Classification of Quasi-Realistic Heterotic String Vacua

Glyn Harries
In collaboration with Alon Faraggi & John Rizos

26/03/2018

Outline

- A brief introduction to the Free Fermionic Formulation (FFF)
- A definition of classification and it's aims will be given
- Previous results of classifications will be outlined
- How machine learning has already been introduced to the FFF will be briefly discussed
- The current project will be outlined along with the plans of how to introduce machine learning into the classification process
- The aim is then to (hopefully) generate a discussion with your thoughts on the project and any considerations to be made
- Conclusions

Introduction to Free Fermions

- The free fermionic construction of string theory offers an interesting way to test the phenomenology of various string models
- To date, the models constructed represent some of the most realistic string models with three chiral generations of matter
- In the free fermionic construction we interpret the extra degrees of freedom as free fermions propagating on the string worldsheet, instead of spacetime dimensions. This allows us to formulate the theory directly in four spacetime dimensions
- We do this by introducing:
- Left moving $c_L = -26 + 11 + D + \frac{D}{2} + N_{f_L} \cdot \frac{1}{2} = 0$
- Right moving $c_R = -26 + D + N_{f_R} \cdot \frac{1}{2} = 0$
- We can then see that to cancel the conformal anomaly, we need:

$$N_{f_L} = 18$$
$$N_{f_R} = 44$$

Introduction to Free Fermions

- The fermions on the worldsheet are

	Label	**Description**
Left–moving	X^μ	Bosonic coordinates with spacetime index, $\mu = 0, \ldots, 3$
	ψ^μ	Majorana–Weyl superpartners of the bosonic coordinates with spacetime index
	$\chi^{1,\ldots,6}$	Majorana–Weyl superpartners to the six compactified dimensions
	$y^{1,\ldots,6}, w^{1,\ldots,6}$	Majorana–Weyl fermions that correspond to the bosons describing the six compactified dimensions in the bosonic formulation
Right–moving	$\overline{X}^\mu$	Bosonic coordinates with spacetime index
	$\overline{y}^{1,\ldots,6}, \overline{w}^{1,\ldots,6}$	Majorana–Weyl fermions that correspond to the bosons describing the six compactified dimensions in the orbifold formulation
	$\overline{\psi}^{1,\ldots,5}, \overline{\eta}^{1,2,3}$	Complex fermions that describe the visible gauge sector
	$\overline{\phi}^{1,\ldots,8}$	Complex fermions that describe the hidden gauge sector

Partition Function

- The partition function is the sum over all the massive and massless string states which have to be included when the string propagates around the vacuum to vacuum amplitude
- The relevent information from the one loop fermionic partition function we concern ourself with for model building is then

$$\sum_{\text{spin structures}} C\binom{\vec{\alpha}}{\vec{\beta}}\, Z\binom{\vec{\alpha}}{\vec{\beta}}$$

- This is what we consider in order to build our models

Basis Vectors

- The complete partition function can be constructed by specifying a set of basis vectors and the one loop phases.
- Basis vectors denote whether each fermion has periodic, antiperiodic or complex boundary conditions
- We write these in the form

$$b_i = \{\alpha(\psi^\mu_{12})\ ,\ \ldots\ ,\ \alpha(w^6) \mid \alpha(\bar{y}^1)\ ,\ \ldots\ ,\ \alpha(\bar{\phi}^8)\}$$

where α is the phase defined in $f \to -e^{i\pi\alpha(f)} f$

Basis Vectors

The standard set of basis vectors used in this talk are the set commonly found in the literature, which generate $SO(10)$ models

