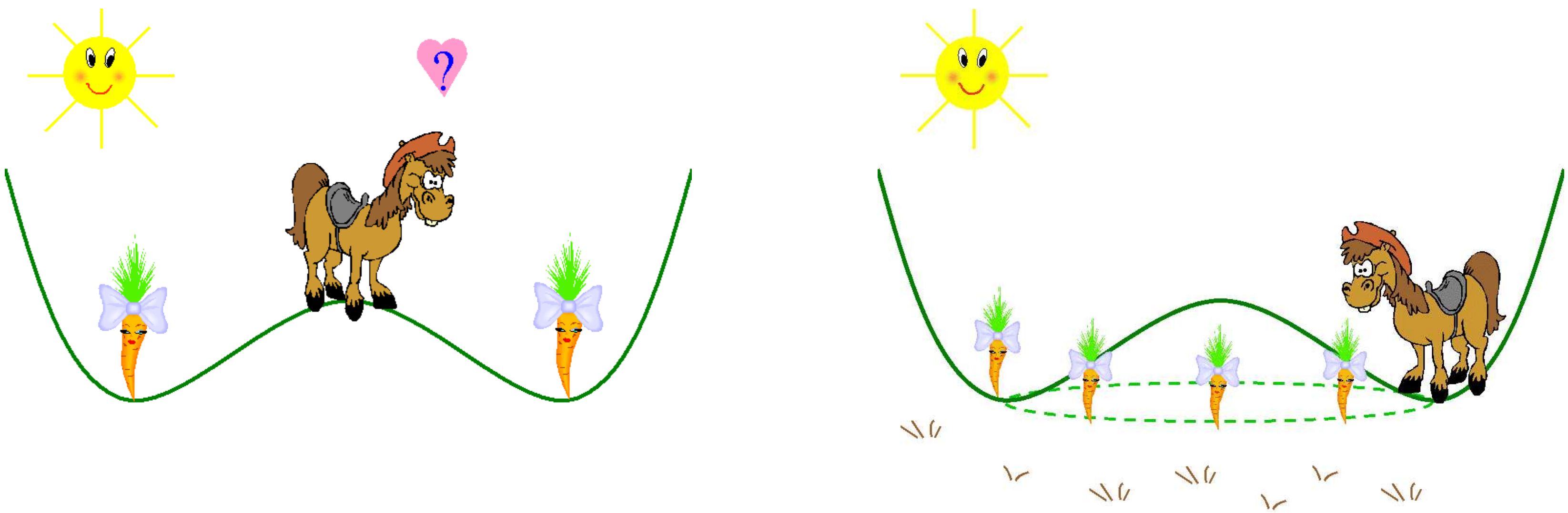


consistent way: Higgs mechanism
= scalar with gauge invariant interactions
and non-invariant ground state

Spontaneous symmetry breaking (SSB)

physical system: has a symmetry

ground state: not symmetric

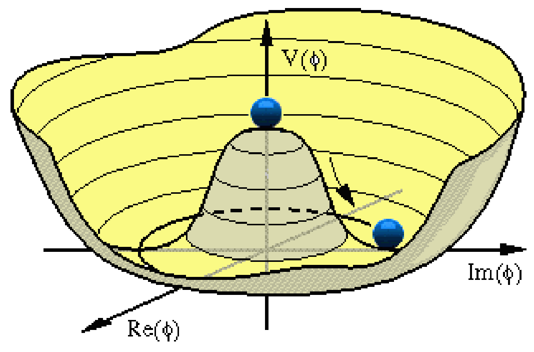


[A. Pich]

example: complex scalar field $\phi \neq \phi^\dagger$

Lagrangian with interaction V (potential), minimum at $\phi_0 = v$

$$\mathcal{L} = |\partial_\mu \phi|^2 \; - \; V(\phi)$$



$V = V(|\phi|)$: *$\mathcal{L}$ symmetric under* $\phi \to e^{i\alpha}\,\phi, \quad U(1)$

$v \neq 0$: $\phi_0' = e^{i\alpha}\, v \neq \phi_0$ *not symmetric*

$V = V(|\phi_0'|) = V(|\phi_0|)$: *vacuum is degenerate* write $\phi(x) = \eta(x) e^{i\theta(x)}$, η and θ real

$V(|\phi|) = V(\eta)$, minimum at $\eta = v:$ $V'(v) = 0$, $V''(v) > 0$

expand around minimum: $\eta(x) = v + \frac{1}{\sqrt{2}} H(x)$

$$V(\eta) = V(v) + \tfrac{1}{2} V''(v) \cdot \tfrac{1}{2} H^2 + \cdots$$

$$\mathcal{L} = \tfrac{1}{2} |\partial_\mu H|^2 - \underbrace{\frac{1}{2} V''(v)}_{m_H^2 > 0} \cdot \tfrac{1}{2} H^2 + v^2 |\partial_\mu \theta|^2 + \cdots$$

$m_H^2 > 0$ *mass of H*

- H field is massive
- θ field is massless, no θ^2 term: Goldstone field
- special case of Goldstone theorem:

  for each broken generator T_a with $T_a \phi_0 \neq 0$
  there is a massless Goldstone field $\theta(x)$

SSB in gauge theories

$$\mathcal{L} = \left(D_\mu\phi\right)^+\left(D^\mu\phi\right) - V(|\phi|) - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}, \quad D_\mu = \partial_\mu - ieA_\mu$$

invariant under local U(1) transformations:

$$\phi'(x) = e^{i\alpha(x)}\,\phi(x) = e^{i\alpha(x)}\,e^{i\theta(x)}\eta(x)$$

$$A'_\mu(x) = A_\mu(x) + \frac{1}{e}\partial_\mu\alpha(x)$$

choose $\alpha(x) = -\theta(x)$: $\quad \phi'(x) = \eta(x)$

$$\mathcal{L} = |(\partial_\mu - ieA'_\mu)\eta|^2 - V(\eta) - \frac{1}{4}F'_{\mu\nu}F'^{\,\mu\nu}$$

massless θ field removed (unphysical)

$$\mathcal{L} = |(\partial_\mu - ieA'_\mu)(v + \tfrac{1}{\sqrt{2}}H)|^2 - \tfrac{1}{4}F'_{\mu\nu}F'^{\mu\nu} - V$$

$$= \underbrace{-\frac{1}{4}F'_{\mu\nu}F'^{\mu\nu} + v^2 e^2\, A'_\mu A'^\mu}_{\textit{massive A-field, } m_A \sim ev} + \underbrace{\frac{1}{2}[(\partial_\mu H)^2 - m_H^2 H^2]}_{\textit{neutral scalar, } m_H \neq 0} + \cdots$$

in this special gauge: no Goldstone field | unitary gauge

A_μ-field propagator: $\frac{i}{k^2 - m_A^2 + i\varepsilon} \underbrace{\left(-g_{\mu\nu} + \frac{k_\mu k_\nu}{m_A^2}\right)}_{\textit{polarization sum of 3 pol. states}}$

massive vector field without spoiling gauge symmetry of $\mathcal{L}$

two different gauges

properties	ϕ field	A_μ field
symmetry manifest	H, θ	2 polarizations (transverse)
physics manifest	H	3 polarizations (2 transverse + 1 longitudinal)

$\theta \quad \rightarrow \quad$ *longitudinal polarization of* A_μ

5. Electroweak Standard Model

preliminaries

<u>Dirac matrices:</u> $\gamma^\mu \; (\mu = 0,1,2,3), \quad \gamma^\mu\gamma^\nu + \gamma^\nu\gamma^\mu = 2\, g^{\mu\nu}$

$\overline{\Gamma} = \gamma^0 (\Gamma)^\dagger \gamma^0, \quad \Gamma$ *any Dirac matrix oder product of matrices*

further Dirac matrix: $\gamma_5 = i\,\gamma^0\gamma^1\gamma^2\gamma^3 = \begin{pmatrix} 0 & \mathbf{1} \\ \mathbf{1} & 0 \end{pmatrix}$

$$\gamma_5\gamma^\mu + \gamma^\mu\gamma_5 = 0, \qquad \overline{\gamma_5} = -\gamma_5, \qquad \gamma_5^2 = \mathbf{1}$$

<u>chiral fermions:</u>

$\psi^L = \frac{1-\gamma_5}{2}\,\psi$ *left-handed spinor, L-chiral spinor*

$\psi^R = \frac{1+\gamma_5}{2}\,\psi$ *right-handed spinor, R-chiral spinor*

projectors on right/left chirality: $\omega_\pm = \frac{1\pm\gamma_5}{2}, \quad (\omega_\pm)^2 = \omega_\pm$

chiral currents:

$$\overline{\psi^L}\,\gamma^\mu\psi^L = \overline{\psi}\,\gamma^\mu\tfrac{1-\gamma_5}{2}\,\psi \equiv J^\mu_L$$ left-handed current

$$\overline{\psi^R}\,\gamma^\mu\psi^R = \overline{\psi}\,\gamma^\mu\tfrac{1+\gamma_5}{2}\,\psi \equiv J^\mu_R$$ right-handed current

$$J^\mu_V = \overline{\psi}\gamma^\mu\psi = J^\mu_L + J^\mu_R$$ vector current

$$J^\mu_A = \overline{\psi}\gamma^\mu\gamma_5\psi = -J^\mu_L + J^\mu_R$$ axialvector current

mass term:

$$m\,\overline{\psi}\psi = m\,(\overline{\psi^L}\,\psi^R + \overline{\psi^R}\,\psi^L)$$

connects L and R !

symmetry group: $SU(2)_I \times U(1)_Y$

$SU(2)_I$: weak isospin, generators $T^a_{\rm I} = \frac{1}{2}\,\sigma_a$ *for* L, $= 0$ *for* R

$U(1)_Y$: weak hypercharge, generator Y $\quad$ $T^3_{\rm I} + Y/2 = Q$

fermion content (ignoring possibe right-handed neutrinos)

					$T^3_{\rm I}$	Y
leptons:	$\Psi^{\rm L}_L =$	$\begin{pmatrix} \nu^{\rm L}_e \\ e^{\rm L} \end{pmatrix}$	$\begin{pmatrix} \nu^{\rm L}_\mu \\ \mu^{\rm L} \end{pmatrix}$	$\begin{pmatrix} \nu^{\rm L}_\tau \\ \tau^{\rm L} \end{pmatrix}$	$\begin{matrix} +\frac{1}{2} \\ -\frac{1}{2} \end{matrix}$	$\begin{matrix} -1 \\ -1 \end{matrix}$
	$\psi^{\rm R}_l =$	$e^{\rm R}$	$\mu^{\rm R}$	$\tau^{\rm R}$	0	-2
quarks:	$\Psi^{\rm L}_Q =$	$\begin{pmatrix} u^{\rm L} \\ d^{\rm L} \end{pmatrix}$	$\begin{pmatrix} c^{\rm L} \\ s^{\rm L} \end{pmatrix}$	$\begin{pmatrix} t^{\rm L} \\ b^{\rm L} \end{pmatrix}$	$\begin{matrix} +\frac{1}{2} \\ -\frac{1}{2} \end{matrix}$	$\begin{matrix} +\frac{1}{3} \\ +\frac{1}{3} \end{matrix}$
	$\psi^{\rm R}_u =$	$u^{\rm R}$	$c^{\rm R}$	$t^{\rm R}$	0	$+\frac{4}{3}$
	$\psi^{\rm R}_d =$	$d^{\rm R}$	$s^{\rm R}$	$b^{\rm R}$	0	$-\frac{2}{3}$

gauge boson content

$SU(2)_I$: generators $T_{\rm I}^1,\ T_{\rm I}^2,\ T_{\rm I}^3$

gauge fields $W_\mu^1,\ W_\mu^2,\ W_\mu^3$

also: $W_\mu^\pm = \frac{1}{\sqrt{2}}\left(W_\mu^1 \mp i\,W_\mu^2\right),\ W_\mu^3$

$U(1)_Y$: generator Y

gauge field B_μ

$$[T_{\rm I}^a,\ T_{\rm I}^b] = i\epsilon_{abc}\,T_{\rm I}^c\,, \qquad [T_{\rm I}^a, Y] = 0$$

gauge boson content

$SU(2)_I$: generators $T_\mathrm{I}^1,\ T_\mathrm{I}^2,\ T_\mathrm{I}^3$

gauge fields $W_\mu^1,\ W_\mu^2,\ W_\mu^3$

also: $W_\mu^\pm = \frac{1}{\sqrt{2}}\left(W_\mu^1 \mp i\,W_\mu^2\right),\ W_\mu^3$

$U(1)_Y$: generator Y

gauge field B_μ

notation: $\not\partial = \gamma^\mu \partial_\mu, \quad \not a = \gamma^\mu a_\mu$

Free Lagrangian of (still massless) fermions:

$$\mathcal{L}_{0,\mathrm{ferm}} = \mathrm{i}\overline{\psi_f}\not\partial\psi_f = \mathrm{i}\overline{\Psi^{\mathrm{L}}_L}\not\partial\Psi^{\mathrm{L}}_L + \mathrm{i}\overline{\Psi^{\mathrm{L}}_Q}\not\partial\Psi^{\mathrm{L}}_Q + \mathrm{i}\overline{\psi^{\mathrm{R}}_l}\not\partial\psi^{\mathrm{R}}_l + \mathrm{i}\overline{\psi^{\mathrm{R}}_u}\not\partial\psi^{\mathrm{R}}_u + \mathrm{i}\overline{\psi^{\mathrm{R}}_d}\not\partial\psi^{\mathrm{R}}_d$$

Minimal substitution:

$$\partial_\mu \;\to\; D_\mu = \partial_\mu - \mathrm{i}g_2 T^a_{\mathrm{I}} W^a_\mu + \mathrm{i}g_1 \tfrac{1}{2} Y B_\mu \;=\; D^{\mathrm{L}}_\mu \omega_- + D^{\mathrm{R}}_\mu \omega_+,$$

$$D^{\mathrm{L}}_\mu = \partial_\mu - \frac{\mathrm{i}g_2}{\sqrt{2}}\begin{pmatrix} 0 & W^+_\mu \\ W^-_\mu & 0 \end{pmatrix} - \frac{\mathrm{i}}{2}\begin{pmatrix} g_2 W^3_\mu - g_1 Y^{\mathrm{L}} B_\mu & 0 \\ 0 & -g_2 W^3_\mu - g_1 Y^{\mathrm{L}} B_\mu \end{pmatrix},$$

$$D^{\mathrm{R}}_\mu = \partial_\mu + \mathrm{i}g_1 \tfrac{1}{2} Y^{\mathrm{R}} B_\mu$$

Photon identification:

" rotation":

$$\begin{pmatrix} Z_\mu \\ A_\mu \end{pmatrix} = \begin{pmatrix} c_{\mathrm{W}} & s_{\mathrm{W}} \\ -s_{\mathrm{W}} & c_{\mathrm{W}} \end{pmatrix}\begin{pmatrix} W^3_\mu \\ B_\mu \end{pmatrix},$$

$c_{\mathrm{W}} = \cos\theta_{\mathrm{W}},\ s_{\mathrm{W}} = \sin\theta_{\mathrm{W}}$,
θ_{W} = mixing angle

$$D^{\mathrm{L}}_\mu\Big|_{A_\mu} = -\frac{\mathrm{i}}{2}A_\mu\begin{pmatrix} -g_2 s_{\mathrm{W}} - g_1 c_{\mathrm{W}} Y^{\mathrm{L}} & 0 \\ 0 & g_2 s_{\mathrm{W}} - g_1 c_{\mathrm{W}} Y^{\mathrm{L}} \end{pmatrix} \overset{!}{=} \mathrm{i}eA_\mu\begin{pmatrix} Q_1 & 0 \\ 0 & Q_2 \end{pmatrix}$$

- charged difference in doublet $Q_1 - Q_2 = 1$ $\quad\to\quad$ $g_2 = \dfrac{e}{s_{\mathrm{W}}}$
- normalize $Y^{\mathrm{L/R}}$ such that $\quad g_1 = \dfrac{e}{c_{\mathrm{W}}}$

$\hookrightarrow$ Y fixed by "Gell-Mann–Nishijima relation": $\quad Q = T^3_{\mathrm{I}} + \dfrac{Y}{2}$

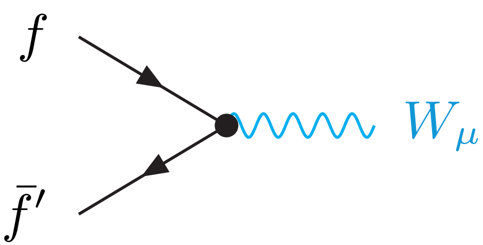
Fermion–gauge-boson interaction:

$$\mathcal{L}_{\mathrm{ferm,YM}} = \frac{e}{\sqrt{2}s_{\mathrm{W}}} \overline{\Psi_F^{\mathrm{L}}} \begin{pmatrix} 0 & \not{W}^{+} \\ \not{W}^{-} & 0 \end{pmatrix} \Psi_F^{\mathrm{L}} + \frac{e}{2c_{\mathrm{W}}s_{\mathrm{W}}} \overline{\Psi_F^{\mathrm{L}}} \sigma^3 \not{Z} \Psi_F^{\mathrm{L}}$$

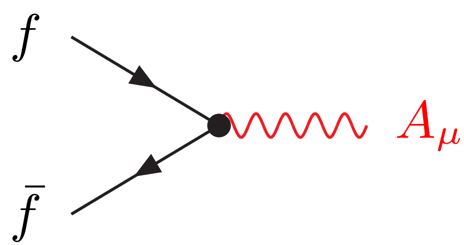
$$- e\frac{s_{\mathrm{W}}}{c_{\mathrm{W}}} Q_f \overline{\psi_f} \not{Z} \psi_f - eQ_f \overline{\psi_f} \not{A} \psi_f$$

(f=all fermions, F= all doublets)

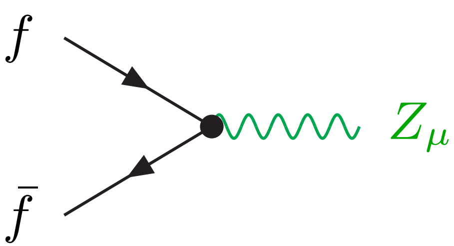
Feynman rules:



$$\frac{\mathrm{i}e}{\sqrt{2}s_{\mathrm{W}}}\gamma_\mu\omega_-$$



$$-\mathrm{i}Q_f e\gamma_\mu$$



$$\mathrm{i}e\gamma_\mu(g_f^+\omega_+ + g_f^-\omega_-) = \mathrm{i}e\gamma_\mu(v_f - a_f\gamma_5)$$

with $$g_f^+ = -\frac{s_{\mathrm{W}}}{c_{\mathrm{W}}}Q_f, \quad g_f^- = -\frac{s_{\mathrm{W}}}{c_{\mathrm{W}}}Q_f + \frac{T^3_{\mathrm{I},f}}{c_{\mathrm{W}}s_{\mathrm{W}}},$$

$$v_f = -\frac{s_{\mathrm{W}}}{c_{\mathrm{W}}}Q_f + \frac{T^3_{\mathrm{I},f}}{2c_{\mathrm{W}}s_{\mathrm{W}}}, \quad a_f = \frac{T^3_{\mathrm{I},f}}{2c_{\mathrm{W}}s_{\mathrm{W}}}$$

gauge field Lagrangian (Yang-Mills Lagrangian)

$$\mathcal{L}_{\rm YM} = -\frac{1}{4} W^a_{\mu\nu} W^{a,\mu\nu} - \frac{1}{4} B_{\mu\nu} B^{\mu\nu}$$

Field-strength tensors:

$$W^a_{\mu\nu} \;=\; \partial_\mu W^a_\nu - \partial_\nu W^a_\mu + g_2 \epsilon^{abc} W^b_\mu W^c_\nu, \qquad B_{\mu\nu} \;=\; \partial_\mu B_\nu - \partial_\nu B_\mu$$

Lagrangian in terms of “physical” fields:

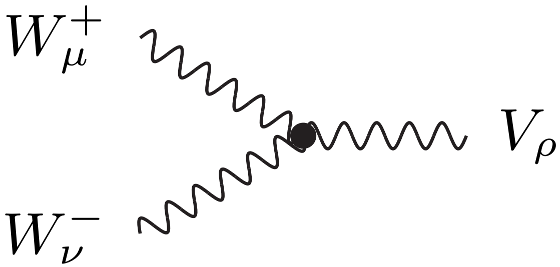
$$\mathcal{L}_{\rm YM} = -\frac{1}{2}(\partial_\mu W^+_\nu - \partial_\nu W^+_\mu)(\partial^\mu W^{-,\nu} - \partial^\nu W^{-,\mu})$$
$$- \frac{1}{4}(\partial_\mu Z_\nu - \partial_\nu Z_\mu)(\partial^\mu Z^\nu - \partial^\nu Z^\mu) - \frac{1}{4}(\partial_\mu A_\nu - \partial_\nu A_\mu)(\partial^\mu A^\nu - \partial^\nu A^\mu)$$

$+$ (trilinear interaction terms involving AW^+W^-, ZW^+W^-)

$+$ (quadrilinear interaction terms involving AAW^+W^-, AZW^+W^-, ZZW^+W^-, $W^+W^-W^+W^-$)

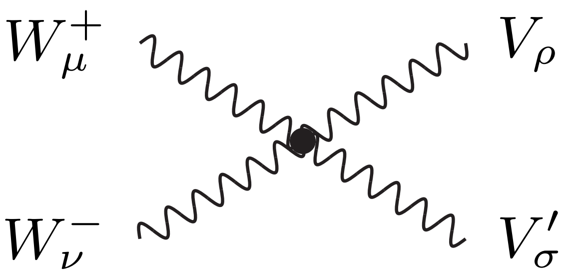
Feynman rules for gauge-boson self-interactions:

(fields and momenta incoming)



$$\mathrm{i}eC_{WWV}\Big[g_{\mu\nu}(k_+ - k_-)_\rho + g_{\nu\rho}(k_- - k_V)_\mu + g_{\rho\mu}(k_V - k_+)_\nu\Big]$$

with $C_{WW\gamma} = 1, \quad C_{WWZ} = -\frac{c_\mathrm{W}}{s_\mathrm{W}}$



$$\mathrm{i}e^2 C_{WWVV'}\Big[2g_{\mu\nu}g_{\rho\sigma} - g_{\mu\rho}g_{\sigma\nu} - g_{\mu\sigma}g_{\nu\rho}\Big]$$

with $C_{WW\gamma\gamma} = -1, \qquad C_{WW\gamma Z} = \frac{c_\mathrm{W}}{s_\mathrm{W}},$

$$C_{WWZZ} = -\frac{c_\mathrm{W}^2}{s_\mathrm{W}^2}, \quad C_{WWWW} = \frac{1}{s_\mathrm{W}^2}$$

