The Number of String Standard Models

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based on: 1202.1757, 1307.4787, 1509.02729, 1706.07688, to appear

How many standard models* does string theory contain?

* At this level: String models with the (MS)SM spectrum.

Outline

- Introduction: line bundle models
- The data
- Counting standard models
- Conclusion

Introduction: line bundle models (Anderson, Gray, Lukas, Palti, 1106.4804)

Data to define a heteritic line bundle model we need:

- A Calabi-Yau 3-fold X
- A line bundle sum $V = L_1 \oplus \cdots \oplus L_5$ on X, $c_1(V) = 0$, so structure group is $S(U(1)^5)$.
- vanishing slopes $\mu(L_a) \equiv c_1(L_a) \wedge J^2 \stackrel{!}{=} 0$
- Anomaly: $c_2(TX) - c_2(V) - c_2(\tilde{V}) = [C]$
  in practice: $c_2(V) \leq c_2(TX)$

N=1, D=4 GUT with gauge group $SU(5) \times S(U(1)^5)$ and matter in $\mathbf{10}, \bar{\mathbf{10}}, \bar{\mathbf{5}}, \mathbf{5}, \mathbf{1}$

- freely acting symmetry Γ on X, so $\hat{X} = X/\Gamma$ is smooth and non simply-connected

- bundle V needs to be equivariant so it descends to a bundle $\hat{V}$ on $\hat{X}$

- complete bundle $\hat{V} \oplus W$ with Wilson line W to break GUT group

standard-like model (hopefully) with gauge group $G_{\rm SM} \times S(U(1)^5)$

The associated 4d GUT theories:

Gauge group $SU(5) \times S(U(1)^5)$

matter multiplets: $\mathbf{10}_a,\ \overline{\mathbf{10}}_a,\ \mathbf{5}_{a,b},\ \bar{\mathbf{5}}_{a,b},\ \mathbf{1}_{a,b}$

	multiplet	$S(U(1)^5)$ charge	associated line bundle L	contained in	
families and mirror families	$\mathbf{10}_{\mathbf{e}_a}$	$\mathbf{e}_a$	L_a	V	$\leftarrow\ = 3\lvert\Gamma\rvert$
	$\overline{\mathbf{10}}_{-\mathbf{e}_a}$	$-\mathbf{e}_a$	L_a^*	V^*	$\leftarrow\ = 0$
	$\bar{\mathbf{5}}_{\mathbf{e}_a+\mathbf{e}_b}$	$\mathbf{e}_a + \mathbf{e}_b$	$L_a \otimes L_b$	$\wedge^2 V$	$\Rightarrow 3\lvert\Gamma\rvert$
	$\mathbf{5}_{-\mathbf{e}_a-\mathbf{e}_b}$	$-\mathbf{e}_a - \mathbf{e}_b$	$L_a^* \otimes L_b^*$	$\wedge^2 V^*$	
C_{ab}^{+} bundle	$\mathbf{1}_{\mathbf{e}_a-\mathbf{e}_b}$	$\mathbf{e}_a - \mathbf{e}_b$	$L_a \otimes L_b^*$	$V \otimes V^*$	
C_{ab}^{-} moduli	$\mathbf{1}_{-\mathbf{e}_a+\mathbf{e}_b}$	$-\mathbf{e}_a + \mathbf{e}_b$	$L_a^* \otimes L_b$		

Number of each multiplet type obtained from $H^1(X,L)$.

