## Vacuum Selection from Cosmology on Networks of String Geometries

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Based on

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## **Physics Summary**

- We construct the largest network(s) of string geometries/ vacua to date, where the geometries are connected by topological transitions (motion in moduli space).
- We argue that such topological transitions correspond to leading-order vacuum transitions, a la Coleman-De Lucialike bubble nucleation.
- Simple models of cosmology using such transitions provide a notion of vacuum selection, where the network structure of the geometries plays a major role.

# string\_data Summary

- Demonstration of interesting physics induced by global structure of non-trivial network of geometries.
- Decently involved computational effort with networks this large, in both constructing the network and performing computations with the adjacency matrix.
- Network has **41,873,645** nodes and **100,036,155** edges.

## Outline

- The string landscape, bubbles, and networks.
- Geometric networks in string theory
- Concrete networks of geometries.
- Examples of geometry selection.

## The String Landscape, Bubbles, and Networks

#### The string landscape

• There is a vast landscape of vacua in string theory; thought to be very large  $\mathcal{O}(10^{500})$  ( $\mathcal{O}(10^{755}), \mathcal{O}(10^{3000}), \mathcal{O}(10^{272,000})$ )

Ashok, Denef, Douglas

Halverson, CL, Sung Taylor, Wang

- Each yields an 4d effective field theory.
- Vast size of the landscape arises from the plethora of possible geometries of the extra compact dimensions and choices of discrete objects on the geometries.

#### The need for vacuum selection

- Why is our universe is selected?
  - 1. Standard Model might be generic, but this is not established.

see e.g. Grassi, Halverson, Shaneson, Taylor

- 2. Anthropic principle does not seem be to be the complete story.
- 3. A (partially) satisfacory explanation: Certain vacua are selected over others, via some early dynamics in the landscape.

#### What does vacuum selection mean?

A few things we know from string theory and effective field theory:

- 1. There are many vacua, with varying physical properties.
- 2. Local vacuum transitions, known as bubble nucleation, can occur. Coleman, De Lucia
- 3. Bubbles can grow, collapse, nucleate other bubbles.

The vacuum distribution is a late-time, steady state solution of such a bubble nucleation model. Vacuum selection would be a distribution that prefers some vacua above others.

#### **Bubble nucleation**



- Original work due to Coleman and De Lucia, in a single effective field theory.
- Universe starts in false vacuum  $\phi_+$  , nucleates bubbles in  $\phi_-$



#### **Bubble nucleation and graphs**

- In general, there can be many vacua.
- Given a vacuum i, bubbles in another vacuum j can form in local patches.
- Bubble nucleation rates  $\Gamma_{ij}$  from vacuum **i** to vacuum **j** depend on microphysics.
- Picture:
  - 1. Start in vacuum i.
  - 2. Nucleate a bubble in **j**.
  - 3. j nucleates a bubble in k, i nucleates another j bubble, and so on.



#### **Bubble nucleation and graphs**

The set of vacua i, and the nucleation rates, define a weighted graph:



Network is input for a cosmological bubble nucleation model.

#### A network of vacua

- A great deal of focus on the vacua (nodes) themselves.
- However, the the connections (edges) between the vacua correspond to tunneling rates. EFT: bubble nucleation will produce these vacua if the rate is non-zero!
- In general we cannot just choose a vacuum by hand and assume it is the end of the story. Need to study the global cosmological structure to see what vacua are selected!

#### A Simple Model of Bubble Nucleation

- Introduced networks, now we can ask what the network structure predicts for bubble nucleation models.
- Simplified model of bubble nucleation (assume bubbles do not collapse). Based on J. Garriga, D. Schwartz-Perlov, A. Vilenkin, and S. Winitzki, see also D. Harlow, S. H. Shenker, D. Stanford, and L. Susskind

 $N_j$  : number of bubbles in vacuum j.

 $\Gamma_{ij}$ : bubble nucleation rate from vacuum j to vacuum i.