$$
\begin{aligned}
v_1 = \mathbb{1} &= \{\psi^\mu,\ \chi^{1,\dots,6}, y^{1,\dots,6}, \omega^{1,\dots,6} | \\
&\qquad \overline{y}^{1,\dots,6}, \overline{\omega}^{1,\dots,6}, \overline{\eta}^{1,2,3}, \overline{\psi}^{1,\dots,5}, \overline{\phi}^{1,\dots,8}\}, \\
v_2 = S &= \{\psi^\mu, \chi^{1,\dots,6}\}, \\
v_{2+i} = e_i &= \{y^i, \omega^i \mid \overline{y}^i, \overline{\omega}^i\},\ i = 1, \dots, 6, \\
v_9 = b_1 &= \{\chi^{34}, \chi^{56}, y^{34}, y^{56} \mid \overline{y}^{34}, \overline{y}^{56}, \overline{\eta}^1, \overline{\psi}^{1,\dots,5}\}, \\
v_{10} = b_2 &= \{\chi^{12}, \chi^{56}, y^{12}, y^{56} \mid \overline{y}^{12}, \overline{y}^{56}, \overline{\eta}^2, \overline{\psi}^{1,\dots,5}\}, \\
v_{11} = z_1 &= \{\overline{\phi}^{1,\dots,4}\}, \\
v_{12} = z_2 &= \{\overline{\phi}^{5,\dots,8}\}.
\end{aligned} \tag{1}
$$

where $i = 1, \dots, 6$ and the fermions which appear in the basis vectors have periodic (Ramond) boundary conditions, where as those not included have antiperiodic (Neveu-Schwarz) boundary conditions.

Basis Vectors

Two notable linear combinations are given the definition x and b_3 and are treated as basis vectors in their own right. They are defined as

$$\begin{aligned} x &= \mathbb{1} + S + \sum_{i=1}^{6} e_i + z_1 + z_2 \\ b_3 &= b_1 + b_2 + x \\ &= \mathbb{1} + S + \sum_{i=1}^{6} e_i + b_1 + b_2 + z_1 + z_2 \end{aligned}$$

GSO Projection

- The equation for the Generalized GSO projection is

$$e^{i\pi b_j \cdot F_\alpha} \left|s\right\rangle_\alpha = \delta_\alpha \, C\begin{pmatrix}\alpha \\ b_j\end{pmatrix}^* \left|s\right\rangle_\alpha$$

  where α is the sector being considered and b_j is the basis vector
- The states $|s\rangle$ which satisfy this equation are 'kept in' and the states which do not satisfy this equation are 'projected out'.
- By performing the GSO projection on sectors we can break the gauge group and remove any unwanted states from the spectrum

Observable Sectors

The chiral matter spectrum arises from the sectors $B^{(1,2,3)}_{pqrs}$ which are given by

$$\begin{aligned} B^{(1)}_{pqrs} &= S + b_1 + pe_3 + qe_4 + re_5 + se_6 \\ &= \{\psi^\mu, \chi^{1,2}, (1-p)y^3\bar{y}^3, pw^3\bar{w}^3, (1-q)y^4\bar{y}^4, qw^4\bar{w}^4, \\ &\quad (1-r)y^5\bar{y}^5, rw^5\bar{w}^5, (1-s)y^6\bar{y}^6, sw^6\bar{w}^6, \bar{\eta}^1, \bar{\psi}^{1,\ldots,5}\} \end{aligned}$$

$$\begin{aligned} B^{(2)}_{pqrs} &= S + b_2 + pe_1 + qe_2 + re_5 + se_6 \\ B^{(3)}_{pqrs} &= S + b_3 + pe_1 + qe_2 + re_3 + se_4 \end{aligned}$$

where $p, q, r, s = 0, 1$.
The Standard Model Higgs states arise from the sectors $B^{(1,2,3)}_{pqrs} + x$.

Projectors

Projectors are linear equations which select whether a sector gives rise to states or projects the states out

The projectors associated with $B_{pqrs}^{(1,2,3)}$ are denoted by $P_{pqrs}^{(1,2,3)}$ and are given by

$$P_{pqrs}^{(1)} = \frac{1}{16}\left(1-C\begin{pmatrix} e_1 \\ B_{pqrs}^{(1)} \end{pmatrix}\right)\cdot\left(1-C\begin{pmatrix} e_2 \\ B_{pqrs}^{(1)} \end{pmatrix}\right)\cdot\left(1-C\begin{pmatrix} z_1 \\ B_{pqrs}^{(1)} \end{pmatrix}\right)\cdot\left(1-C\begin{pmatrix} z_2 \\ B_{pqrs}^{(1)} \end{pmatrix}\right)$$