Arena: complete intersection CY manifolds (CICYs)

ambient space: $$\mathcal{A} = \bigotimes_{r=1}^{m} \mathbb{P}^{n_r}$$

CICY: $$X = \{p_i = 0\} \subset \mathcal{A}$$

for example: $$[\mathbb{P}^4|5] , \quad \left[\begin{array}{c|c} \mathbb{P}^2 & 3 \\ \mathbb{P}^2 & 3 \end{array} \right] , \quad \left[\begin{array}{c|c} \mathbb{P}^1 & 2 \\ \mathbb{P}^1 & 2 \\ \mathbb{P}^1 & 2 \\ \mathbb{P}^1 & 2 \end{array} \right] , \quad \left[\begin{array}{c|cc} \mathbb{P}^1 & 0 & 2 \\ \mathbb{P}^1 & 1 & 1 \\ \mathbb{P}^1 & 1 & 1 \\ \mathbb{P}^1 & 1 & 1 \\ \mathbb{P}^1 & 1 & 1 \end{array} \right] \begin{array}{l} \leftarrow J_1 \\ \leftarrow J_2 \\ \vdots \\ \\ \\ \end{array}$$

Complete classification of about 8000 spaces

(Hubsch, Green, Lutken, Candelas 1987)

Classification of freely-acting discrete symmetries

(Braun, 2010)

Line bundle cohomology can be computed.

(Anderson, He, Lukas, 2008)

Focus on favourable Cicys: $H^{1,1}(X) = \mathrm{Span}(J_i)$, $J = t^i J_i$

line bundles $L = \mathcal{O}_X(\mathbf{k})$ where $c_1(L) = k^i J_i$

basic data specifying a model:

$X \sim [\mathcal{A}\,|\,Q]$ $\longrightarrow$ $h := h^{1,1}(X)$, $c_i := c_{2i}(TX)$, d_{ijk}

$V \sim \left(k_a^i\right)_{a=1,\ldots,5}^{i=1,\ldots,h}$ $\longrightarrow$ $(2k_{\max}+1)^{4h}$ choices for $|k_a^i| \leq k_{\max}$

symmetry Γ $\longrightarrow$ $\Gamma = \mathbb{Z}_2$ most common subject to:

$c_1(V) = 0$ →	$\sum_a k_a^i = 0\,, \quad i = 1, \ldots, h$
$c_2(V) \leq c_2(TX)$ →	$-\frac{1}{2} d_{ijk} \sum_a k_a^j k_a^k \leq c_i\,, \quad i = 1, \ldots, h$
$\mu(L_a) = 0$ →	$d_{ijk} k_a^i t^j t^k = 0\,, \quad a = 1, \ldots, 5\,,\ \mathbf{t} \in \mathcal{K}(X)$
$\mathrm{ind}(V) = -6$ →	$\frac{1}{6} d_{ijk} \sum_a k_a^i k_a^j k_a^k = -6$

essentially, set of diophantine equations

The data

(Anderson, Gray, Constantin, Lukas, Palti, 1307.4787, 1202.1757)

An exhaustive scan over favourable Cicys with $h^{1,1} \leq 6$: 68 manifolds

Requires scanning over $\sim 10^{40}$ bundles (k_a^i)

How do we know we have found all viable models?

Table 6: *Number of models as a function of k_{max} on CICYs with $h^{1,1}(X) = 6$. Total number of models: 41036*

X, $\|\Gamma\|$	$k_{\rm m} = 1$	$k_{\rm m} = 2$	$k_{\rm m} = 3$	$k_{\rm m} = 4$	$k_{\rm m} = 5$	$k_{\rm m} = 6$	$k_{\rm m} = 7$	$k_{\rm m} = 8$	$k_{\rm m} = 9$	$k_{\rm m} = 10, 11, 12, 13$
3413, 3	0	2278	2897	2906	2906	2906				
4190, 2	11	766	1175	1243	1246	1247	1249	1249	1249	
5273, 2	29	4895	7149	7738	7799	7810	7810	7810		
5302, 2	0	4314	5978	6360	6369	6369	6369			
5302, 4	0	11705	16988	17687	17793	17838	17868	17868	17868	
5425, 2	0	2381	3083	3305	3337	3337	3337			
5958, 2	0	148	224	240	253	253	253			
6655, 5	0	92	178	189	194	194	198	201	202	203
6738, 2	1	2733	4116	4346	4386	4393	4399	4399	4399	

Results:

Number of consistent SU(5) GUT models with correct indices:

$h^{1,1}(X)$	1	2	3	4	5	6	total
#models	0	0	6	552	21731	41036	63325

After demanding absence of $\bar{10}$ and presence of $5-\bar{5}$ pair:

34989 models

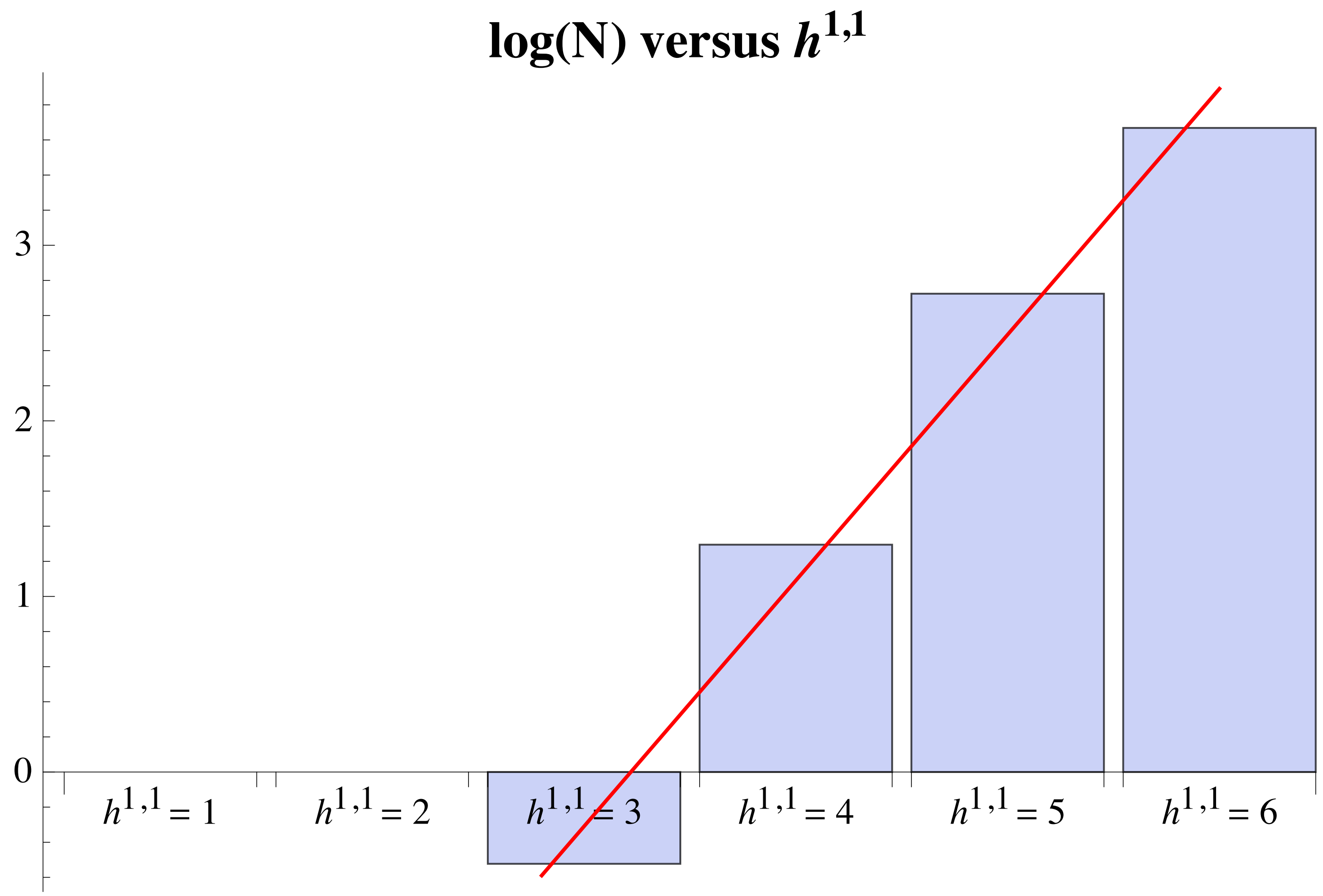
Experience with sub-set of models indicates practically all of these will lead to (MS)SM spectra.