Higgs mechanism $\Rightarrow$ masses of W and Z bosons

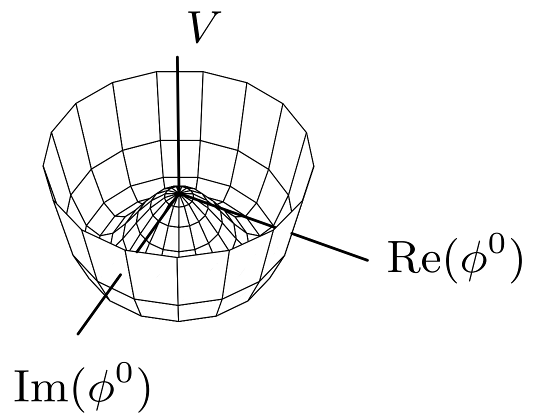
spontaneous breaking $SU(2)_I \times U(1)_Y \to U(1)_Q$
unbroken em. gauge symmetry, massless photon

Minimal scalar sector with complex scalar doublet $\Phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}$, $Y_\Phi = 1$

Scalar self-interaction via Higgs potential:

$$V(\Phi) = -\mu^2 \Phi^\dagger \Phi + \frac{\lambda}{4}(\Phi^\dagger \Phi)^2, \quad \mu^2, \lambda > 0,$$

$= \mathrm{SU(2)_I \times U(1)_Y}$ symmetric



$V(\Phi) =$ minimal for $|\Phi| = \sqrt{\frac{2\mu^2}{\lambda}} \equiv \frac{v}{\sqrt{2}} > 0$

ground state Φ_0 (=vacuum expectation value of Φ) not unique

specific choice $\Phi_0 = \begin{pmatrix} 0 \\ \frac{v}{\sqrt{2}} \end{pmatrix}$ not gauge invariant $\Rightarrow$ spontaneous symmetry breaking

elmg. gauge invariance unbroken, since $Q\Phi_0 = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \Phi_0 = 0$

Field excitations in Φ:

$$\Phi(x) = \begin{pmatrix} \phi^+(x) \\ \frac{1}{\sqrt{2}}\Big(v + H(x) + \mathrm{i}\chi(x)\Big) \end{pmatrix}$$

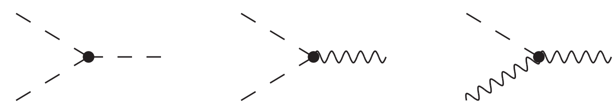
Gauge-invariant Lagrangian of Higgs sector: $(\phi^- = (\phi^+)^\dagger)$

$$\mathcal{L}_\mathrm{H} = (D_\mu\Phi)^\dagger(D^\mu\Phi) - V(\Phi) \qquad \text{with } D_\mu = \partial_\mu - \mathrm{i}g_2\frac{\sigma^a}{2}W^a_\mu + \mathrm{i}\frac{g_1}{2}B_\mu$$

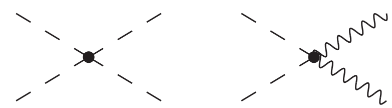
$$= (\partial_\mu\phi^+)(\partial^\mu\phi^-) - \frac{\mathrm{i}ev}{2s_\mathrm{W}}(W^+_\mu\partial^\mu\phi^- - W^-_\mu\partial^\mu\phi^+) + \frac{e^2v^2}{4s^2_\mathrm{W}}W^+_\mu W^{-,\mu}$$

$$+\frac{1}{2}(\partial\chi)^2 + \frac{ev}{2c_\mathrm{W}s_\mathrm{W}}Z_\mu\partial^\mu\chi + \frac{e^2v^2}{4c^2_\mathrm{W}s^2_\mathrm{W}}Z^2 + \frac{1}{2}(\partial H)^2 - \mu^2H^2$$

$+$ (trilinear SSS, SSV, SVV interactions)



$+$ (quadrilinear $SSSS$, $SSVV$ interactions)



Implications:

- gauge-boson masses: $M_\mathrm{W} = \frac{ev}{2s_\mathrm{W}}, \quad M_\mathrm{Z} = \frac{ev}{2c_\mathrm{W}s_\mathrm{W}} = \frac{M_\mathrm{W}}{c_\mathrm{W}}, \quad M_\gamma = 0$
- physical Higgs boson H: $M_\mathrm{H} = \sqrt{2\mu^2}$ = free parameter
- would-be Goldstone bosons $\phi^\pm$, χ: unphysical degrees of freedom

general gauge: Goldstone fields $\phi^{\pm}$, χ are present

required: gauge fixing term $\mathcal{L}_{fix}$

R_ξ gauge:

$$\mathcal{L}_{fix} = -\frac{1}{2\xi_\gamma}\,(F^\gamma)^2 - \frac{1}{2\xi_Z}\,\left(F^Z\right)^2 - \frac{1}{2\xi_W}\,\left(F^\pm\right)^2$$

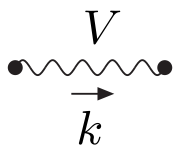
with the gauge-fixing functionals F^a: (ξ_V = arbitrary gauge-fixing parameters)

$$F^{\pm} = \partial W^{\pm} \mp \mathrm{i}\xi_{\mathrm{W}} M_{\mathrm{W}} \phi^{\pm}, \qquad F^{\mathrm{Z}} = \partial Z - \xi_{\mathrm{Z}} M_{\mathrm{Z}} \chi, \qquad F^{\gamma} = \partial A$$

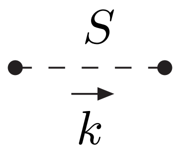
(*notation:* $\partial A = \partial_\mu A^\mu\,,\ \cdots$)

- elimination of mixing terms $(W_\mu^\pm \partial^\mu \phi^\mp)$, $(Z_\mu \partial^\mu \chi)$ in Lagrangian
  $\hookrightarrow$ decoupling of gauge and would-be Goldstone fields (no mix propagators)

- boson propagators:



$$D^{VV}_{\mu\nu}(k) = -\mathrm{i}\left[\frac{g_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}}{k^2 - M_V^2} + \frac{k_\mu k_\nu}{k^2}\,\frac{\xi_V}{k^2 - \xi_V M_V^2}\right], \qquad V = W, Z, \gamma$$



$$D^{SS}(k) = \frac{\mathrm{i}}{k^2 - \xi_V M_V^2}, \qquad S = \phi, \chi$$

- important special cases:
  - $\xi_V = 1$: 't Hooft–Feynman gauge
    $\hookrightarrow$ convenient gauge-boson propagators $\dfrac{-\mathrm{i} g_{\mu\nu}}{k^2 - M_V^2}$
  - $\xi_\mathrm{W}, \xi_\mathrm{Z} \to \infty$: "unitary gauge"
    $\hookrightarrow$ elimination of would-be Goldstone bosons

Fermion masses

fermions in chiral representations of gauge symmetry

$$\begin{pmatrix} \nu_e \\ e \end{pmatrix}_L , \quad e_R \quad \Rightarrow \text{mass term } m_e(\bar{e}_L e_R + \bar{e}_R e_L) = m_e\,\bar{e}e$$

not gauge invariant

solution of the SM: introduce <u>Yukawa interaction</u>

= new interaction of fermions with the Higgs field

gauge invariant interaction, g_e = Yukawa coupling constant

$$\mathcal{L}_{\text{Yuk}} = g_e\,[\,\overline{\psi^L}\,\Phi\,e_R + \overline{e_R}\,\Phi^\dagger\psi^L\,]$$

most transparent in unitary gauge

$$\Phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} \to \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + H \end{pmatrix}$$

apply to the first lepton generation $\psi^L = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix}, \quad e_R$:

$$\frac{g_e}{\sqrt{2}} \left[(\overline{\nu_L}, \overline{e_L}) \begin{pmatrix} 0 \\ v + H \end{pmatrix} e_R + \overline{e_R} \, (0, v + H) \begin{pmatrix} \nu_L \\ e_L \end{pmatrix} \right]$$

$$= \underbrace{\frac{g_e}{\sqrt{2}}}_{m_e} v \left[\overline{e_L} e_R + \overline{e_R} e_L \right] + \underbrace{\frac{g_e}{\sqrt{2}}}_{m_e/v} H \left[\overline{e_L} e_R + \overline{e_R} e_L \right]$$

$$= m_e \, \overline{e} e + \frac{m_e}{v} H \, \overline{e} e$$

3 generations of leptons and quarks

- for massless neutrinos: no generation mixing for leptons

  repeat construction for μ and τ with g_μ and g_τ

  $\Rightarrow \quad m_\mu,\ m_\tau$

- quark sector: generation mixing

  Yukawa couplings not generation-diagonal

$$U = \begin{pmatrix} u \\ c \\ t \end{pmatrix}, \quad D = \begin{pmatrix} d \\ s \\ b \end{pmatrix}, \qquad \overline{U} = (\overline{u}, \overline{c}, \overline{t}), \quad \overline{D} = (\overline{d}, \overline{s}, \overline{b})$$

unitary gauge:

$$\Phi = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + H(x) \end{pmatrix}, \quad \tilde{\Phi} = \frac{1}{\sqrt{2}} \begin{pmatrix} -v - H(x) \\ 0 \end{pmatrix},$$

$$\mathcal{L}_{\mathrm{Y}} = -\left(\overline{U}_L, \overline{D}_L\right) G_d \, \Phi \, D_R + \left(\overline{U}_L, \overline{D}_L\right) G_u \, \tilde{\Phi} \, U_R + \text{h.c.}$$

mass term for $H(x) = 0$:

$$-\overline{D}_L \underbrace{G_d \frac{v}{\sqrt{2}}}_{M_d} D_R - \overline{U}_L \underbrace{G_u \frac{v}{\sqrt{2}}}_{M_u} U_R + \text{h.c.}$$

mass matrices, non-diagonal

diagonalization by unitary matrices $V^u_{L,R}$, $V^d_{L,R}$:

$$U_{L,R} = V^u_{L,R}\,\hat{U}_{L,R}\,, \qquad \overline{U}_{L,R} = \overline{\hat{U}}_{L,R}\,(V^u_{L,R})^\dagger$$

$$D_{L,R} = V^d_{L,R}\,\hat{D}_{L,R}\,, \qquad \overline{D}_{L,R} = \overline{\hat{D}}_{L,R}\,(V^d_{L,R})^\dagger$$

$\Rightarrow$ mass eigenstates $\hat{U}$, $\hat{D}$

$$\overline{D}_L M_d D_R + \overline{U}_L M_u U_R =$$

$$\overline{\hat{D}}_L \underbrace{(V^d_L)^\dagger M_d V^d_R}_{M_d^{\rm diag}} \hat{D}_R + \overline{\hat{U}}_L \underbrace{(V^u_L)^\dagger M_u V^u_R}_{M_u^{\rm diag}} \hat{U}_R$$

diagonalization by unitary matrices $V^u_{L,R}$, $V^d_{L,R}$:

$$U_{L,R} = V^u_{L,R}\,\hat{U}_{L,R} \qquad \overline{U}_{L,R} = \overline{\hat{U}}_{L,R}\,(V^u_{L,R})^\dagger$$

$$D_{L,R} = V^d_{L,R}\,\hat{D}_{L,R} \qquad \overline{D}_{L,R} = \overline{\hat{D}}_{L,R}\,(V^d_{L,R})^\dagger$$

$\Rightarrow$ mass eigenstates $\hat{U}$, $\hat{D}$

$$\overline{D}_L M_d D_R + \overline{U}_L M_u U_R =$$

$$\overline{\hat{D}}_L \underbrace{(V^d_L)^\dagger M_d V^d_R}_{M_d^{\text{diag}}} \hat{D}_R + \overline{\hat{U}}_L \underbrace{(V^u_L)^\dagger M_u V^u_R}_{M_u^{\text{diag}}} \hat{U}_R$$

$$\begin{pmatrix} m_d & & \\ & m_s & \\ & & m_b \end{pmatrix} \qquad \begin{pmatrix} m_u & & \\ & m_c & \\ & & m_t \end{pmatrix}$$

- Yukawa interactions in terms of mass eigenstates:

$$\begin{aligned}\mathcal{L}_{\rm Y} &= -\,(\overline{U}_L,\overline{D}_L)\, G_d\, \Phi\, D_R + (\overline{U}_L,\overline{D}_L)\, G_u\, \tilde{\Phi}\, U_R \;\; + \text{h.c.} \\ &= -\overline{\hat{D}}_L\, M_d^{\rm diag}\, \hat{D}_R \left(1+\frac{H}{v}\right) - \overline{\hat{U}}_L\, M_u^{\rm diag}\, \hat{U}_R \left(1+\frac{H}{v}\right) \;\; + \text{h.c.}\end{aligned}$$

flavor-diagonal interactions with H field

- neutral weak and electromagnetic currents:

$$\overline{U}_{L(R)}\gamma^\mu U_{L(R)} = \overline{\hat{U}}_{L(R)}\gamma^\mu \hat{U}_{L(R)}\,, \quad \overline{D}_{L(R)}\gamma^\mu D_{L(R)} = \overline{\hat{D}}_{L(R)}\gamma^\mu \hat{D}_{L(R)}$$

flavor-diagonal because $V^{u,d}_{L,R}$ are unitary matrices

- charged current:

$$\overline{U}_L\gamma^\mu D_L + \overline{D}_L\gamma^\mu U_L = \overline{\hat{U}}_L V \gamma^\mu \hat{D}_L + \overline{\hat{D}}_L V^\dagger \gamma^\mu \hat{U}_L$$

remnant: $V \equiv V_{\rm CKM} = (V_L^u)^\dagger V_L^d$

Features of the CKM mixing:

- V = 3-dim. generalization of Cabibbo matrix $U_{\rm C}$
- V is parametrized by 4 free parameters: 3 real angles, 1 complex phase

  $\hookrightarrow$ complex phase is the only source of CP violation in SM

  counting:

  $$\begin{pmatrix}\text{\#real d.o.f.}\\ \text{in } V\end{pmatrix} - \begin{pmatrix}\text{\#unitarity}\\ \text{relations}\end{pmatrix} - \begin{pmatrix}\text{\#phase diffs. of}\\ u\text{-type quarks}\end{pmatrix} - \begin{pmatrix}\text{\#phase diffs. of}\\ d\text{-type quarks}\end{pmatrix} - \begin{pmatrix}\text{\#phase diff. between}\\ u\text{- and } d\text{-type quarks}\end{pmatrix}$$

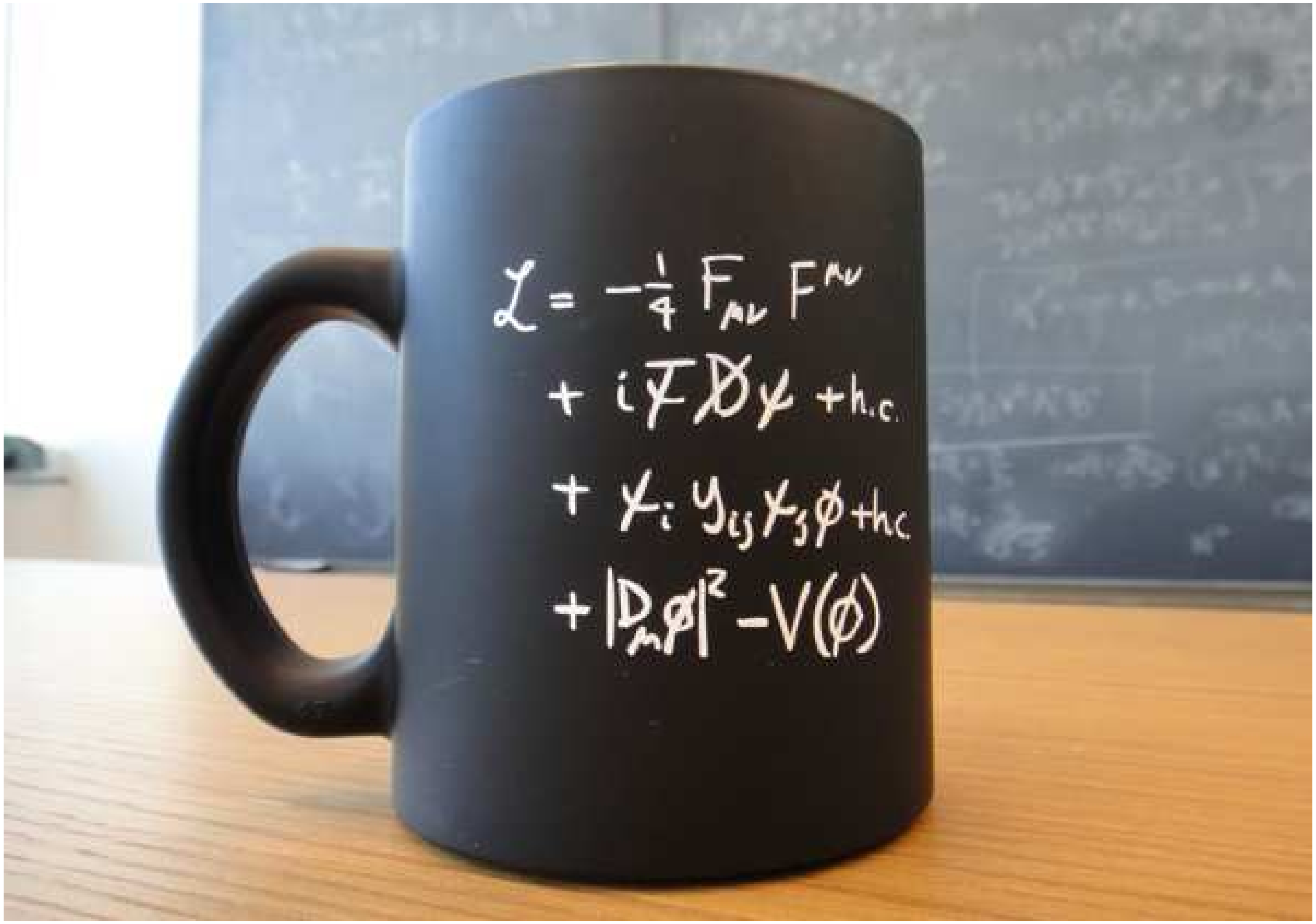
  $$= 18 - 9 - 2 - 2 - 1 = 4$$

- no flavour-changing neutral currents in lowest order,

  flavour-changing suppressed by factors $G_\mu(m_{q_1}^2 - m_{q_2}^2)$ in higher orders

  ("Glashow–Iliopoulos–Maiani mechanism")

The Standard Model Lagrangian



- renormalizable $\Rightarrow$ precision calculations
- quantum effects in precision observables detectable
- involve Higgs mass dependence

6. Phenomenology of W and Z bosons and precision tests

Cross sections and decay widths

scattering process: $a + b \quad \rightarrow \quad b_1 + b_2 + \cdots + b_n$

$$|a(p_a), b(p_b)> = |i>, \quad |b_1(p_1), \cdots b_n(p_n)> = |f>$$

matrix element = probability amplitude for $i \rightarrow f$:

$$S_{fi} =< f|S|i>$$

for $i \neq f$: $S_{fi} = (2\pi)^4 \, \delta^4(P_i - P_f) \, \mathcal{M}_{fi} \left[\frac{1}{(2\pi)^{3/2}}\right]^{n+2}$

$P_i = p_a + p_b \ = \ P_f = p_1 + \cdots + p_n$ momentum conservation

factors $(2\pi)^{-3/2}$ from wave function normalization (plane waves)

$\mathcal{M}_{fi}$ from Feynman graphs and rules probability for scattering into phase space element $\mathrm{d}\Phi$:

$$\mathrm{d}W_{fi} = |S_{fi}|^2 \, \mathrm{d}\Phi, \qquad \mathrm{d}\Phi = \frac{\mathrm{d}^3 p_1}{2p_1^0} \cdots \frac{\mathrm{d}^3 p_n}{2p_n^0}$$

$\frac{\mathrm{d}^3 p_i}{2p_i^0} = \mathrm{d}^4 p_i \, \delta(p_i^2 - m_i^2)$ Lorentz invariant phase space

differential cross section:

$$\mathrm{d}\sigma = \frac{(2\pi)^4}{4\sqrt{(p_a \cdot p_b)^2 - m_a^2 m_b^2}} \; |\mathcal{M}_{fi}|^2 \; (2\pi)^{-3n} \, \delta^4(P_i - P_f) \, \frac{\mathrm{d}^3 p_1}{2p_1^0} \cdots \frac{\mathrm{d}^3 p_n}{2p_n^0}$$

decay process: $a \quad \rightarrow \quad b_1 + b_2 + \cdots + b_n$

$$|a(p_a)> = |i>, \quad |b_1(p_1), \cdots b_n(p_n)> = |f>$$

$$S_{fi} = (2\pi)^4 \, \delta^4(p_a - P_f) \, \mathcal{M}_{fi} \, \left[\frac{1}{(2\pi)^{3/2}}\right]^{n+1}$$

decay width (differential):

$$\mathrm{d}\Gamma = \frac{(2\pi)^4}{2\, m_a} \; |\mathcal{M}_{fi}|^2 \, (2\pi)^{-3n} \, \delta^4(p_a - P_f) \, \frac{\mathrm{d}^3 p_1}{2p_1^0} \cdots \frac{\mathrm{d}^3 p_n}{2p_n^0}$$

special case: 2-particle phase space

$$a + b \to b_1 + b_2, \qquad a \to b_1 + b_2$$

- **cross section**

in the CMS, $\vec{p}_a + \vec{p}_b = 0 = \vec{p}_1 + \vec{p}_2$

$$\boxed{\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} = \frac{1}{64\pi^2 s} \frac{|\vec{p}_1|}{|\vec{p}_a|} \, |\mathcal{M}_{fi}|^2}$$

$$\mathrm{d}\Omega = \mathrm{d}\cos\theta \, \mathrm{d}\phi, \qquad \theta = \sphericalangle(\vec{p}_a, \vec{p}_1)$$

$$s = (p_a + p_b)^2 = E^2_{\mathrm{CMS}}$$

- **decay rate**

for final state masses $m_1 = m_2 = m$

$$\boxed{\frac{\mathrm{d}\Gamma}{\mathrm{d}\Omega} = \frac{1}{64\pi^2 \, m_a} \sqrt{1 - \frac{4m^2}{m_a^2}} \, |\mathcal{M}_{fi}|^2}$$

features of the ew Standard Model

- Higgs boson probably found, all other particles confirmed
- consistent quantum field theory
  - in accordance with unitarity
  - renormalizable $\Rightarrow$ predictions at higher orders
- formal parameters: $g_2,\ g_1,\ v,\ \lambda,\ g_f,\ V_{\mathrm{CKM}}$
  physical parameters: $\alpha,\ M_W,\ M_Z,\ M_H,\ m_f,\ V_{\mathrm{CKM}}$

Basic parameters and relations

ew mixing angle: $s_{\mathrm{W}} \equiv \sin\theta_{\mathrm{W}}, \quad c_{\mathrm{W}} \equiv \cos\theta_{\mathrm{W}}$

gauge coupling constants: $g_2 = \frac{e}{s_{\mathrm{W}}}, \quad g_1 = \frac{e}{c_{\mathrm{W}}}$

vector boson masses:

$$M_W = \frac{1}{2} g_2 v = \frac{ev}{2s_{\mathrm{W}}}$$

$$M_Z = \frac{ev}{2s_{\mathrm{W}}c_{\mathrm{W}}} = \frac{M_W}{c_{\mathrm{W}}}$$

$$s_{\mathrm{W}}^2 = 1 - \frac{M_W^2}{M_Z^2}$$

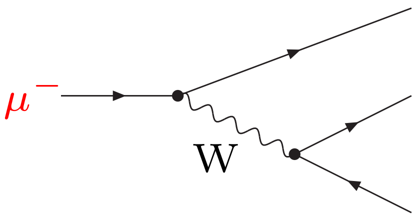
neutral current (NC) couplings:

$$a_f = \frac{g_2}{2c_{\mathrm{W}}}\, T_3^f$$

$$v_f = \frac{g_2}{2c_{\mathrm{W}}}\, (T_3^f - 2Q_f s_{\mathrm{W}})$$

observables and experiments

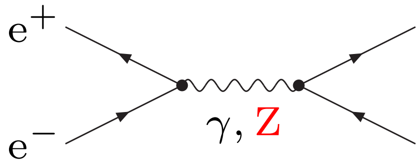
- Muon decay: $\mu^- \to \nu_\mu e^- \bar{\nu}_e$



determination of the Fermi constant

$$G_\mu = \frac{\pi \alpha M_Z^2}{\sqrt{2} M_W^2 (M_Z^2 - M_W^2)} + \ldots$$

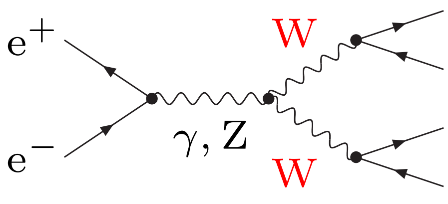
- Z production (LEP1/SLC): $e^+e^- \to Z \to f\bar{f}$

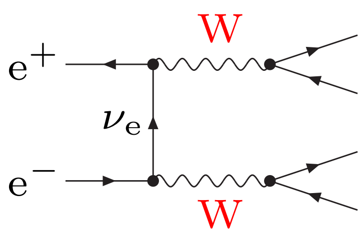


various precision measurements at the Z resonance: $M_Z, \Gamma_Z, \sigma_{had}, A_{FB}, A_{LR}$, etc.

