

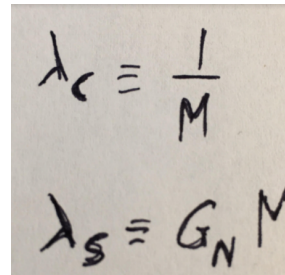
our “quantum-gravity phenomenological models” will turn out to be (at best) like the Bohr-Sommerfeld quantization...

even the assumption that the quantum-gravity scale should coincide with the Planck scale should be viewed as just a weak guess:

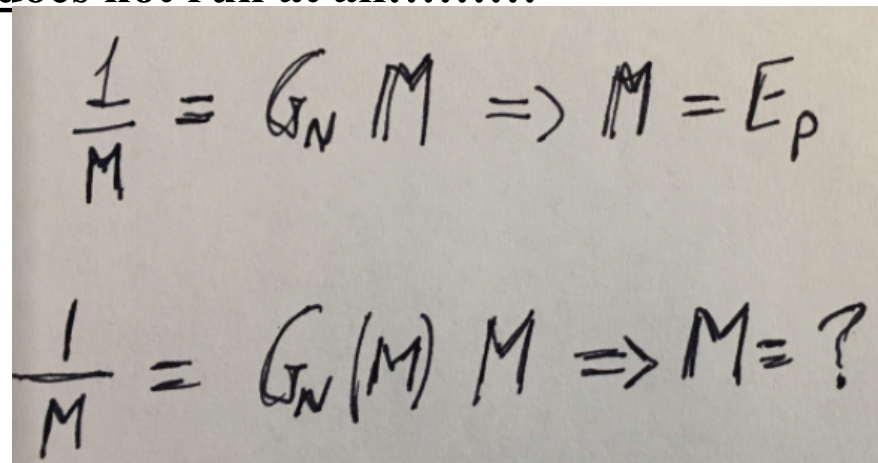
$$E_{\text{QG}} \sim E_{\text{Planck}} = 1.2 \cdot 10^{19} \text{ GeV} = \left(\frac{\hbar c^5}{G} \right)^{\frac{1}{2}} \quad \text{i.e. } 10^{-35} \text{ meters (“Planck length”)}$$

mainly comes from observing that at the **Planck scale**

$$\lambda_C \sim \lambda_S$$


$$\lambda_C \equiv \frac{1}{M}$$
$$\lambda_S \equiv G_N M$$

Note that it is only rough order-of-magnitude estimate at best in particular this estimate assumes that G does not run at all!!!!!! it most likely does run!!!


$$\frac{1}{M} = G_N M \Rightarrow M = E_p$$
$$\frac{1}{M} = G_N(M) M \Rightarrow M = ?$$

expected many new structures for the quantum gravity realm...

of particular interest for phenomenology the possible implications for relativistic symmetries (Lorentz, Poincarè,...)

Planck length as the minimum allowed value for wavelengths:

- suggested by several indirect arguments combining quantum mechanics and GR
- found in some detailed analyses of formalisms in use in the study of the QG problem

But the minimum wavelength is the Planck length for which observer?

GAC, ModPhysLettA (1994)
PhysLettB (1996)

Other results from the 1990s (mainly from spacetime noncommutativity and LoopQG) provided “theoretical evidence” of **Planck-scale modifications of the on-shell relation**, in turn inviting us to scrutinize the fate of relativistic symmetries at the Planck scale

Toward the mid 1990s these observations led several researchers to work at the hypothesis that in order to address the quantum-gravity problem one should give up the relativity of observers (preferred-frame picture)

GAC+Ellis+Nanopoulos+Sarkar, Nature(1998)
Alfaro+Tecotl+Urrutia, PhysRevLett(1999)
Gambini+Pullin, PhysRevD(1999)
Schaefer, PhysRevLett(1999)

This would be “**Planck-scale broken Lorentz symmetry**”

[notice difference with SME (talk by Stecker): here looking for specific scenarios of Planck-scale Lorentz-symmetry breaking, in SME most general scenario of Lorentz-symmetry breaking is considered]

but from 2000 onwards together with broken Lorentz symmetry
there starts to be a literature on the possibility
of “Planck-scale deformations of relativistic symmetries”
[jargon: “DSR”, for “doubly-special”, or “deformed-special”, relativity]

GAC, grqc0012051, IntJournModPhysD11,35

hep-th/0012238, PhysLettB510,255

Kowalski-Glikman, hep-th/0102098, PhysLettA286,391

Maguero+Smolin, hep-th/0112090, PhysRevLett88,190403

grqc/0207085, PhysRevD67,044017

GAC, grqc/0207049, Nature418,34

change the laws of transformation between observers so that the new properties
are observer-independent

- * a law of minimum wavelength can be turned into a DSR law
- * could be used also for properties other than minimum wavelength,
such as deformed on-shellness, deformed uncertainty relations...

The notion of DSR-relativistic theories is best discussed in analogy with the transition
from Galileian Relativity to Special Relativity

analogy with Galilean-SR transition

introduction to DSR case is easier starting from reconsidering the Galilean-SR transition (the SR-DSR transition would be closely analogous)

Galilean Relativity

on-shell/dispersion relation $E = \frac{p^2}{2m} \quad (+m)$

linear composition of momenta $p_\mu^{(1)} \oplus p_\mu^{(2)} = p_\mu^{(1)} + p_\mu^{(2)}$

linear composition of velocities $\vec{V} \oplus \vec{V}_0 = \vec{V} + \vec{V}_0$

Special Relativity

special-relativistic law of composition of momenta is still linear

$$p_{\mu}^{(1)} \oplus p_{\mu}^{(2)} = p_{\mu}^{(1)} + p_{\mu}^{(2)}$$

but the on-shell/dispersion relation takes the new form

$$E = \sqrt{p^2 + m^2}$$

of course (since c is invariant of the new theory) the special-relativistic boosts act nonlinearly on velocities (whereas Galilean boosts acted linearly on velocities)

and the special-relativistic law of composition of velocities is nonlinear, noncommutative and nonassociative

$$\mathbf{w} = \mathbf{v} \oplus \mathbf{u} \quad \begin{pmatrix} w_1 \\ w_2 \\ w_3 \end{pmatrix} = \frac{1}{1 + \frac{v_1 u_1 + v_2 u_2 + v_3 u_3}{c^2}} \left\{ \left[1 + \frac{1}{c^2} \frac{\gamma_{\mathbf{v}}}{1 + \gamma_{\mathbf{v}}} (v_1 u_1 + v_2 u_2 + v_3 u_3) \right] \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix} + \frac{1}{\gamma_{\mathbf{v}}} \begin{pmatrix} u_1 \\ u_2 \\ u_3 \end{pmatrix} \right\}$$

much undervalued in most textbooks, which only give composition of parallel velocities:

$$\frac{v + u}{1 + (vu/c^2)}$$

from Special Relativity to DSR

If there was an observer-independent scale E_P (inverse of length scale ℓ) then, for example, one could have a modified on-shell relation as relativistic law

$$m^2 = \Lambda(E, p; E_P) = E^2 - p^2 - \frac{E}{E_P} p^2 + O\left(\frac{E^4}{E_P^2}\right)$$

For suitable choice of $\Lambda(E, p; E_P)$ one can easily have a **maximum allowed value of momentum**, **i.e. minimum wavelength**

($p_{\max} = E_P$ for $\ell = -1/E_P$ in the formula here shown)

$$\cosh(\ell m) = \cosh(\ell p_0) - \frac{\ell^2}{2} e^{-\ell p_0} p_1^2$$

it turns out that such laws could still be relativistic, part of a relativistic theory where not only c (“speed of massless particles in the infrared limit”) but also E_P would be a nontrivial relativistic invariant

action of boosts on momenta must of course be deformed so that

$$[N_k, \Lambda(E, p; E_P)] = 0$$

then it turns out to be necessary to correspondingly deform the law composition of momenta

$$P_\mu^{(1)} \oplus P_\mu^{(2)} \neq P_\mu^{(1)} + P_\mu^{(2)}$$

minimum wavelength from noncommutativity:
the kappaMINKOWSKI noncommutative spacetime

$$[x_j, t] = i\lambda x_j \quad [x_j, x_m] = 0$$

Lukierski+Nowicki+Ruegg+Tolstoy,PLB(1991)
Nowicki+Sorace+Tarlino,PLB(1993)
Majid+Ruegg,PLB (1994)
Lukierski+Ruegg+Zakrzewski, AnnPhys(1995)

evidently not invariant under «classical translations»

$$[x_j + a_j, x_0 + a_0] = [x_j, x_0] = i\lambda x_j \neq i\lambda(x_j + a_j)$$

but adding commutative numbers to the noncommutative coordinates of kappa-Minkowski is evidently not a reasonable thing....