$$P_{pqrs}^{(2)} = \frac{1}{16}\left(1-C\begin{pmatrix} e_3 \\ B_{pqrs}^{(2)} \end{pmatrix}\right)\cdot\left(1-C\begin{pmatrix} e_4 \\ B_{pqrs}^{(2)} \end{pmatrix}\right)\cdot\left(1-C\begin{pmatrix} z_1 \\ B_{pqrs}^{(2)} \end{pmatrix}\right)\cdot\left(1-C\begin{pmatrix} z_2 \\ B_{pqrs}^{(2)} \end{pmatrix}\right)$$

$$P_{pqrs}^{(3)} = \frac{1}{16}\left(1-C\begin{pmatrix} e_5 \\ B_{pqrs}^{(3)} \end{pmatrix}\right)\cdot\left(1-C\begin{pmatrix} e_6 \\ B_{pqrs}^{(3)} \end{pmatrix}\right)\cdot\left(1-C\begin{pmatrix} z_1 \\ B_{pqrs}^{(3)} \end{pmatrix}\right)\cdot\left(1-C\begin{pmatrix} z_2 \\ B_{pqrs}^{(3)} \end{pmatrix}\right)$$

These projectors can be expressed as a system of linear equations where p, q, r, s are unknowns. The solutions to the equations give the combinations of p, q, r, s for which these sectors survive.

Classification

When moving to a computer based analysis, the following notation can be introduced

$$C\binom{b_i}{b_j} = e^{i\pi(b_i|b_j)} \qquad \text{where } (b_i|b_j) = 0, 1, \pm\frac{1}{2}$$

Classification

Using the new notation defined on the previous slide, the projectors $P^{(i)}_{pqrs}$, where $i = 1, 2, 3$, can be written (respectively) in a matrix form $\Delta^i W^i = Y^i$

$$\begin{pmatrix} (e_1|e_3) & (e_1|e_4) & (e_1|e_5) & (e_1|e_6) \\ (e_2|e_3) & (e_2|e_4) & (e_2|e_5) & (e_2|e_6) \\ (z_1|e_3) & (z_1|e_4) & (z_1|e_5) & (z_1|e_6) \\ (z_2|e_3) & (z_2|e_4) & (z_2|e_5) & (z_2|e_6) \end{pmatrix} \begin{pmatrix} p \\ q \\ r \\ s \end{pmatrix} = \begin{pmatrix} (e_1|b_1) \\ (e_2|b_1) \\ (z_1|b_1) \\ (z_2|b_1) \end{pmatrix}$$

$$\begin{pmatrix} (e_3|e_1) & (e_3|e_2) & (e_3|e_5) & (e_3|e_6) \\ (e_4|e_1) & (e_4|e_2) & (e_4|e_5) & (e_4|e_6) \\ (z_1|e_1) & (z_1|e_2) & (z_1|e_5) & (z_1|e_6) \\ (z_2|e_1) & (z_2|e_2) & (z_2|e_5) & (z_2|e_6) \end{pmatrix} \begin{pmatrix} p \\ q \\ r \\ s \end{pmatrix} = \begin{pmatrix} (e_3|b_2) \\ (e_4|b_2) \\ (z_1|b_2) \\ (z_2|b_2) \end{pmatrix}$$

$$\begin{pmatrix} (e_5|e_1) & (e_5|e_2) & (e_5|e_3) & (e_5|e_4) \\ (e_6|e_1) & (e_6|e_2) & (e_6|e_3) & (e_6|e_4) \\ (z_1|e_1) & (z_1|e_2) & (z_1|e_3) & (z_1|e_4) \\ (z_2|e_1) & (z_2|e_2) & (z_2|e_3) & (z_2|e_4) \end{pmatrix} \begin{pmatrix} p \\ q \\ r \\ s \end{pmatrix} = \begin{pmatrix} (e_5|b_3) \\ (e_6|b_3) \\ (z_1|b_3) \\ (z_2|b_3) \end{pmatrix}$$