$\Rightarrow$ good knowledge of the $Zf\bar{f}$ sector

- W-pair production (LEP2/ILC): $e^+e^- \to WW \to 4f(+\gamma)$





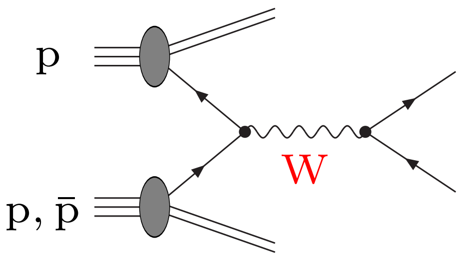
– measurement of M_W

– γWW/ZWW couplings

– quartic couplings: $\gamma\gamma WW$, γZWW

experiments at hadron colliders

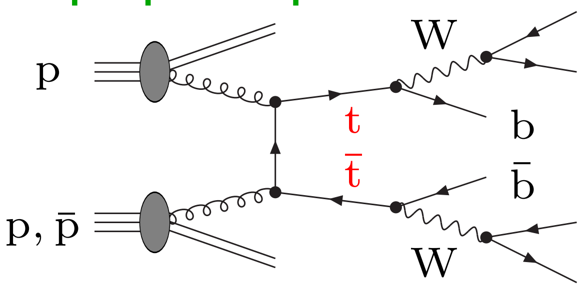
- W production (Tevatron/LHC): $\mathrm{pp}, \mathrm{p\bar{p}} \to \mathrm{W} \to l\nu_l(+\gamma)$



  – measurement of M_{W}

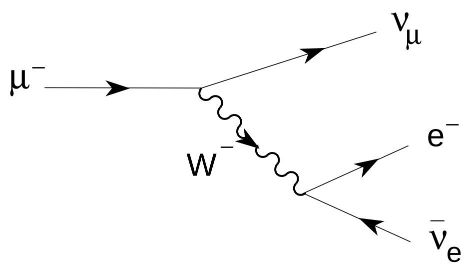
  – bounds on $\gamma\mathrm{WW}$ coupling

- top-quark production (Tevatron/LHC): $\mathrm{pp}, \mathrm{p\bar{p}} \to \mathrm{t\bar{t}} \to 6f$



  – measurement of m_{t}

μ decay



$$\mathcal{M} = \left(\frac{ig_2}{2\sqrt{2}}\right)^2 J_\rho^{(\mu)} \frac{-ig^{\rho\sigma}}{q^2 - M_W^2} J_\sigma^{(e)}$$

$|q|^2 \simeq m_\mu^2 \ll M_W^2$: $$\mathcal{M} = -\frac{g_2^2}{8M_W^2} J_\rho^{(\mu)} J^{\rho\,(e)}$$

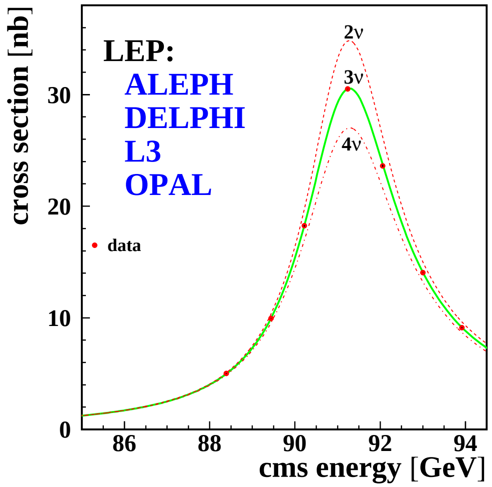
Fermi model *with point-like 4-fermion interaction:*

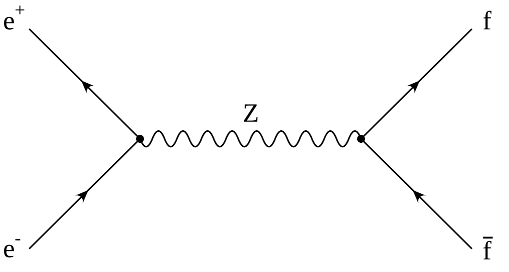
$$\mathcal{M} = \frac{G_F}{\sqrt{2}} J_\rho^{(\mu)} J^{\rho\,(e)}$$ *low-energy limit of SM*

$$\Rightarrow \quad \boxed{\frac{G_F}{\sqrt{2}} = \frac{g_2^2}{8M_W^2} = \frac{e^2}{8s_W^2 c_W^2 M_Z^2} = \frac{\pi\alpha}{2s_W^2 c_W^2 M_Z^2} = \frac{\pi\alpha}{2(1 - M_W^2/M_Z^2)M_W^2}}$$

$$G_F = 1.1663787(6) \cdot 10^{-5}\,\mathrm{GeV}^{-2}$$

Z resonance





$$\mathcal{M} = J_\mu^{(e)} \, \frac{-ig^{\mu\nu}}{s - M_Z^2 + iM_Z\Gamma_Z} \, J_\nu^{(f)}$$

propagator with finite width Γ_Z *(unstable particle)*

$$\Gamma_Z = \sum_f \Gamma(Z \to f\bar{f}), \quad \Gamma(Z \to f\bar{f}) = \frac{M_Z}{12\pi}\,(v_f^2 + a_f^2)$$

differential cross section at $s = M_Z^2$:

$$\frac{d\sigma}{d\Omega} \sim (v_e^2 + a_e^2)(v_f^2 + a_f^2)\,(1 + \cos^2\theta) \;+\; (2v_e a_e)(2v_f a_f) \cdot 2\cos\theta$$

$\Rightarrow$ *forward-backward asymmetry* $A_{FB} = \frac{\sigma_F - \sigma_B}{\sigma_F + \sigma_B} = \frac{3}{4} A_e A_f$

$$A_f = \frac{2 v_f a_f}{v_f^2 + a_f^2}$$

polarized cross section for $e^-_{L,R}$:

$\Rightarrow$ *left-right asymmetry* $A_{LR} = \frac{\sigma_L - \sigma_R}{\sigma_L + \sigma_R} = A_e$

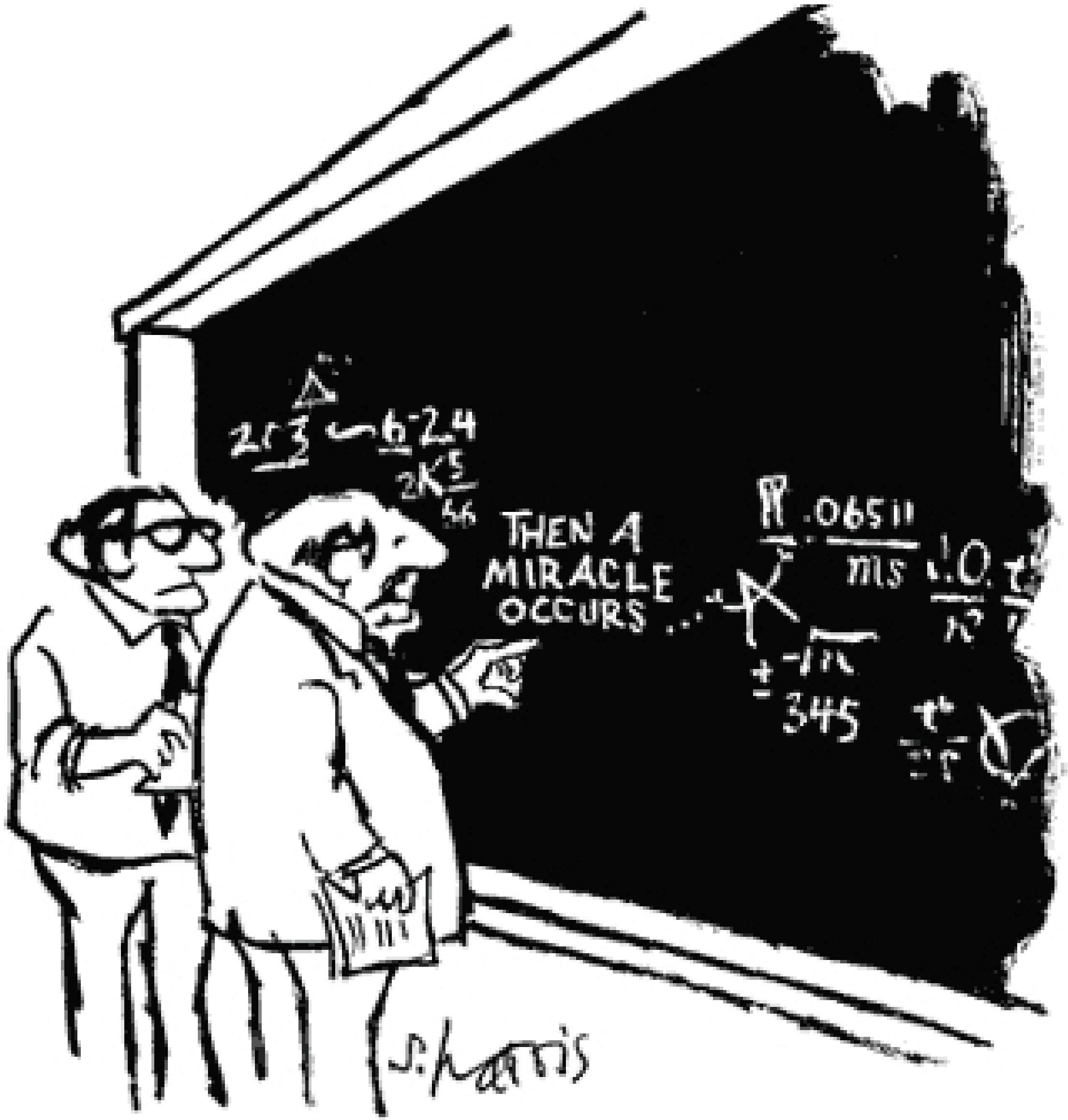
asymmetries determine $\sin^2\theta_W$

experimental results (selection)

M_{Z} [GeV]	$= 91.1875 \pm 0.0021$	0.002%
Γ_{Z}[GeV]	$= 2.4952 \pm 0.0023$	0.09%
$\sin^2\theta_{\mathrm{eff}}^{\mathrm{lept}}$	$= 0.23148 \pm 0.00017$	0.07%
M_{W} [GeV]	$= 80.385 \pm 0.015$	0.02%
m_{t} [GeV]	$= 173.2 \pm 0.9$	0.52%
G_{F} [GeV^{-2}]	$= 1.16637(1)10^{-5}$	0.001%

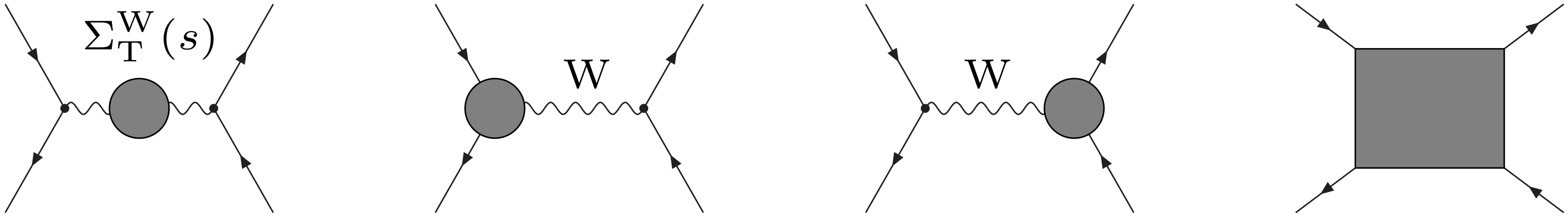
loop effects are at least one order of magnitude larger than experimental uncertainties

precise experiments need precise calculations

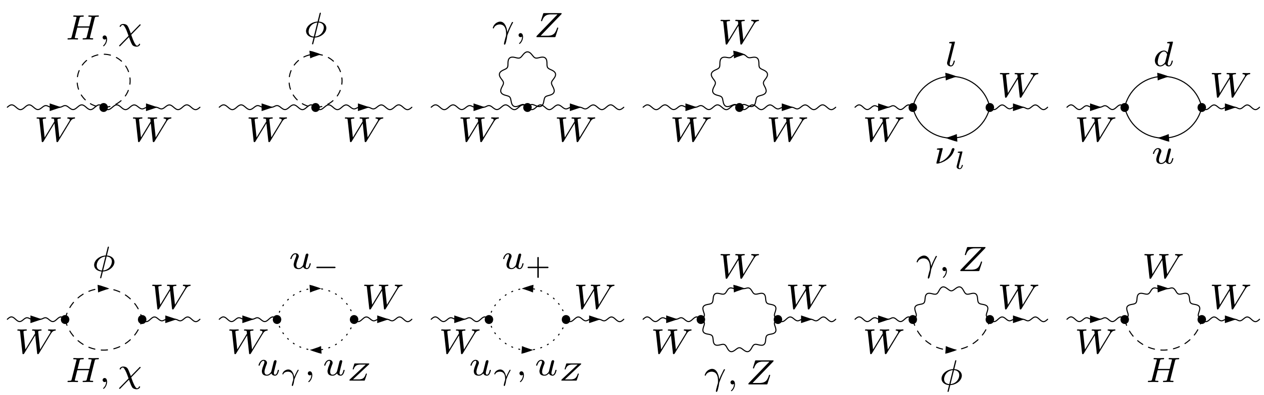


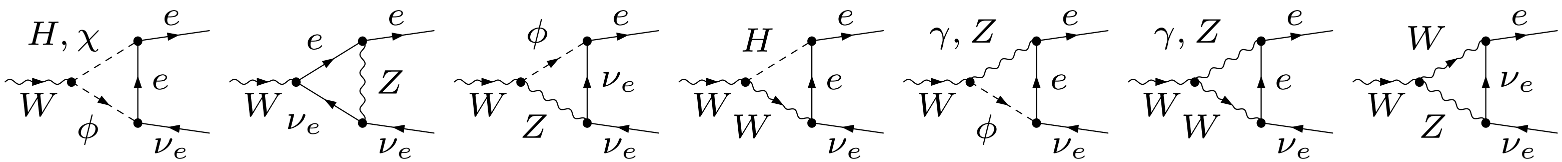
"I think you should be more explicit here in step two."

example: 1-loop diagrams for μ decay amplitude

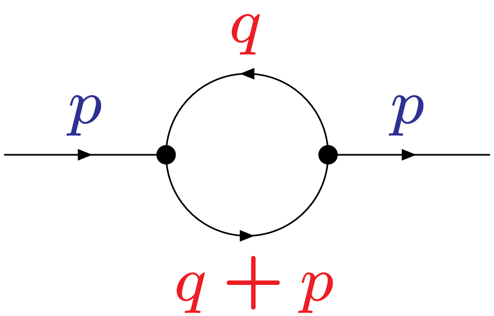


W self-energy $\quad$ $Wl\nu_l$ vertex correction $\quad$ box diagrams





Example of loop integral:



$$\sim \int d^4q \frac{1}{(q^2 - m_1^2)\left[(q+p)^2 - m_2^2\right]}$$

$$q \to \infty : \quad \sim \int^\infty \frac{q^3 dq}{q^4} = \int^\infty \frac{dq}{q} \quad \to \infty$$

$\Rightarrow$ integral diverges for large q

$\Rightarrow$ theory in this form not physically meaningful

needs (i) regularization

(ii) renormalization

Regularization:

theory modified such that expressions become mathematically meaningful

⇒ "regulator" introduced, removed at the end

e.g. cut-off in loop integral

$$\int_0^{\infty} d^4q \;\rightarrow\; \int_0^{\Lambda} d^4q; \quad \Lambda \rightarrow \infty \text{ at the end}$$

technically more convenient: dimensional regularization

$$\int d^4q \;\rightarrow\; \int d^Dq, \quad D = 4 - \varepsilon; \quad D \rightarrow 4 \text{ at the end}$$

Renormalization:

- absorption of divergencies
- determination of physical meaning of parameters order by order in perturbation theory

add counterterms that absorb divergent parts

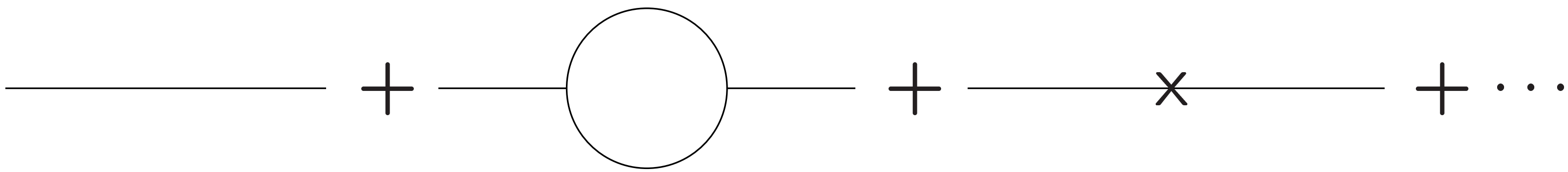
- parameters in $\mathcal{L}$ are formal, "bare parameters"

  $g_0 = g + \delta g$ *for a coupling,* $m_0 = m + \delta m$ *for a mass*
- $g,\ m$ are "physical", *i.e.* measurable

mass renormalization, $m_0^2 = m^2 + \delta m^2$

Physical mass: pole of propagator

inverse propagator up to 1-loop order:

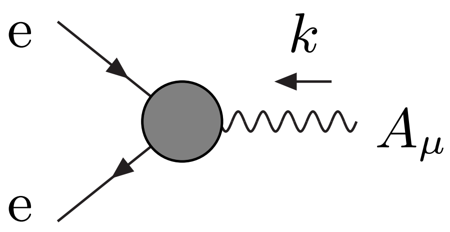


$p^2 - m^2$ $\qquad \Sigma(p^2) \qquad - \delta m^2$

on-shell renormalization: $\delta m^2 = \mathrm{Re}\,\Sigma(m^2)$

charge renormalization: $e_0 = e + \delta e$

δe *cancels loop contributions to* $ee\gamma$ *vertex in the Thomson limit*



$\xrightarrow{k\to 0}$ $\mathrm{i}e\gamma_\mu$ for on-shell electrons

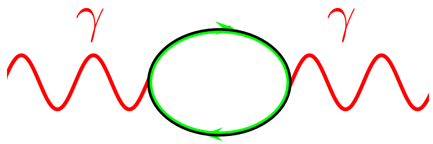
$\Rightarrow$ $e =$ elementary charge of classical electrodynamics

fine-structure constant $\alpha(0) = \dfrac{e^2}{4\pi} = 1/137.03599976$

δe *contains photon vacuum polarization* $\Pi^\gamma(k^2 = 0)$:

$$\Pi^\gamma(0) = \underbrace{\Pi^\gamma(0) - \Pi^\gamma(M_Z^2)}_{\text{non-perturbative}} + \underbrace{\Pi^\gamma(M_Z^2)}_{\text{perturbative}}$$

photon vacuum polarization



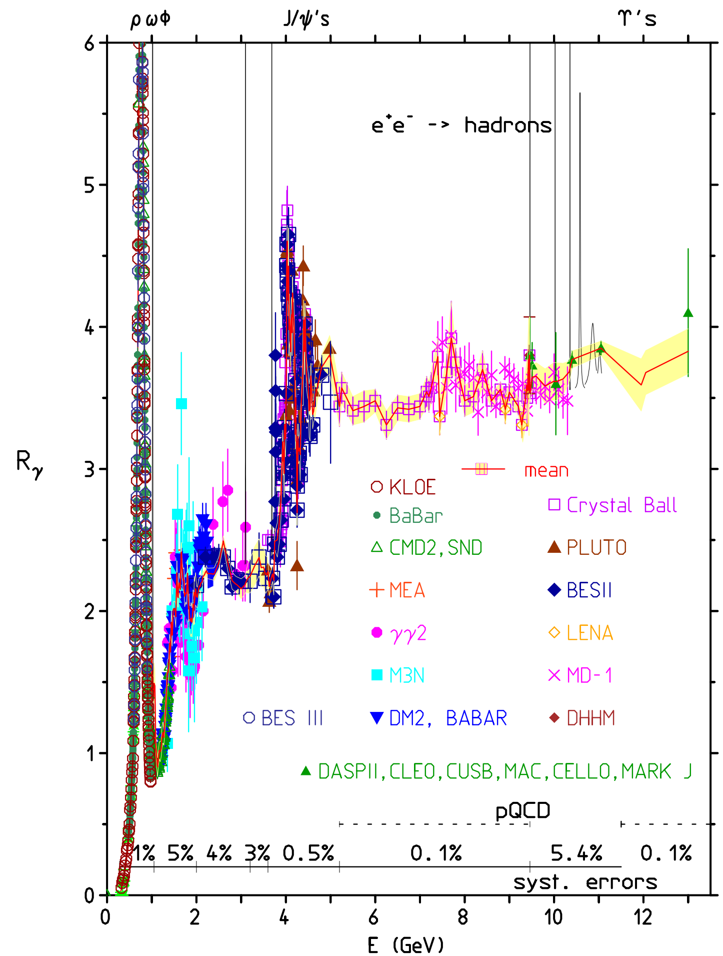
$$\Pi^{\gamma}_{\rm ferm}(M_Z^2) - \Pi^{\gamma}_{\rm ferm}(0) \equiv \Delta\alpha \quad \rightarrow \quad \alpha(M_Z) = \frac{\alpha}{1-\Delta\alpha} \simeq \frac{1}{129}$$

$$\begin{aligned}
\Delta\alpha &= \Delta\alpha_{\rm lept} + \Delta\alpha_{\rm had}\,, \\
\Delta\alpha_{\rm lept} &= 0.031498\ (4-\text{loop}) \qquad \text{Steinhauser 1998; Sturm 2013} \\
\Delta\alpha_{\rm had} &= 0.02757 \pm 0.00010 \qquad \text{Davier et al. 2010} \\
&= 0.027626 \pm 0.000103 \qquad \text{Hagiwara et al. 2011} \\
&= 0.027504 \pm 0.000118 \qquad \text{Jegerlehner 2015}
\end{aligned}$$

significant parametric uncertainty

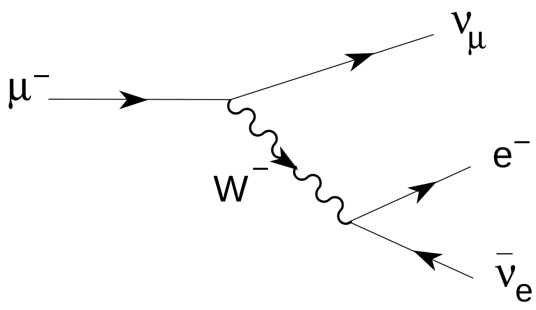
$$\Delta\alpha_{\rm had} = -\frac{\alpha}{3\pi} M_Z^2 \; {\rm Re} \int_{4m_\pi^2}^{\infty} ds' \, \frac{R_{\rm had}(s')}{s'(s' - M_Z^2 - i\epsilon)}$$

$$R_{\rm had} = \frac{\sigma(e^+e^- \to \gamma^* \to {\rm hadrons})}{\sigma(e^+e^- \to \gamma^* \to \mu^+\mu^-)}$$