#### A Simple Model of Bubble Nucleation

 $\frac{d\mathbf{N}}{dt} = \Gamma\mathbf{N}$ 

Solution: 
$$\mathbf{N} = e^{\Gamma t} \mathbf{N}_0 = \sum_p a_p e^{\gamma_p t} \mathbf{v}_p$$

- $\mathbf{N}_0$  : initial vacuum numbers
- $\gamma_p$  ,  $\mathbf{v}_p$  : eigenvalues, eigenvectors of  $\Gamma$
- $a_p$  : initial conditions

#### A Simple Model of Bubble Nucleation

#### Late time solution:

- Let  $\ \gamma_0$  ,  $\mathbf{v}_0$  largest eigenvalue, eigenvector of  $\ \Gamma$
- As  $t 
  ightarrow \infty$   $\, {f N}\,$  is dominated by the largest eigenvector of  $\, \Gamma \,$

$$\mathbf{N} \to a_0 e^{\gamma_0 t} \mathbf{v}_0$$

However, the entries become infinite as  $\,t
ightarrow\infty$ 

Define the fractional distribution of vacua:  $~~{f p}={f N}/|{f N}|$ 

**p** is well-defined, and independent of initial condition.

#### A non-trivial distribution in p indicates vacuum selection!

To solve we need to determine  $\Gamma$  in our model, and ensure that the answer makes sense (i.e. no negative entries in **p**).

#### A Better Model of Bubble Nucleation

- The last model was a simplified one, designed to capture interesting dynamics without messy details.
- However, it is probably too simple: fails to capture the fact that a larger bubble has more volume, and therefore can produce more bubbles.
- More realistic model: original one of Garriga, Schwartz-Perlov, Vilenkin, and Winitzki (GSVW).
- This is (very recent) work in progress, so we'll use the toy model to demonstrate how the non-trivial network structure plays a role, and report initial progress using the GSVW model.

#### The GSVW Model

 $f_j$  fractional (comoving) volume distribution of jth vacuum

$$\frac{df_j}{dt} = \sum_i (-\kappa_{ij}f_j + \kappa_{ji}f_i), \qquad \kappa_{ij} = \Gamma_{ij}\frac{4\pi}{3}H_j^{-4},$$

Can be written in matrix notation

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$$egin{aligned} rac{d\mathbf{f}}{dt} &= \mathbf{M}\mathbf{f}, & M_{ij} &= \kappa_{ij} - \delta_{ij} \sum_r \kappa_{ri}. & \mathbf{Consistency: requires the} \ \mathbf{f}(t) &= \mathbf{f^{(0)}} + \mathbf{s}e^{-qt} + ... & \mathbf{M}\mathbf{f_0} &= \mathbf{0}. \end{aligned}$$
 $p_j &\propto \sum_{lpha} H^q_{lpha} \kappa_{j lpha} s_{lpha}. \end{aligned}$ 

 $s_{lpha}$  eigenvector corresponding to least-negative non-zero eigenvalue

We will keep both models in consideration.

# Steps for vacuum selection in string theory:

- 1. Construct the network of vacua in the string landscape.
- 2. Model cosmological evolution using  $\Gamma_{ij} \rightarrow$  differential equation.
- 3. Solve for the largest eigenvector of  $\Gamma$  (or **M**), which provides a notion of vacuum selection.

First step: construct the nodes (vacua) and edges (nucleation rates).

## Geometric networks in string theory

# What data defines a metastable vacuum in string theory?

- 1. Choice of compact manifold, perhaps with special holonomy.
- 2. Choices of objects: flux and branes.
- 3. Choice of solution to equations of motion.

These are what I would call "typical string vacua", at least in the corners of the landscape that we understand best.

These gives the **nodes** in the network.

In general these vacua are hard to construct explicitly, so we will consider a coarse-grained toy model: **nodes = geometries**.

#### Bubble nucleation between vacua

Transitions between vacua in the landscape seem rather complicated. One generically expects tunneling between many, if not all, of the vacua, and the tunneling is outside of the realm of effective field theory.

However, there is a set of transitions, **geometric transitions**, that might play a special role.

These geometric transitions give the geometries a graph structure.

These are completely general in string theory, but to discuss them I will specialize to F-theory.

#### F-theory (see Timo's talk)

- IIb with 7-branes, varying axiodilaton.
- More general than IIb with D7-branes. Allows for strong coupling.
- Mathematically described by a Calabi-Yau elliptic fibration over base B, where B is the internal space.

$$y^2 = x^3 + fx + g$$

• Singularities of elliptic fibration specify location and type of 7-brane in B.

$$\Delta = 4f^3 + 27g^2 = 0$$

• The type of 7-brane determines a gauge theory localized on the 7-brane. Matter lives at intersections of 7-branes.