Available at:

http://www-thphys.physics.ox.ac.uk/projects/CalabiYau/linebundlemodels/index.html

Counting Standard Models

Number of SU(5) GUT models per CY: $N = N(h, c_i, d_{ijk})$

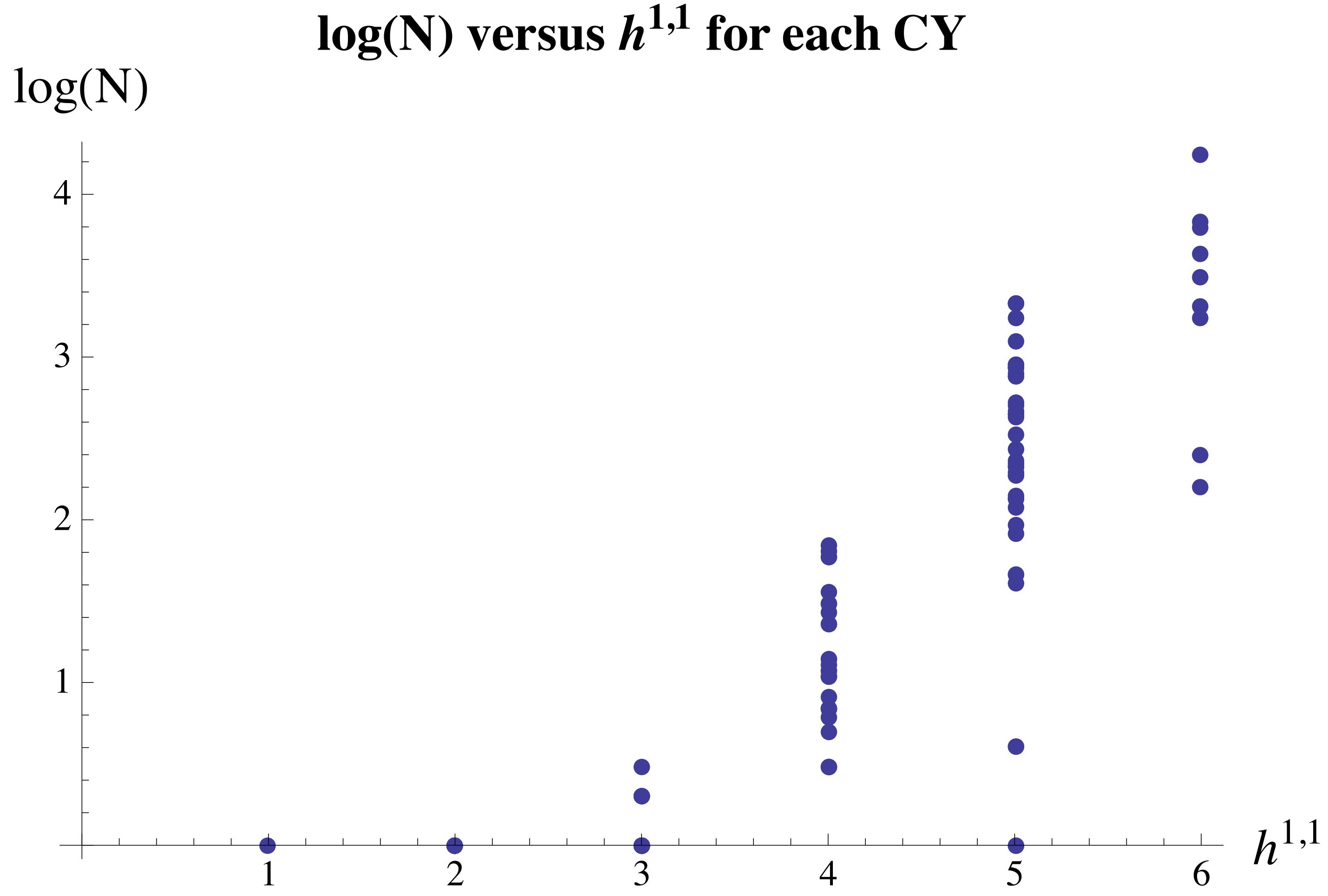


$$\log N(h) \simeq -4.5 + 1.4\, h$$

For CICYs: $h_{\max} = 19$ $\log N(h_{\max}) \simeq 22$

All known CYs: $h_{\max} = 491$ $\log N(h_{\max}) \simeq 683$

however . . .



. . . strong dependence of N on c_i and d_{ijk}

Why is the number of models even finite? (Constantin, Lukas, Mishra, 1509.02729)

moduli space metric: $G_{ij} = -3\left(\frac{\kappa_{ij}}{\kappa} - \frac{2\kappa_i\kappa_j}{3\kappa^2}\right)$

$$\kappa = d_{ijk}t^i t^j t^k$$
$$\kappa_i = d_{ijk}\, t^j\, t^k$$
$$\kappa_{ij} = d_{ijk}\, t^k$$

$\kappa_i k_a^i = 0$

$$0 < \sum_a \mathbf{k}_a^T G\, \mathbf{k}_a = -\frac{3}{\kappa} d_{ijk} \sum_a k_a^i\, k_a^j\, t^k = \frac{6}{\kappa} t^i\, c_{2i}(V) \leq \frac{6}{\kappa} t^i\, c_{2i}(TX) \leq \frac{6}{\kappa}|\mathbf{t}||c_2(TX)|$$

bound on line bundle sums:

$$\sum_a \mathbf{k}_a^T \tilde{G}\, \mathbf{k}_a \leq |c_2(TX)| \qquad \tilde{G} = \frac{\kappa}{6|\mathbf{t}|} G$$

Shows finiteness, provided we stay away from the Kahler cone boundaries.

Volume in k_a^i space: $V \sim \left(\frac{|c_2(TX)|}{\bar{d}} \right)^{5h/2}$

Asymptotic count of solutions goes with a power of the volume.