- These matrices are implemented by the code by a series of if statements
- If $P_{pqrs}^{(1,2,3)} = 1$ then the state survives and if $P_{pqrs}^{(1,2,3)} = 0$ then the state is projected out
- The 4×4 matrix Δ^i and the right hand side of the equation Y^i call to the GGSO projection coefficient matrix in order to set the values in the brackets $(b_i|b_j)$
- The code then runs through the 16 distinct possibilities for p, q, r, s and records which combinations of p, q, r, s survive

Chirality Conditions

- In a similar manner to the projectors, the chirality of the surviving states are found
- The GSO projections of each basis vector in the model is imposed on the sector in order to find the string states which survive and with what chirality
- All possible states after GSO projecting on the sector are currently calculated by hand and the phases of the GSO projection are encoded using a ladder of if-else statements
- *I.e* The if-else ladder performs the GSO projections by calling to the relevent phases in the GGSO projection coefficient matrix of the vacua

GGSO Projection Coefficients

An example of the GGSO projection coefficients for a Left-Right Symmetric (LRS) model is

$$
(b_i|b_j) = \begin{array}{c} \\ \mathbb{1} \\ S \\ e_1 \\ e_2 \\ e_3 \\ e_4 \\ e_5 \\ e_6 \\ b_1 \\ b_2 \\ z_1 \\ z_2 \\ \alpha \end{array}
\begin{array}{c}
\begin{array}{ccccccccccccc} \mathbb{1} & S & e_1 & e_2 & e_3 & e_4 & e_5 & e_6 & b_1 & b_2 & z_1 & z_2 & \alpha \end{array} \\
\left(\begin{array}{ccccccccccccc}
1 & 1 & 1 & 1 & 0 & 1 & 0 & 0 & 0 & 1 & 1 & 1 & 1.5 \\
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
1 & 1 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 \\
1 & 1 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 1 & 0 \\
0 & 1 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
1 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 \\
0 & 1 & 1 & 1 & 0 & 0 & 0 & 1 & 0 & 1 & 1 & 1 & 1 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
1 & 0 & 1 & 1 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 \\
1 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 1 \\
1 & 1 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 \\
1 & 1 & 0 & 0 & 1 & 0 & 1 & 1 & 0 & 1 & 0 & 1 & 1
\end{array}\right)
\end{array}
$$

Aims of Classification

The classification of a model then refers to finding vacua consistent with our phenomenological constraints, such as

- Three chiral generations of observable matter
- $\mathcal{N} = 1$ SUSY
- No exotic states at the level of the Standard Model
- No observable gauge group enhancements
- Constraints on the number and type of Higgs particles in the spectrum

The classification of the model then refers to enumerating the number of vacua in the model consistent with these constraints

Previous Results - Pati-Salam Classification - arXiv:1007.2268

- Previous classifications have been performed for Pati-Salam models
- The models discussed here use the basis vectors shown previously along with the basis vector

$$\alpha = \{\overline{\psi}^{4,5}\ ,\ \overline{\phi}^{1,2}\}$$

Previous Results - Pati-Salam Classification - arXiv:1007.2268

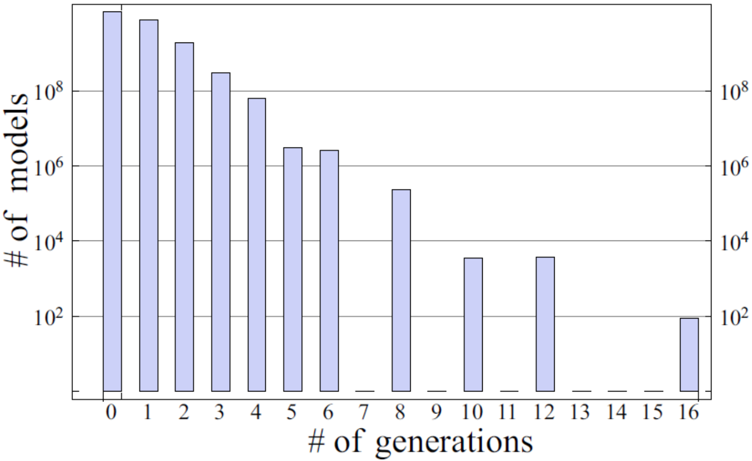

Figure: Number of models versus number of generations in a random sample of 10^{11} GGSO configurations