Adopting in particular noncommutative translation parameters such that

$$[\varepsilon_j, \varepsilon_0] = 0 \quad [\varepsilon_j, x_\mu] = 0 \quad [\varepsilon_j, x_k] = 0 \quad [\varepsilon_j, x_0] = i\lambda \varepsilon_j$$

then

Sitarz, PhysLettB349(1995)42 Majid+Oeckl, math.QA/9811054

$$[x_j + \varepsilon_j, x_0 + \varepsilon_0] = [x_j, x_0] + [\varepsilon_j, x_0] = i\lambda(x_j + \varepsilon_j)$$

boosts must adapt to these deformed translations, resulting in deformed mass Casimir

deformed boosts are such that there is a maximum momentum (minimum wavelength)

minimum wavelength from discreteness:
the simple case of a one-dimensional polymer

$$X = X_0 + n\lambda \quad n \in \mathbb{Z}$$

evidently, because of discreteness, translation transformations reflect the fact that

$$f(\mathbf{X}) \longrightarrow f(\mathbf{X}) + \mathbf{d}f(\mathbf{X})$$

with

$$\frac{f(X + \lambda)dX - f(X)dX}{\lambda} = df(X)$$

boosts must adapt to these deformed translations, resulting in deformed mass Casimir

deformed boosts are such that there is a maximum momentum (minimum wavelength)

It was recently realized that this sort of theoretical frameworks (with DSR-deformed relativistic laws) may be connected to an old idea advocated by Max Born

one of the first papers on the quantum gravity problem was a paper by Max Born [*Proc.R.Soc.Lond.*A165,29(1938)] centered on the dual role within quantum mechanics between momenta and spacetime coordinates (Born reciprocity)

$$P_{\mu} \leftrightarrow x^{\mu}$$

Born argued that it might be impossible to unify gravity and quantum theory unless we make room for curvature of momentum space

this idea of curvature of momentum space had no influence on quantum-gravity research for several decades, but recently:

momentum space for certain models based on spacetime noncommutativity was shown to be curved

some “perspectives” on Loop Quantum Gravity have also advocated curvature of momentum space

and perhaps most importantly we now know that **the only quantum gravity we actually can solve, which is 3D quantum gravity, definitely has curved momentum space**

GAC+Matassa+Mercati+Rosati, PhysicalReviewLetters106,071301 (2011)

GAC+Freidel+KowalskiGlikman+Smolin, PhysRevD84,084010 (2011)

Carmona+Cortes+Mercati, PhysRevD84,084010 (2011)

GAC, PhysicalReviewLetters111,101301 (2013)

in 3D quantum gravity

see, e.g., **Freidel+Livine**,
PhysRevLett96,221301(2006)

consider a matter field ϕ coupled to gravity,

$$Z = \int Dg \int D\phi e^{iS[\phi,g]+iS_{GR}[g]}, \quad (1)$$

where g is the space-time metric, $S_{GR}[g]$ the Einstein gravity action and $S[\phi, g]$ the action defining the dynamics of ϕ in the metric g .

integrate out
the quantum gravity fluctuations and derive an *effective action* for ϕ taking into account the quantum gravity correction:

$$Z = \int D\phi e^{iS_{eff}[\phi]}.$$

the effective action obtained through this constructive procedure gives matter fields in a noncommutative spacetime (similar to, but not exactly given by, kappa-Minkowski) and with curved momentum space, as signalled in particular by the deformed on-shellness

(anti-deSitter momentum space) $\cos(E) - e^{\ell E} \frac{\sin(E)}{E} P^2 = \cos(m)$

...and is proving valuable for phenomenology.

Much studied opportunity for phenomenology comes from fact that several pictures of quantum spacetime predict that the speed of photons is energy dependent.

Calculation of the energy dependence in a given model used to be lengthy and cumbersome. We now understand those results as dual redshift on Planck-scale-curved momentum spaces:

these results so far are fully understood for the case of
[maximally symmetric curved momentum space] \otimes [flat spacetime]

it turns out that there is a duality between this and the familiar case of
[maximally-symmetric curved spacetime] \otimes [flat momentum space]

In particular,
ordinary redshift in deSitter spacetime implies that massless particles emitted with same energy but at different times from a distant source reach the detector with different energy

dual redshift in deSitter momentum space implies that massless particles emitted simultaneously but with different energies from a distant source reach the detector at different times

GAC+**Barcaroli**+**Gubitosi**+**Loret**,
Classical&QuantumGravity30,235002 (2013)
GAC+**Matassa**+**Mercati**+**Rosati**,
PhysicalReviewLetters106,071301 (2011)

dual redshift on Planck-scale-curved momentum spaces (but with flat spacetime) produces time-of-arrival effects which at leading order are of the form ($n \in \{1,2\}$)

$$\Delta T = \left(\frac{E}{E_P} \right)^n T$$

and could be described in terms of an energy-dependent “physical velocity” of ultrarelativistic particles

$$v = c + s_{\pm} \left(\frac{E}{E_P} \right)^n c$$

these are very small effects but (at least for the case $n=1$) they could cumulate to an observably large ΔT if the distances travelled T are cosmological and the energies E are reasonably high (GeV and higher)!!!

GRBs are ideally suited for testing this:

cosmological distances (established in 1997)

photons (and neutrinos) emitted nearly simultaneously

with rather high energies (GeV.....TeV...100 TeV...)

GAC+Ellis+Mavromatos+Nanopoulos+Sarkar, Nature393,763(1998)

GAC, NaturePhysics10,254(2014)

problem:

solid theory is for (curved momentum space and) flat spacetime

phenomenological opportunities are for propagation over cosmological distances, whose analysis requires curved spacetime

study of theories with both curved momentum space and curved spacetime still in its infancy

GAC+Rosati, PhysRevD86,124035(2012)

KowalskiGlikman+Rosati, ModPhysLettA28,135101(2013)

Heckman+Verlinde, arXiv:1401.1810(2014)

Jacob and Piran [JCAP0801,031(2008)] used a compelling heuristic argument for producing a formula of energy-dependent time delay applicable to FRW spacetimes, which has been the only candidate so far tested

$$\Delta T = -s_{\pm} \frac{E}{M_{QG}} \frac{c}{H_0} \int_0^z d\zeta \frac{(1 + \zeta)}{\sqrt{\Omega_{\Lambda} + (1 + \zeta)^3 \Omega_m}}$$

where as usual H_0 is the Hubble parameter, Ω_{Λ} is the cosmological constant and Ω_m is the matter fraction.

Jacob-Piran formula is surely not the most general possibility.

It is important for phenomenology to understand this issue, but it requires handling the interplay between curvature of spacetime and curvature of momentum space in subtle ways

GAC+Rosati, PhysRevD

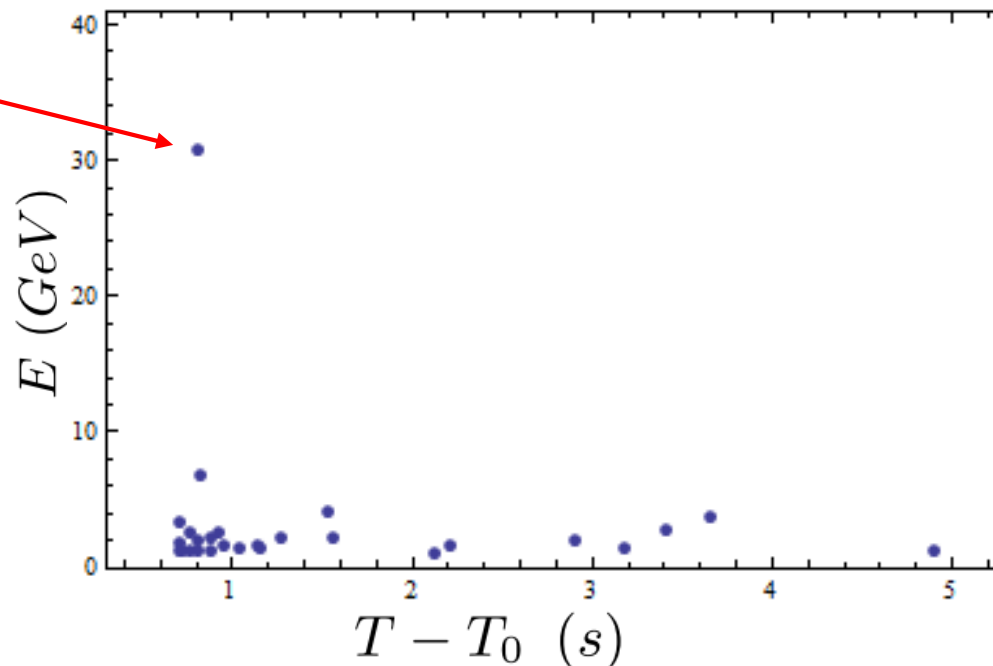
testing Jacob-Piran formula:

focus on $n=1$ case (sensitivity to the $n=2$ case still far beyond our reach presently
but potentially within reach of future neutrino astrophysics)

first came GRB080916C data providing a limit of $M_{QG} > 10^{-1} M_{\text{planck}}$ for
hard spectral lags and $M_{QG} > 10^{-2} M_{\text{planck}}$ for soft spectral lags

analogous studies of blazars lead to comparable limits

then came GRB090510 (magnificent short burst) allowing to establish a
limit at M_{planck} level on both signs of dispersion (soft and hard spectral lags)



a test with accuracy of
about one part in 10^{20} !!!

this Planck-scale sensitivity is illustrative of how we have learned over this past decade that there are ways for achieving in some cases sensitivity to Planck-scale-suppressed effects, something that was thought to be impossible up to the mid 1990s

Quantum-Gravity Phenomenology exists!!!

a collection of other plausible quantum-gravity effects and of some associated data analyses where Planck-scale sensitivity was achieved (or is within reach) can be found in my “living review”

GAC, LivingRev.Relativity16,5(2013)

<http://www.livingreviews.org/lrr-2013-5>

still makes sense to test in-vacuo dispersion statistically...
 our “quantum-gravity phenomenological models” will turn out
 to be (at best) like the Bohr-Somerfeld quantization...

in order to best setup the statistical analysis it is convenient to notice that we are testing
a linear relationship between Δt
and the product of energy and the redshift-dependent function $D(z)$

$$\Delta t = \eta \frac{E}{M_P} D(z) \quad \text{with} \quad D(z) = \int_0^z d\zeta \frac{(1 + \zeta)}{H_0 \sqrt{\Omega_\Lambda + (1 + \zeta)^3 \Omega_m}}$$

we can absorb the redshift dependence into an “accordingly rescaled energy”,
 which we call E^*

$$E^* \equiv E \frac{D(z)}{D(1)}$$

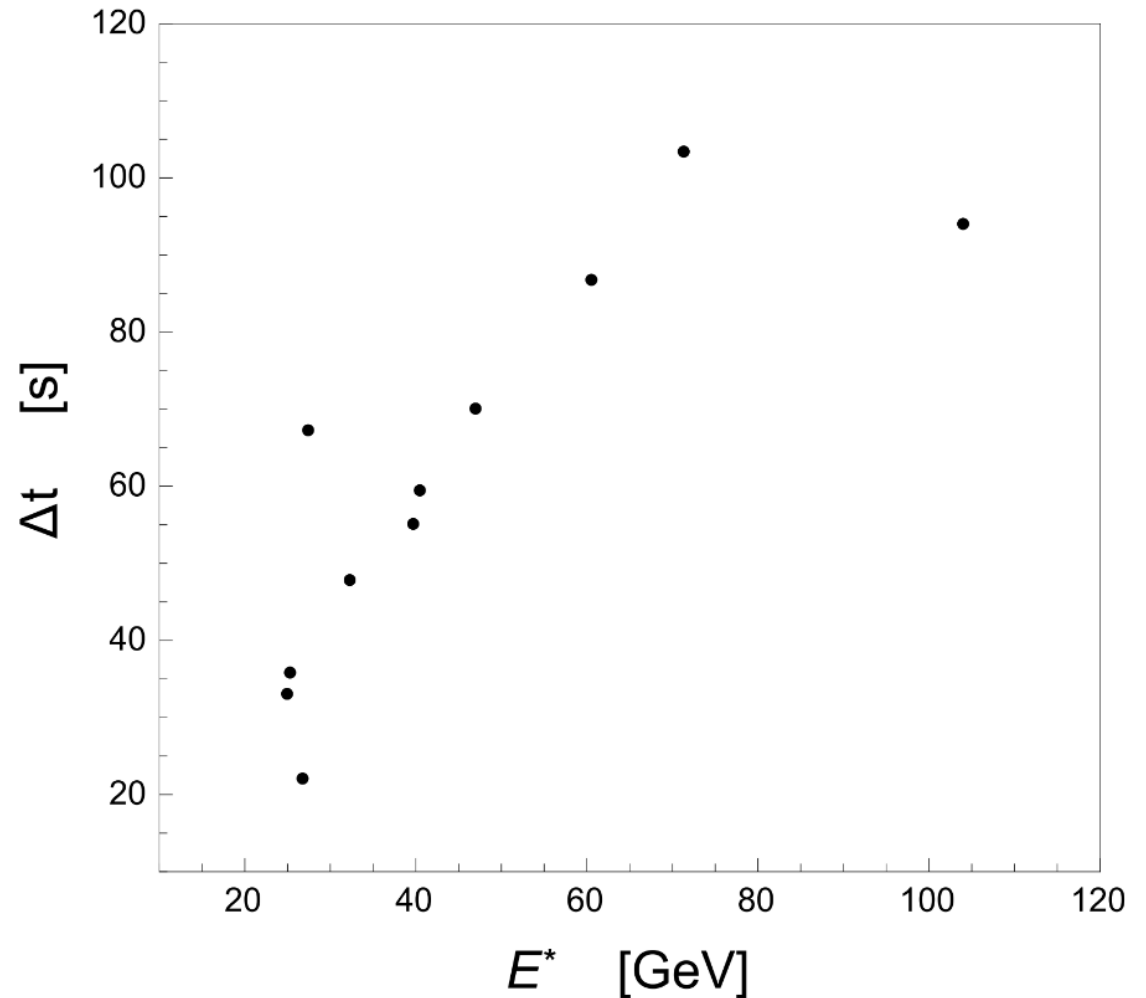
This then affords us the luxury of analysing data in terms of a linear relationship
 between Δt and E^*

$$\Delta t = \eta \cdot D(1) \frac{E^*}{M_P}$$

criteria:

- focus on photons whose energy at emission was greater than 40 GeV
- take as Δt the time-of-observation difference between such high-energy photons and the first peak of the (mostly low-energy) signal

[note that this makes sense only for photons which were emitted in (near) coincidence with the first peak...not all those with >40GeV will ...and surely only a rather small percentage of all photons...]