#### A few details

• Elliptic fibration parametrized by Weierstrass equation

$$y^2 = x^3 + f(z_i)x + g(z_i)$$

f and g are global sections of line bundle:

 $f \in \Gamma(\mathcal{O}(-4K_B))$   $g \in \Gamma(\mathcal{O}(-6K_B))$ 

• Singularities of the elliptic fibration play an important role. The subloci where the elliptic fiber becomes singular is given by the discriminant locus:

$$\Delta = 4f^3 + 27g^2 = 0$$

Physically, the vanishing of the discriminant marks the location of 7-branes, which are divisors in **B**. Let **z** be a local coordinate where the fiber becomes singular. The MOV (a, b, c) along **z**= **0** of (**f**, **g**, Δ), respectively, determine, by Kodaira's classification of singular fibers, a geometric gauge group, which is some of the data for our physical theory.

$$f = z^a F \quad g = z^b G \quad \Delta = z^c \tilde{\Delta}$$

#### **Non-Higgsable 7-branes**

		1		d.	a	
F	a	b	c	Sing.	G	
$I_0$	$\geq 0$	≥ 0	0	none	none	
$I_n$	0	0	$n \ge 2$	$A_{n-1}$	$SU(n)$ or $Sp(\lfloor n/2 \rfloor)$	
II	$\geq 1$	1	2	none	none	
III	1	$\geq 2$	3	$ A_1 $	SU(2)	
IV	$\geq 2$	2	4	$A_2$	SU(3) or $SU(2)$	Kodaira
$I_0^*$	$\geq 2$	$\geq 3$	6	$D_4$	$SO(8)$ or $SO(7)$ or $G_2$	Roualia
$I_n^*$	2	3	$n \ge 7$	$D_{n-2}$	SO(2n-4) or $SO(2n-5)$	
$ IV^* $	$\geq 3$	4	8	$E_6$	$E_6$ or $F_4$	
$ III^* $	3	$\geq 5$	9	$E_7$	$E_7$	
$II^*$	$\geq 4$	5	10	$E_8$	$E_8$	

 In some cases, we may have a > 0 and b > 0 for all choices of complex structure moduli.

#### **Morrison, Taylor**

 The locus z = 0 then has a <u>non-Higgsable 7-brane</u>. We say these are non-Higgsable because the fiber type is independent of complex structure, and the gauge group therefore cannot be broken/changed by a complex structure deformation (Higgsing).

#### Some selective progress: Halverson, Grassi, Morrison, Shaneson, Taylor, Wang

#### Data for a compactification



Many choices for this data gives many vacua. We want to understand the possible transitions between them.

#### The numbers

- Original estimate of number of flux vacua on a single geometry:  $\mathcal{O}(10^{500})_{\rm Ashok, \, Denef, \, Douglas}$
- Select geometries can support many more:  $\mathcal{O}(10^{272,000})$  Taylor, Wang

- On the other hand, there is an explicit lower bound on the number of geometries (this talk):  $\mathcal{O}(10^{755})_{\text{Halverson, CL, Sung}}$
- Estimate for the actual number is:  $\mathcal{O}(10^{3000})$   $^{\rm Taylor, Wang}$

There is a large landscape of vacua!

#### **Geometric Transitions**

- When 7-branes stack up enough and intersect, a new branch of moduli space appears.
- Blowing up along this cycle separates the 7-branes, and changes the topology of the compact directions in space.



• Technically,  $MOV_C(f,g) \ge (4,6)$  or  $MOV_P(f,g) \ge (8,12)$ 

Hayakawa, Wang, Morrison

- This corresponds to a crepant resolution of the corresponding fourfold, and is therefore motion in Calabi-Yau moduli space.
- This is a **topological transition** in the extra spatial dimensions, that breaks apart interesting branes.

#### **Base transitions**

- Starting with an elliptically fibered Calabi-Yau X -> B, one can crepantly pass to another elliptically fibered Calabi-Yau X" -> B' by a base-change, and pass to a minimal Weierstrass model.
- This procedure is
  - Perform a blowup B' -> B in the base along a subvariety C and perform a base change

$$X' = X \times_{B} B' \to B'$$

 Perform a change of coordinates and pass to a minimal Weierstrass model X" -> B'.