Previous Results - Pati-Salam Classification - arXiv:1007.2268

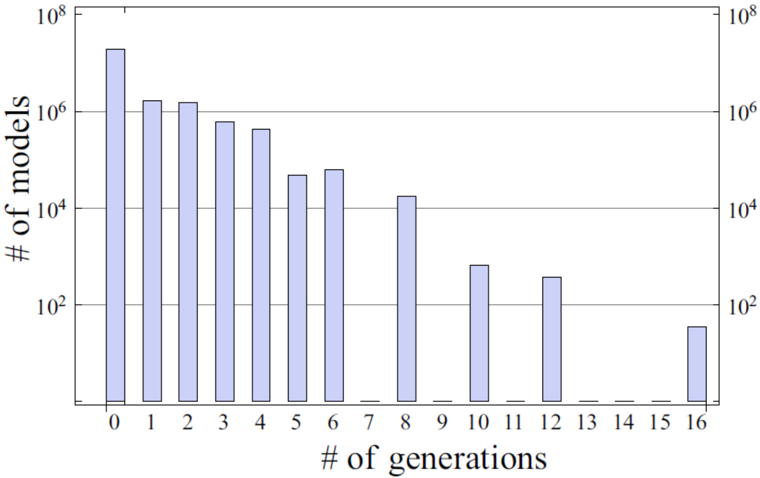

Figure: Number of exotic free models versus the number of generations in a random sample of 10^{11} GGSO configurations

Previous Results - Pati-Salam Classification - arXiv:1007.2268

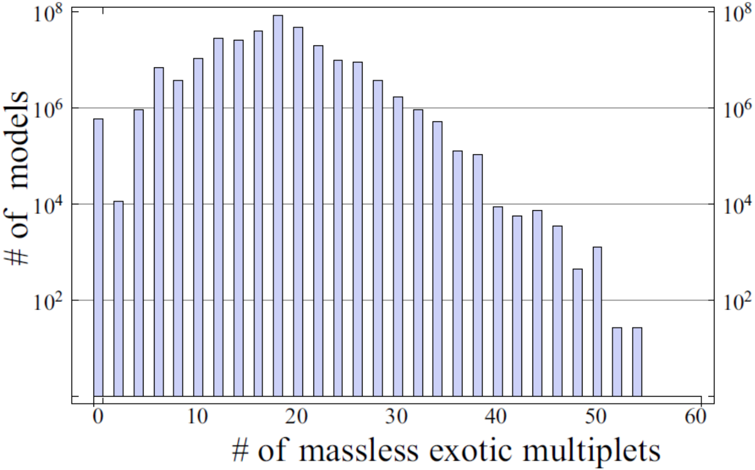

Figure: Number of 3-generation models versus the total number of exotic multiplets in a random sample of 10^{11} GGSO configurations

Previous Results - Pati-Salam Classification - arXiv:1007.2268

	constraint	# of models in sample	probability	estimated # of models in class
	None	100000000000	1	2.25×10^{15}
(a)	+ No gauge group enhancements.	78977078333	7.90×10^{-1}	1.78×10^{15}
(b)	+ Complete families	22497003372	2.25×10^{-1}	5.07×10^{14}
(c)	+ 3 generations	298140621	2.98×10^{-3}	6.71×10^{12}
(d)	+ PS breaking Higgs	23694017	2.37×10^{-4}	5.34×10^{11}
(e)	+ SM breaking Higgs	19191088	1.92×10^{-4}	4.32×10^{11}
(f)	+ No massless exotics	121669	1.22×10^{-6}	2.74×10^{9}
(g)	+ Minimal PS Higgs	31804	3.18×10^{-7}	7.16×10^{8}

Figure: Pati-Salam models statistics with respect to phenomenological constraints imposed on massless spectrum