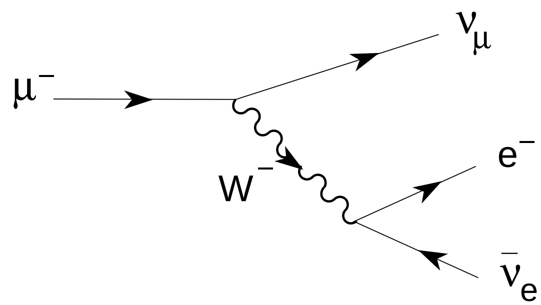
Jegerlehner 2015

$M_W - M_Z$ correlation



$$\frac{G_F}{\sqrt{2}} = \frac{\pi\alpha}{2M_W^2\left(1 - M_W^2/M_Z^2\right)}$$

$M_W - M_Z$ correlation



$$\frac{G_F}{\sqrt{2}} = \frac{\pi\alpha}{2M_W^2\left(1 - M_W^2/M_Z^2\right)}$$

$M_W = 80.939 \pm 0.002\,\text{GeV}$ *from* $G_F,\ \alpha,\ M_Z$

$M_W = 79.965 \pm 0.005\,\text{GeV}$ *with* $\alpha \to \alpha(M_Z)$

$M_W = 80.385 \pm 0.015\,\text{GeV}$ *exp.* $37\sigma\ /\ 28\,\sigma$

with loop contributions

$$\frac{G_F}{\sqrt{2}} = \frac{\pi\alpha}{2M_W^2\left(1 - M_W^2/M_Z^2\right)} \cdot (1 + \Delta r)$$

Δr : quantum correction

$$\Delta r = \Delta r(m_t, M_H)$$

$$\Delta r = \Delta\alpha - \frac{c_W^2}{s_W^2}\Delta\rho + \cdots$$

$$\Delta\rho \sim \frac{m_t^2}{M_W^2}$$

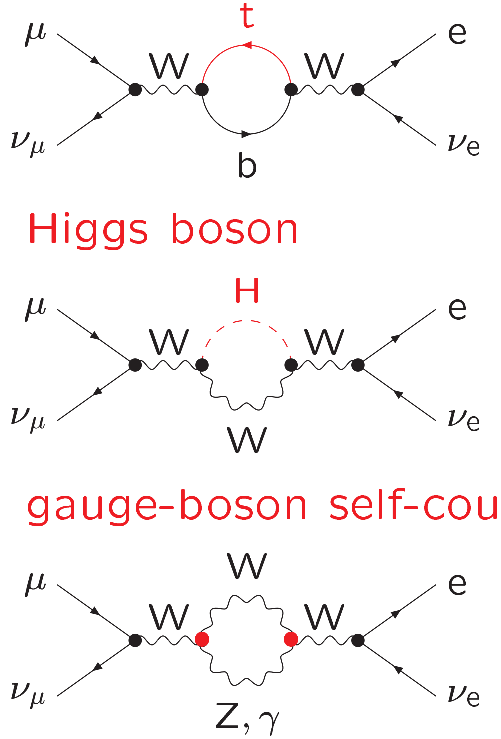
determines W mass

$$M_W = M_W(\alpha, G_F, M_Z, m_t, M_H)$$

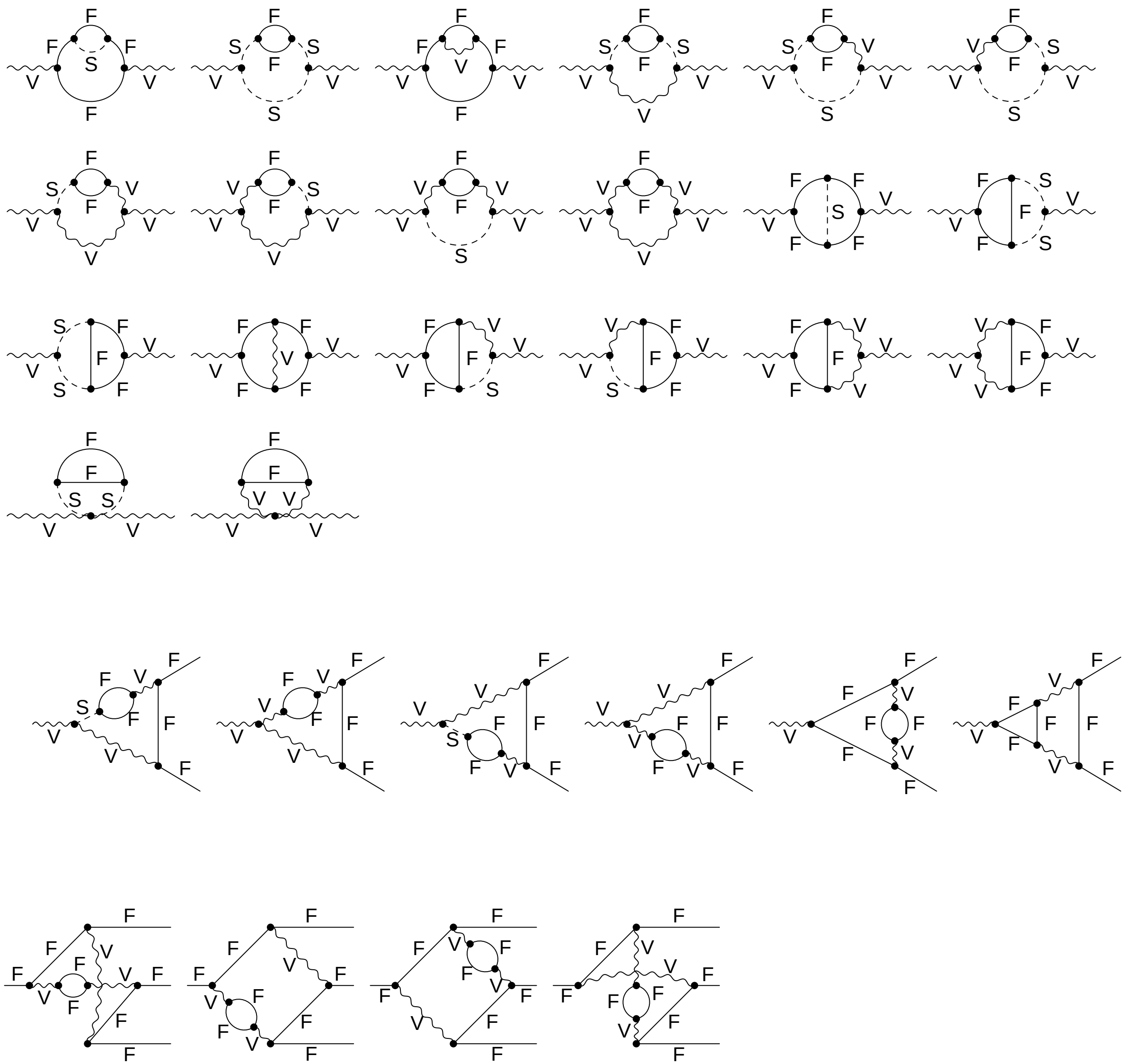
complete at 2-loop order

1-loop examples

- top quark
- Higgs boson
- gauge-boson self-couplings

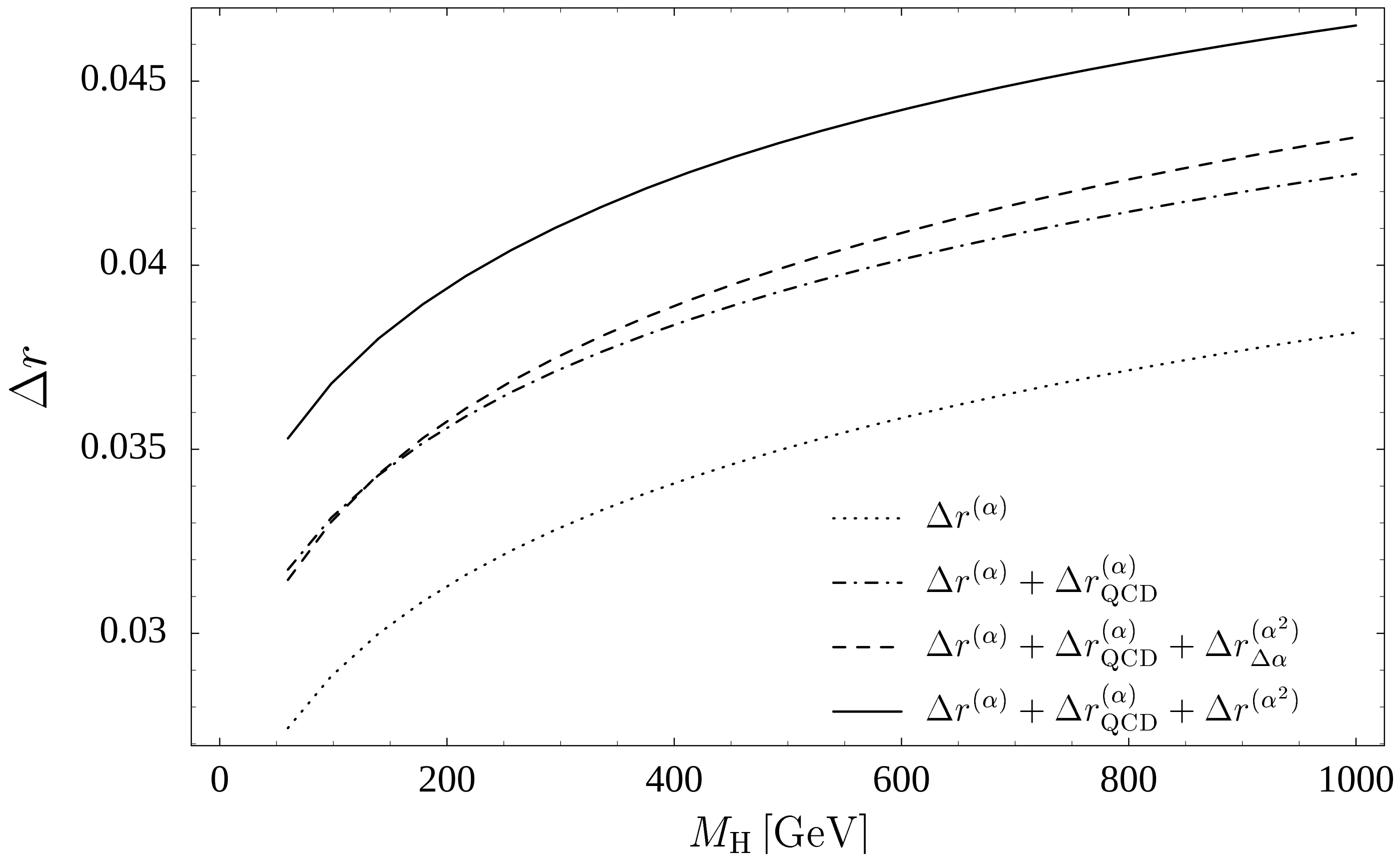


full structure of SM



2-loop examples

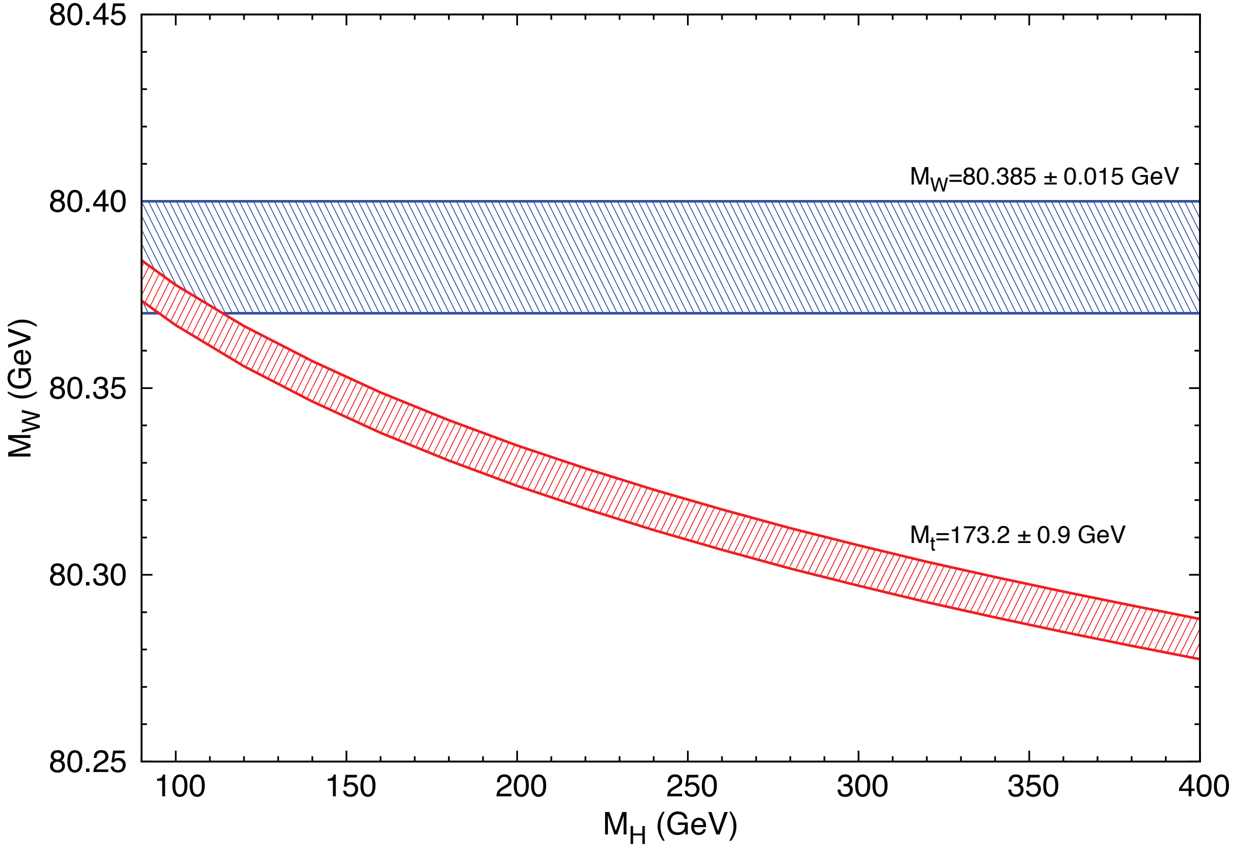
effects of higher-order terms on Δr



variation of Δr by 0.001 $\Rightarrow$ $\delta M_W = 18\,\mathrm{MeV}$

3-loop ($\Delta\rho$) $\Rightarrow$ $\delta M_W = 12\,\mathrm{MeV}$

present exp. error: $\Delta M_W = 15\,\mathrm{MeV}$ / theo: $4\,\mathrm{MeV}$

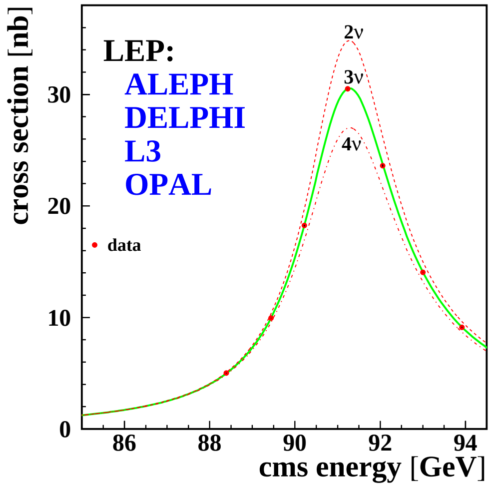


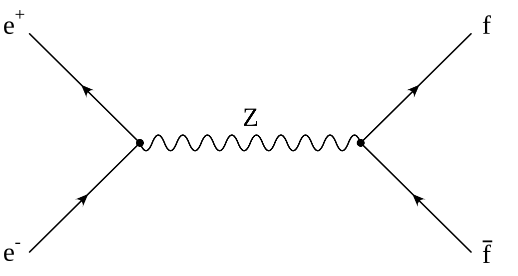
variation of Δr by 0.001 $\quad \Rightarrow \quad \delta M_W = 18\,\mathrm{MeV}$

present exp. error: $\quad \Delta M_W = 15\,\mathrm{MeV}$

$\delta(\Delta\alpha) = 10^{-4}$: $\quad \Delta M_W = 1.8\,\mathrm{MeV}$

Z resonance



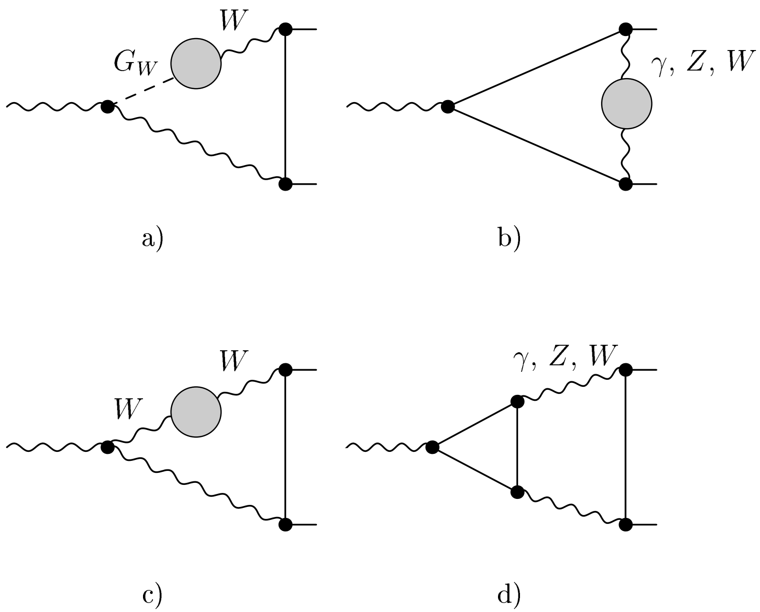


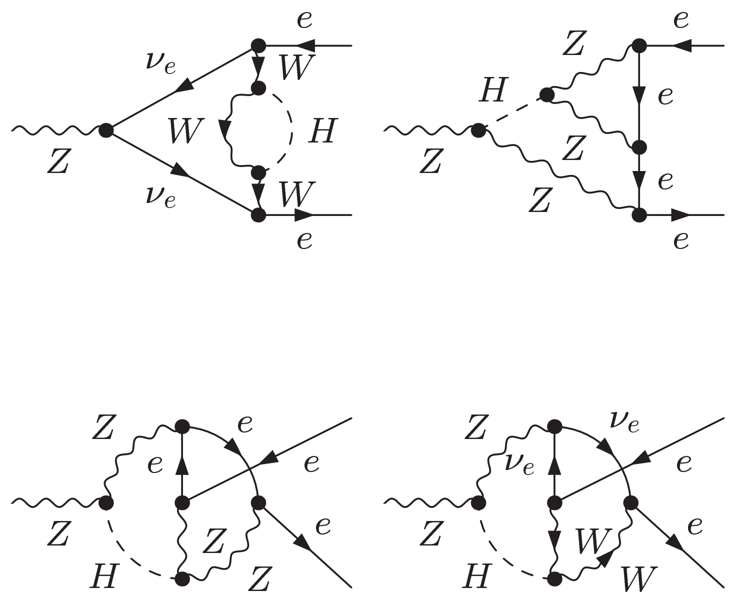
- effective Z boson couplings with higher-order $\Delta g_{V,A}$

$$v_f \to g_V^f = v_f + \Delta g_V^f, \qquad a_f \to g_A^f = a_f + \Delta g_A^f$$

- effective ew mixing angle (for $f = e$):

$$\sin^2\theta_{\text{eff}} = \frac{1}{4}\left(1 - \text{Re}\,\frac{g_V^e}{g_A^e}\right) = \kappa \cdot \left(1 - \frac{M_W^2}{M_Z^2}\right)$$