#### Candelas, Diaconescu, Florea, Morrison, Rajesh

• For this procedure to be crepant we need

 $MOV_C(f,g) \ge (4,6)$  if C is a curve in B  $MOV_C(f,g) \ge (8,12)$  if C is a point in B

 This produces a new elliptic Calabi-Yau X" -> B', with a new base B' which is a blowup of B.

#### **Geometric Transitions**

- Geometric transitions are possible, and we expect them to happen quantum mechanically via instantons.
- At leading order there is a moduli space M, and the different geometries correspond to different points/ regions of the moduli space.
- Expectation: Coleman-De Lucia type instanton mediates between geometries.



#### **Geometric Transitions**

- However, the situation is more complicated:
  - 1. The theory is N = 1, so the general expectation is that the moduli space will be lifted, so the vacua are discrete. This seems to be true in examples.
  - 2. The geometries in general do not correspond to the same effective field theory, and so the relevant instantons must be a generalization of the usual ones of Coleman-De Lucia (generalization of distance on field space).
  - 3. The geometries are separated by regions with complicated, non-perturbative physics (tensionless strings, CFT points).



## Modeling $\Gamma$

- To calculate  $\Gamma\,$  we need detailed information about the microphysics.
- In general these vacua are not even in the same effective 4d field theory, so analysis goes beyond the original Coleman-De Lucia story.
- Main assumption: the dominant bubble nucleation process is between geometries that are directly connected by a single topological transition.

## Motivation for nearby geometries

- We want to generalize the Coleman-De Lucia result, which involves a measure of distance between vacua.
- The network structure provides such a distance, somewhat natural as the network captures motion in moduli space.
- Multiple topological transitions require tuning to higher codimension in moduli space, so we expect nearby geometric transitions to be preferential.



 (Before moduli stabilization/quantum effects, these transitions correspond to motion in moduli space. Finite temperature (for instance) could allow for such fluctuations to occur dynamically).

## Modeling $\Gamma$

- Missing information about the vacua themselves: fluxes, vacuum energies, etc.
- First step will be to isolate how the graph structure affects vacuum selection, and so we consider a simplified model:

$$\Gamma = \alpha \mathbf{A}$$

- $\alpha$  constant that determines overall transition rate
- A is the adjacency matrix of the graph: has entry 1 if two geometries are connected, zero otherwise.

## The simplified model

 $\Gamma = \alpha \mathbf{A}$ 

Let  $\ \gamma_0$  ,  $\mathbf{v}_0$  largest eigenvalue, eigenvector of  $\ \Gamma$ 

As  $t 
ightarrow \infty$   $\, {f N}\,$  is dominated by the largest eigenvector of  $\, \Gamma \,$ 

 $\mathbf{N} \to a_0 e^{\gamma_0 t} \mathbf{v}_0$ 

Define the fractional distribution of vacua:  $~~{f p}={f N}/|{f N}|$ 

In this case, p is the largest eigenvector of the adjacency matrix of the network, also called the eigenvector centrality of the network.

Perron-Frobenius theorem: **p** is strictly positive if **A** is the adjacency matrix of a connected graph, so the interpretation as a fractional vacuum distribution is sensible!

## The GSVW Model $M_{ij} = \kappa_{ij} - \delta_{ij} \sum_{r} \kappa_{ri}.$ $\kappa_{ij} = \Gamma_{ij} \frac{4\pi}{3} H_j^{-4},$ $p_j \propto \sum_{\alpha} H_{\alpha}^q \kappa_{j\alpha} s_{\alpha}.$

 $S_{lpha}$  eigenvector corresponding to least-negative eigenvalue

• We want to isolate the effect of the graph structure, so for now set

$$H_a = H = 1$$

- Consistency requires terminal vacuum, so we assign one terminal and one non-terminal vacuum per geometry.
- Need to compute smallest-magnitude eigenvalue of A 2D.
   In both models the vacuum selection is determined by some graph property.

# Concrete networks of geometries

#### Networks of geometries

 Both are ensembles of Calabi-Yau's associated with toric varieties. The one we are interested in admit a combinatorial description as triangulations of polytopes.

 The Tree ensemble: Calabi-Yau Elliptic fibrations over toric 3-folds.