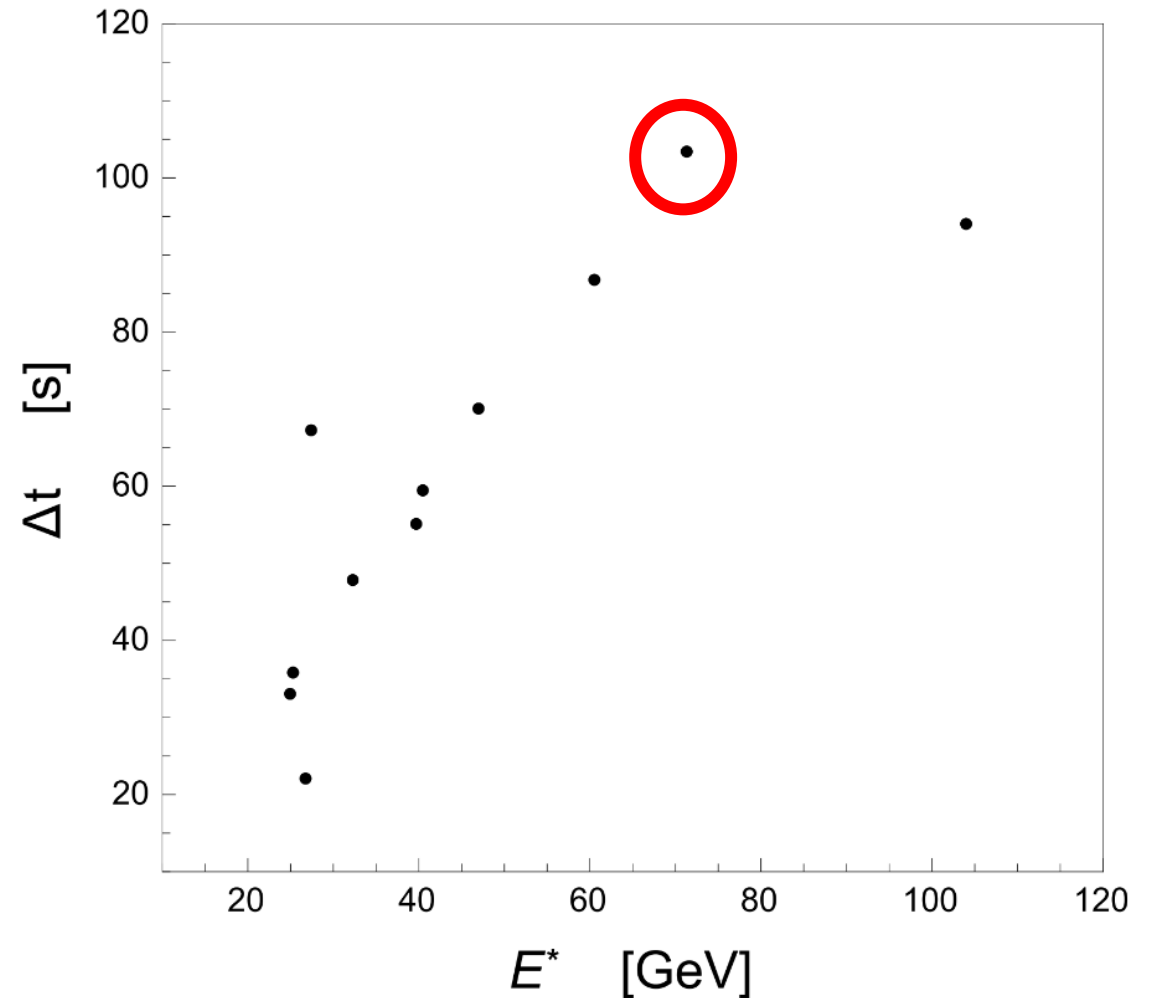


in order to get a sense of how striking this data situation is one can ask how often such high correlation between Δt and E^* would occur if the pairing of values of Δt and E^* was just random: overall having such high correlation would happen in less than 0.1% of cases, and correlation as high as seen for the best 8 out of 11 in 0.0013% of cases

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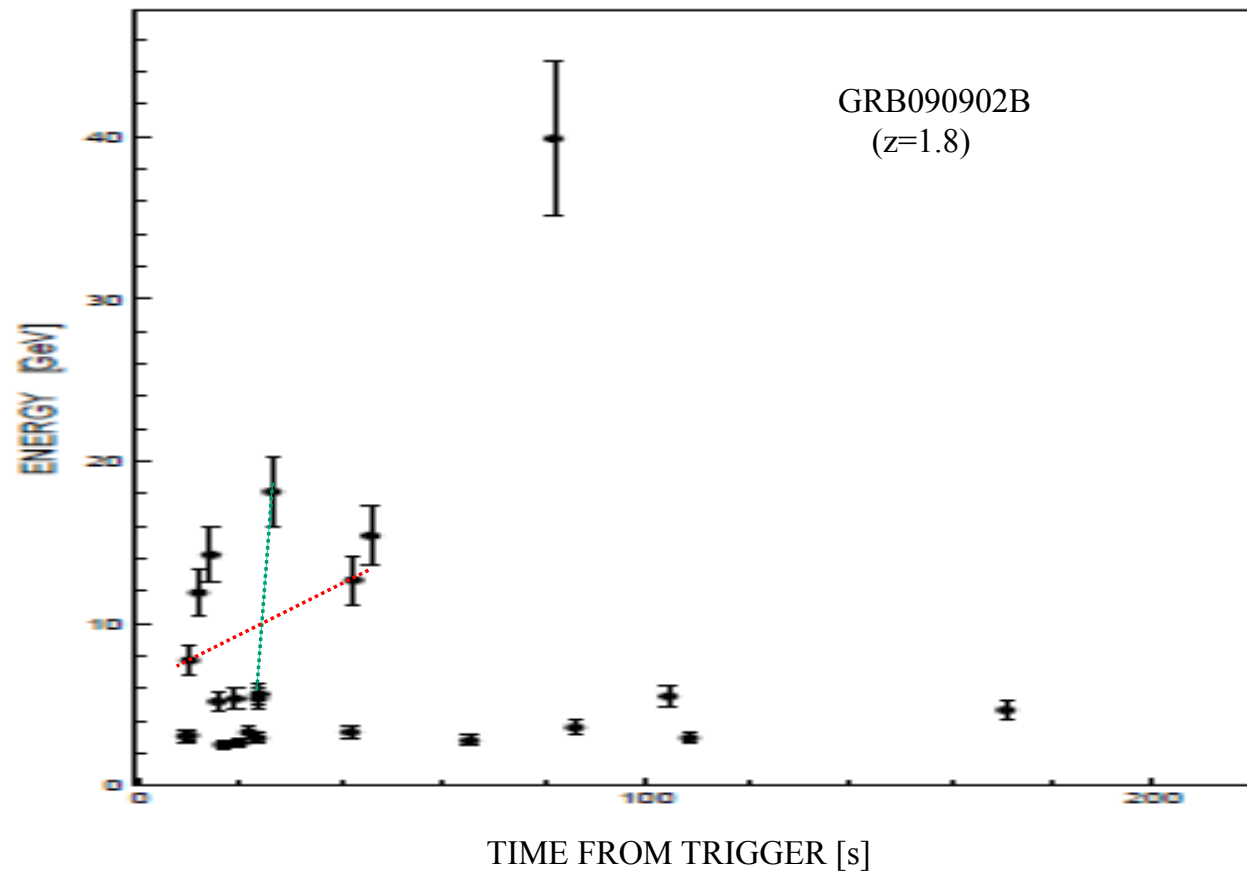
there is no reason to dwell much on statistical significance since more data will be available in a rather near future...

actually we already have more data to analyse, the GRB photons with energy at emission lower than 40 GeV, but for those it would be absurd to assume emission in near coincidence with the first peak of the GRB

previous graph gives η_γ of 30 ± 6

and note that each pair of photons in a GRB nominally determines a value of η_γ , though the large majority of them will be “spurious” for our analysis (photons emitted in different phases of the GRB)

we can still see if the frequency of occurrence of η_γ of about 30 is particularly high



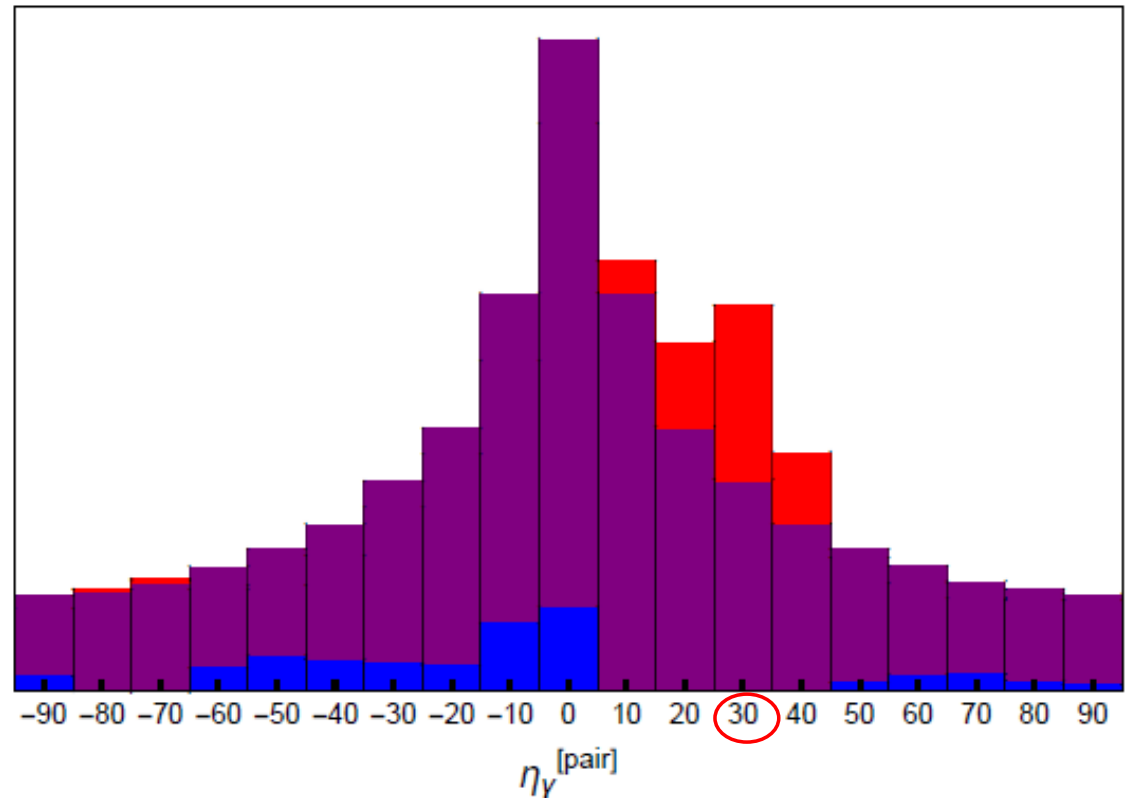
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for bins where the observed population is higher than expected we color the bar in purple up to the level expected, showing then the excess in red;