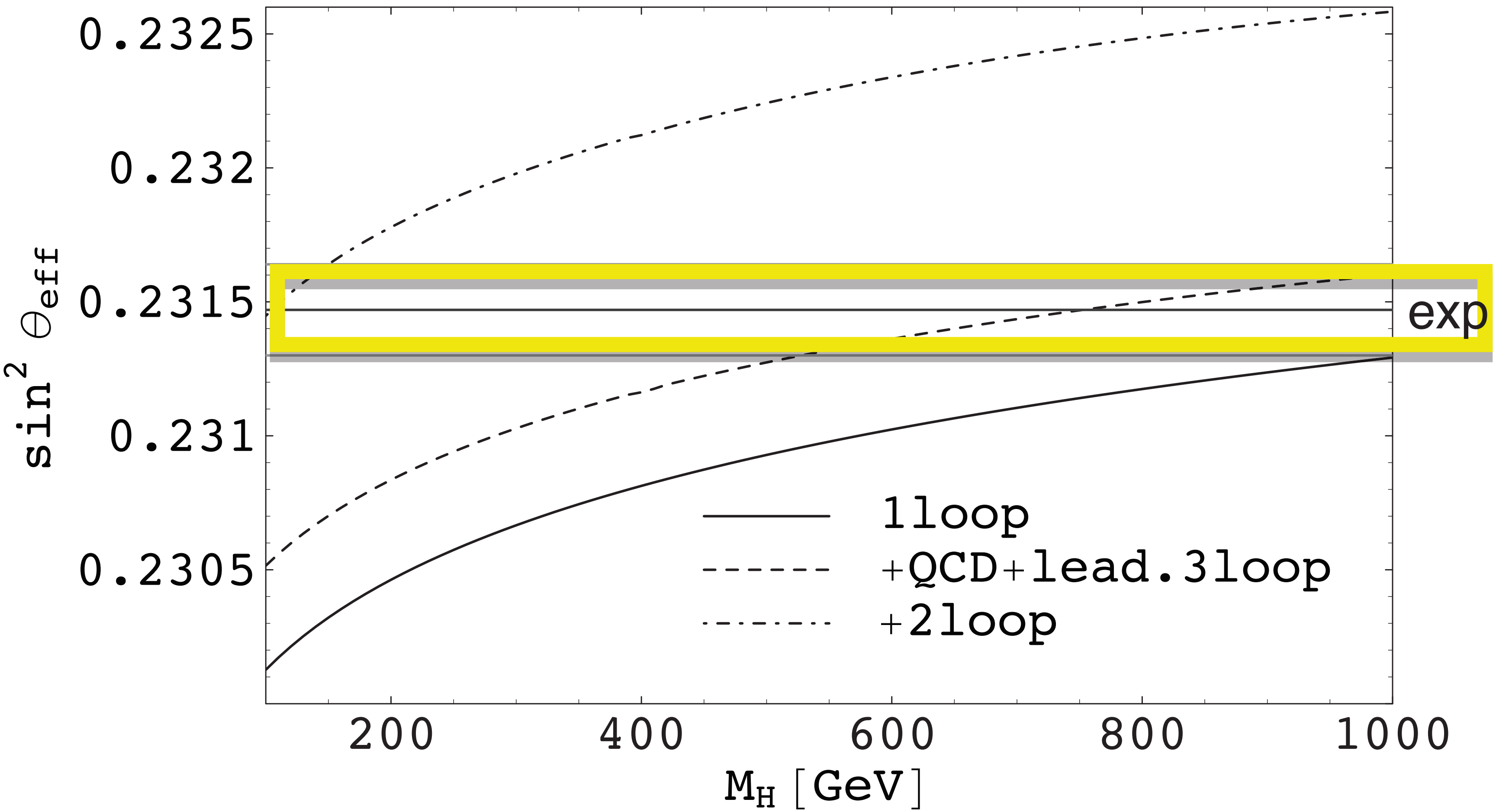




2-loop examples for Z couplings

complete 2-loop calculation available for $\sin^2\theta_{\rm eff}$

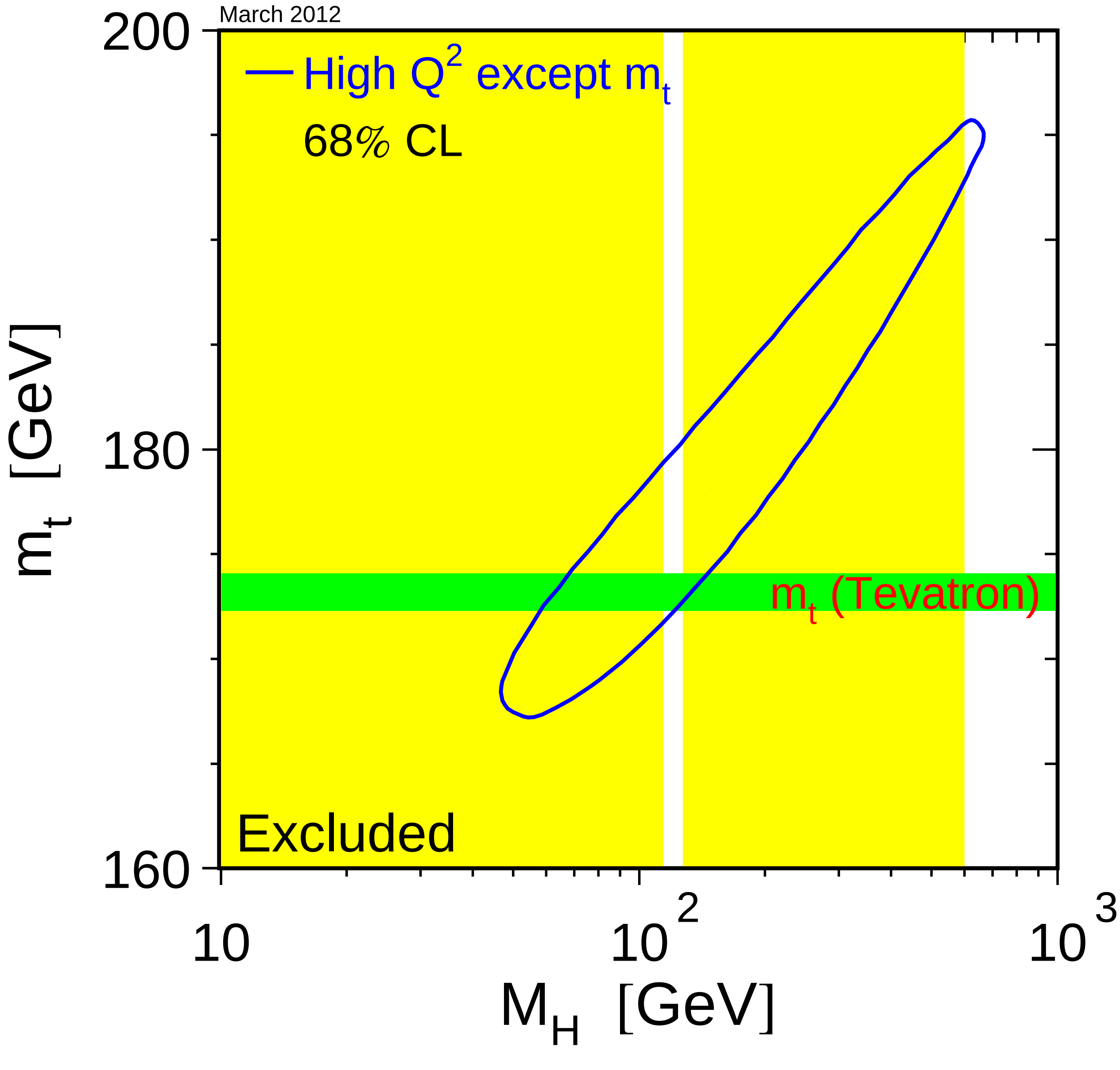
importance of two-loop calculations



lowest order: $\sin^2\theta_W = 1 - \frac{M_W^2}{M_Z^2} = 0.22290 \pm 0.00029$

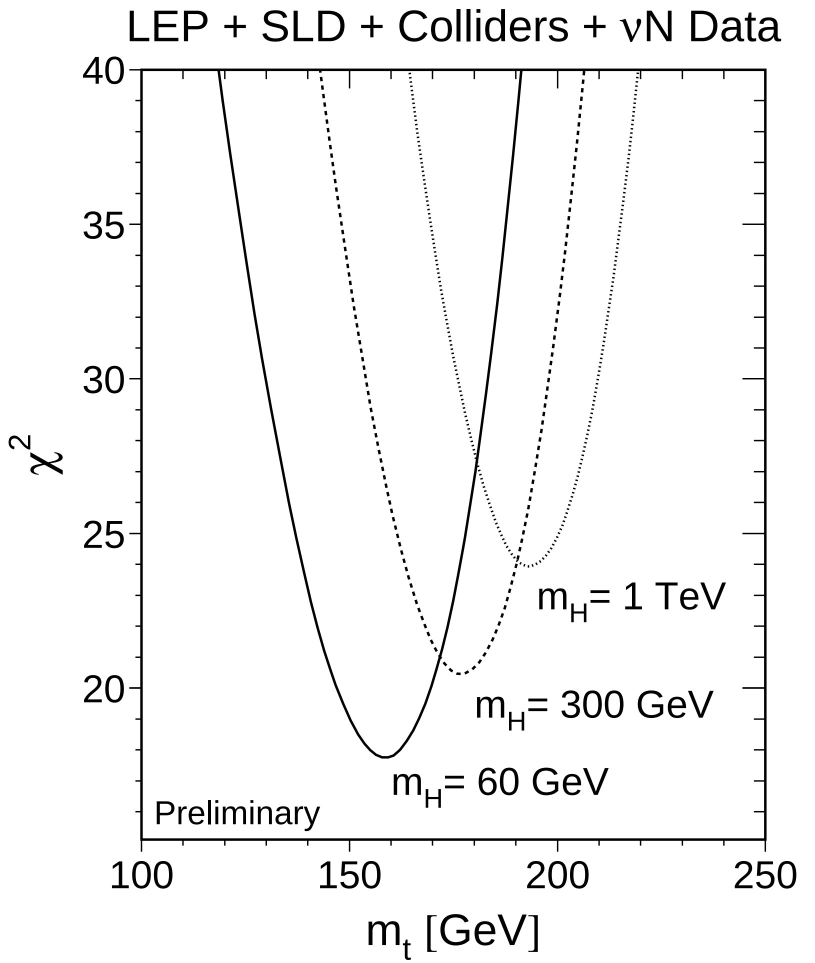
exp. value: $\sin^2\theta_{\text{eff}} = \quad 0.23153 \pm 0.00016$

Global analysis within the SM



before the top quark was discovered ($<$ 1995):

indirect mass determination $\Rightarrow$ $m_t = 178 \pm 8\,^{+17}_{-20}\,\mathrm{GeV}$



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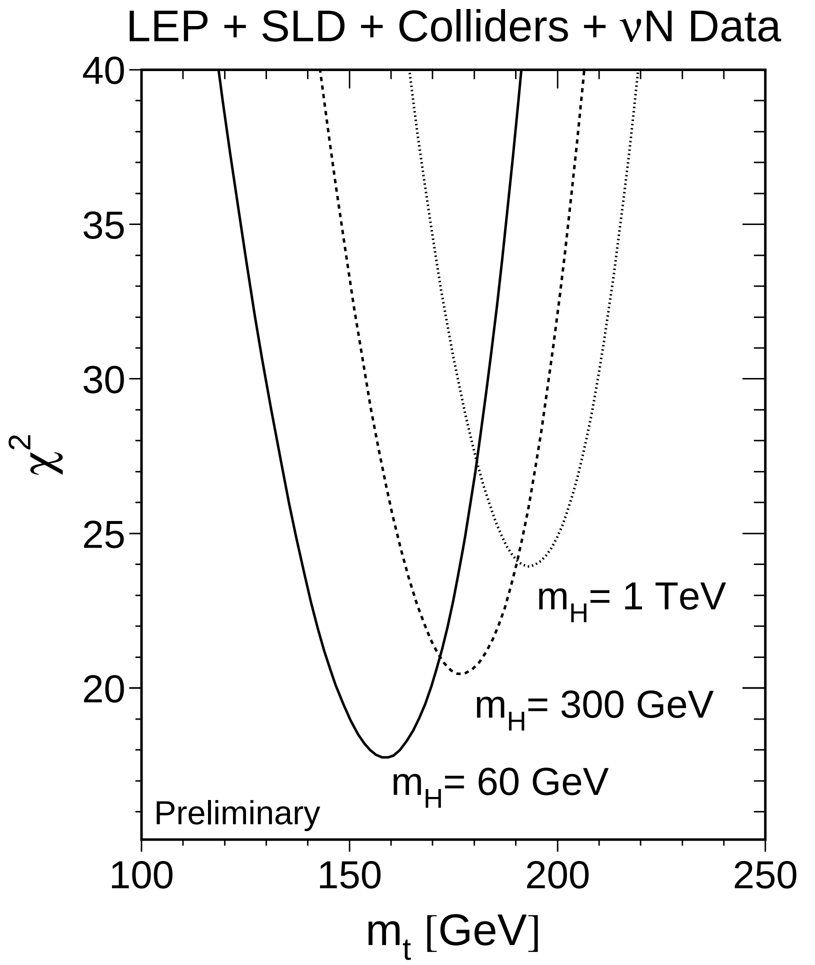

top discovery: *Tevatron 1995* $\mathrm{m_t} = 180 \pm 12\,\mathrm{GeV}$

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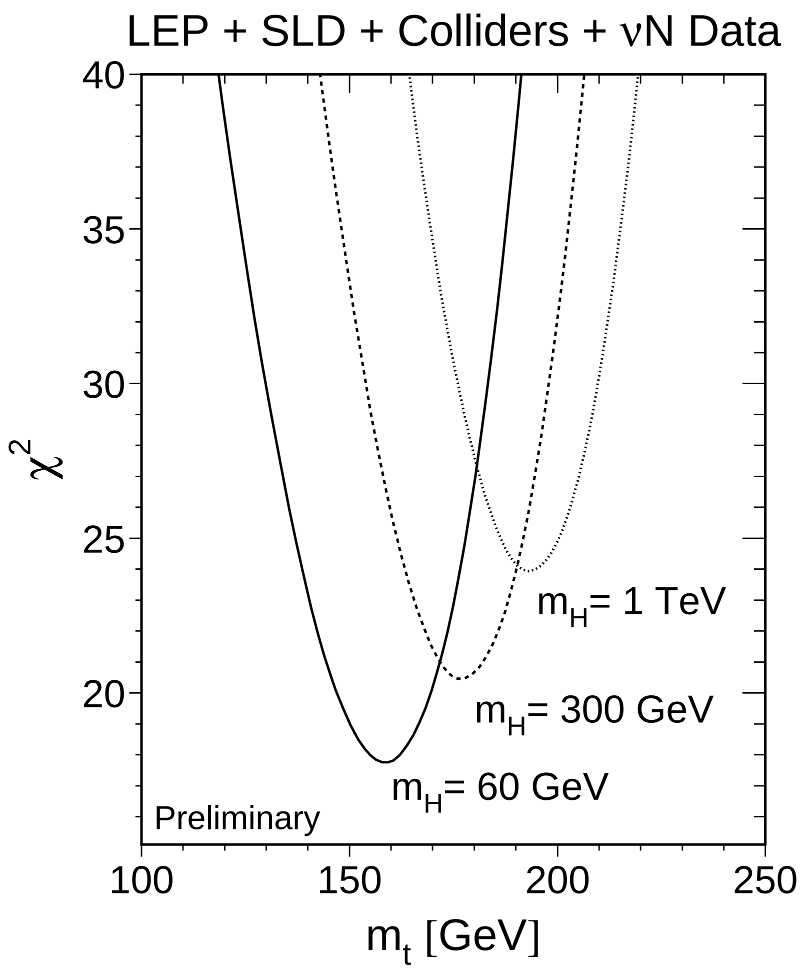
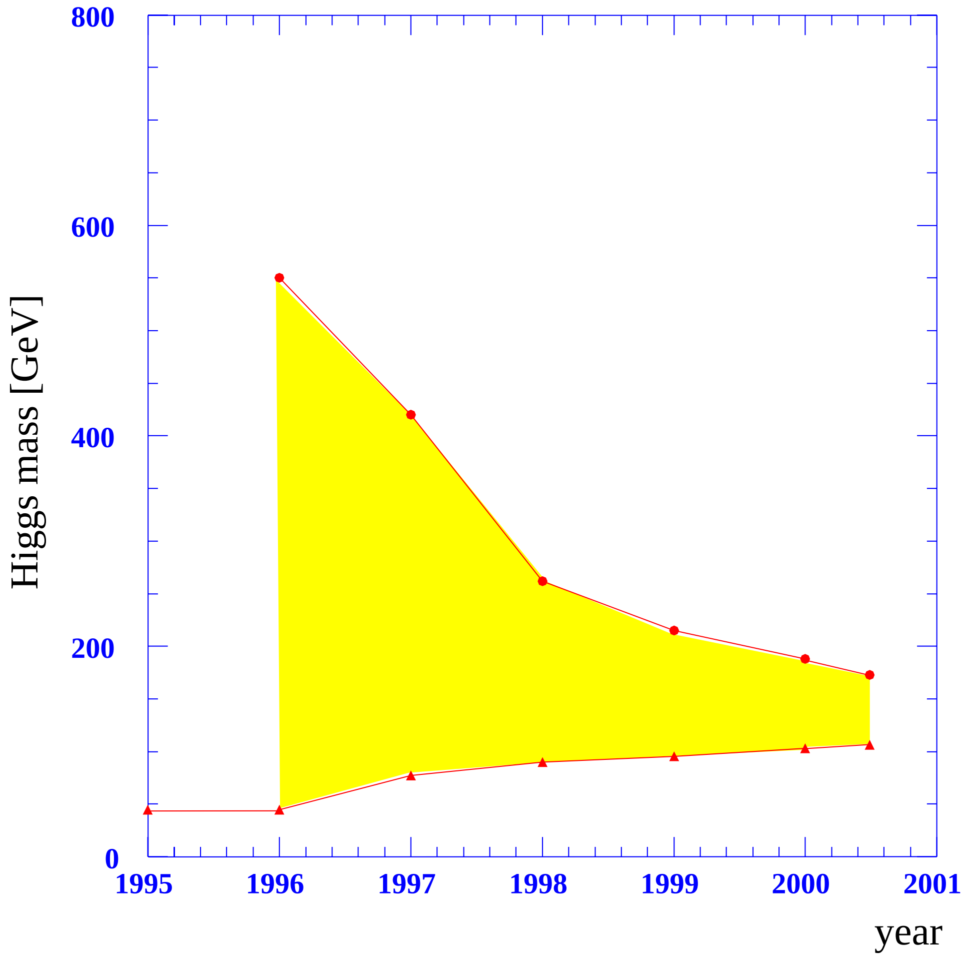


top discovery: *Tevatron 1995* $\mathrm{m_t} = 180 \pm 12\,\mathrm{GeV}$

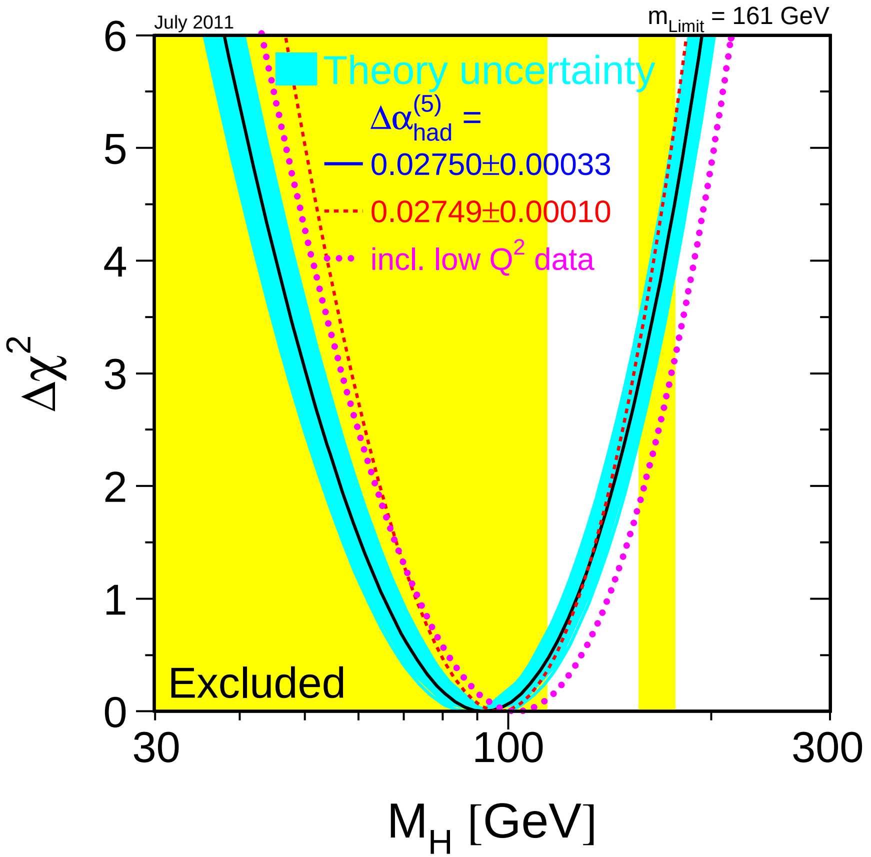
today: $\mathrm{m_t} = 173.2 \pm 0.9\,\mathrm{GeV}$

The way to the Higgs boson

development of bounds from direct and indirect searches



Global fit to the Higgs boson mass

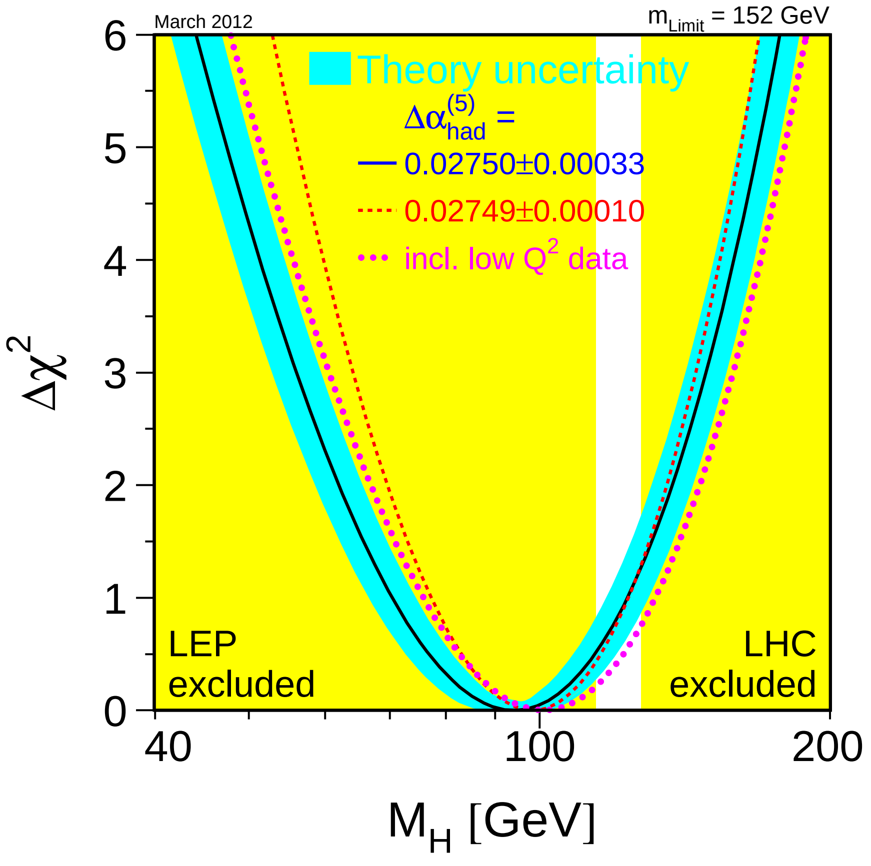


blueband: Theory uncertainty

"Precision Calculations at the Z Resonance"
CERN 95-03

[Bardin, WH, Passarino (eds.)]