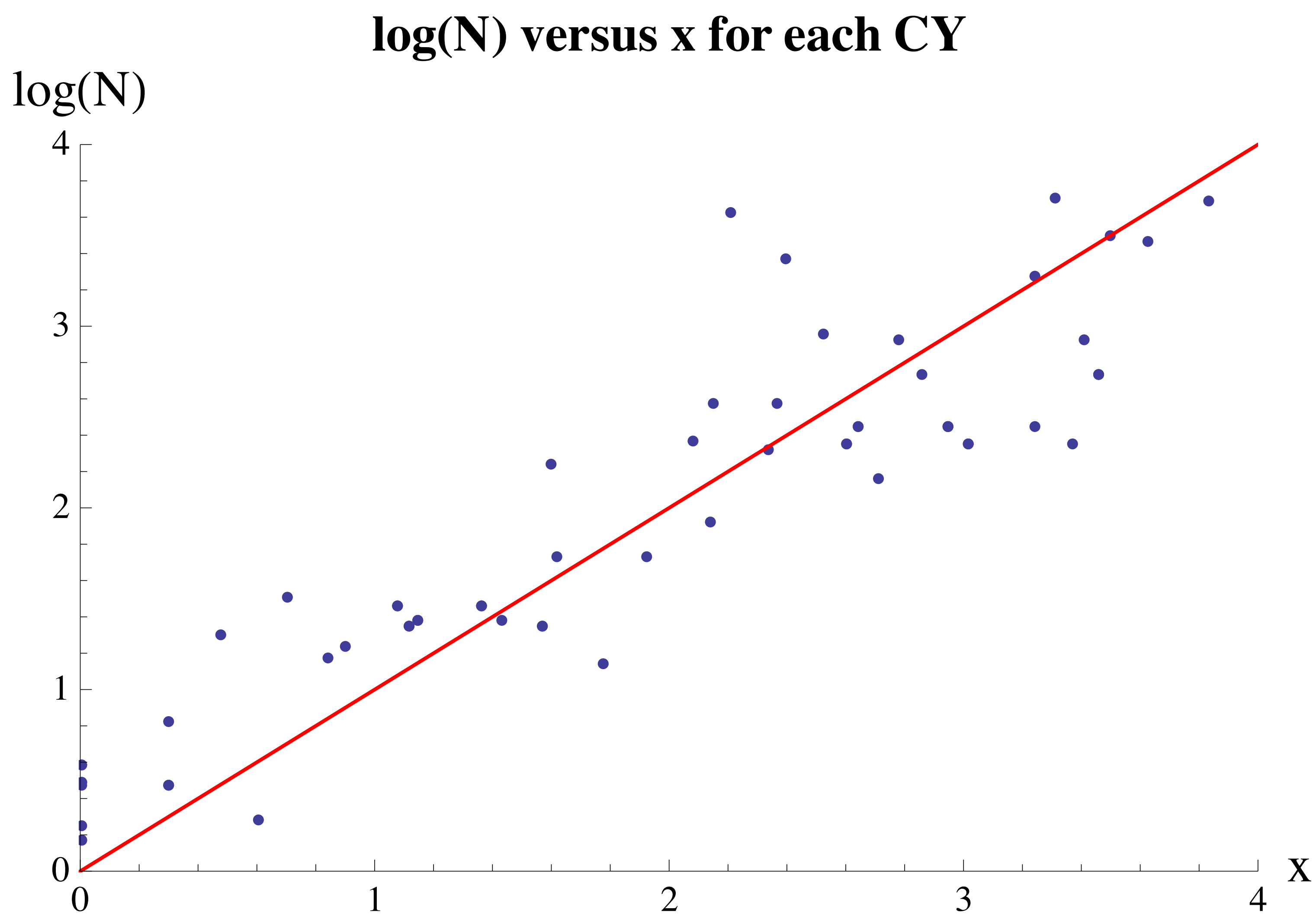
(Browning, Heath-Brown, Salberger, math/0410117)

Assume $\log N = x$ where

$$x = A_0 + A_1 h + (A_2 + A_3 h) \log |c_2(TX)| + (A_4 + A_5 h) \log \bar{d}$$

A fit of the data leads to

$$x \simeq 8 + \frac{3}{4} h + \left(-7 + \frac{1}{3} h \right) \log |c_2(TX)| + \left(2 - \frac{1}{4} h \right) \log \bar{d}$$



For CICYs: $h_{\max} = 19$ $\log N_{\max} \simeq 19$

All known CYs: $h_{\max} = 491$ $\log N_{\max} \simeq 470$

How seriously should this be taken?

Result for CICYs - $\sim 10^{19}$ standard models - should be trusted.

There are caveats extrapolating to $h^{1,1} = 491$.

- Low $h^{1,1}$ data scattered, so probably large error at $h^{1,1} = 491$. (But there will still be 10^{hundreds} standard models.)

- There may be few discrete symmetries for CYs with large $h^{1,1}$.

  The Kreuzer-Skarke list contains only 16 cases with toric symmetries, all of them for $h^{1,1} \leq 7$.
  (Batyrev, Kreuzer, math/0505432)

  More general symmetries for Kreuzer-Skarke CYs have only been explored for low $h^{1,1}$.
  (Braun, Lukas, Sun, 1704.07812)
  (Altman, Gay, He, Jejjala, Nelson, 1411.1418)

- There are other obstructions to a physical model.