for bins where the observed population is lower than expected the bar height gives the expected population, while the blue portion of the bar quantifies the amount by which the observed population is lower than expected

large variety of phenomenological models

- * quantum-gravity scale could be bigger or smaller than E_{planck}
- * can be brokenSR or deformedSR
 - notice that no quantum-spacetime picture has been shown rigorously to lead to brokenSR
 - notice that threshold anomalies (e.g. anomalous transparency... $\gamma\gamma\rightarrow e^+e^-$) are only possible with brokenSR (protected by a theorem in any deformedSR scenario, GAC, PhysRevD85,084034)
 - for time-of-flight analyses techniques borrowed from propagation of light in media might not apply to deformedSR
- *the redshift dependence may be different from the Jacob-Piran ansatz
- *the effects can be spin/helicity/polarization dependent
- *the effects can be particle-type dependent (different for photons and neutrinos)
- *the effects should be fuzzy but theory work at present only provides essentially the deformation of the lightcone, without being able to establish the fuzziness of the deformed lightcone

CLOSING REMARKS

the “preliminary statistical evidence” is strong enough to encourage us to think about alternative phenomenological models, giving a better description of the data situation...would have to be a case such that my simple-minded in-vacuo-dispersion formula is like the Bohr-Sommerfeld description of atoms:

- what about the 31GeV event from GRB090510? Should we ascribe it to a remarkable conspiracy?
is the effect intrinsically statistical/non-systematic? Does the effect depend on polarization?
Does the effect depend on direction? Do we need to look beyond the Jacob-Piran formula?
(most of the data that give more strength to the statistical evidence are from very distant GRBs)**
- 4 out of our 9 neutrinos are “early neutrinos”...are they background? Or does the effect for neutrino have both signs? If so why does the effect have only one sign for photons?**

**working on quantum gravity
one cannot avoid getting the
feeling that Nature might have
hidden very well some of its
most fascinating secrets**

**still we have no other
option but to keep looking**

**and maybe we are wrong and
the secrets are not so well
hidden**



GAC+Barcaroli+D'Amico+Loret+Rosati, arXiv1605.00496, PhysicsLettersB761,318

GAC+D'Amico+Rosati +Loret, arXiv:1612.02765, NatureAstronomy1,0139

[these use latest data release by IceCube....also see previous exploratory analysis on 2008-2010 IceCube data GAC+Guetta+Piran, Astrophys.J.806,269]

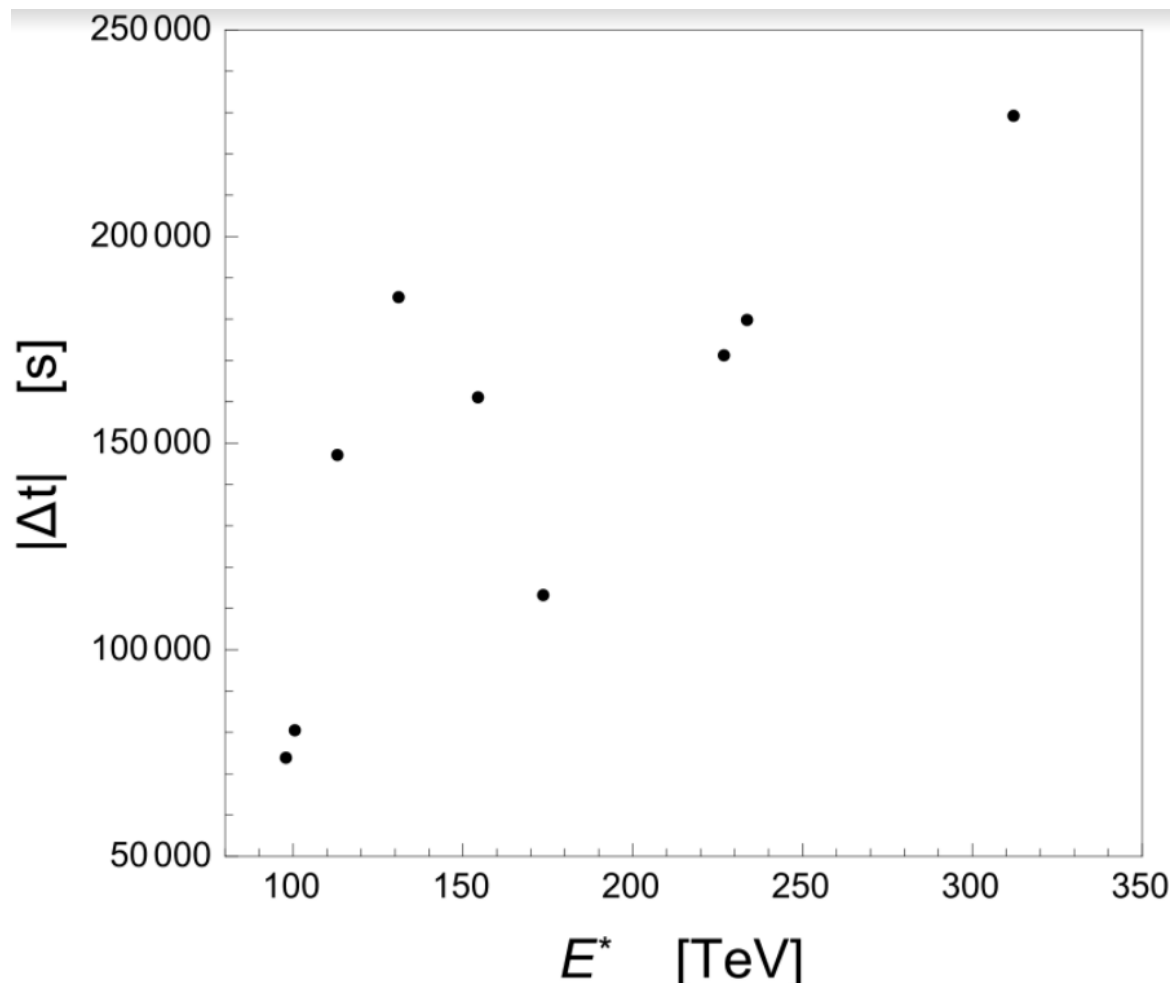
IceCube still found no GRB neutrinos (expected at least a dozen at this point)

If effect is of seconds for GeV photons it can be very large for 300TeV neutrinos...the time window adopted by IceCube would never catch such GRB neutrinos...