$$M_{\mathrm{H}} < 161 \text{ GeV} \quad (\text{at } 95\% \text{ C.L.})$$



after the 2011 results
from the LHC
on the Higgs boson mass

$$M_{\mathrm{H}} < 152\ \mathrm{GeV} \quad (95\%\mathrm{C.L.})$$

$$M_{\mathrm{H}} = 94^{+29}_{-24}\,\mathrm{GeV}$$

7. Higgs bosons

Higgs potential: $$\boxed{V = -\mu^2 \left(\Phi^\dagger\Phi\right)^2 + \frac{\lambda}{4}\left(\Phi^\dagger\Phi\right)^4}$$

Higgs field in unitary gauge: $$\Phi(x) = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v + H(x) \end{pmatrix}$$

$H(x)$: *real scalar field, describes neutral spin-0 bosons*

minimum of V: $$v = \frac{2\mu}{\sqrt{\lambda}}, \quad M_H = \mu\sqrt{2}$$

$$\Rightarrow \quad \lambda = \frac{4\mu^2}{v^2} = \frac{2M_H^2}{v^2}$$

$$\boxed{V = \frac{M_H^2}{2} H^2 + \frac{M_H^2}{2v} H^3 + \frac{M_H^2}{8v^2} H^4}$$

M_H *is the only free parameter*

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M_H *is the only free parameter*

general gauge: $$\Phi(x) = \begin{pmatrix} \phi^+(x) \\ \frac{1}{\sqrt{2}}\left[v + H(x) + i\chi(x)\right] \end{pmatrix}$$

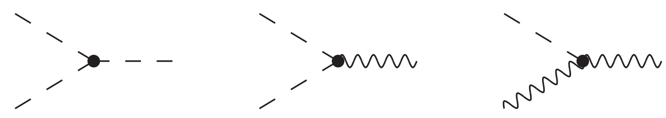
gauge invariant Lagrangian of the Higgs sector

$$\mathcal{L}_\mathrm{H} = (D_\mu \Phi)^\dagger (D^\mu \Phi) - V(\Phi) \qquad \text{with } D_\mu = \partial_\mu - \mathrm{i} g_2 \frac{\sigma^a}{2} W_\mu^a + \mathrm{i} \frac{g_1}{2} B_\mu$$

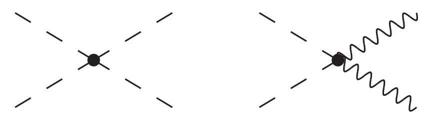
$$= (\partial_\mu \phi^+)(\partial^\mu \phi^-) - \frac{\mathrm{i}ev}{2s_\mathrm{W}} (W_\mu^+ \partial^\mu \phi^- - W_\mu^- \partial^\mu \phi^+) + \frac{e^2 v^2}{4 s_\mathrm{W}^2} W_\mu^+ W^{-,\mu}$$

$$+ \frac{1}{2}(\partial\chi)^2 + \frac{ev}{2c_\mathrm{W} s_\mathrm{W}} Z_\mu \partial^\mu \chi + \frac{e^2 v^2}{4 c_\mathrm{W}^2 s_\mathrm{W}^2} Z^2 + \frac{1}{2}(\partial H)^2 - \mu^2 H^2$$

$+$ (trilinear SSS, SSV, SVV interactions)



$+$ (quadrilinear $SSSS$, $SSVV$ interactions)



$\Rightarrow$ **H-V-V gauge interactions, V=W and Z**

gauge invariant Lagrangian of the Higgs sector

$$\mathcal{L}_{\mathrm{H}} = (D_\mu \Phi)^\dagger (D^\mu \Phi) - V(\Phi) \qquad \text{with } D_\mu = \partial_\mu - \mathrm{i} g_2 \frac{\sigma^a}{2} W_\mu^a + \mathrm{i} \frac{g_1}{2} B_\mu$$

$$= (\partial_\mu \phi^+)(\partial^\mu \phi^-) - \frac{\mathrm{i}ev}{2s_{\mathrm{W}}} (W_\mu^+ \partial^\mu \phi^- - W_\mu^- \partial^\mu \phi^+) + \frac{e^2 v^2}{4 s_{\mathrm{W}}^2} W_\mu^+ W^{-,\mu}$$

$$+ \frac{1}{2} (\partial \chi)^2 + \frac{ev}{2 c_{\mathrm{W}} s_{\mathrm{W}}} Z_\mu \partial^\mu \chi + \frac{e^2 v^2}{4 c_{\mathrm{W}}^2 s_{\mathrm{W}}^2} Z^2 + \frac{1}{2} (\partial H)^2 - \mu^2 H^2$$

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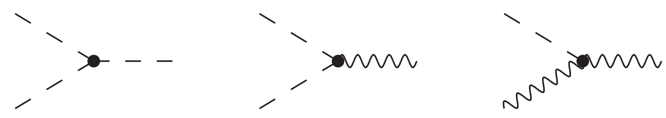

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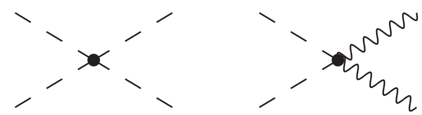
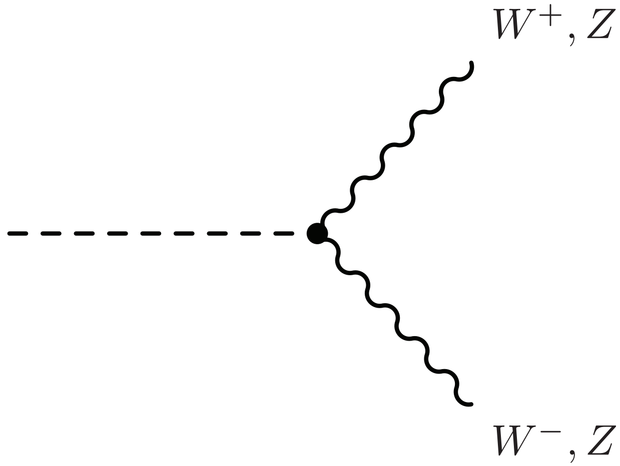


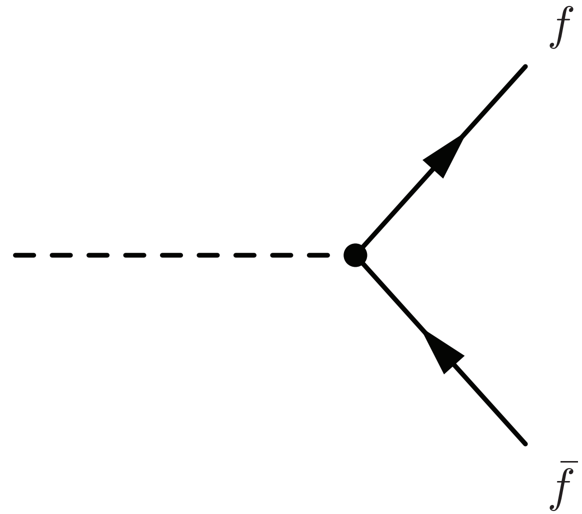
$\Rightarrow$ H-V-V gauge interactions, V=W and Z

$$\mathcal{L}_{\mathrm{Yuk}} = -\sum_f \left(m_f + \frac{m_f}{v} H \right) \overline{\psi}_f \psi_f + \cdots (f f \chi, \phi^\pm)$$

$\Rightarrow$ H-f-f Yukawa interactions



$$g_2 M_W \ , \quad g_2 \frac{M_Z}{c_W}$$



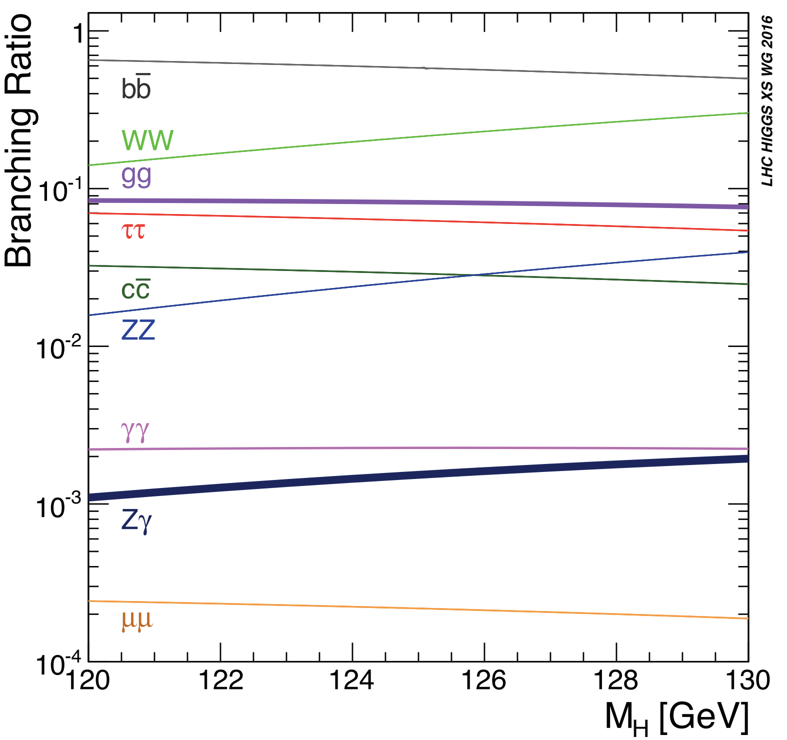
$$\frac{m_f}{v} = \frac{g_2 \, m_f}{2M_W}$$

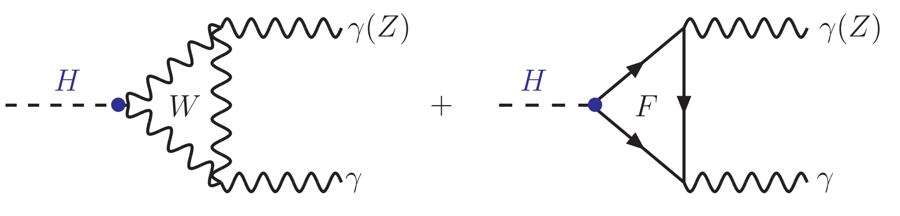
$$\Gamma(H \to f\bar{f}) = N_c \frac{G_F M_H}{4\pi\sqrt{2}} \, m_f^2, \quad N_C = 3\,(1) \text{ for quarks (leptons)}$$

$$\Gamma(H \to VV) = \frac{G_F M_H^3}{8\pi\sqrt{2}} \, F(r) \left(\frac{1}{2}\right)_Z, \quad r = \frac{M_V}{M_H}$$

Higgs boson decay channels

branching ratios $$BR(H \to X) = \frac{\Gamma(H \to X)}{\Gamma(H \to \text{all})}$$

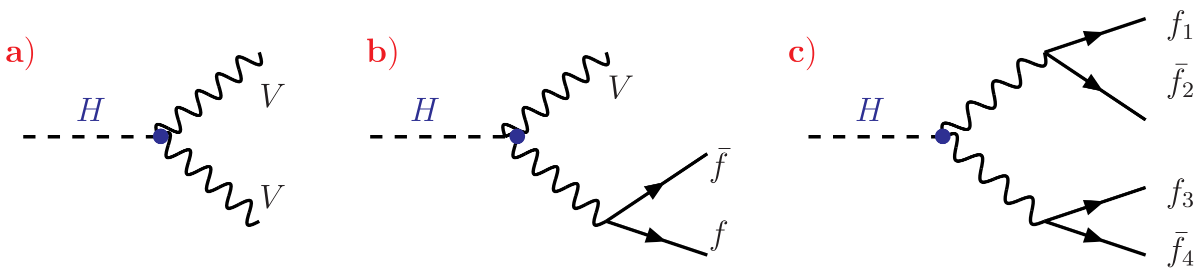


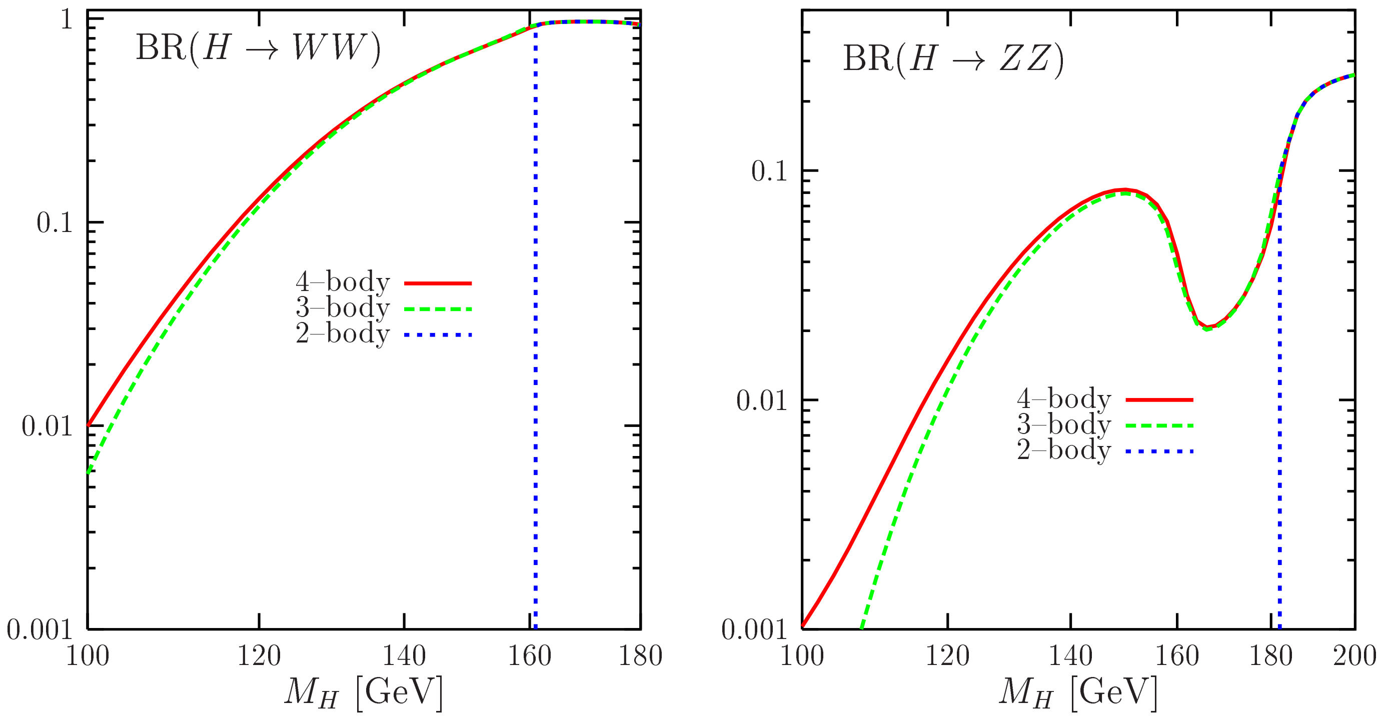


loop-induced (rare) decays

Higgs decays into 4 fermions

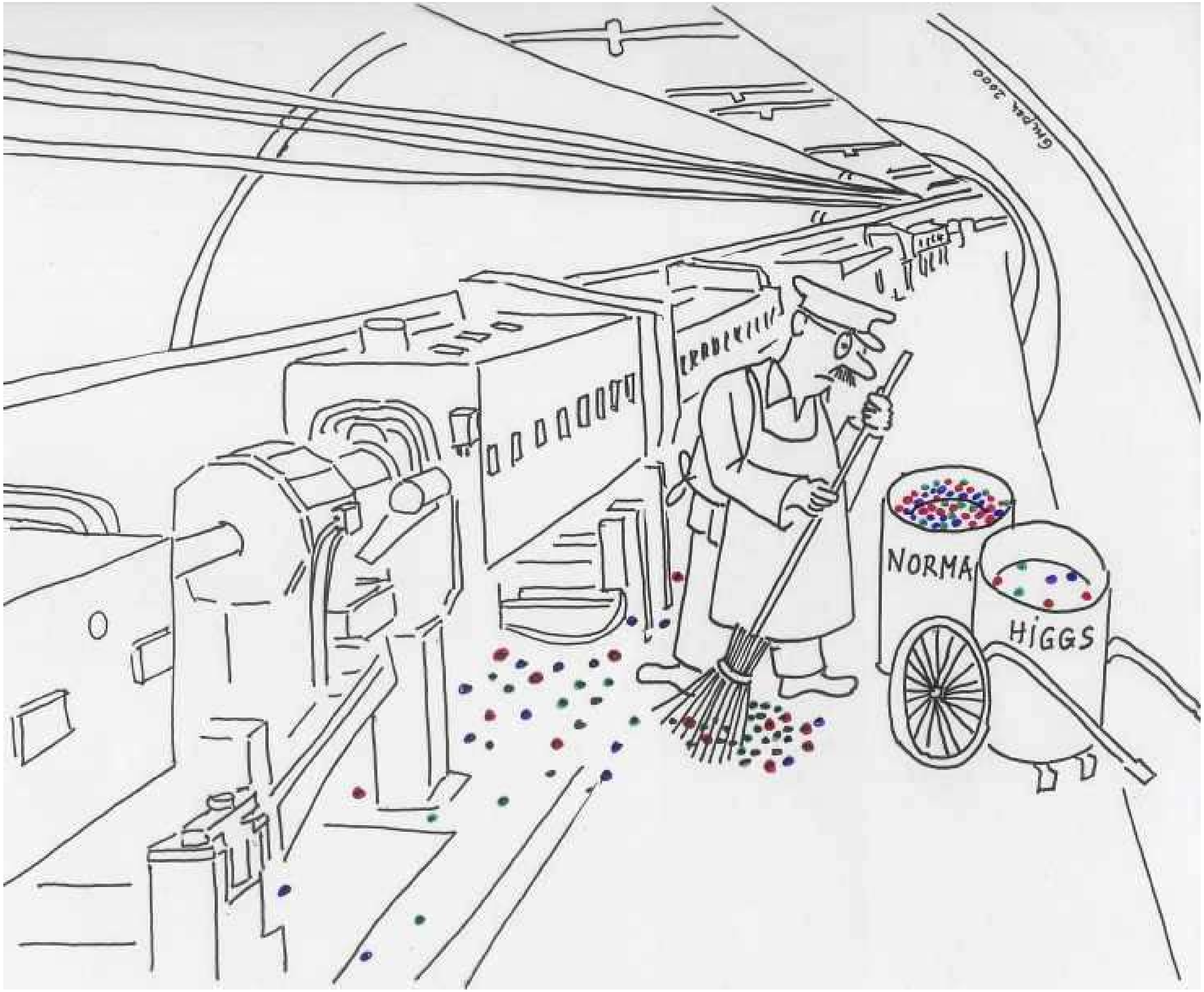
also below VV threshold with one or two V off-shell





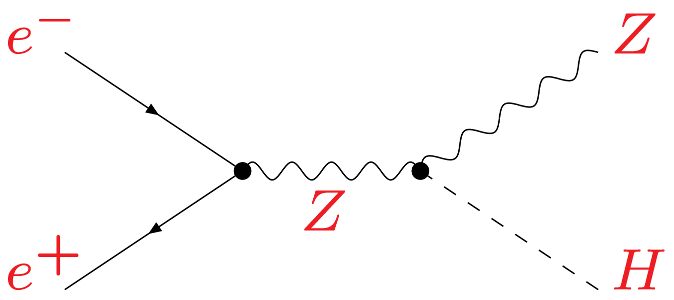
[Djouadi]

The direct search for the Higgs boson



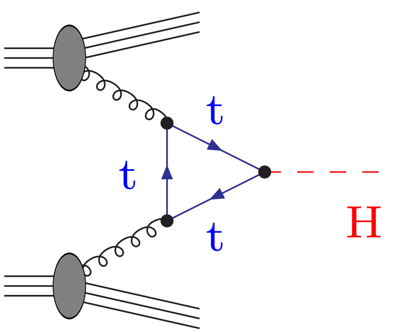
The direct search for the Higgs boson

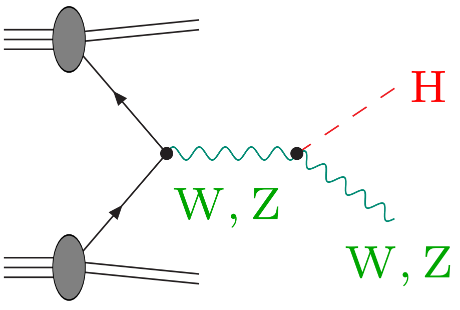
Higgs production at LEP:



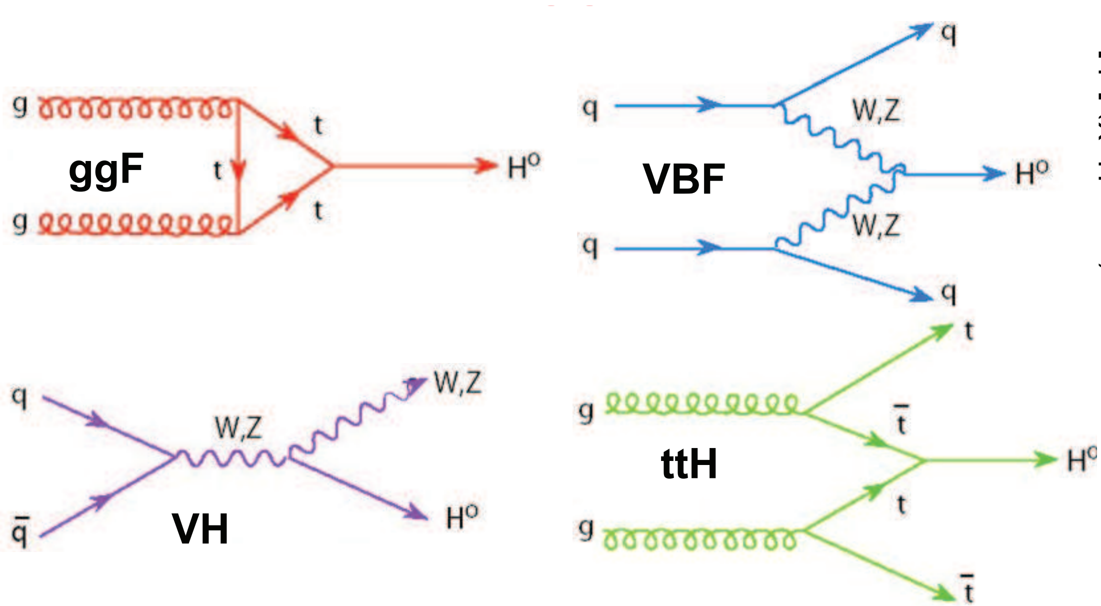
excluded $M_H < 114\,\mathrm{GeV}$

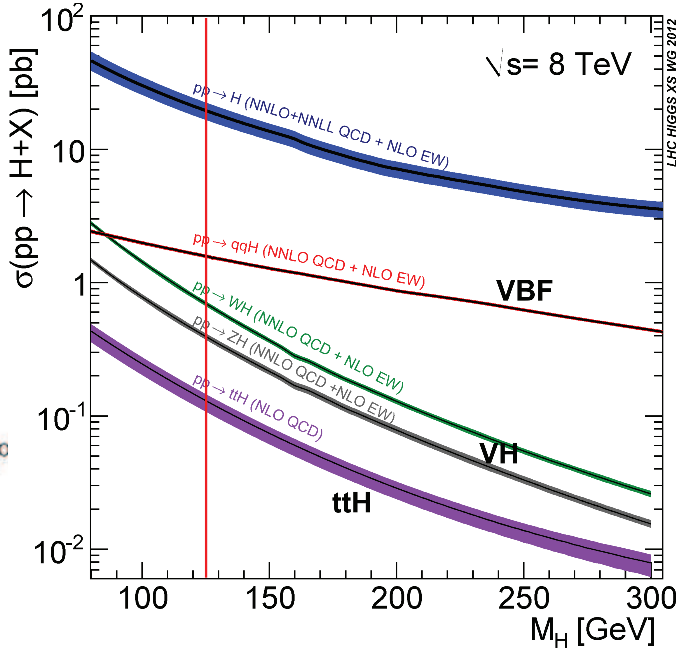
Higgs production at the Tevatron:





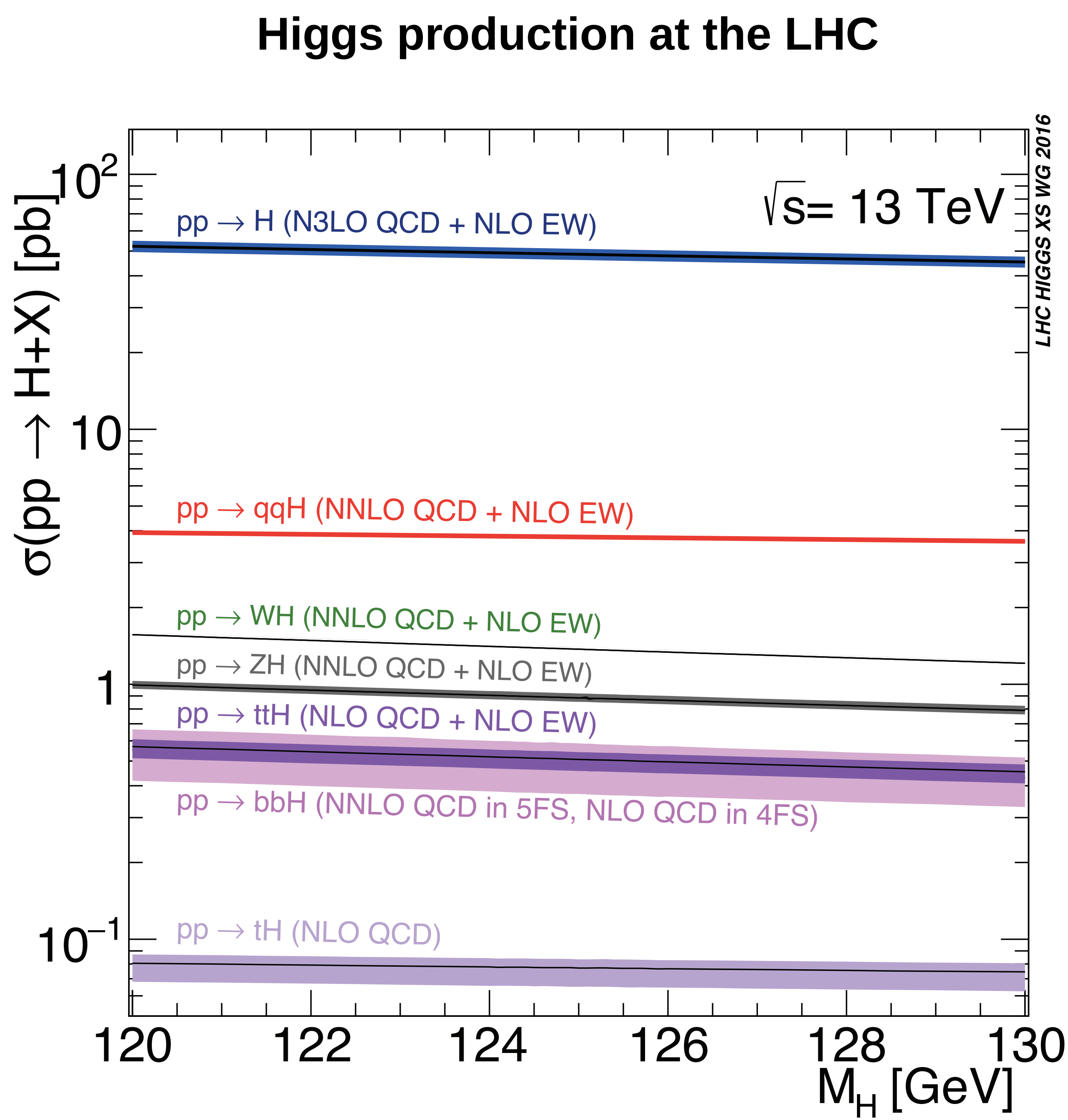
Higgs production at the LHC



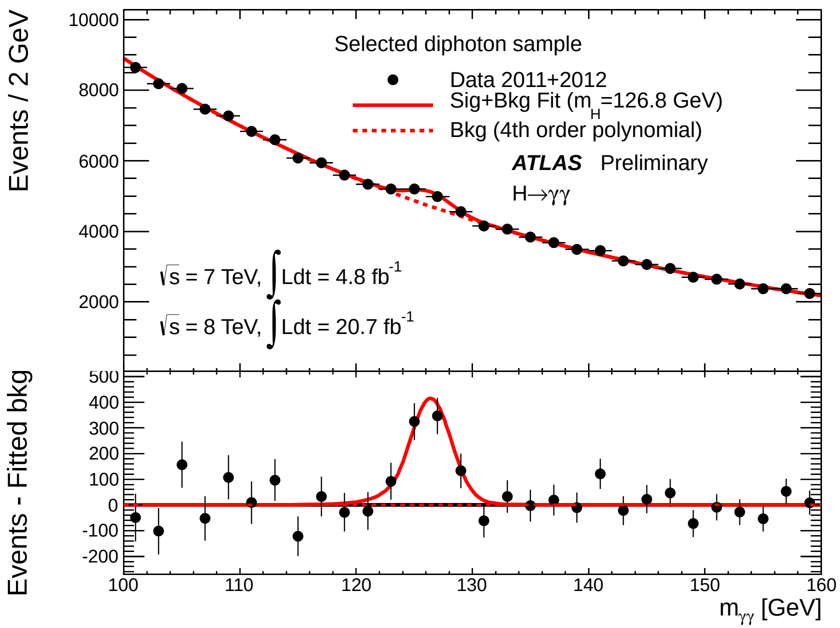


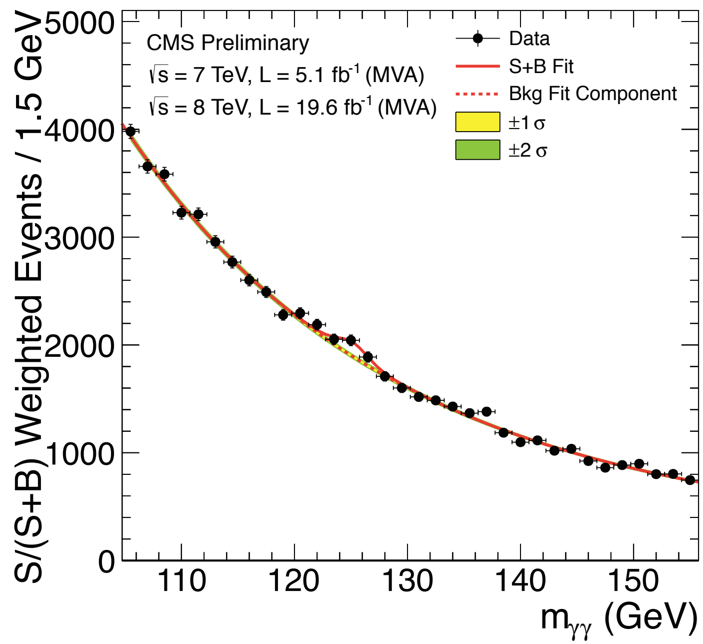
Handbook of Higgs Cross sections, Vol. 1, 2, 3, 4

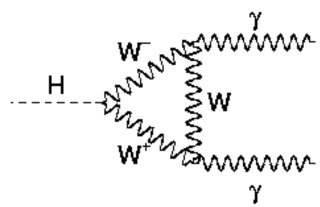
arXiv:1101.0593, arXiv:1201.3084, arXiv:1307.1347, arXiv:1610.07922

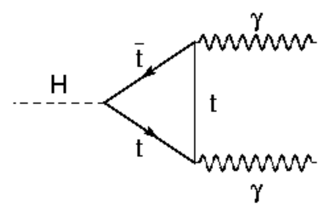

Higgs production at the LHC
$\sqrt{s}$= 13 TeV
LHC HIGGS XS WG 2016
pp → H (N3LO QCD + NLO EW)
pp → qqH (NNLO QCD + NLO EW)
pp → WH (NNLO QCD + NLO EW)
pp → ZH (NNLO QCD + NLO EW)
pp → ttH (NLO QCD + NLO EW)
pp → bbH (NNLO QCD in 5FS, NLO QCD in 4FS)
pp → tH (NLO QCD)
σ(pp → H+X) [pb]
10^2
10
1
10^{-1}
120
122
124
126
128
130
M_H [GeV]

$H \to \gamma\gamma$

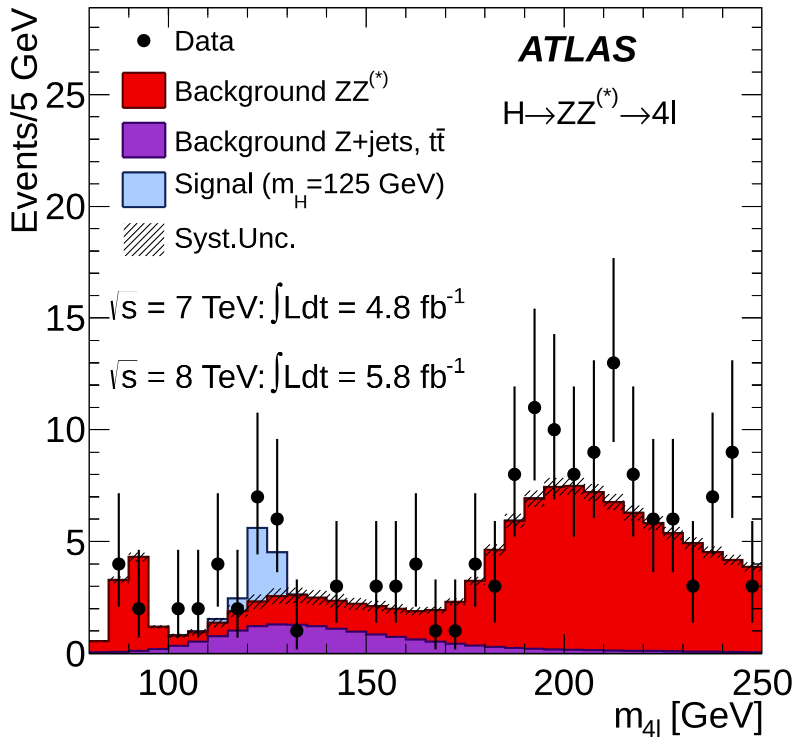


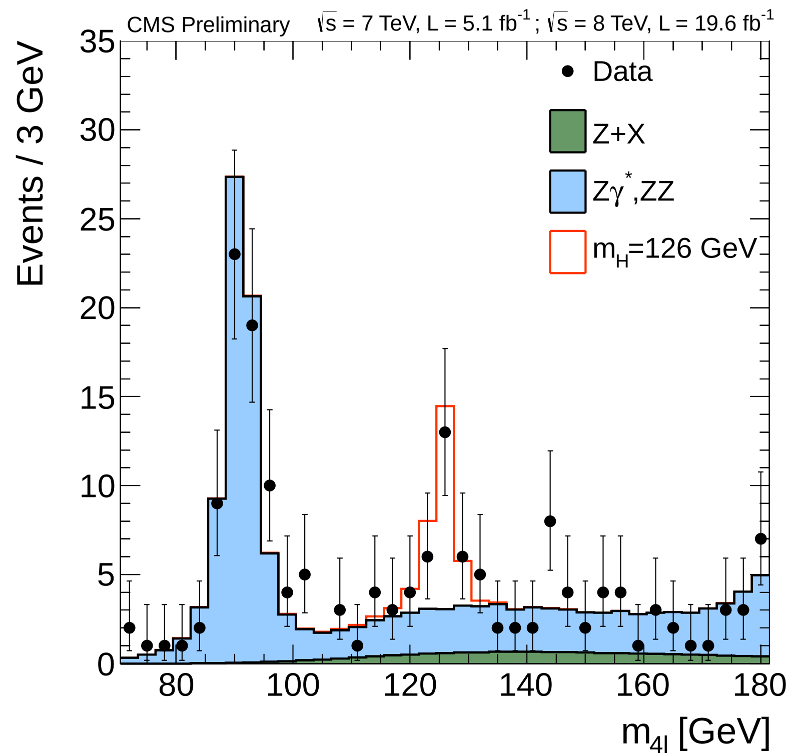


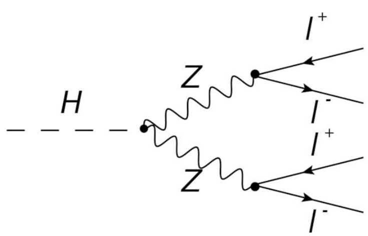




$$H \to ZZ \to l^+l^- \; l^+l^-$$







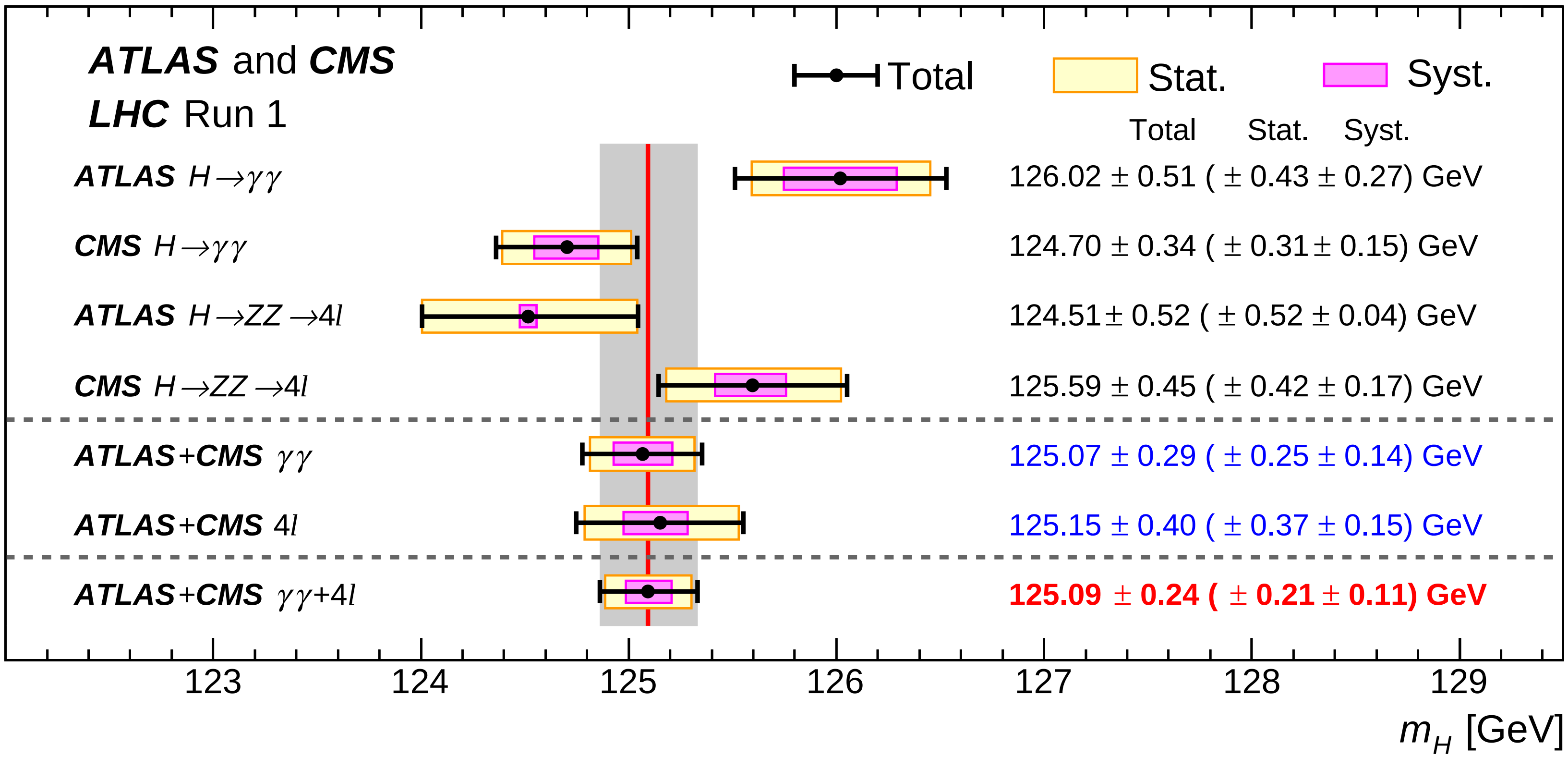
signal + background

the Higgs – or not the Higgs?

CERN, July 2012

Oviedo, October 2013

combined ATLAS/CMS Higgs-boson mass determination



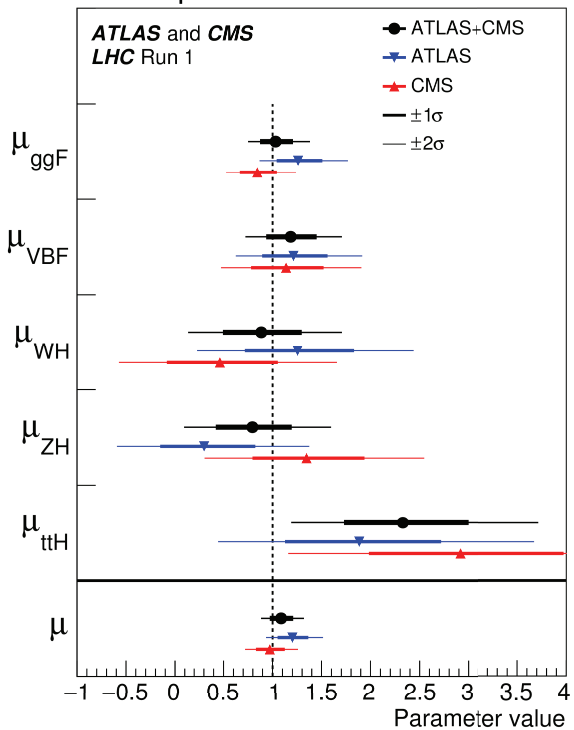
arXiv:1503.07589

A Standard Model Higgs boson at the LHC?