For example: Line bundle models on elliptically fibered CYs (with a toric base). (Braun, Brodie, Lukas, 1706.07688)

Number of physical models:

base B	$k_{\max}$	k_{mod}	#models
F_2	10	–	0
F_4	10	–	0
F_7	10	4	54
F_9	7	6	22
F_{13}	3	3	≥ 46
$F_{15}^{(a)}$	3	3	≥ 236
$F_{15}^{(b)}$	3	3	≥ 84
total	–	–	≥ 442

but . . .

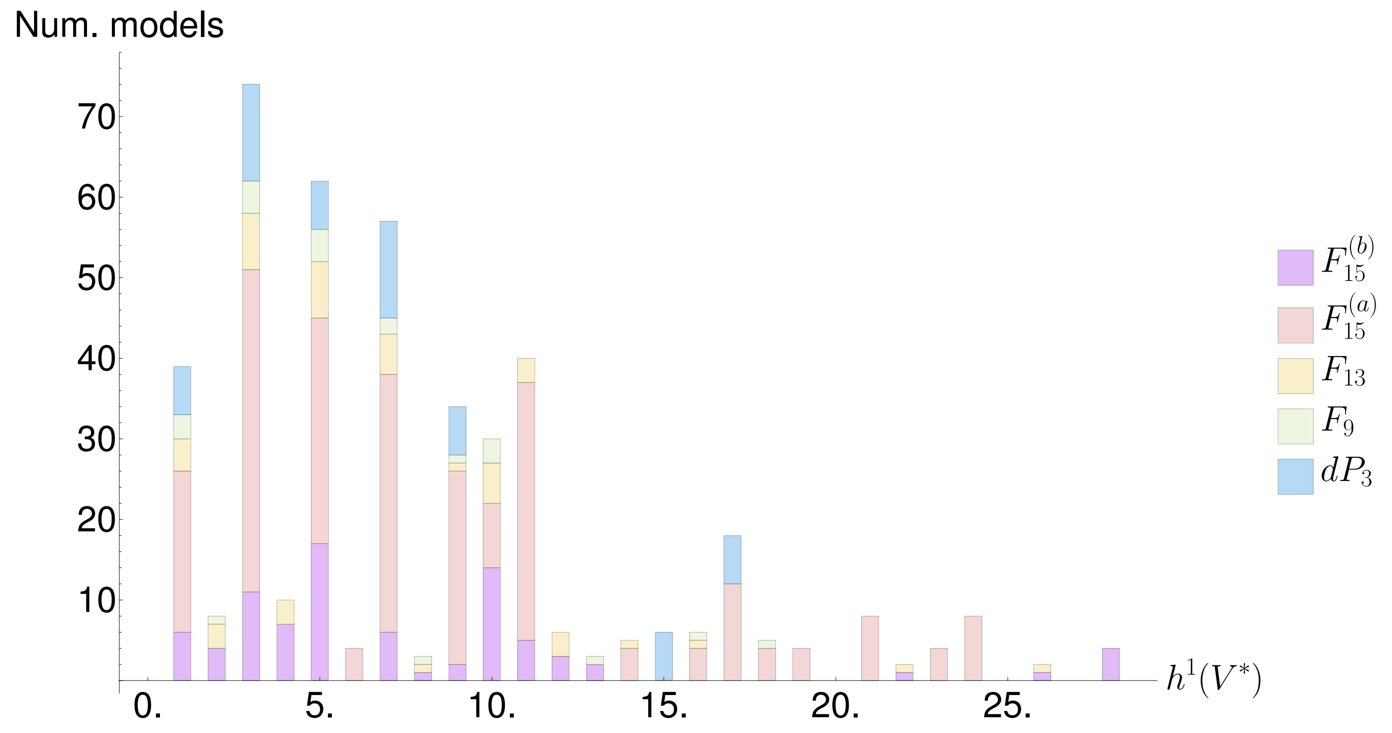


Figure 2: Frequency plot of $h^1(V^*)$ which gives the number of $\overline{\mathbf{10}}$ multiplets, combined for all base spaces.