IceCube has reported so far 21 shower neutrinos with energy between 60 and 500 TeV

we found that 9 of them could be “GRB-neutrino candidates” (direction compatible with the GRB direction and time of observation within 3 days of the GRB)

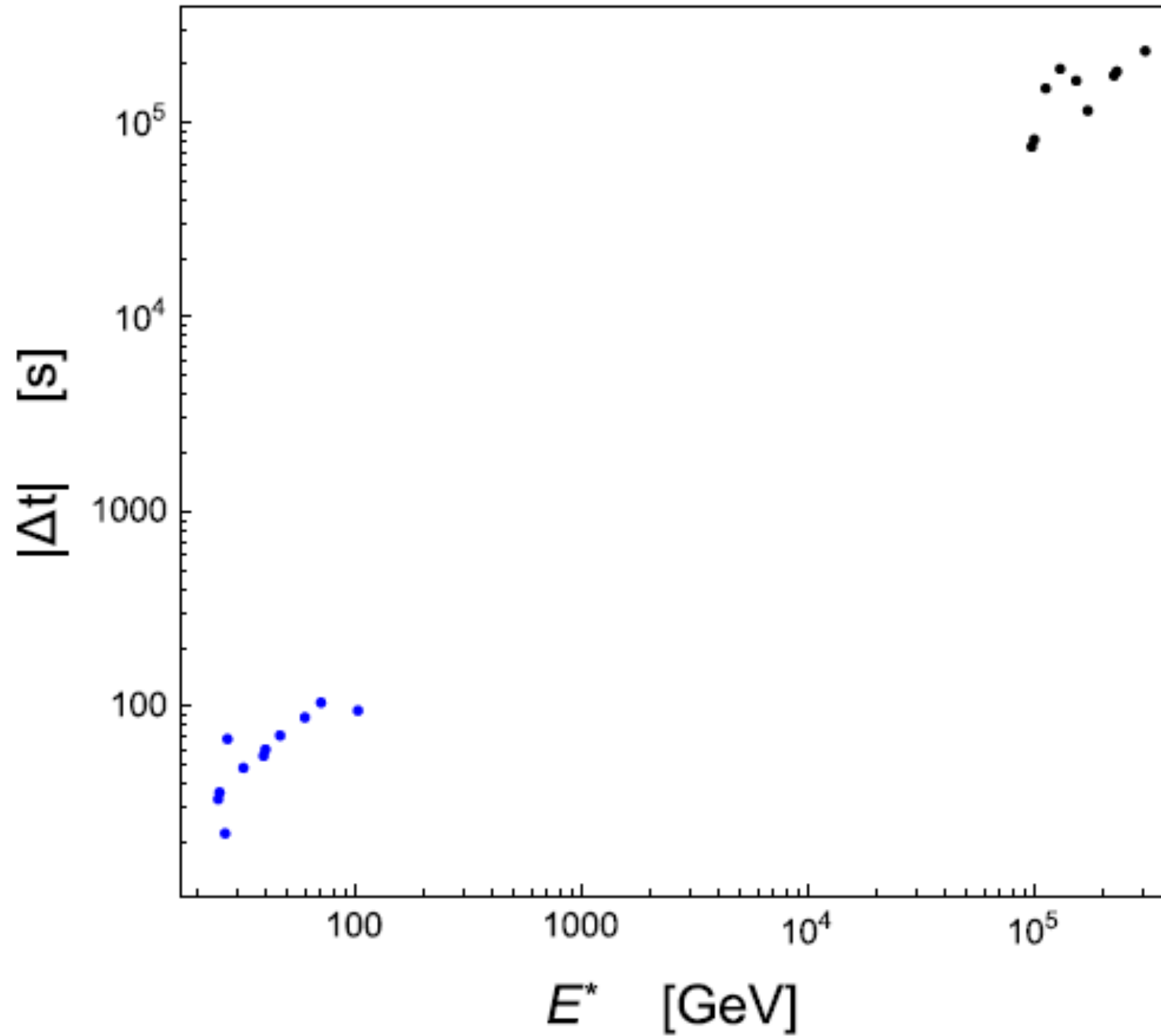
so let's see if they provided some support for the linear dependence between Δt and E^*



the correlation found in data is 0.95

particularly amazing considering that we can independently estimate (even if there was in-vacuo dispersion, and therefore some of these are GRB neutrinos) that most likely 3 or 4 of our 9 neutrinos must be background neutrinos, unrelated to GRBs

the false alarm probability is 0.5% (probability of finding such a high correlation if all neutrinos are background neutrinos that happened to fit by accident our GRB-neutrino selection criteria)



mass of a particle with four-momentum p_μ is determined by the metric geodesic distance on momentum space from p_μ to the origin of momentum space

$$m^2 = d_\ell^2(p, 0) = \int dt \sqrt{g^{\mu\nu}(\gamma^{[A;p]}(t)) \dot{\gamma}_\mu^{[A;p]}(t) \dot{\gamma}_\nu^{[A;p]}(t)}$$

where $\gamma^{[A;p]}_\mu$ is the metric geodesic connecting the point p_μ to the origin of momentum space

$$\frac{d^2 \gamma_\lambda^{[A]}(t)}{dt^2} + A^{\mu\nu}{}_\lambda \frac{d\gamma_\mu^{[A]}(t)}{dt} \frac{d\gamma_\nu^{[A]}(t)}{dt} = 0 \quad \text{with } A^{\mu\nu}{}_\lambda \text{ the Levi-Civita connection}$$

the affine connection on momentum space determines the law of composition of momenta, and it might not be the Levi-Civita connection of the metric on momentum space (it is not in 3D quantum gravity and in all cases based on noncommutative geometry, where momentum space is a group manifold)

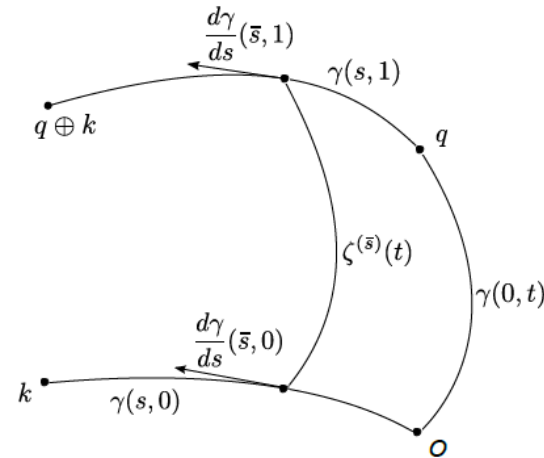


Figure 1. We determine the law of composition of momenta from the affine connection by associating to the points q and k of momentum space the connection geodesics $\gamma^{(q)}$ and $\gamma^{(k)}$ which connect them to the origin of momentum space. We then introduce a third curve $\bar{\gamma}(s)$, which we call the parallel transport of $\gamma^{(k)}(s)$ along $\gamma^{(q)}(t)$, such that for any given value \bar{s} of the parameter s one has that the tangent vector $\frac{d}{ds}\bar{\gamma}(\bar{s})$ is the parallel transport of the tangent vector $\frac{d}{ds}\gamma^{(k)}(\bar{s})$ along the geodesic connecting $\gamma^{(k)}(\bar{s})$ to $\bar{\gamma}(\bar{s})$. Then the composition law is defined as the extremal point of $\bar{\gamma}$, that is $q \oplus k = \bar{\gamma}(1)$.

This could have been just a futile “geometric interpretation” but it is proving useful

It establishes valuable similarities between different theories.

**In particular theories with curved momentum spaces can still be relativistic,
but this requires that momentum space is maximally symmetric
(dS/anti-dS cases discussed above)**

GAC,arXiv:11105081, PhysRevD85,084034

**and the relativistic symmetries are a “deformation”
of ordinary special-relativistic symmetries,
examples of the above-mentioned
DSR-relativistic theories**