Today

- 2. The hypersurface ensemble: Calabi-Yau hypersurfaces in toric 4-folds.
- In both cases, nodes represent Calabi-Yau geometries, and edges represent blowups between the geometries.

#### **Toric combinatorics**

- Toric varieties are combinatorial: they admit a description in terms of a fan of rational polyhedral cones.
- Each ray of the fan  $v_i$  corresponds to a homogeneous (toric) coordinate  $x_i$  and therefore each ray corresponds to a divisor

$$D_i = \{x_i = 0\}$$

- n-dimensional cones then correspond to codimension-n subvarieties, by setting the corresponding toric coordinates to zero.
- Example:  $\mathbb{P}^2$



#### **Toric varieties and polytopes**

 Some fans correspond to face fans of triangulations of boundaries of reflexive polytopes. Such reflexive polytopes give a rich class of toric varieties.

Batyrev, Kreuzer, Skarke



A fine, regular, star triangulation (FRST) of a 3d reflexive polytope corresponds to a smooth projective toric 3-fold.

- These toric varieties are weak Fano toric varieties (WFTV), and the generic CY 4-fold elliptic fibrations over them are smooth, which implies there are no non-Higgsable 7-branes, and no gauge groups generically.
- There are 4319 3d reflexive polytopes, and ~10^15 triangulations total, and so these are a rich class of toric threefolds.

Halverson, Tian, Carifio, Kriokov, Nelson

Halverson, CL, Sung

1. Start with a weak Fano toric 3-fold base, corresponding to a triangulated 3d reflexive polytope. Defines a **Fan** with rays  $v_i$ .



2. Blowup subvarieties to reach a new toric base. Combinatorially described by adding new ray  $v_e$  to the fan, corresponding to a new exceptional divisor  $D_e$ .

$$v_e = \sum_i a_i v_i$$



• Define the height of a blowup as

$$h = \sum_{i} a_{i}$$

- In general, can blow up along
  - 1. Toric curves  $\langle \rangle$  edges in the triangulated polytope.



Growing a tree above the edge! Disclaimer: not a graph theory tree.

2. Toric points  $\langle - \rangle$  faces (triangles) in the triangulated polytope.





 To visualize, it's easier to project all rays back onto the polytope, so 'growing a tree' corresponds to subdividing edges and faces.





 Calabi-Yau elliptic fibrations over these bases form a connected moduli space, related by topological transitions, if a technical condition is satisfied, which is

#### $MOV_{D_e}(g) < 6 \mbox{ or } MOV_{D_e}(f) < 4 \mbox{ Hayakawa, Wang}$

 A sufficient condition to ensure that each Calabi-Yau is connected in moduli space limits the possible blowups in a given local patch to a finite set, rendering the ensemble finite.

 $MOV_{D_e}(g) < 6 \leftrightarrow h(v_e) \leq 6 \ \text{for all} \ v_e \ \text{ Halverson, CL, Sung}$ 

• The topological transitions give this ensemble a network structure: **geometries are nodes**, and **topological transitions are edges**.

#### Factorizing blowups

 Easiest to describe in combinatorial language with a picture. Consider this FRST of a face of a 3d reflexive polytope, and the following observation about toric blowups:

Subdivision of edges does not respect original FRST of polytope.

Subdivision internal to a face (2-simplex) DOES respect original FRST of polytope!

 If we first consider sequences of subdivisions (blowups) internal to each face on the original triangulation, then we can perform such blowups without affecting the toric fan elsewhere, and so can work locally with each face. We can then subdivide each edge, corresponding to blowups of toric curves.



#### The Edge Network $\ensuremath{N_E}$

• First consider blowup of curves. Toric curves correspond to edges in the triangulation.



• A single toric curve, corresponding to an edge in the triangulation, admit 82 configurations of blowups.

These configurations form a network  $N_E$  with 82 nodes and 1386 edges.

#### The Face Network $N_F$



A toric point corresponds to a triangle in the triangulation.



These configurations form a network  $N_F$  with **41,873,645** nodes and **100,036,155** edges.

#### The Tree Network



Ensemble of tree geometries overwhelmingly generated by blowups of toric varieties corresponding to two reflexive polytopes with the most edges and triangles.

Each has 108 toric curves (edges) and 72 toric points (triangles) when triangulated.