Previous Results - FSU(5) Classification - arXiv:1403.4107

The flipped $SU(5)$ models which have been classified used the same basis vectors discussed previously along with the basis vector

$$\alpha = \{\overline{\psi}^{1,\ldots,5} = \frac{1}{2}\ ,\ \overline{\eta}^{1,2,3} = \frac{1}{2}\ ,\ \overline{\phi}^{1,2,3,4} = \frac{1}{2}\ ,\ \overline{\phi}^{5}\}$$

Previous Results - FSU(5) Classification - arXiv:1403.4107

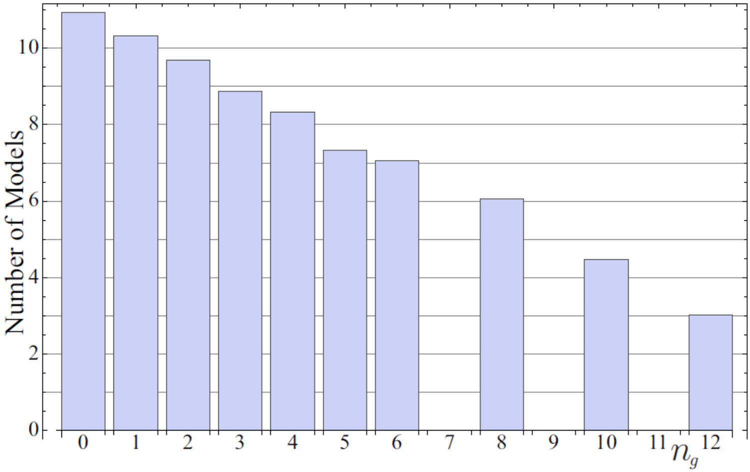

Figure: Logarithm of the number of models against the number of generations in a random sample of 10^{12} flipped $SU(5)$ configurations

Previous Results - FSU(5) Classification - arXiv:1403.4107

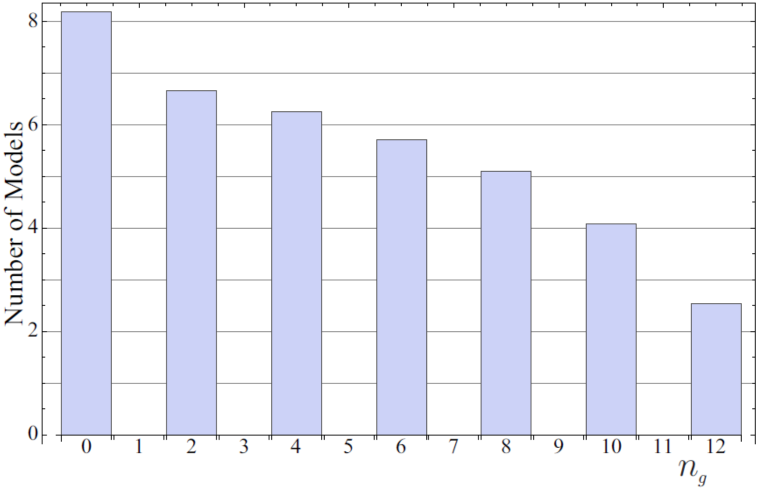

Figure: Logarithm of the number of exophobic models against the number of generations in a random sample of 10^{12} flipped $SU(5)$ configurations

Previous Results - FSU(5) Classification - arXiv:1403.4107

	Constraints	Total models in sample	Probability	Estimated number of models in class
	No Constraints	1000000000000	1	1.76×10^{13}
(1)	+ No Enhancements	762269298719	7.62×10^{-1}	1.34×10^{13}
(2)	+ Anomaly Free Flipped $SU(5)$	139544182312	1.40×10^{-1}	2.45×10^{12}
(3)	+ 3 Generations	738045321	7.38×10^{-4}	1.30×10^{10}
(4a)	+ SM Light Higgs	706396035	7.06×10^{-4}	1.24×10^{10}
(4b)	+ Flipped $SU(5)$ Heavy Higgs	46470138	4.65×10^{-5}	8.18×10^{8}
(5)	+ SM Light Higgs + & Heavy Higgs	43624911	4.36×10^{-5}	7.67×10^{8}
(6a)	+ Minimal Flipped $SU(5)$ Heavy Higgs	42310396	4.23×10^{-5}	7.44×10^{8}
(6b)	+ Minimal SM Light Higgs	25333216	2.53×10^{-5}	4.46×10^{8}
(7)	+ Minimal Flipped $SU(5)$ Heavy Higgs + & Minimal SM Light Higgs	24636896	2.46×10^{-5}	4.33×10^{8}
(8)	+ Minimal Exotic States	1218684	1.22×10^{-6}	2.14×10^{7}