GAC, grqc0012051, IntJournModPhysD11,35

GAC, hepth0012238,PhysLettB510,255

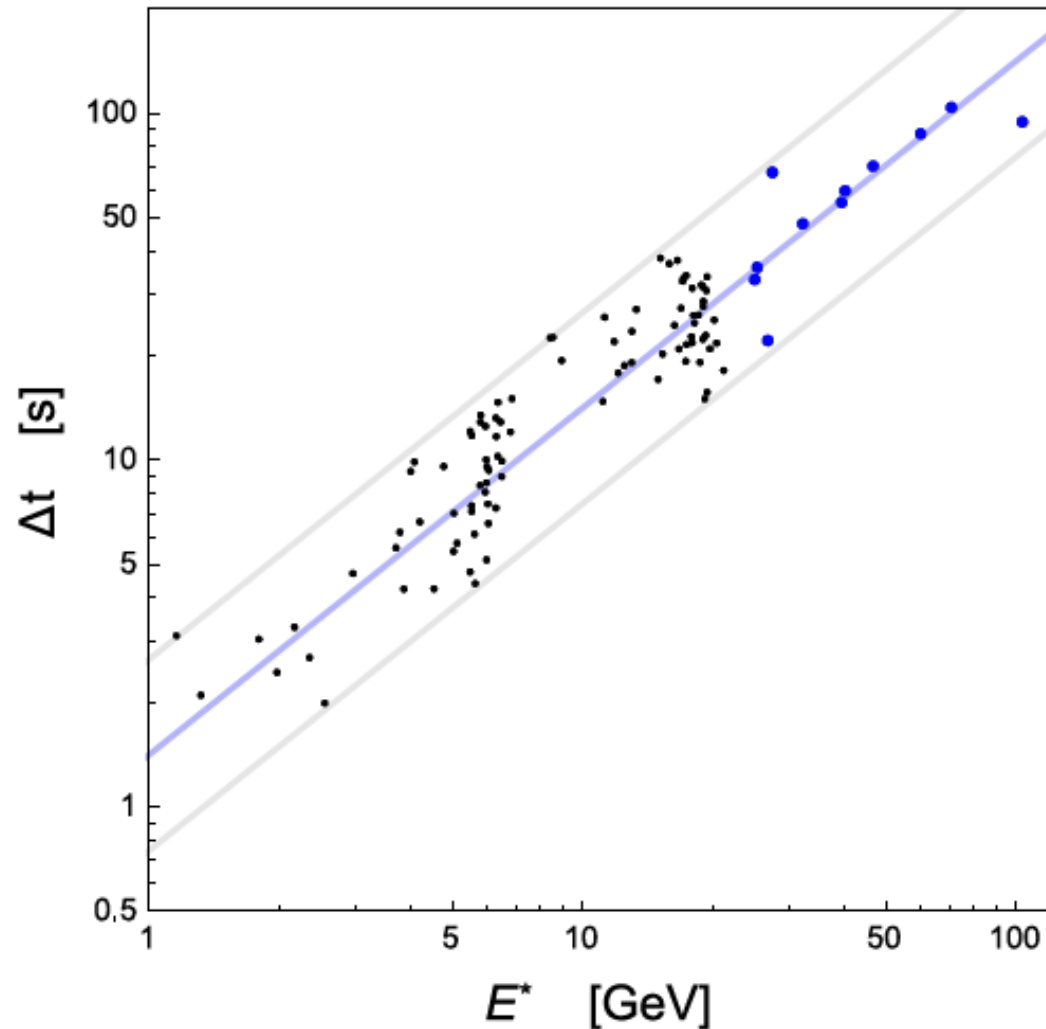
KowalskiGlikman,hepth0102098,PhysLettA286,391

Magueijo+Smolin,hepth0112090,PhysRevLett88,190403

Magueijo+Smolin,grqc0207085,PhysRevD67,044017

GAC,grqc0207049,Nature418,34

histogram quantifies but the consistency between situation for photons with energy at emission greater than 40 GeV and situation for photons with energy at emission between 5 and 40 GeV goes even beyond what is suggested by histogram



Relative locality limit

$$\hbar \rightarrow 0, \quad G_N \rightarrow 0, \quad \text{but with fixed } \sqrt{\frac{\hbar}{G_N}} = M_p$$

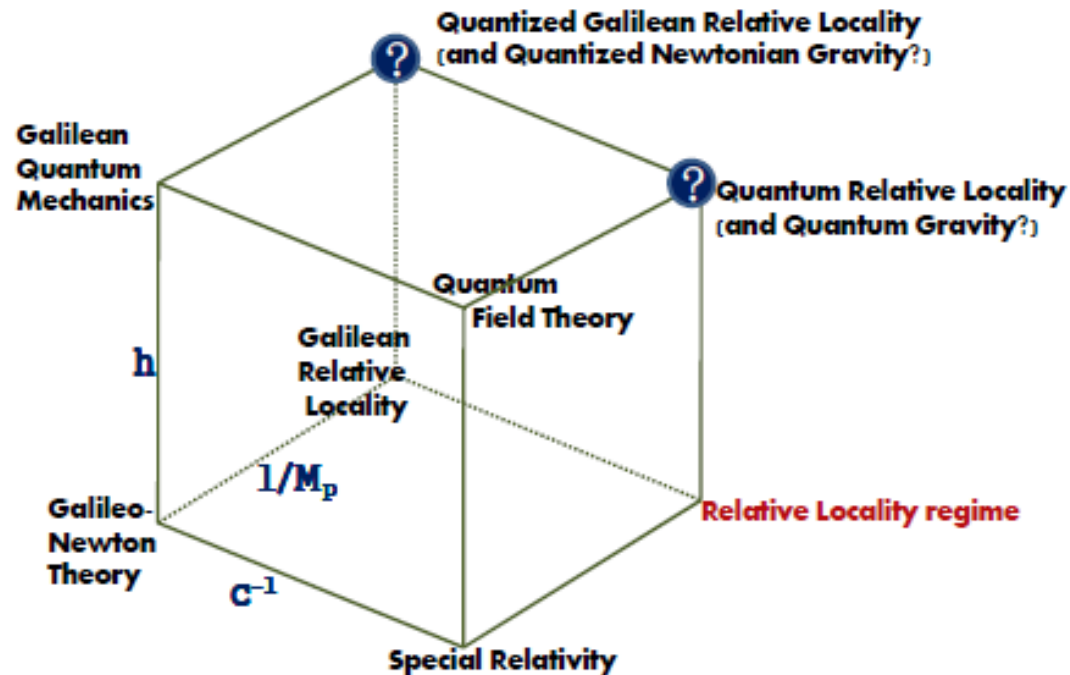
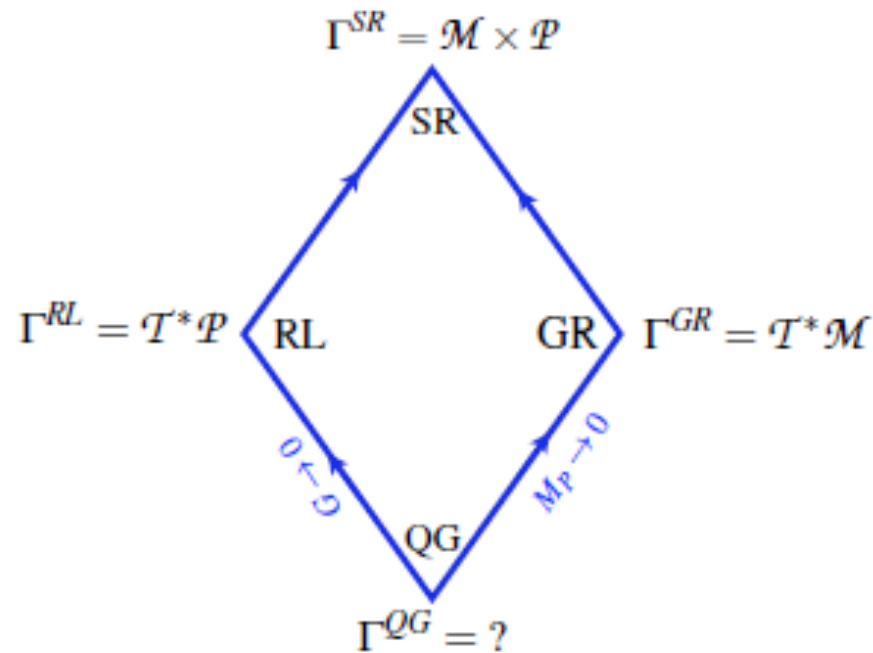


Figure 4: If the Planck scale is primitive (together with \hbar and c) while Newton constant is derived, the Bronstein cube would have to be redrawn as here shown. I am assuming that the relative-locality regime is described by the relative-locality framework or something similar to it. This may also suggest that that the theory encompassing all these regimes (“quantum gravity”) could be obtained as a quantum theory on the relative-locality momentum space.



These novel phenomena have a consistent mathematical description in which the notion of spacetime gives way to an invariant geometry formulated in a phase space. In special relativity, the phase space associated with each particle is a product of spacetime and momentum space, *i.e.* $\Gamma^{SR} = \mathcal{M} \times \mathcal{P}$.