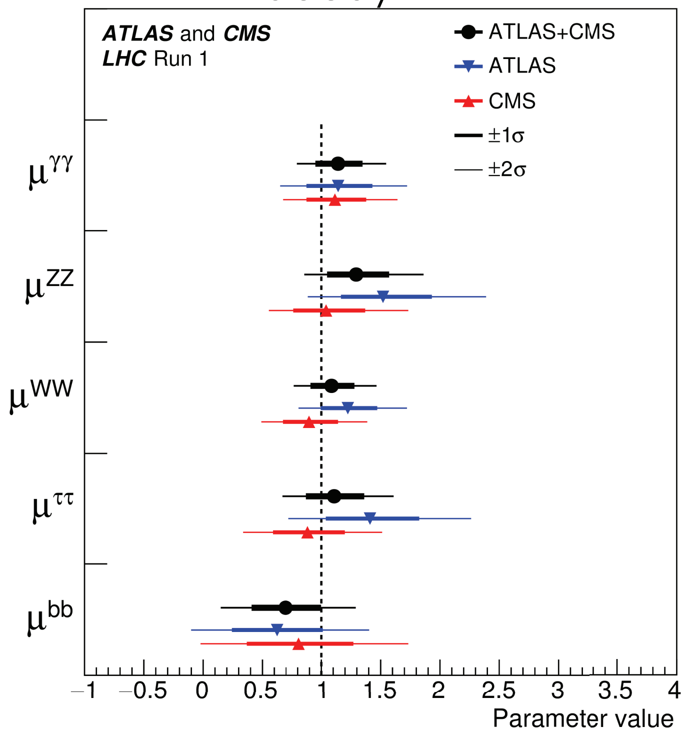
signal strength $\mu=\sigma/\sigma_{SM}$

ATLAS+CMS JHEP 08 (2016) 045

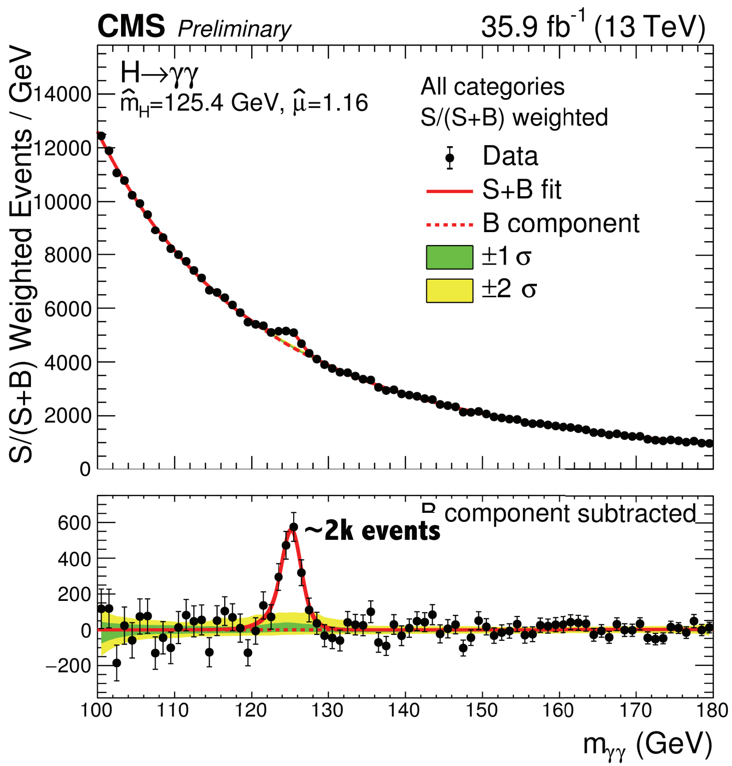
production



decay

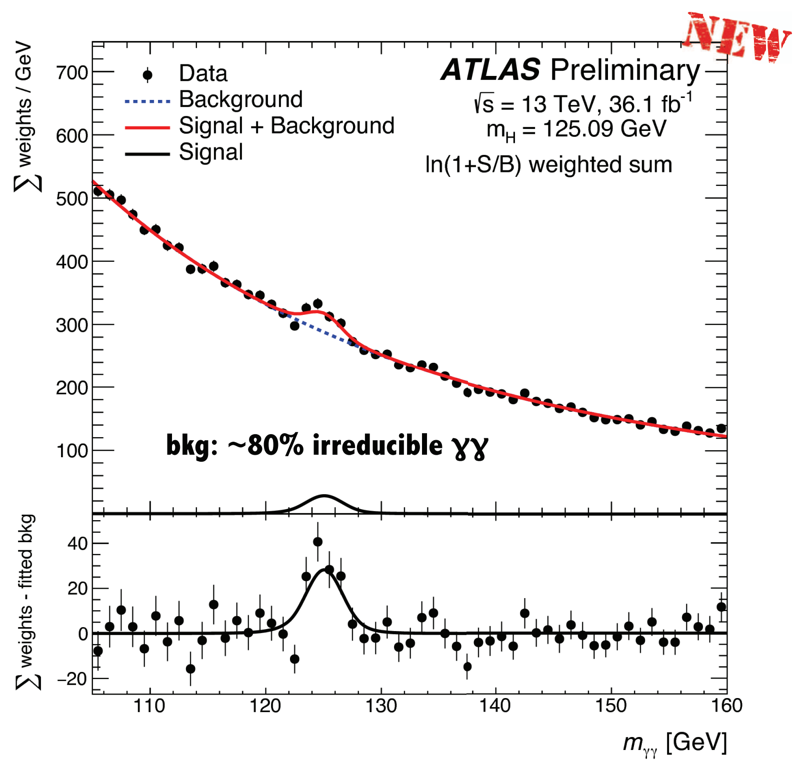


CMS HIG-16-040



$$\mu = 1.16^{+0.15}_{-0.14} = 1.16^{+0.11}_{-0.10}(\mathrm{stat})^{+0.09}_{-0.08}(\mathrm{exp})^{+0.06}_{-0.05}(\mathrm{theo})$$

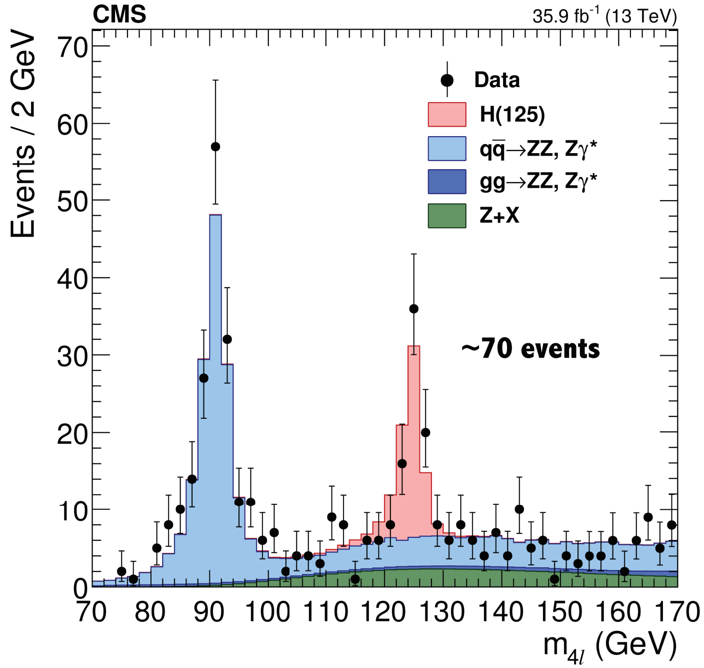
ATLAS-CONF-2017-045



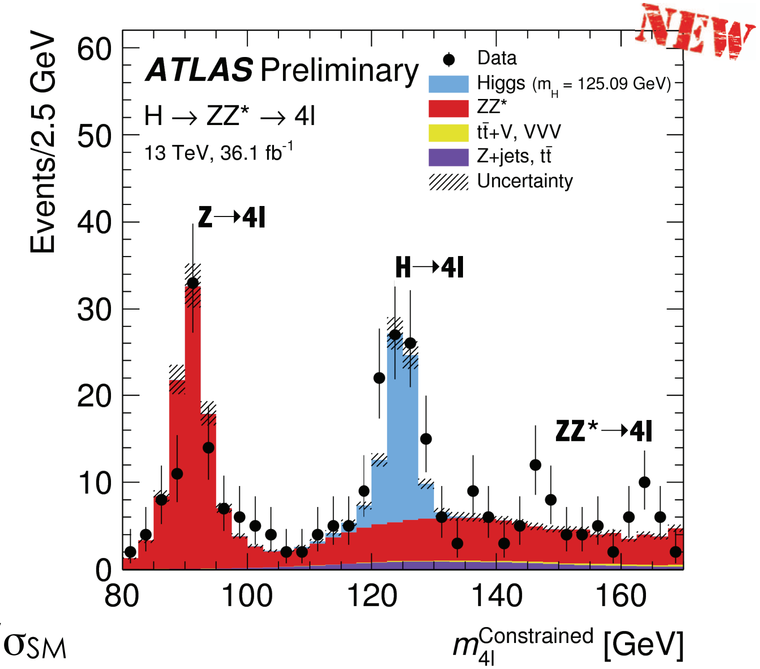
$$\mu = 0.99\ ^{+0.14}_{-0.14} = 0.99\ ^{+0.12}_{-0.11}\,(\mathrm{stat.})\ ^{+0.06}_{-0.05}\,(\mathrm{exp.})\ ^{+0.06}_{-0.05}\,(\mathrm{theory})$$

EPS 2017

CMS arXiv:1706.09936 Submitted to JHEP



ATLAS-CONF-2017-043



$\mu=\sigma/\sigma_{SM}$

$$\mu = 1.05^{+0.15}_{-0.14}(\text{stat})^{+0.11}_{-0.09}(\text{syst})$$

$$\mu = 1.28^{+0.18}_{-0.17}(\text{stat})^{+0.08}_{-0.06}(\text{exp})^{+0.08}_{-0.06}(\text{theo})$$

EPS 2017

Theoretical bounds on Higgs boson mass

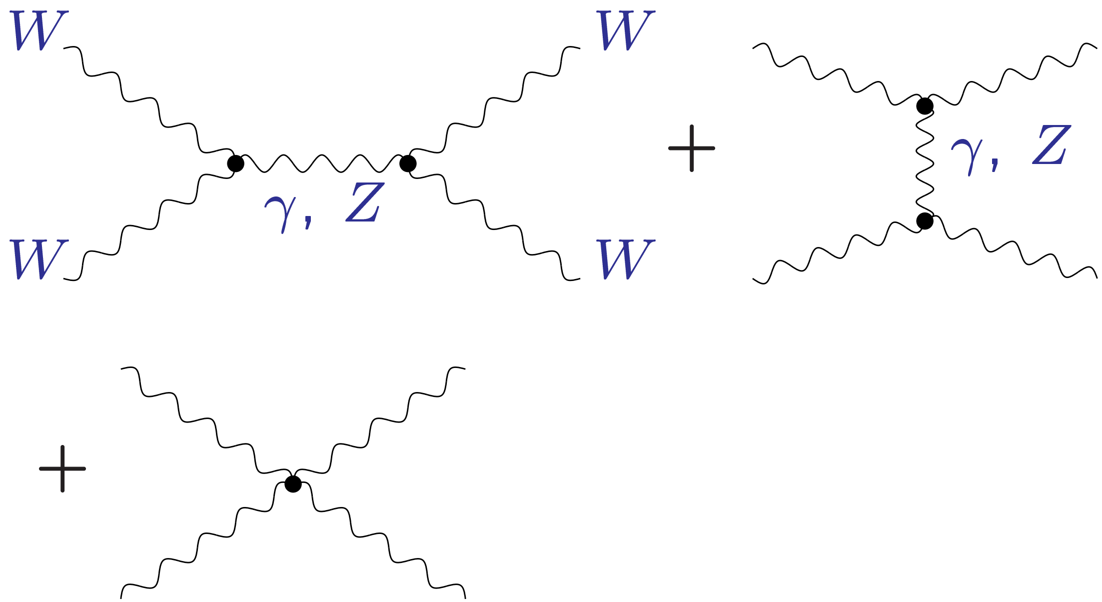
- unitarity $\rightarrow$ upper bound
- Landau pole $\rightarrow$ upper bound
- vacuum stability $\rightarrow$ lower bound

unitarity

scattering of longitudinally polarized W bosons:

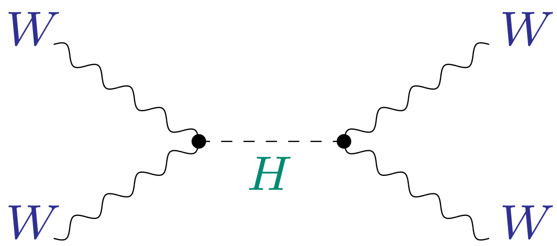
$$W_L W_L \to W_L W_L$$

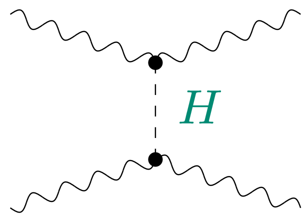
$\mathcal{M}_V =$



$$= -g^2 \frac{E^2}{M_\text{W}^2} + \mathcal{O}(1) \qquad \text{for } E \to \infty$$

Extra contribution from scalar particle:

$\mathcal{M}_S =$ 

$+$ 

$$= g_{WWH}^2 \frac{E^2}{M_{\mathrm{W}}^4} + \mathcal{O}(1) \qquad \text{for } E \to \infty$$

$$\mathcal{M} \quad = \mathcal{M}_V + \mathcal{M}_S$$

$\Rightarrow$ terms with bad high-energy behavior cancel for

$$g_{WWH} = g\, M_{\mathrm{W}}$$

for $s >> M_W^2$, with $t = -\frac{s}{2}\,(1 - \cos\theta)$,

$$\mathcal{M} \approx \frac{M_H^2}{v^2}\left(2 + \frac{M_H^2}{s - M_H^2} + \frac{M_H^2}{t - M_H^2}\right)$$

partial wave expansion:

$$\mathcal{M}(s,t) = 8\pi \sum_{l=0}^{\infty} (2l+1)\, P_l(\cos\theta)\, a_l$$

unitarity condition: $|a_l| < 1$

project on $l = 0$ partial wave:

$$a_0 = \frac{1}{16\pi} \int_{-1}^{1} d\cos\theta\, \mathcal{M}(s,t)$$

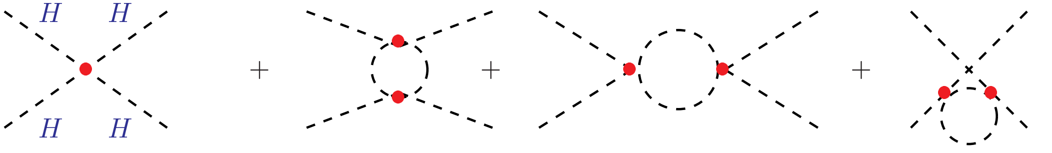
$$= \frac{M_H^2}{8\pi v^2} \left[2 + \frac{M_H^2}{s - M_H^2} - \frac{M_H^2}{s} \log\left(1 + \frac{s}{M_H^2}\right)\right]$$

$$\approx \frac{M_H^2}{4\pi v^2} \quad \text{for} \quad s >> M_H^2$$

$$\boxed{a_0 < 1 \quad \Rightarrow \quad M_H < 872\,\mathrm{GeV}}$$

Landau pole

Higgs self coupling is scale dependent, $\lambda(Q)$



variation with scale Q described by RGE

$$Q^2 \frac{d\lambda}{dQ^2} = \beta(\lambda) = \frac{3}{4\pi^2}\,\lambda^2$$

solution:

$$\lambda(Q) = \frac{\lambda(v)}{1 - \frac{3}{4\pi^2}\lambda(v)\log\frac{Q^2}{v^2}} \quad \text{with} \quad \lambda(v) = \frac{M_H^2}{2v^2}$$

diverges at scale $Q = \Lambda_C$ (Landau pole)

$$\Lambda_C = v\,\exp\left(\frac{4\pi^2 v^2}{3M_H^2}\right)$$

self-coupling diverges at

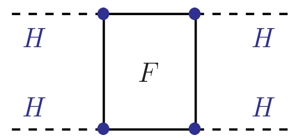
$$\Lambda_C = v \, \exp\left(\frac{4\pi^2 v^2}{3M_H^2}\right)$$

maximum Higgs mass by condition $\Lambda_C > M_H$

$$\Rightarrow \quad M_H < 800\,\mathrm{GeV}$$

vacuum stability

top-quark Yukawa coupling $g_t \sim m_t$ contributes to the running Higgs self coupling $\lambda(Q)$ through top loop $\sim g_t^4$



variation with scale Q described by RGE

$$Q^2 \frac{d\lambda}{dQ^2} = \frac{3}{4\pi^2} \left(\lambda^2 - \frac{m_t^4}{v^4} \right)$$

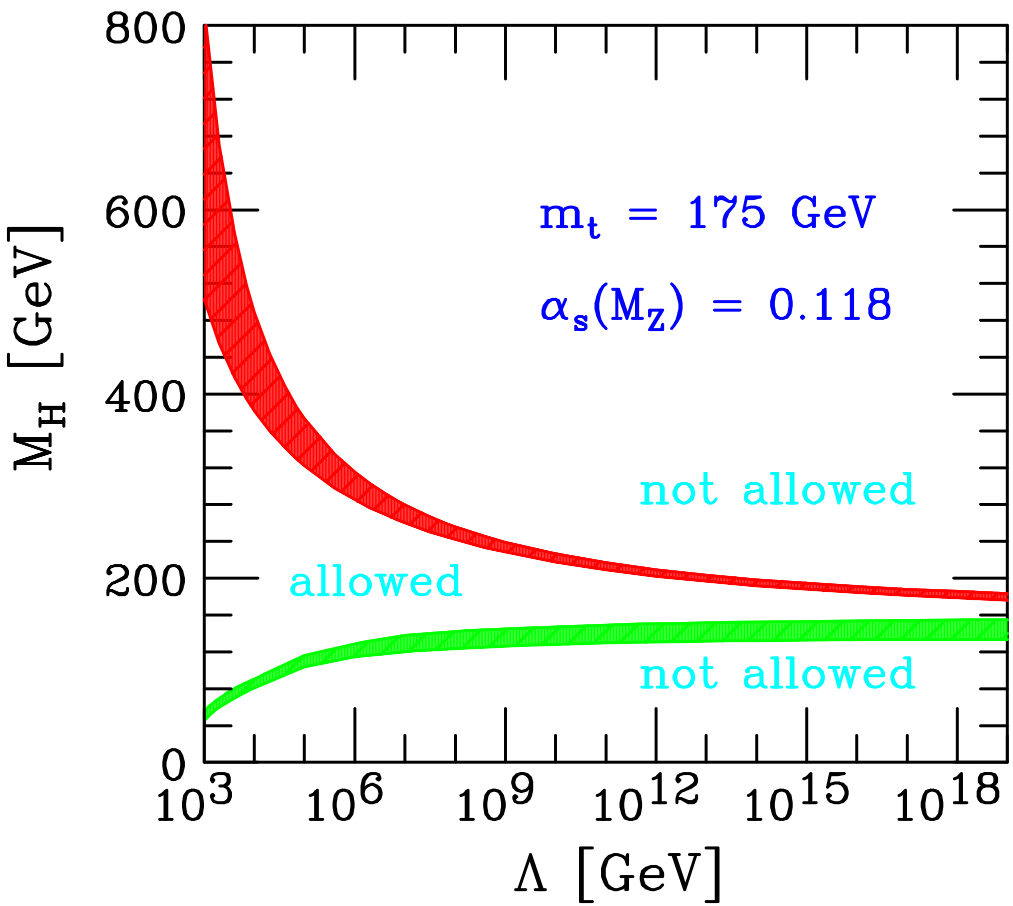
approximate solution:

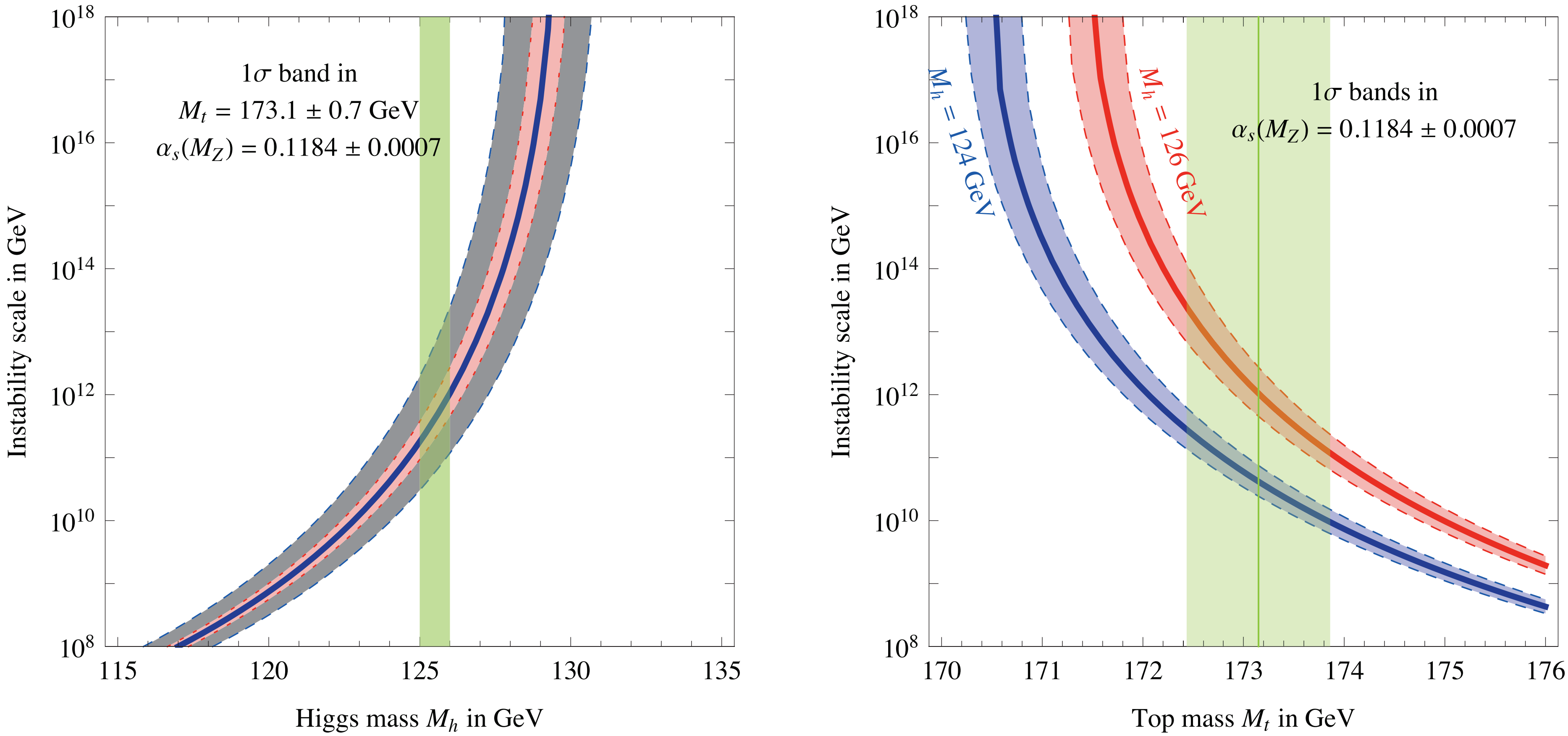
$$\lambda(Q) = \lambda(v) - \frac{3m_t^4}{2\pi^2 v^4} \log \frac{Q}{v}$$

$\lambda(Q) < 0$ for $Q > \Lambda_C$ $\rightarrow$ vacuum not stable

high value of Λ_C needs M_H large enough combined effects:

$$\frac{d\lambda}{dt} = \frac{1}{16\pi^2}\,\left(12\lambda^2 - 3\,g_t^4 + 6\,\lambda\,g_t^2\, + \cdots\right)$$





[Degrassi et al. 2012]

The success of the Standard Model

- impressive confirmation by a huge data sample from low to high energies, no significant deviations
- quantum effects have been established at many σ
- perfect indirect and direct determination of the top quark
- now being repeated for the Higgs boson
- new particle at 125 GeV strong candidate for the Higgs boson
- if confirmed: Standard Model closed

Happy End of a successful story ?

Shortcomings of SM

- no mass terms for neutrinos [introduce ν_R $\ldots$]

- hierarchy problem $v \ll M_{\rm Pl}, \quad M_H \ll M_{\rm Pl}$
- large number of free parameters $g_1,\ g_2,\ v,\ m_f,\ V_{\rm CKM}$
- no further unification of forces
- missing link to gravity

- nature of dark matter?
- baryon asymmetry of the universe?

- next steps with LHC and upgrades
  - – confirm the Higgs boson properties
  - – check versus electroweak precision measurements
  - – or find deviations, new structures:
    - more Higgs bosons (doublets, singlet, ..)
    - supersymmetry (minimal or non-minimal)
    - new strong sector, substructure
    - . . .


RESEARCH INSTITUTE
UNANSWERED QUESTIONS
UNQUESTIONED ANSWERS
S. Harris