Figure: Statistics for the flipped $SU(5)$ models with respect to phenomenological constraints

Left-Right Symmetric

- Currently, there is a classification running on a LRS model
- This uses the basis vector

$$\alpha = \{\overline{\psi}^{1,2,3} = \frac{1}{2}\,,\ \overline{\eta}^{1,2,3} = \frac{1}{2}\,,\ \overline{\phi}^{1,\ldots,6} = \frac{1}{2}\,,\ \overline{\phi}^{7}\}$$

- The sample size is 1×10^{11} vacua
- Models have been found which are not enhanced, 3 generation, have a light Standard Model and heavy Higgs, are anomaly free and have a top quark mass coupling
- So far, there have been no 3 generation exophobic vacua found and it looks unlikely that this model will yield them
- The results will be be presented in an upcoming publication

Previous Machine Learning Applications - arXiv:1404.7359

- Genetic algorithms have already been applied to the Free Fermionic Formulation by Steve Abel and John Rizos
- In their paper they successfully showed that using genetic algorithms makes searches for phenomenologically viable vacua much more efficient
- An example search for vacua with 3 generation exophobic Pati-Salam models with a top Yukawa coupling was used
- Conventionally, one of these models occurs in every 10^{10} scanned
- Using a genetic algorithm one can be found in every 10^{5}, which is drastically more efficient

Reinforcement Learning Project

- The project to involve machine learning in the classification analysis has begun
- Currently a parallel code is being written to analyse the observable sectors of a LRS model, meaning the number of generations of a model can be calculated and the Higgs states found
- This code will be able to run to find 3 generation models with a light and heavy Higgs which would compromise a training set of data
- The aim of the project is to adapt this code to use reinforcement learning technqiues in order to find patterns in the linear combinations of GGSO projection coefficients which give rise to 3 generation models and / or Higgs particles

Reinforcement Learning Project

- It has been found in previous cases that if certain linear combinations of the GGSO projection coefficients are fixed then ‘fertile regions’ of 3 generation models can be found
- The idea is to use reinforcement learning to find what linear combinations of GGSO projections lead to these fertile regions
- Currently, the GGSO projection coefficients are set randomly and the resulting models analysed by the code. Reinforcement learning would be used to discover patterns in fixing the GGSO projection coefficients which lead to phenomenologically desirable results

Reinforcement Learning Project

- The idea is to think of the GGSO projection matrix analogously to a game board
- The reward function would gain a positive value for leading to models with phenomenologically desirable results (such as being 3 generations or having a Higgs)
- If the model has 'bad' phenomenology, such as not having complete generations, then the reward function would take a negative value
- In this way, this reinforcement learning project can work similarly to programs such as alphaGO
- In the same way as alphaGO having a winning game and therefore assigning a positive reward value to the board states it used to get there, a GGSO projection matrix configuration which leads to 'good' phenomenology would be assigned a positive value and vice versa in a negative case

Conclusions

- The Free Fermionic Formulation provides a robust framework in which to study the phenomenological properties of string vacua
- Classifications of Pati-Salam and $FSU(5)$ vacua have been performed to find the statistics of the number of 3 generation models, number of exotic multiplets and the number of exophobic vacua
- The classification of Left-Right Symmetric models will be presented in a future publication
- Previously, genetic algorithms have been applied to the FFF with good results
- The current work aims to use reinforcement learning to find fertile regions of 3 generation models of LRS vacua

Thank you for listening

I welcome any input on ideas about how this can be achieved or what limitations this method may have