In general relativity, the spacetime manifold \mathcal{M} has a curved geometry. The particle phase space is no longer a product. Instead, there is a separate momentum space, \mathcal{P}_x associated to each spacetime point $x \in \mathcal{M}$. This is identified with the cotangent space of \mathcal{M} at x , so that $\mathcal{P}_x = \mathcal{T}_x^*(\mathcal{M})$. The whole phase space is the cotangent bundle of \mathcal{M} , *i.e.* $\Gamma^{GR} = \mathcal{T}^*(\mathcal{M})$

Within the framework of relative locality, it is the momentum space \mathcal{P} that is curved. There then must be a separate spacetime, \mathcal{M}_p for each value of momentum, $\mathcal{M}_p = \mathcal{T}_p^*(\mathcal{P})$. The whole phase space is then the cotangent bundle over momentum space, *i.e.* $\Gamma^{RL} = \mathcal{T}^*(\mathcal{P})$.

Special Relativity (continued)

special-relativistic law of composition of momenta is still linear

$$p_{\mu}^{(1)} \oplus p_{\mu}^{(2)} = p_{\mu}^{(1)} + p_{\mu}^{(2)}$$

but the on-shell/dispersion relation takes the new form

$$E = \sqrt{p^2 + m^2}$$

and time is relative:

simultaneity of events occurring in the same place
(fully coincident events) still is objective



but simultaneity of distant events is “relative”,
i.e. observer dependent



relative locality

proper introduction to the notion of relative locality would require a full talk...

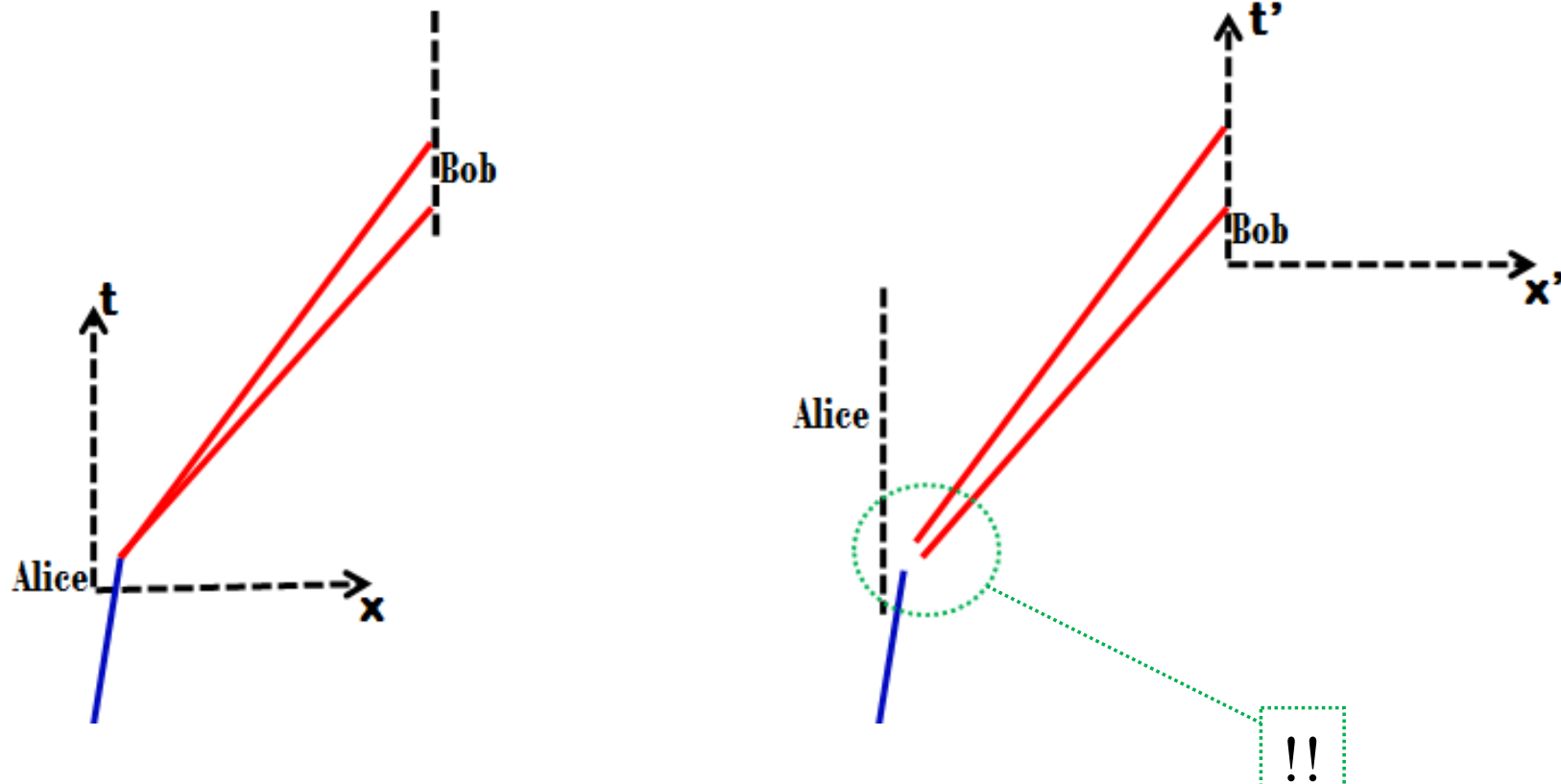
famously from viewpoint of a “Galileian physicist” the fact that special relativity introduces an invariant velocity scale produces artifacts about the simultaneity (time coincidence) of events, that is “relative simultaneity”...

from viewpoint of “Einsteinian physicist” a relativistic theory which introduces also an invariant energy scale produces artifacts about the spacetime coincidence (locality) of events

GAC+Matassa+Mercati+Rosati, [PhysicalReviewLetters106,07301](#)

GAC+Freidel+Kowalski+Smolin, [PhysicalReviewD84,087702](#)

illustrative example:



today I focused on phenomenology of “in-vacuo dispersion”, a systematic effect “fuzziness” (as resulting for example from spacetime noncommutativity) is at least equally interesting but more difficult to model reliably

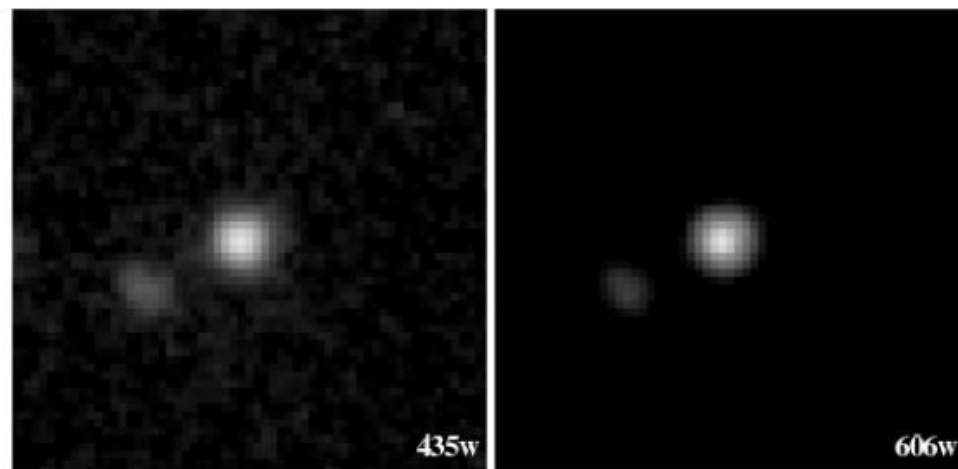
early attempts of model-building phenomenology:
blurring of wave front in precision interferometry

GAC, Nature(1999)
PhysRevD(2000)
Ng+VanDam, FoundPhys(2000)

latest studies of fuzziness tested effects
that could blur images of distant quasars

phenomenology of fuzziness
is improving very quickly

Christiansen+Ng+VanDam, PhysRevLett(2006)
Tamburini+Cuofano+DellaValle+Gilmozzi,A&A (2011)
GAC+Astuti+Rosati,PhysRevD(2013)
GAC,PhysRevLett(2013)



Here shown are copies of Hubble telescope Ultra-Deep-Field images for quasar 6732 (redshift 3.2), with left panel showing a B-filter (‘blue’) image and right panel showing a V-filter (‘visible’) image. From the original images one finds (quantitatively) that this quasar is somewhat blurred in the shorter-wavelength B filter. This illustrates the nature of the effect expected in some quantum-spacetime models: propagation of photons in the fuzzy spacetime should produce blurring of images, with more blurring found for larger distances (greater ‘accumulation’ of tiny Planck-length effects) and for shorter wavelengths (more sensitive to the fundamental short-distance structure of spacetime).