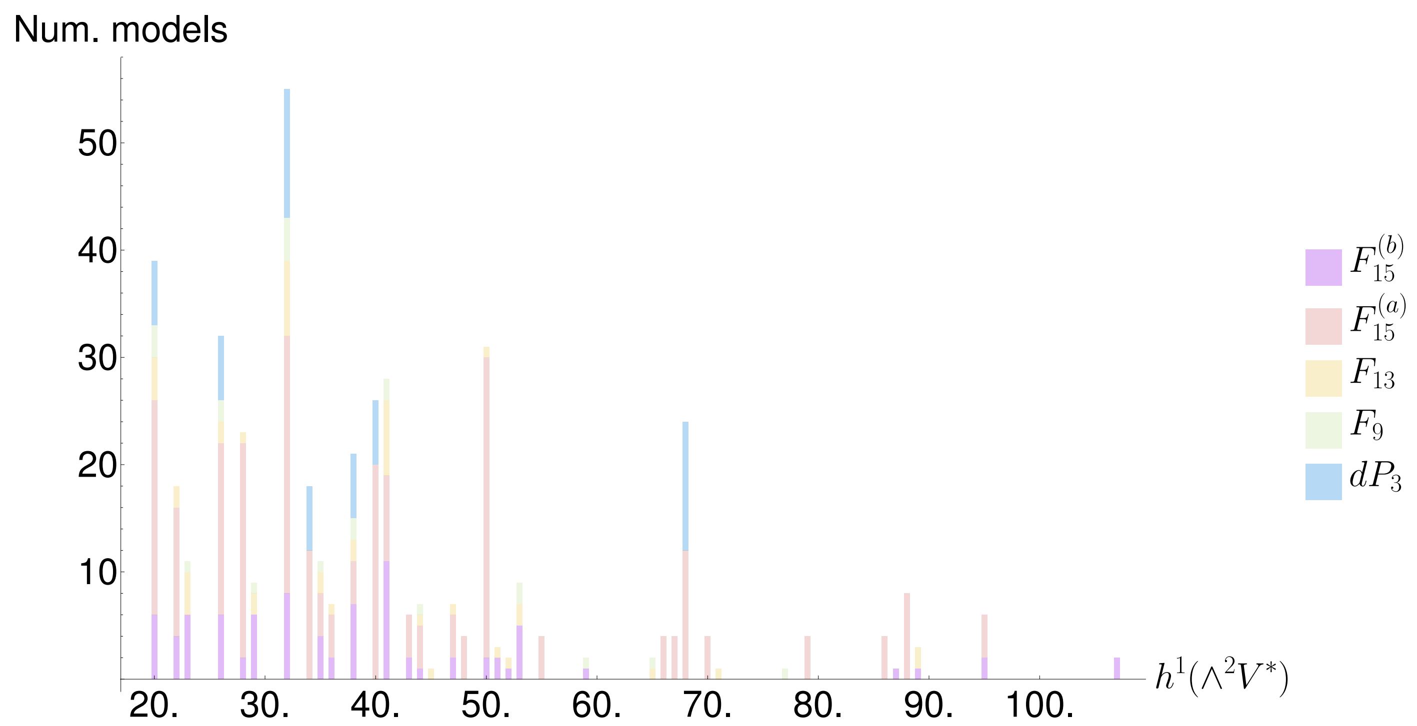


Figure 3: Frequency plot of $h^1(\wedge^2 V^*)$ which gives the number of $\mathbf{5}$ multiplets, combined all base space spaces.

No example without mirror family and always ≥ 19 Higgs pairs!

Conclusion

- Algorithms (e.g. to compute cohomology) too slow at large $h^{1,1}$.
  (need better algorithms, machine learning?)

- Size of model spaces increases exponentially with $h^{1,1}$, so systematic search is not possible.
  (need more sophisticated search methods, genetic algorithms?)

- Even on conservative estimates the number of models with the MSSM spectrum amounts to more data than currently stored in total.
  (need better discriminators than spectrum: coupling constants)

Thanks and good luck!