#### The Tree Network



• A generic network with  $10^{755}$  nodes would be completely intractable, but this network factorizes into a cartesian product of graphs:



Cartesian product =  $\Box$ 

#### The Tree Network



- The tree network  $N_{\mathrm{tree}}$  factorizes as

$$N_{\text{tree}} = N_E^{\square \, 108} \,\square \, N_F^{\square \, 72}$$

• Simply put, two geometries in the Cartesian product are adjacent if they are related by a single blowup in a single local patch.

#### By understanding $N_E$ and $N_F$ we can learn about $N_{\rm tree}$ !

#### Universality



- Enormous number of geometries, too many to scan.
- However, understanding the construction algorithm allows us to read off the minimal geometric gauge group in terms of simple combinatorial data, with probability  $\geq 0.999995$

$$G \ge E_8^{10} \times F_4^{18} \times U^9 \times F_4^{H_2} \times G_2^{H_3} \times A_1^{H_4} \qquad U \in \{G_2, F_4, E_6\}$$
$$rk(G) \ge 160 + 4H_2 + 2H_3 + H_4$$

 $H_2, H_3, H_4$  are number of height 2,3,4 blowups.

There actually are, definitively, over 10^500 string geometries. and we have an exact lower bound.

We can actually control this ensemble, through precise knowledge of the construction algorithm.

### Strong Coupling



- Non-Higgsable clusters, so generic points in moduli space are strongly coupled. Do any of these geometries admit a Sen limit?
- In recent work, we worked out requirements for the existence of a Sen limit in the following cases: Halverson, CL, Sung
  - 1. Toric bases.
  - 2. Algebraic bases constructed from gluing local patches, where the local patches are crepant resolutions of orbifold singularities.
- Applied to the tree ensemble, the fraction that admits a Sen limit is  $3 \times 10^{-391}$  (See Jim's talk)

Not only do generic points in moduli space have strong coupling points, but all subloci do as well!

# Examples of geometry selection

## The Simplified Model



• We will look at the simplified cosmological model for now, as the GSVW model is in progress.

Recall 
$$N_{\text{tree}} = N_E^{\Box \, 108} \,\Box \, N_F^{\Box \, 72}$$

A useful fact is that  $\mathbf{p}(N_{\text{tree}}) = \mathbf{p}(N_E)^{\otimes 108} \otimes \mathbf{p}(N_F)^{\otimes 72}$ 

We therefore need to calculate  $\ \mathbf{p}(N_E)$  ,  $\ \mathbf{p}(N_F)$ 







Largest entry is 0.07, 98 percent of the entries are at least a factor of 1000 smaller. Ratio of largest to smallest is ~  $10^7\,$ 

# The Simplified Model

 $N_E$  a much smaller, less interesting network, but still a non-flat distribution.

Full tree network: Ratio of largest to smallest eigenvector centrality ~

$$7 \times 10^{1457}$$

## This is a measure of maximal geometry selection in the tree network.

While it is a toy model, it is a coarse grained toy model of actual huge string networks!

Main lesson: non-trivial graph structure in networks of F-theory geometries gives rise to geometry selection in a simple bubble nucleation cosmology.



Selected node in  $N_F$  :





Full tree network:

 $E_8^{37} \times F_4^{85} \times G_2^{220} \times SU(2)^{320}$ 



 $s_{lpha}$  eigenvector corresponding to least-negative eigenvalue

Full tree network: Ratio of largest to smallest eigenvector centrality ~

 $10^{1555}$ 

#### Recap

- The landscape of string vacua naturally has a network associated with it: nodes = vacua, edges = bubble nucleation rates.
- Introduced large networks of string geometries.
- We considered a toy model for vacuum selection using network science that can give rise to large selection factors, and we demonstrated that is does in concrete geometric networks.
- GSVW model is standard model for bubble nucleation, still has large selection factors from network structure!

### Work in progress

- Consider a distribution of Hubble constants/CCs on each geometry, allow for multiple vacua per geometry, estimate using flux/geometric techniques.
- Allow for the graph to be weighted via flux vacua estimates.

## Musings

- Ways to to move away from the toy regime: fluxes, mobile branes, vacuum energies.
- Transitioning between vacua in string theory interpolates between different effective field theories, important to understand better.
- Dynamics of geometric transitions and relevant instantons need to be calculated.
- Would be interesting to expand the graphs (non-toric, etc.).

## Thanks!